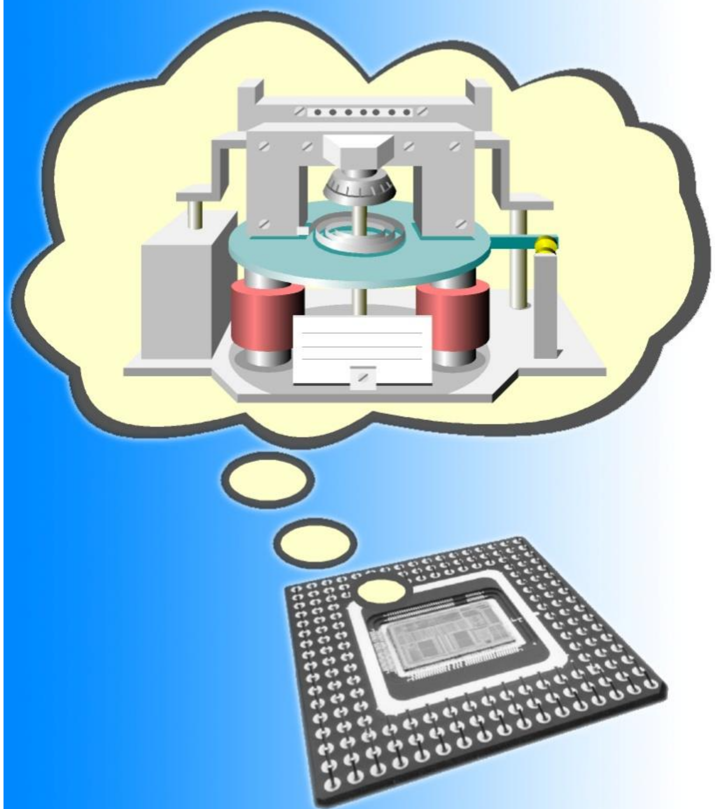

Power System Relaying,

An Introduction

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Acknowledgments

To be completed.

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from

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1.0 Introduction

Electric power systems are subject to faults and failures that result in unsafe conditions and can damage equipment if the faulty conditions persist. It is imperative to disconnect the faulty equipment as fast as practicable without interrupting the capability of the system to serve the electric loads. Since the early days of electric power systems, technology and procedures were developed to achieve this goal. The key components of the technology are fuses, relays and breakers and associated control circuits. The technology and methodologies have evolved over the years to very sophisticated protection components with remarkable capabilities. A key component of this system is the protective relay that performs the monitoring of the system, the logic for identifying faulty or intolerable conditions and making the decision as to when to interrupt what circuits or initiate shut down procedures. We refer to these procedures as protective relaying. The objective of protective relaying is to selectively and reliably isolate a faulty power system component in the minimum possible time so that (a) the exposure of the system to fault conditions will be minimized, (b) damage will be avoided and (c) safety hazard to persons in and around the faulty power systems will be minimized. Lewis Blackburn [???] has defined protective relaying as follows.

The science, skill, and art of applying and setting relays and/or fuses to provide maximum sensitivity to faults and undesirable conditions, but to avoid their operation on all permissible or tolerable conditions.

There are two main issues associated with the objectives of the above definition of protective relaying: (a) monitoring and identification of an intolerable condition for which protection must be provided and (b) selection of protective action so that the minimum possible part of the system will be affected. These issues form the core of protective relaying. Hardware, methods, logic, philosophy, etc. employed for these two core issues have evolved over the years. Experience has dictated many protection practices. The power system must be protected against two types of phenomena: (a) fast transients that normally last very short time but have the potential of damaging the system, such as lightning transients and (b) faulty conditions and subsequent disturbed conditions that may persist in the system until action is taking to remove the cause of the faulty/disturbed conditions. The first type of phenomena is characterized with high transient voltages with relatively short durations (microseconds, milliseconds) of relatively low energy while the latter are characterized with high current and relatively longer duration (tens of milliseconds and longer) and therefore high energy content. It is important to note that the two types of phenomena are interrelated. For example, a fast voltage transient (from lightning) may cause insulation breakdown and therefore a permanent follow-up fault condition on the system.

A layered system is utilized to protect the power system. The first layer, referred to as the overvoltage protection system, provides for the protection of the system against the fast transients. The second layer, referred to as the relaying protection system, provides for the protection of the system against faulty/disturbed conditions. This book focuses on the second layer.

The relaying protection problem is very complex. To simplify the problem, the overall protection problem is partitioned into smaller problems with the use of the protection zone concept. A protection zone is a small part of a power system (for example a single transmission line, a single generator, etc.). The design of a protection system for one protection zone (**zone** protection) is a relatively simpler problem. Assuming that any part of a power system will belong to one protection zone, then the overall power system is protected. The design of a protection system for a protection zone is focused only in abnormal and intolerable conditions that are caused by faulty conditions in the protection zone alone. Therefore, the zone protection system is designed to respond to abnormal and intolerable conditions in its own zone alone. This protection system design approach covers the majority of the abnormal and intolerable conditions in the power system for which protection should be provided. However, there are some phenomena that involve system abnormal and intolerable conditions that are not local to any single protection zone or may not involve a fault, for example system wide oscillations triggered by a fault that has been successfully identified and cleared. These oscillations (or follow-up disturbed conditions) may or may not be tolerable. In case that they are not tolerable, the protection system should be designed to protect the system. For these cases, information is needed from the overall system together with the ability to determine that the oscillation will be intolerable and then action should be taken. We refer to these types of problems as "**system** protection". Therefore the protection design problem can be viewed as consisting of two sub-problems: (a) design of the protection system for a protection zone (**zone** protection), and (b) design of system protection schemes for system abnormal conditions (**system** protection).

2.0 The Power System

The electric power system is occasionally disturbed with faults, failed equipment and other abnormal operating conditions. The number of possible types of disturbances is extremely large. Disturbances may be temporary and tolerable while occurring or permanent and intolerable or temporary and intolerable. A protection system monitors specific quantities of the power system, typically three voltages and three currents, and it is expected to determine from the collected data the status of the system and whether an action should be taken. Some conditions are easily determined (for example a short circuit) while others require more sophisticated processing of the data (for example transformer over-excitation, generator out of step, etc.). Because the power system is a complex dynamical system, the classification of the system condition using limited information (such as three voltages and three currents at a specific location only) may be difficult in certain cases.

The first component in the design of the algorithms to classify the condition of the system and whether a relay should act is an in-depth understanding of the electric power system and its response to disturbances. It is imperative that the response and behavior of the system to disturbances and faults for which we need to protect is well understood. This knowledge is necessary for the design of proper monitoring and prediction methods that when applied properly, they will provide for the proper initiation of protective action. The system protection functions require a deep understanding of the operating characteristics of the system. "Know what and for what you need to protect" is a general principle that vividly applies to the protection of the electric power system.

The second component in the design of protective systems is the philosophy of protection. Why do we need a "protection philosophy?" For the simple reason that we may have an "overly conservative" protection system that will disrupt service to the customers for each disturbance to the system or a "overly tolerable" system that will allow unsafe conditions to persist in the system for a longer periods of time. The protection philosophy "draws the line" between these two extremes cases by making sound choices that minimize service disruptions, maximize equipment life and minimizes exposure to unsafe conditions for humans. Specifically, there are many hard constraints that must be met (safety, equipment operating limits, etc.) but there is also room for many choices. In general, the prevailing protection philosophy is to isolate the minimum possible part of the system for all intolerable conditions. The term intolerable conditions implies a condition that will do harm to people and/or equipment.

The third component that must be addressed is what phenomena affect the performance of the protective system. The protective system comprises the relays as well as interrupting devices and all the controls associated with the interrupting devices. The reliable operation of the entire system depends on the proper design of the integrated system, possible self-checking procedures, fail safe schemes and maintenance procedures. One obvious requirement is that the interrupting system (breaker, fuse, etc.) must be able to interrupt the fault current successfully. For example if a breaker is designed to operate reliable for fault currents below 40 kAs and the actual fault current is 50 kAs then the breaker may not be able to interrupt the fault current and isolate the faulty device leading to breaker failure and more serious abnormal conditions than the original fault. We will address some of these issues in appropriate sections.

3.0 Classification of Relays

Relays can be classified in accordance to their intended function as well as in accordance to the technology they use. Table 1.1 provides a classification of relays based on their intended function. In this book we focus on protective relays, even though we will many times make reference to other types of relays. It is also important to note that with the introduction of the numerical relays, many functions can be "packaged" within one relay. As a result the classification of relays into protective, monitoring, reclosing, etc. have become obsolete since a single relay may perform many of these functions.

Table 1.1 Relay Classifications by Function

Protective
Monitoring
Reclosing
Regulating
Auxiliary
Synchronizing

The technology of relay construction has changed over the years as technology has changed. Table 1.2 provides a list of the various relay technologies over the years.

Table 1.2 Relay Technologies

Electromechanical	1900s
Solid State	1960s
Digital or Numerical	1980s

Initially, electromechanical relays were introduced at the early stages of the electric power industry. Electromechanical relays are electromechanical systems that are designed to perform a logic function based on specific inputs of voltages and currents. This technology started with the very simple plunger type relay and evolved into highly sophisticated systems that performed complex logical operations, for example the modified mho relay is a system that monitors the impedance of the system as "seen" at a specific point in the system and acts whenever the impedance moves into a pre-specified region (relay characteristic). In the early years of the electric power industry, the inverse time delay overcurrent relay was developed based on the induction disk (Westinghouse) or the induction cup (GE). The induction disk relay is illustrated in Figure 1.1. The time overcurrent protection function is one of the main protection functions provided in practically all protection schemes. Over the years the electromechanical relays developed into sophisticated analog logic devices with great selectivity and operational reliability.

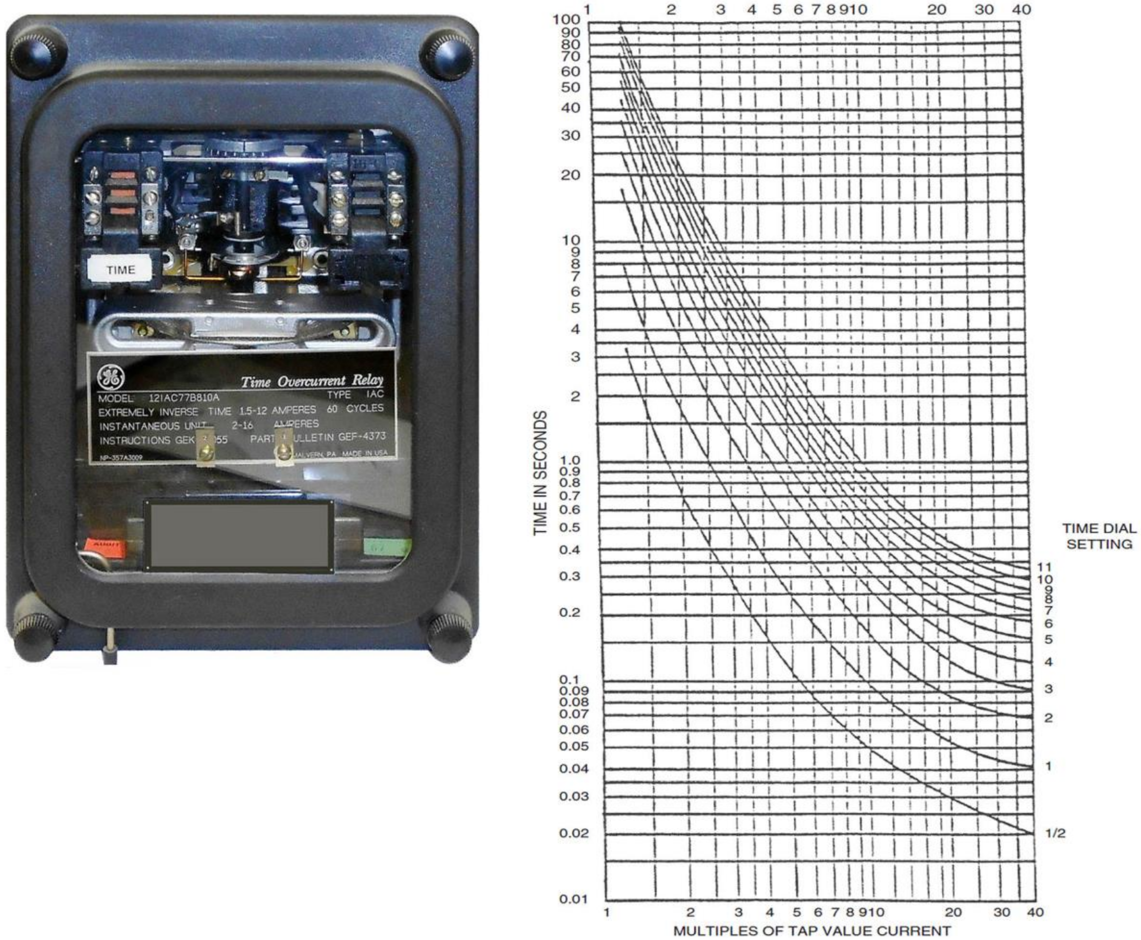


Figure 1.1a Electromechanical Relay. Technology: Induction Disk, Principle of Operation: Extremely Inverse Time Overcurrent

The invention of the transistor in the late 40's and the subsequent solid state technology provided an opportunity to replace the bulky electromechanical relays with solid state based relays. The attractiveness was not only size reduction but the ability to implement even more complex logic functions. The development of solid state relays was very slow because of concerns of how solid state technology will perform in the harsh electromagnetic environment of electrical installations. By the time that solid state relays started becoming acceptable to the industry, the first effort to develop digital (numerical relays) was introduced in the late sixties and early seventies. The first computer relay was developed in 1970 with a trial implementation in a substation in California. Computer relays appeared to be expensive at that time. However, the computer relaying efforts coupled with the introduction of the microprocessor in the early 1980s led to the development of the microprocessor based relay (numerical relay). The microprocessor provided the capability to implement extremely complex logic functions in a very small package and at a low cost. In addition, it provided the capability to implement multiple logic functions with only a single microprocessor. This capability was recognized in the early stages of the microprocessor technology. However, two issues have kept the development at very slow pace.

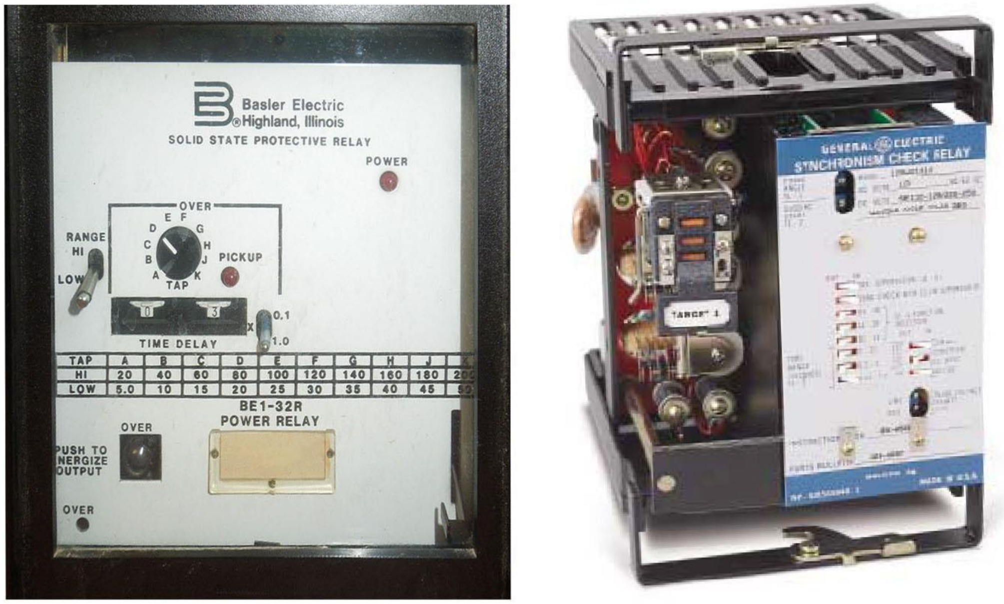


Figure 1.1b Solid State Relays.

Technology: Transistor Based Analog & Logic Circuits

The first issue was that of reliability. Microprocessors operate at very low voltages. A microprocessor based relay installed in the harsh electromagnetic environment of an electrical installation may be vulnerable and therefore the reliability of the relay may be jeopardized. While this was true with the early systems, designs were developed that hardened and shielded microprocessor systems for safe and reliable operation in the harsh electromagnetic environment of substations and electrical installations. The second issue was the fact that early microprocessors did not have the required computing power to perform the required computations in real time. Both of these issues have been resolved and forgotten. The first commercial microprocessor based relay was introduced in 1984 (Schweitzer Laboratories) and its many advantages propelled the technology to remarkable capabilities. The main advantage of these relays is the ability to add protective functions by simply augmenting the software in the microprocessor thus enabling multifunctional relays in one single box assuming that the microprocessor has enough computing capability to handle the computations. Today the microprocessor based relay (or digital or numerical relay) is a well-designed component with very high reliability, capable of operating in the harsh electromagnetic environment of an electrical installation and with computing power that is remarkable. A digital relay is typically a multifunctional relay, i.e. it performs several relaying functions within a single device. The multi-functionality capability of the digital relay has established it as the choice device from both the economic and performance points of view. The advantages of the digital relay are so many that today any new installation or upgrades are based almost exclusively on digital relays. A more detailed historical perspective will be provided in another chapter.

Figure 1.2 illustrates samples of numerical relays from different manufacturers. Note that numerical relays are now just "boxes" with few displays. However, the state of the art has evolved to the point where each one of the numerical relays can be interfaced with a personal computer and via sophisticated user interfaces, the entire relay can be viewed and programmed. The complexity of a multifunctional relay is quite high requiring training in its use, but the sophistication and the capabilities are enormous. It is important to point out that the multifunctional numerical relays have reduced the space requirements in the substation control house dramatically with associated cost reductions. Specifically, while in the past the protection of a transmission line may needed several single function electromechanical relays, taking several racks in a substation control house, now the entire set of protective relays is typically housed in a 19 inch by 8 inch device. Furthermore, the numerical relays are equipped with communication ports and they can interface to communication networks so that engineers can access the relay from the comfort of their office, review settings, download system conditions recorded by the relay, enter new settings, etc. This is a truly new environment with tremendous flexibility and potential for new innovative ways for protecting the system.

A typical numerical relay includes a data acquisition system consisting of a front analog input circuit followed by analog to digital converters. This system converts the monitored quantities into digitized waveforms that are fed to the microprocessor where all the analysis and logic is performed. As the technology advances we have seen in recent years the separation of the data acquisition system from the relays. Specifically, data acquisition system have been developed that can be placed directly in the field (near a breaker, near a transformer, etc.) under the generic name of "merging units". The merging unit digitizes the data in the field. Subsequently the digitized data can be transmitted to the relay via fiber optic lines. The digital relay now is simply a computer that receives the data directly in digitized form and performs the analysis and the logic. The evolution of the technology and the implication to protection and system automation will be discussed in later sections.

Figure 1.2 Digital (Numerical) Multifunctional Relays – Courtesy of Manufacturers

4.0 Protective Relaying System Components

The protective system consists of four discrete components: (a) the instrumentation subsystem that consists of instrument transformers that generate low voltage and low current outputs for input to the relays or logic subsystem. Ideally, these voltages and currents should be scaled replicas of the high voltages and currents of the electric power system. Practically, however, the instrumentation channels introduce errors that slightly distort the waveforms of the high voltages and currents. (b) the logic subsystem, i.e. the relay, which processes the voltages and currents (and possibly status inputs) and makes decisions. Present relay systems are microprocessor based and are capable of performing complex computations and logic. The purpose of these computations is to identify and characterize the operating condition of the subsystem that they monitor and through some logic to determine whether action is required to remedy an intolerable condition, (c) the control subsystem, consisting of discrete inputs and discrete outputs to the logic subsystem and logic circuits to activate the relay decisions. For example the status of the interrupting subsystem may be instrumented as an input to the logic subsystem. In this case, the protective relay monitors the status of the interrupting device and the protection logic may take this information into consideration. There are two types of status indicators of interrupting devices (mainly breaker): a 52a contact (normally open switch or open “on the shelf” – it will be closed when the breaker is energized and in closed position) and a 52b contact (normally closed switch or closed “on the shelf” – it will be open when the breaker is energized and in closed position). These contacts are mechanically controlled by the location of the breaker for maximum reliability, and (d) the interrupting subsystem. The interrupting subsystem consists of a breaker, a motor operated switch, etc.

Figure 1.3 illustrates the basic components of such a system. Figure 1.4 illustrates the symbolic representation of such a system. Figure 1.5 illustrates a sample control circuit for a protective relaying system. Figure 1.6 illustrates an example monitoring system for faults in the battery circuit of the control system. The technologies involved in each subsystem may vary. Table 1.3 lists the monitoring and decision components and Table 1.4 list the interrupting devices. These components will be reviewed next.

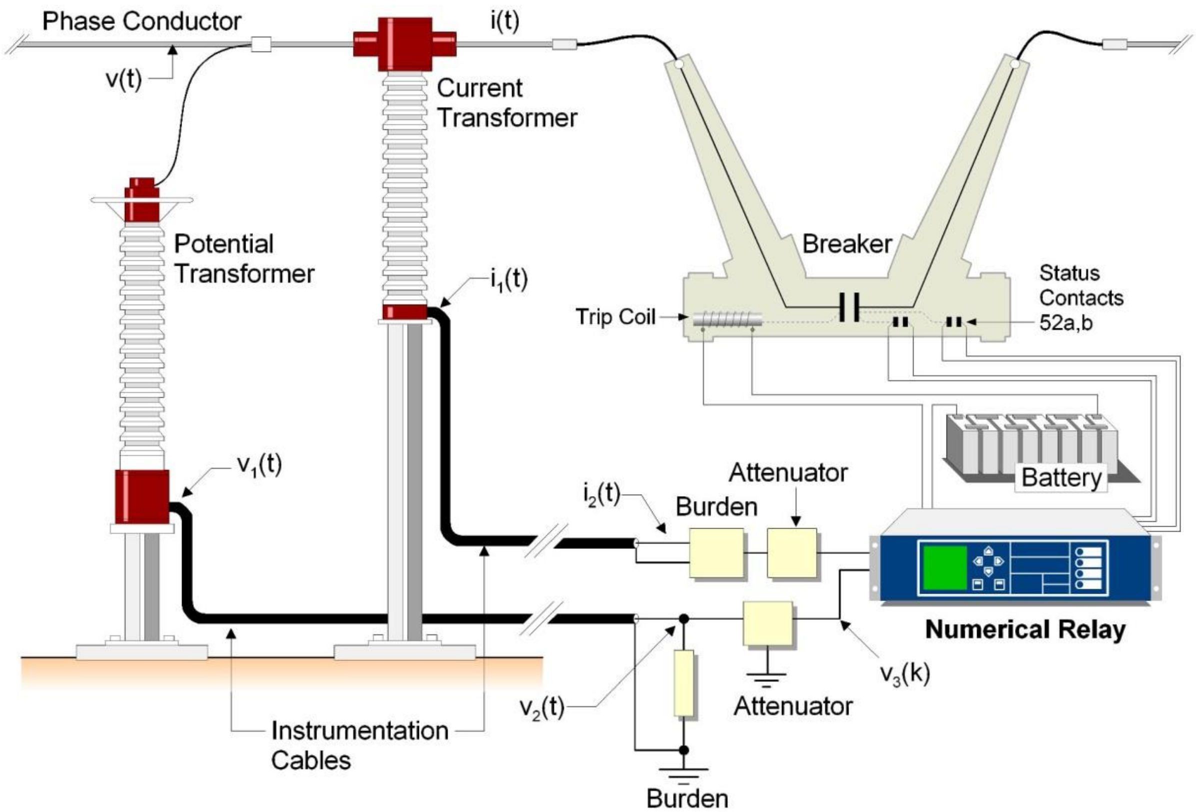


Figure 1.3 Illustration of a Protective Relaying System

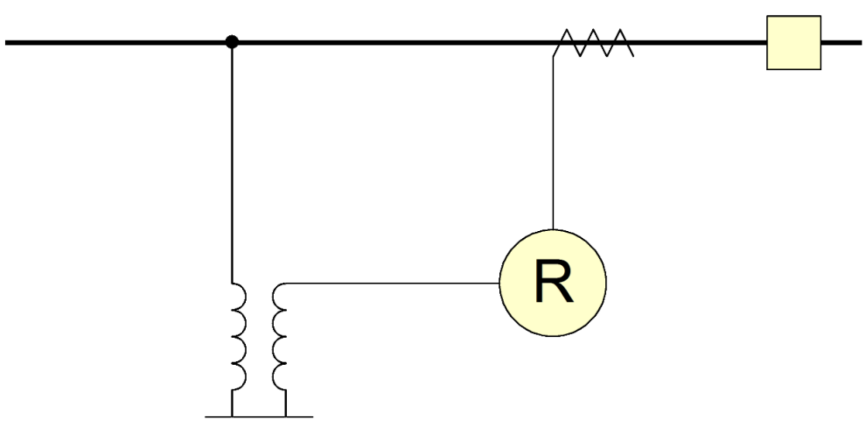


Figure 1.4 Symbolic Representation of a Protective Relaying System - Relay with Current and Voltage Input

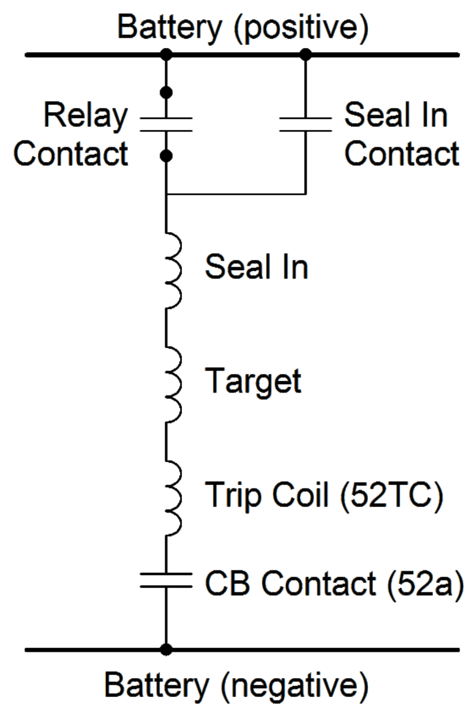


Figure 1.5 Example Control Circuit of a Protective Relaying System

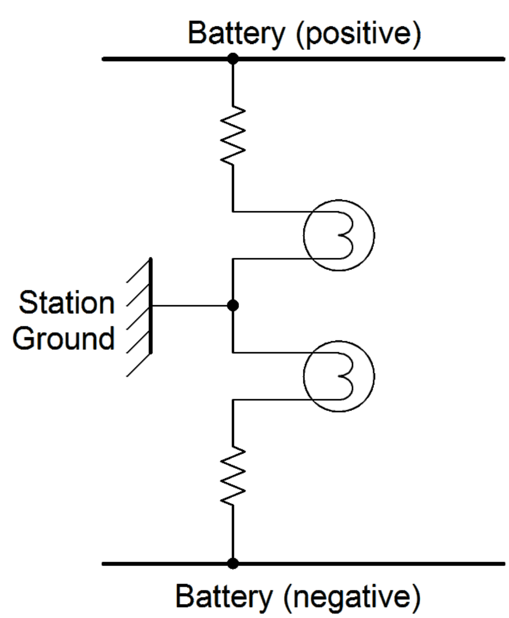


Figure 1.6 Example Monitoring System for Battery Faults in the Control Circuit of a Protective Relaying System

Table 1.3 Monitoring and Decision Subsystem Components

Instrument Transformers
Merging Units
Relays

Table 1.4 Interruption Devices

Fuses
Circuit Breakers

4.1 Instrumentation Subsystem

The objective of the instrumentation subsystem is to provide the proper interface between the high voltage electric power system and the relays that operate at relatively low voltage. The instrumentation consists of instrument transformers that convert the high voltage and high current of the power system into instrumentation level voltages and currents that can be fed into the relay. The electromechanical relays typically require relatively substantial electric energy to operate as compared to the numerical relays. For this the reason the input voltages and currents into the relays were standardized at relatively high levels as compared to the today's numerical relay needs. Standard voltages are 69 V and 115 Volts and standard currents are 5 Amperes and 1 Ampere. The instrumentation subsystem is connected to the high voltage system and generates replicas of the voltages and currents at relay standard levels. In general the instrumentation subsystem consists of voltage transformers and current transformers. We refer to these transformers as instrumentation transformers. The instrumentation subsystem for numerical relays also comprises an analog to digital conversion stage. Traditional numerical relays they package the analog to digital conversion unit within the relay. In recent years we have seen the separation of the analog to digital conversion unit from the relay with the introduction of the merging units. The two alternative approaches are show in Figure 1.7. The new technology of merging units has some very important advantages that will be discussed in later sections.

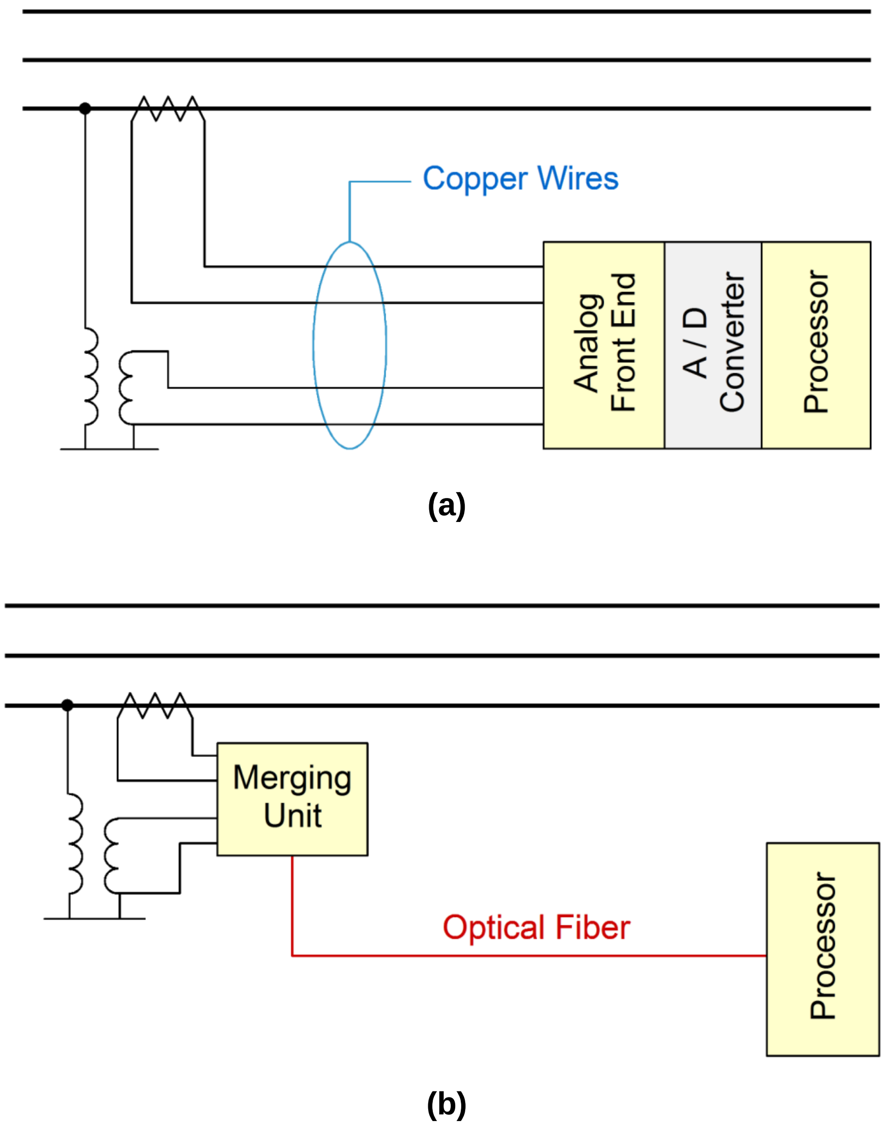


Figure 1.7 Numerical Relay Instrumentation

(a) Traditional Approach, (b) Merging Unit Approach

The standard types of voltage transformers are listed in Table 1.5. Specifically, there are four types of **V**oltage **T**ransformers (VT). (a) wound type potential transformer (PT), (b) capacitor coupled voltage transformer (CCVT), (c) resistive voltage divider (VD) and (d) optical voltage transformer, optical VT (EOVT). In the past, only the first two technologies were used and the standard secondary voltages were 69.3 V or 115 V. Figure 1.8 provides photographs of typical voltage transformers. The details of the various voltage transformer technologies will be examined in Chapter 6 in detail.

Figure 1.8 Photographs of Typical Voltage Transformers
(From Left to Right: EHV Potential Transformer, Two-Bushing Potential Transformer, Medium Voltage Potential Transformer, CCVT, Optical VT (EOVT))

Electric current is sensed with current transformers. There are four types of current transformers used for this purpose. These are listed in Table 1.6. Note that current transformers with current output CT/C or simply CT represent the workhorse for relaying applications. These transformers are applied in such a way that the relay is part of the secondary circuit of the CT as it is illustrated in Figure 1.9a. Note that the "burden", i.e. the impedance on the secondary circuit is the impedance of the relay or the sum of the impedances of all relays connected to the secondary circuit of the CT. The relay impedance is typically a very small resistance, typically 0.1 ohms. In general several relays can be connected (in series) to the same secondary circuit of a single CT. The CT/V devices are equipped with an impedance at the secondary and the output is the voltage across this impedance as it is illustrated in Figure 1.9b. The output of these CTs is normally connected to a high input impedance device. In many applications the primary current may be of very high value. Iron core CTs for these very high current may saturate and thus jeopardize the proper operation of relays. In these cases it is desirable to have CTs that are linear and do not saturate. The air core CTs (otherwise known as Rogowski coils) can be used. These CTs do not saturate. The issue with these CTs is their precision and they may need calibration every time they are installed in a new position. Recently optical CTs have been introduced (MOCT). Figure 1.10 provides photographs of typical current transformers.

We have mentioned two issues with CTs, saturation and calibration. Both of these issues are very important for relaying application. Chapter 6 provides additional information for the instrument transformers.

Table 1.5: Voltage Transformer Technologies

(Standard: Secondary Voltage: 69.3 Volts, 115 Volts)
PT
CCVTs
VDs
Optical VT - EOVT

Table 1.6: Current Transformer Technologies

(Standard: 5 Amperes or 1 Ampere)
CT/C
CT/V
Air Core CT
Optical CT – MOCT
(xxTyy or xxCyy, xx: max error, yy sec. voltage)

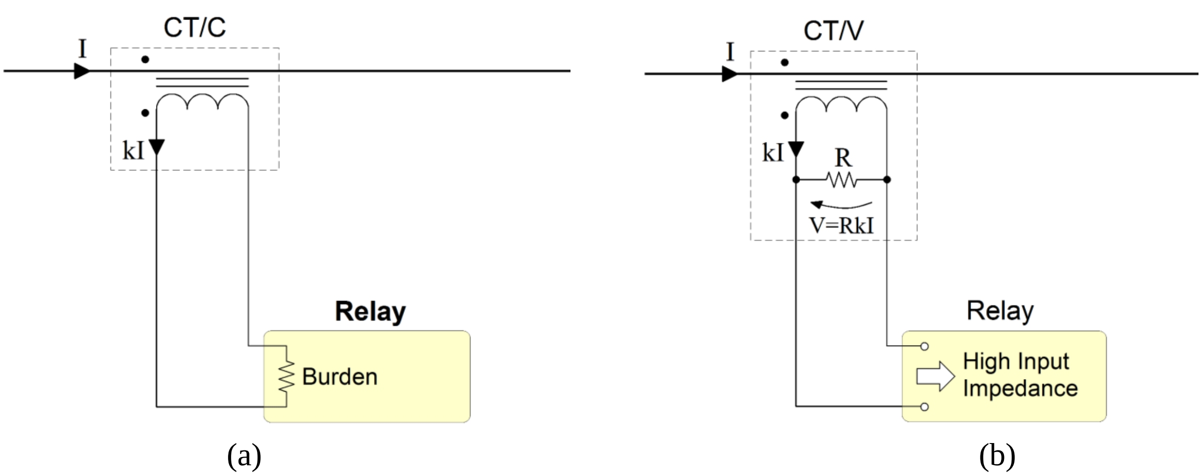


Figure 1.9 Illustrations of CT Connections to a Relay
((a) CT/C Connection: Burden is typically a Low Impedance, 0.1 Ohms or 0.14 Ohms, (b) CT/V Connection: High Input Impedance Relay)

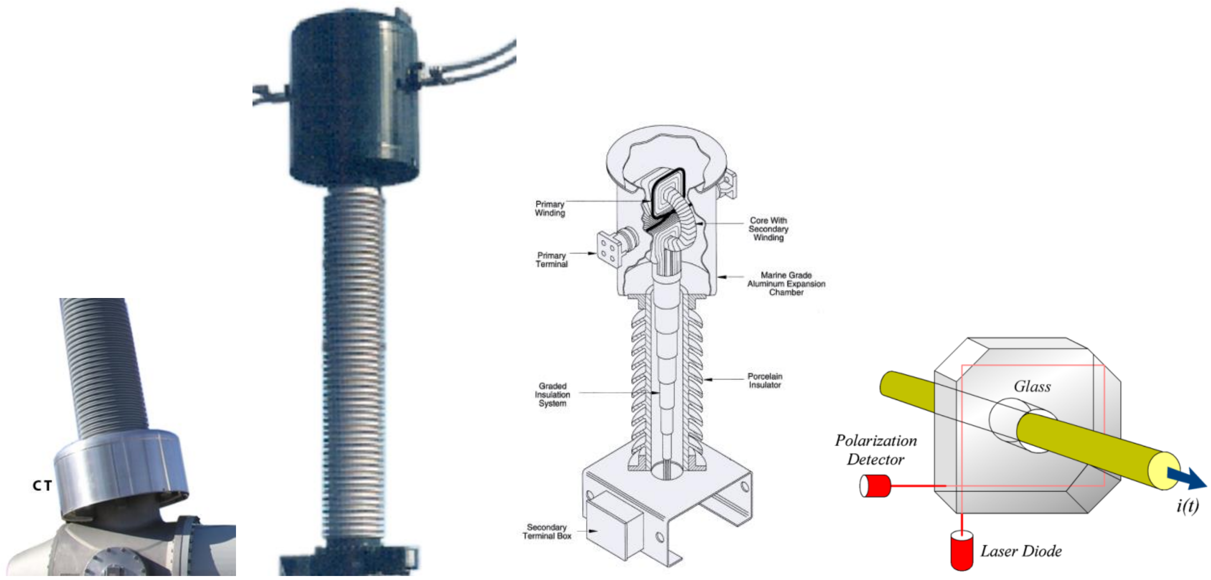


Figure 1.10 Photographs of Typical Current Transformers
(From Left to Right: CT mounted on Breaker Bushing, Free Standing CTs, Perspective Cutout View of a CT, Optical CT-MOCT)

Finally, relaying diagrams typically show the connections of instrument transformers to the relays. There are two types of diagrams: (a) wire diagrams and (b) schematic diagrams. The schematic diagrams show symbolically the CTs and VTs. The symbols used for symbolic diagrams are shown in Figure 1.11. The symbols on the left hand side of the diagram are typical in US practice and the symbols shown in the right hand side are typical of European practice.

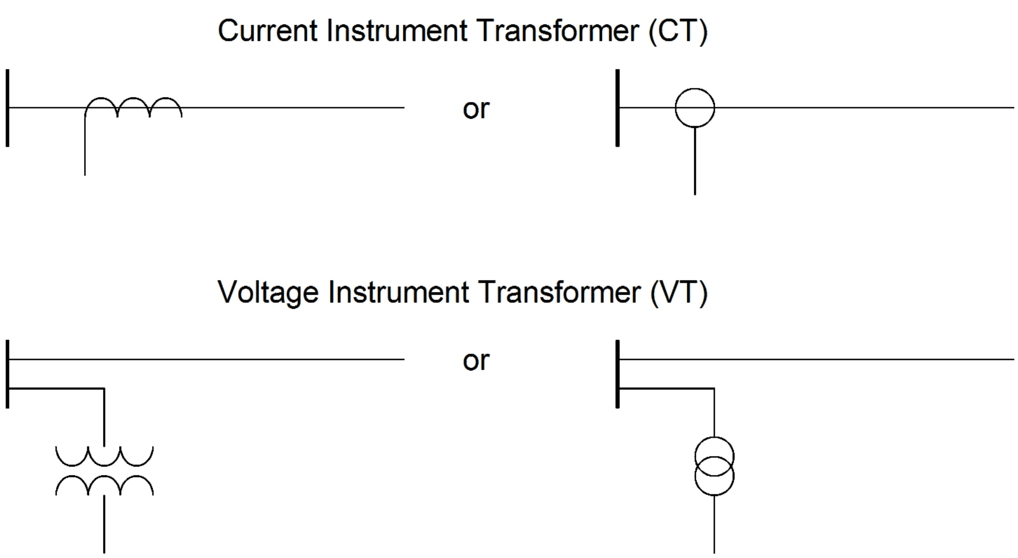
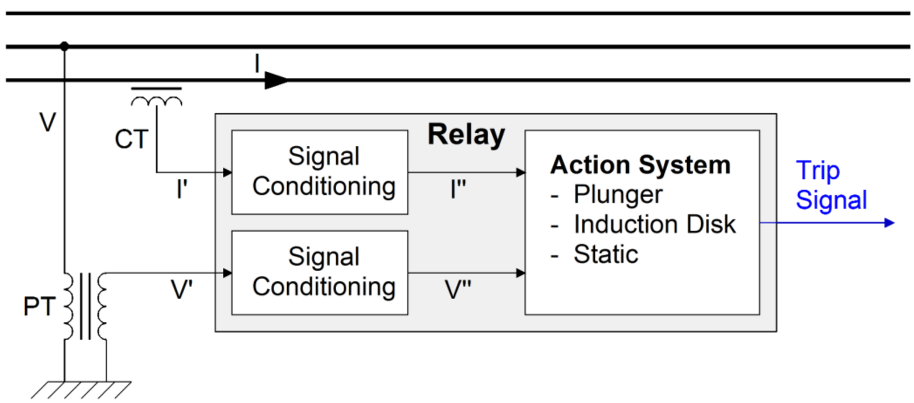


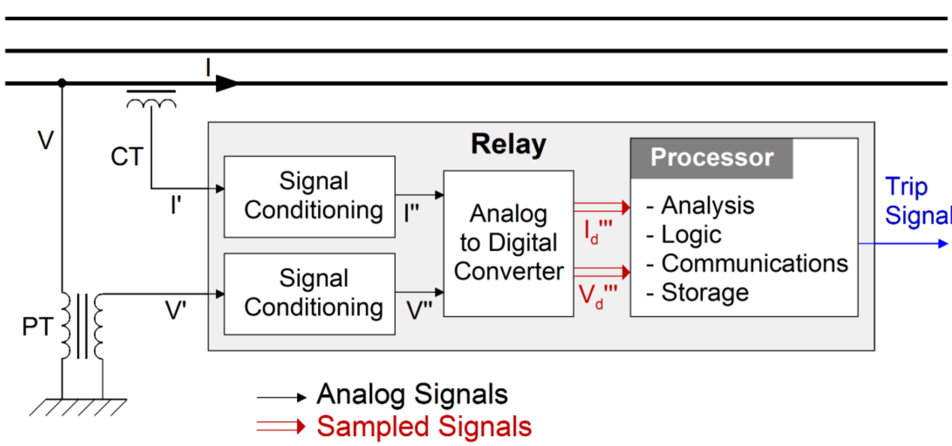
Figure 1.11 Representations of Instrument Transformers in Schematic Diagrams

4.2 Protective Relays

The generic structure of a protective relay (electromechanical or numerical) is illustrated in Figure 1.12. The relay takes inputs from instrument transformers. Typically, a signal conditioning circuit will be employed for filtering any unwanted transients and spikes in the signal. The filtered signals are fed to (a) the logic circuit of an electromechanical relay which may consists of an induction disk, plunger, solid state circuit, etc., see Figure 1.12a, or (b) to an analog to digital converter that samples the waveform of the voltages and currents and transmits the sampled data to the processor for analysis and protection logic, see Figure 1.12b. The logic circuit (electromechanical or numerical) will generate a control action which most times may be the trip signal for one or more breakers. There are many relay types, some of them with quite complex functions. The best source of information for specific relays is the manufacturer. Manufacturers of relays maintain web sites where someone can download manuals and other information pertinent to the relays.



(a)



(b)

Figure 1.12: Generic Structure of a Relay

((a) Electromechanical or Solid State Relay, (b) Numerical Relay)

Prior to the transistor and computer era, relay technology was based on electromechanical elements to perform the relay logic. The most common electromechanical components are the plunger relay, the induction disk/cup relay, and the balancing beam relay, see Figure 1.13. These electromechanical components have been used in combination with additional circuitry to perform remarkably complex logic functions, such as time delay overcurrent protection, directional protection, differential protection, distance protection, etc. Examples of these protection schemes will be examined in Chapter 5.

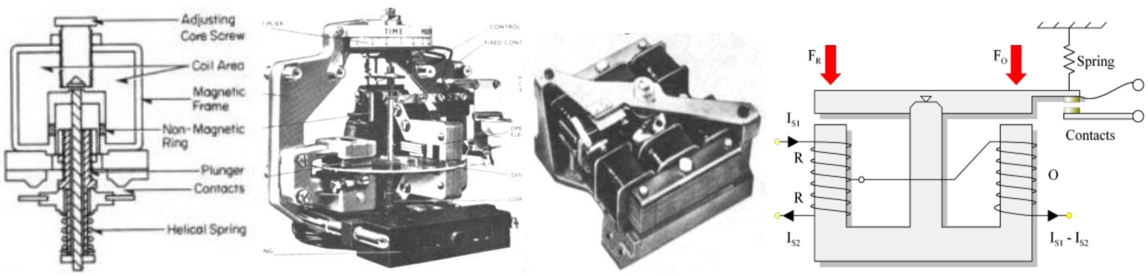


Figure 1-13: Various Types of Electromechanical Protective Relays

(From Left to Right: Plunger, Induction Disk, Induction Cup, Balancing Beam)

The invention of the transistor enabled the development of solid state relays. Solid state relays (static relays) are devices that include discrete component electronic circuits that mimic the logic of the electromechanical relays. The advantages of the solid state relays are (a) smaller size and (b) ability to perform more complex logic than the electromechanical relays. The era of solid state relays was short lived because of the development of the microprocessor based relay (or numerical relay).

Numerical relays consist of an input section where the analog signals of voltages and currents are conditioned by appropriate analog filters (protection & isolation) and then digitized by means of analog to digital converters (DAQ), see Figure 1.14. The digitized signals are processed by microprocessors via algorithms that perform identification of the system conditions and the protective function. In addition numerical relays can be directly interfaced with the control circuit via input contacts and output contacts as shown in Figure 1.14. For an example of where the input and output contacts may be connected see Figure 1.3. Other major advantages of numerical relays is their ability to (a) display results, operational status, logic status, etc. on display screens via user interfaces, (b) store data when necessary (by appropriate triggering options) for later viewing and analysis (oscillography), and (c) communicate information with the rest of the world via different communication channels.

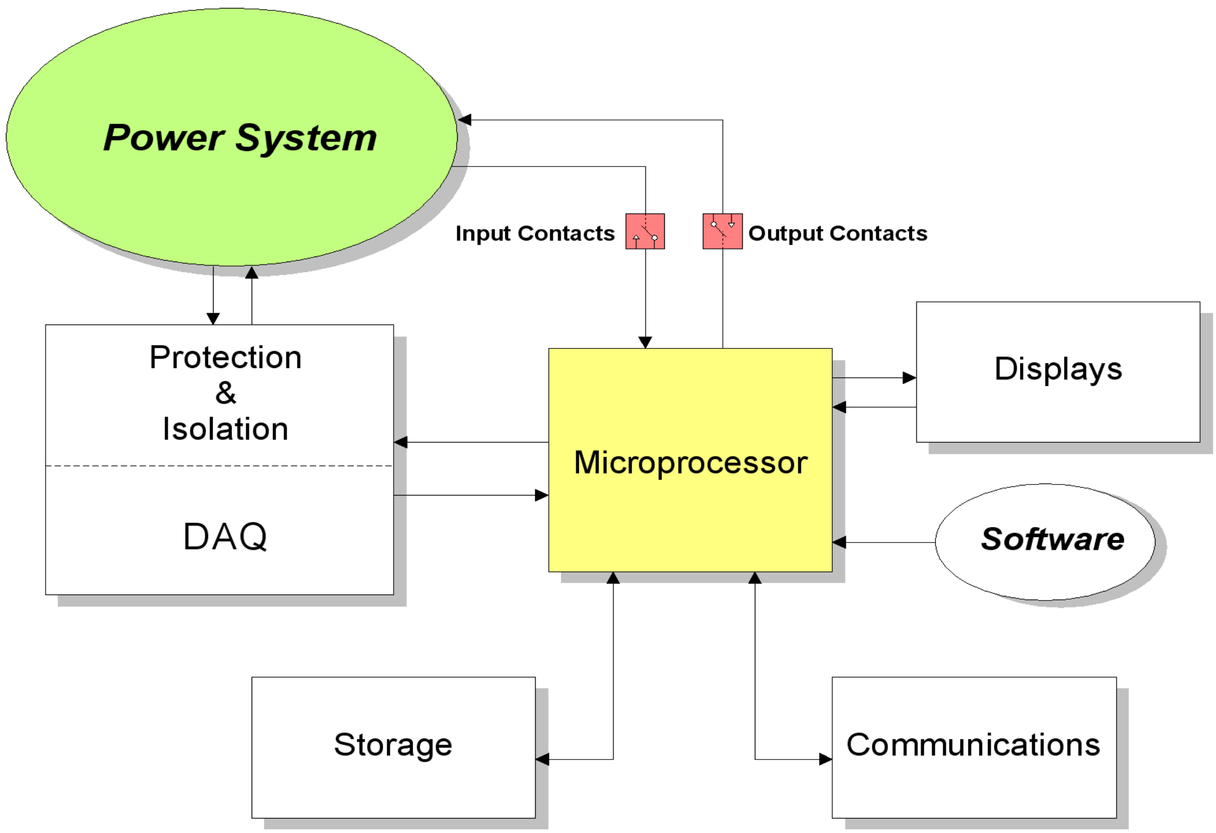


Figure 1.14: Block Diagram of a Numerical Relay

In general electromechanical relays have an analog circuit for each protection function. This makes electromechanical relays large by necessity. In addition because of packaging considerations typically each electromechanical relay has only one or a couple of functions packed in one box. This means that a protection scheme that may require several protection functions it may require many relays creating the need for larger real estate.

Numerical relays are simpler from this point of view. Since each protection function is simply a computer program the size of the numerical relay does not increase with the addition of another protection function. Therefore it is customary to design the software of numerical relays so that a single relay will perform several protection functions. For this reason, we many times refer to numerical relays as multi-functional relays.

4.3 Fuses

Fuses are the simplest protective devices. A fuse consists of a filament which is designed to melt when the electric current through it exceeds a certain value. This simple design allows a fuse to be inserted into the circuit that is supposed to protect. One major disadvantage of the fuse is that once it operates, the circuit remains open until a human replaces the fuse.

4.4 Breakers

Breakers are the main technology for interrupting high voltage electric circuits. Over the years many different circuit breaker technologies have been developed for high voltage circuits. The various technologies are listed in Table 1.7. Figure 1.15a illustrates an 230 kV oil circuit breaker (OCB) and Figure 1.15b illustrates a 500 kV SF6 circuit breaker. Figure 1.16 illustrates the internal construction of a medium voltage breaker. A brief discussion of the characteristics of each technology is given in Chapter 5. From the application point of view, the important parameters of a breaker are (a) interrupting capability rating, (b) capability to withstand transient voltage recovery expressed in microseconds, and (c) maintenance requirements. These parameters are defined in standards.

Figure 1.15: High Voltage Circuit Breakers
((a) 230 kV Oil Circuit Breaker (OCB) With Switches – Courtesy New York Power Authority, (b) 500 kV SF6 Breaker – CTs are Mounted on the Bushings of the Breaker)

Figure 1.16: The Internal Construction of a 13.8 kV Circuit Breaker

Table 1.7 Breaker Technologies

Oil Circuit Breaker
Air Blast Circuit Breaker
Vacuum
SF6

5.0 Zones of Protection

The protection problem of an electric power system is a very complex problem. To manage this problem, we partition the problem into smaller problems. Specifically, we use the concept of protection zones. A protection zone is a small portion of a large complex power system. The problem of protecting this smaller portion of the system is much simpler than the problem of protecting the entire power system. The protection system of a protection zone is responsible for protecting this portion of the system for any faulty condition that initiate in the protection zone. Note that as long as any component of the system belongs to at least one protection zone, then this component will be protected. A typical partition of a power system into protection zones is illustrated in Figure 1.17.

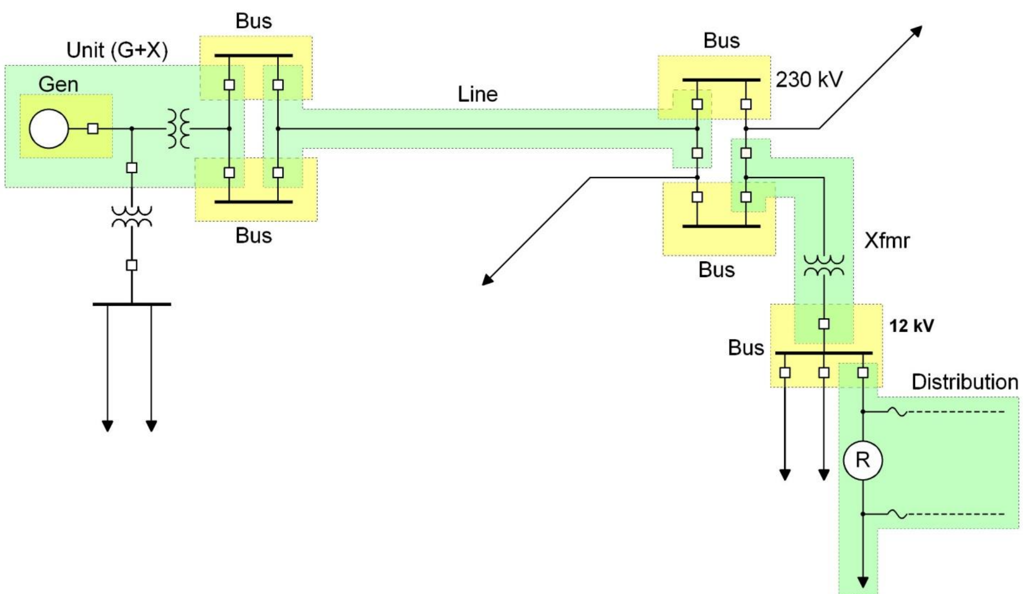


Figure 1.17: Example of System Partition into Protection Zones

One may define a protection zone for any power system component or a cluster of power system components. Table 1.8 tabulates some of the usual protection zones.

Table 1.8: Typical Protection Zones

Generator
Transformer
Unit (Generator and Step-Up Transformer)
Bus
Transmission Line
Motor
Capacitor Bank
Reactor
Other

6.0 Objectives of Protective Relaying

The protective relaying system of an electric power system has to meet a number of objectives such as load conservation, selectivity, safety, speed, etc. Some of the objectives may be conflicting for example load conservation requires that we should delay any protection action so that we serve load while safety requires that we take protection action as soon as possible to maximize safety. This section presents a discussion of these objectives.

Safety: Safety of personnel near and around electrical installation is of paramount importance. Any person in the vicinity of an electric power system and/or touching and properly handling electrical equipment should be safe under any adverse conditions. When a fault occurs, equipment cabinets, support structures etc. may develop voltage that may be applied to the persons touching these equipment. The design of the system and the protection infrastructure should be such as the persons in this situation should be safe. This is achieved by a combination of: (a) proper design of grounding and bonding systems that will limit the voltages to safe levels and (b) by designing the protection system to clear the fault condition as fast as practicable so that the exposure time is minimized and the safety of personnel is maximized. In this book we address the protection system design. For additional information of problem (a) – design of safe grounding systems, consult reference [???].

Equipment Damage Avoidance: A fault in the system may cause flow of excessive current in parts of the system, overvoltages in other parts of the system or speeding-up or down of rotating equipment and mechanical forces on conductors. These conditions may damage equipment and therefore the source of the problem should be removed as fast as practicable so that damage is avoided.

Load Conservation: Another objective is to continue to provide power to customers that may need electric power. This requirement suggests that the protective system should act in such a way that customers are not affected if possible. For networked systems, such as transmission systems, protective relays act to disconnect faulted devices in relatively fast times so that the system continues to provide power to customers. This means that the system remains stable, frequency deviations are minimal and the entire system continues to operate in synchronism and provides electric service to customers.

Protection Reliability: It is defined as the ability of the protective system to operate correctly whenever the conditions require operation (dependability) and to not operate on tolerable conditions or conditions that are the responsibility of another protective system (security). Thus protection reliability requires (a) dependability and (b) security. These two concepts are defined as follows: **Dependability**: Dependability is defined as the degree of certainty that the relay system will operate correctly. Thus dependability refers to the ability of a relaying system to correctly identify abnormal and intolerable conditions that are originated within its protection zone and to act and successfully interrupt the abnormal condition. **Security**: It is defined as the degree of certainty that a relay will not operate incorrectly. Thus security refers to the ability of the relaying system to correctly identify whether the cause of an abnormal condition is external to the zone that it is protecting and therefore not to operate for this condition. These definitions are schematically represented in Figure 1.18. Note that dependability means that when the relay operates, the operation is the correct action. Security means that when the relay does not operate, the non-operation is the correct action. Note that Figure 1.18 defines a reliable protection system as one that always takes the correct action, whether this action is “no-operation” or “operation”. An unreliable protection system is one that may not-operate when operation is required or it will operate when no-operation is the correct action.

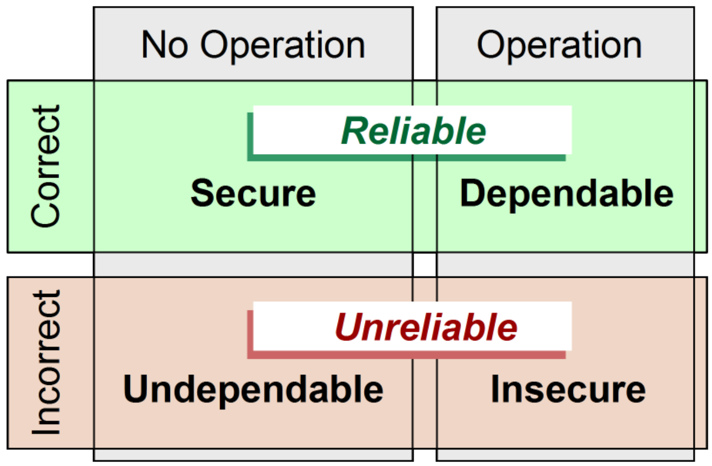


Figure 1.18: Definition of Protective System Reliability

Selectivity: It is defined as the ability of the protective relay system to disconnect the minimum possible part of the system so that service continuity is maximized, i.e. minimum number of customers is interrupted. What is minimum in a particular system depends on the number and location of interrupting devices (breakers, etc.). Obviously the more interrupting devices there are in a system the more selective the protective system can be. There is always a tradeoff between cost and selectivity. Selectivity can be also affected by the arrangement of the breakers, for example, breaker and a half, double breaker, ring bus, etc.

Speed: The speed by which the protective system disconnect a faulted device. The speed is many times dictated by (a) the time it is required to detect the condition, (b) the time delays required for coordination with other protective devices and (c) the time it takes for the breakers to clear the fault once they have received the tripping signal. The speed of operation of the protective system is the sum of all of these times.

Economy: As in any engineering system, the cost of the protective system is always a factor. The cost typically includes the cost of the interrupting devices, type and number, the instrumentation and the cost of relays and controls.

Many of these objectives are conflicting. For example selectivity and speed are conflicting objectives. For proper coordination among protective devices it is necessary to introduce delays in the response time of relays and thus sacrificing speed of operation. The cost of the protective system is conflicting with all other technical objectives of the protective system. For example to maximize selectivity, one needs to include more interrupting devices thus increasing the cost.

Similar arguments can be made for the other technical objectives of a protective system. In general past practices and experience play a very big role at the selection of the protective system. The importance of the particular protective zone also plays a big role in determining the technical selections. In the past, the use of single function electromechanical relays made these choices difficult. Today, the use of numerical multifunctional relays have simplified the process of selecting the protective functions but the issue of selecting and designing the number and location of interrupting devices remains the same.

7.0 Know the Power System

Judicious application and design of protective systems requires that the operating characteristics of the electric power system as well as the response of the system to abnormal conditions are well understood. The basic principle of protective relaying is to be able to identify all possible intolerable conditions from a limited number of measurements (typically three voltages and three currents) and disconnect the source of the problem. This is achieved by extensive study of the system to be protected so that the correct prognosis be performed and the root cause of the problem be identified so that the correct device is disconnected. Many times, oversights result in mis-diagnosed problems that may lead to relay operation on healthy parts of the system (insecure operation). The power system is a complex system and many complex responses can occur during abnormal conditions. In the past the task of developing knowledge about the system to be protected was with rudimentary tools and diagnosis by symptoms. Today we have excellent tools for comprehensive investigations not only by post mortem analysis of recorded system performance but also with excellent analytical tools that can predict system response and relay performance. It is impossible in a text book to cover all the possible complex behavior of a power system that affect protection reliability. We provide a couple of examples that demonstrate known problems.

Example E1.1: Consider the electric power system of Figure E1.1a. The system consists of five transmission lines, three of them on the same right of way and thus mutually coupled. One of the mutually coupled lines is protected with a ground fault relay that is set at 0.5 amperes. Assume a ground fault on the other mutually coupled line. Compute the zero sequence current on the secondary of the CTs on the unfaulted line.

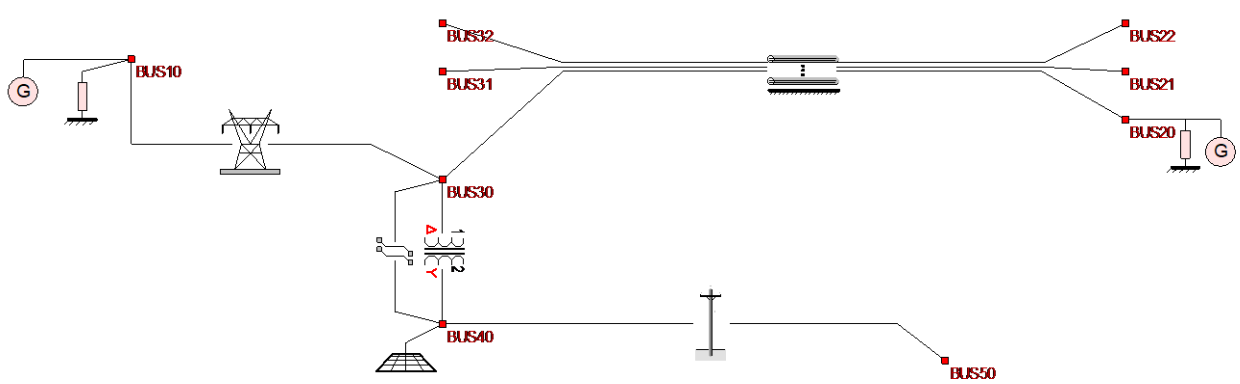


Figure E1.1a

Solution: to be continued.

Example E1.2: Consider the distribution circuit of Figure E1.2. Assume that the phase B conductor has broken and fell on the soil where it makes conduct with the soil for a length of 35 feet. Compute the total fault current. The distribution line is protected with a ground fault relay that is set to 0.5 A. The CT is 1200:5. The soil resistivity is 85.0 ohm-meters.

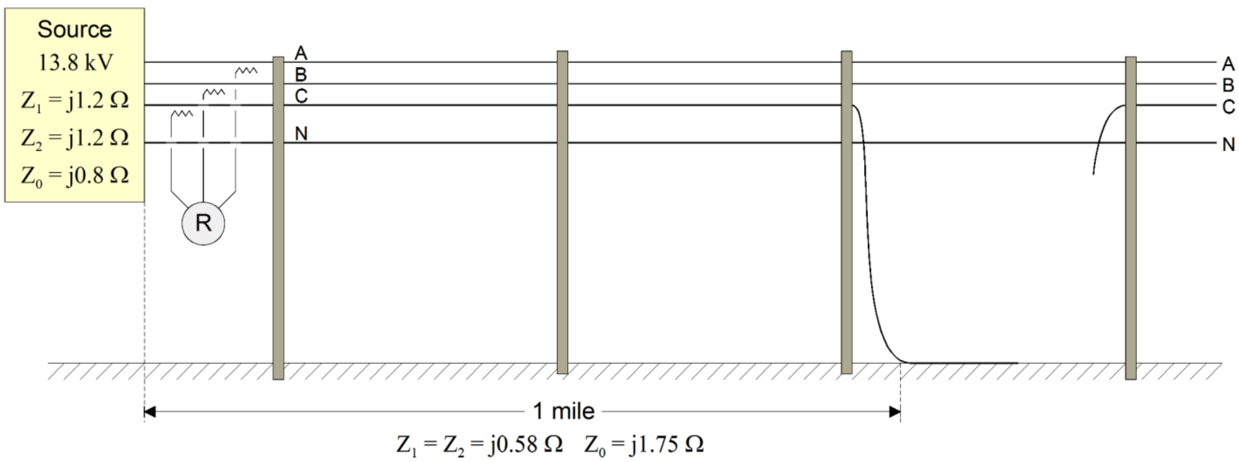


Figure E1.2

Solution: to be added

8.0 Standards and Books

The area of protective relaying is rich in bibliography. An extensive bibliography is provided at the end of this book. In addition there are many standards that address the recommended protective relaying practice as well as standardization of terms and definitions. For example, standard [???] provides the standard numbers for identification the various protective relaying functions and devices. A standard notation is very important. The designation numbers listed in Table 1.9 are typically used to refer to relaying functions. It is important to note that combinations do exist. For example a time overcurrent function (51) with voltage control is typically referred to as function 51V.

Table 1.9 Device Numbers - IEEE C37.2-1996 (1987)

No.	Device Description	No.	Device Description
1	Master Element	51	AC Inverse Time Overcurrent Relay
2	Time Delay Relay (starting or closing)	52	AC Circuit Breaker
3	Checking or Interlocking Relay	53	Field Excitation Relay
4	Master Contactor	54	Turning Gear Engaging Device
5	Stopping Device	55	Power Factor Relay
6	Starting Circuit Breaker	56	Field Application Relay
7	Rate of Change	57	Short-Circuiting or Grounding Device
8	Control Power Disconnecting Device	58	Rectification Failure Relay

9	Reversing Device	59	Overvoltage Relay
10	Unit Sequence Switch	60	Voltage or Current Balance Relay
11	Multifunction Device	61	Density Switch or Sensor
12	Overspeed Device	62	Time-Delay Stopping or Opening Relay
13	Synchronous Speed Device	63	Pressure Switch
14	Underspeed Device	64	Ground Detector Relay
15	Speed or Frequency Matching Device	65	Governor
16	Data Communication Device (new for Ethernet switch)	66	Notching or Jogging Device
17	Shunting or Discharge Switch	67	AC Directional Overcurrent Relay
18	Accelerating or Decelerating Device	68	Blocking or “Out-of-Step” Relay
19	Starting-to-Running Transition Switch	69	Permissive Control Device
20	Electrically Operated Valve	70	Rheostat
21	Distance Relay	71	Liquid Level Switch
22	Equalizer Circuit Breaker	72	DC Circuit Breaker
23	Temperature Control Device	73	Load-Resistor Contactor
24	Volts per Hertz Relay	74	Alarm Relay
25	Synchronizing or Synchronism-Check Relay	75	Position Changing Mechanism
26	Apparatus Thermal Device	76	DC Overcurrent Relay
27	Undervoltage Relay	77	Telemetering Device
28	Flame Detector	78	Phase-Angle Measuring Relay
29	Isolating Contactor or Switch	79	AC Reclosing Relay
30	Annunciator Relay	80	Flow Switch
31	Separate Excitation Device	81	Frequency Relay
32	Directional Power Relay	82	DC Load-Measuring Reclosing Relay
33	Position Switch	83	Automatic Selective Control or Transfer Relay
34	Master Sequence Device	84	Operating Mechanism
35	Brush-Operating or Slip-Ring Short-Circuiting	85	Pilot Communications, Carrier or Pilot-Wire Relay
36	Polarity or Polarizing Voltage Device	86	Lockout Relay
37	Undercurrent or Underpower Relay	87	Differential Protective Relay
38	Bearing Protective Device	88	Auxiliary Motor or Motor Generator
39	Mechanical Condition Monitor	89	Line Switch
40	Field (over/under excitation) Relay	90	Regulating Device
41	Field Circuit Device	91	Voltage Directional Relay
42	Running Circuit Breaker	92	Voltage and Power Directional Relay
43	Manual Transfer or Selector Device	93	Field-Changing Contactor
44	Unit Sequence Starting Relay	94	Tripping or Trip-Free Relay
45	Atmospheric Condition Monitor	95	Spare
46	Reverse-Phase or Phase-Balance Current Relay	96	Spare
47	Phase-Sequence or Phase-Balance Voltage Relay	97	Spare
48	Incomplete Sequence Relay	98	Spare
49	Machine or Transformer Thermal Relay	99	Spare
50	Instantaneous Overcurrent Relay		

Since the industry has moved almost 100% to digital relays, standards have been developed to define how the various quantities should be computed and how the relays should respond. As an example, the IEEE Std C37.112-1996 entitled “IEEE Standard Inverse-Time Characteristic Equations for Overcurrent Relays”, defines the time overcurrent function. Specifically, the standard provides analytical expressions that match the characteristics of time overcurrent electromechanical relays. These analytic expressions enable the design of numerical relays that match the characteristics of time overcurrent electromechanical relays.

The number of relaying standards is very large covering all the protective relaying functions, the application to various component protection, communications among relays, storing and retrieving disturbance data, etc. As we proceed in the discussion of various topics, some of these standards will be mentioned. It should be also understood that as technology evolves, some of these standards become dated.

9.0 Power System Relaying and Automation

Protective relays, recorders, meters, etc. are presently almost exclusively digital. Each of these devices is a computer (microprocessor) interfaced with data acquisition systems, control circuits, communications, etc. in other words it is an **embedded system**. As such they have the capability to communicate with other digital devices, to be calibrated (locally or remotely), to exchange information, to be reprogrammed, etc. There has been tremendous amount of work to standardize digital protective devices and in general IEDs in terms of functions, communications, interoperability, etc. The objective of the work is to increase the automation in the system and using the more recently introduced term to develop the smart grid.

The objectives of automation are:

1. Decrease labor cost in protective system installations
2. Renter all protective relays interoperable
3. Standardize relaying functions
4. Enable remote protective relaying settings
5. Enable remote calibration
6. Enable model validation
7. Enable post fault data retrieval and analysis
8. Enable communication and sharing of data with all entities involved.

Efforts to increase automation have evolved to a nice advanced state. As with any automation, breakthroughs in technology, render past systems obsolete while the new systems are equipped with many more new functions and capabilities. Figure 1.19 illustrates the evolution of the automation levels. Figure 1.19a shows an electromechanical relay with add-on meters for monitoring the system. Note that separate devices are needed for the meters. The arrangement includes one relay and four meters to provide one single protection function and displays of (a) real power, (b) reactive power, (c) electric current, and (d) voltage. The photograph on the left shows one such system in existence today (there are many such systems in existence). This type of technology represents the state of art in the mid 1980's.

Figure 1.19b shows a numerical relay with point to point communication. Note the relay has a display that in general can display a user selected quantity (voltage, current, real power, reactive power, harmonics, imbalance, etc.). Therefore there is no need for separate meter devices. The arrangement includes only one relay. The relay may have multiple protective functions. In addition it has a communication port for transferring the data to another location (i.e. central office). This type of technology emerged in the late 1980's and early 90's. The photo on the left side of the figure illustrates such a system. Note the presence of meters (analog or digital) as well - in the transition years the meters are still used for redundancy. Note that the digital relay can provide the metering function as well.

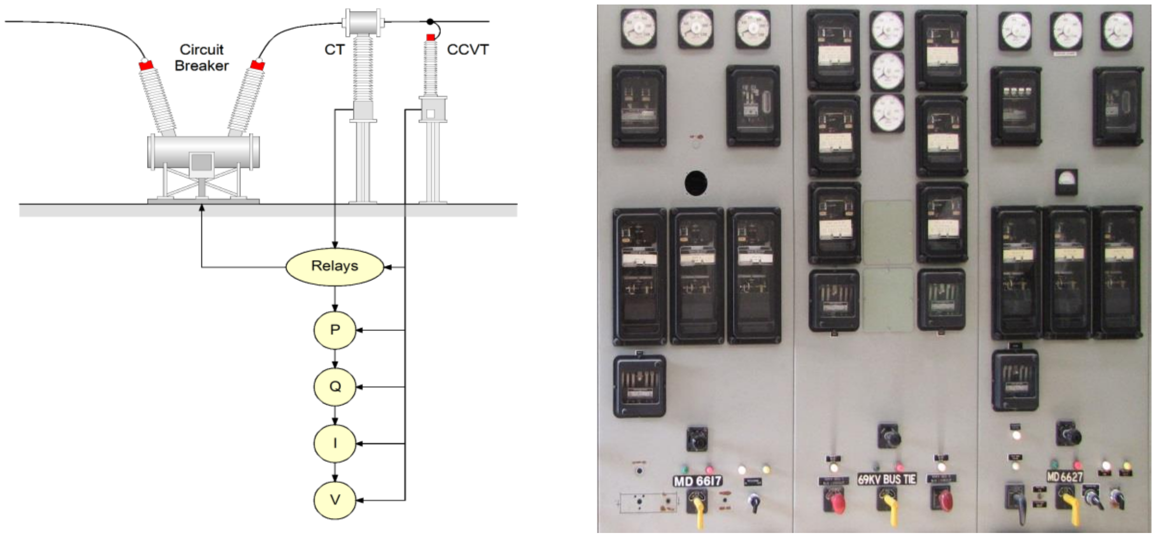


Figure 1.19a: Evolution of Automation in Protection and Control
(Protective Relaying and Displays with Electromechanical Technology (circa 1980))

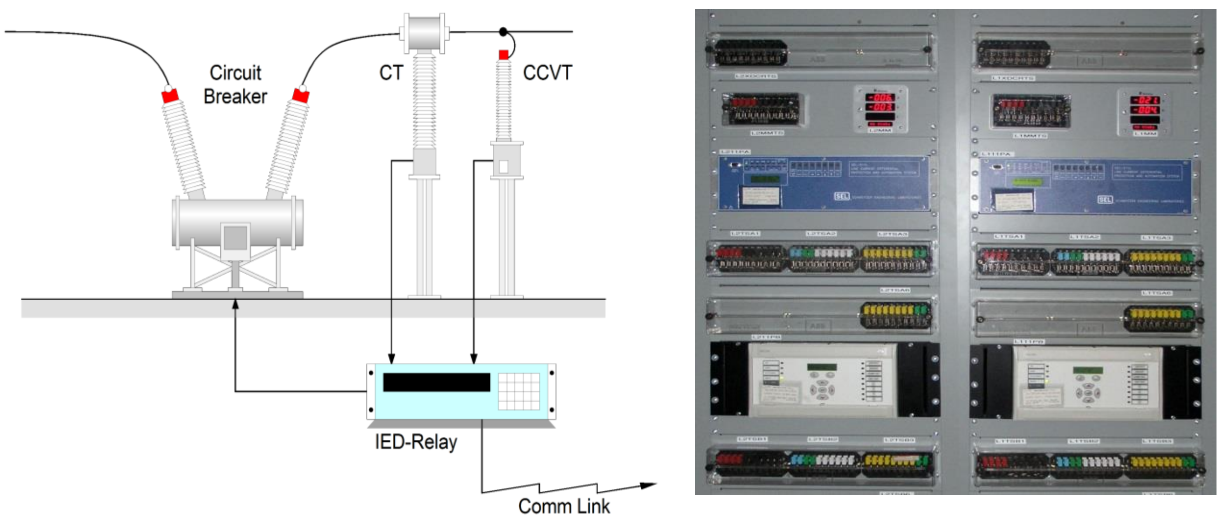


Figure 1.19b: Evolution of Automation in Protection and Control
(Protective Relaying, Displays and Communications with Early Generation Numerical Relays (circa 1990))

Figure 1.19c shows a present day approach to substation automation. The approach includes merging units (i.e. data acquisition systems that are placed in the field next to instrument transformers and convert the measurements into digital form), a process bus that receives data from the merging units via fiber optic lines, numerical relays that are attached to the process bus, a substation bus that receives processed data from the relays (for example, voltage/current magnitude, phasors, power, etc.), communications with typically multiple points/entities and a man machine interface (Human Interface) to monitor the operation of the system. Note that this automation approach also integrates other intelligent electronic devices (meters, data concentrators, etc.) into the system thus providing a fully integrated approach to data management and utilization.

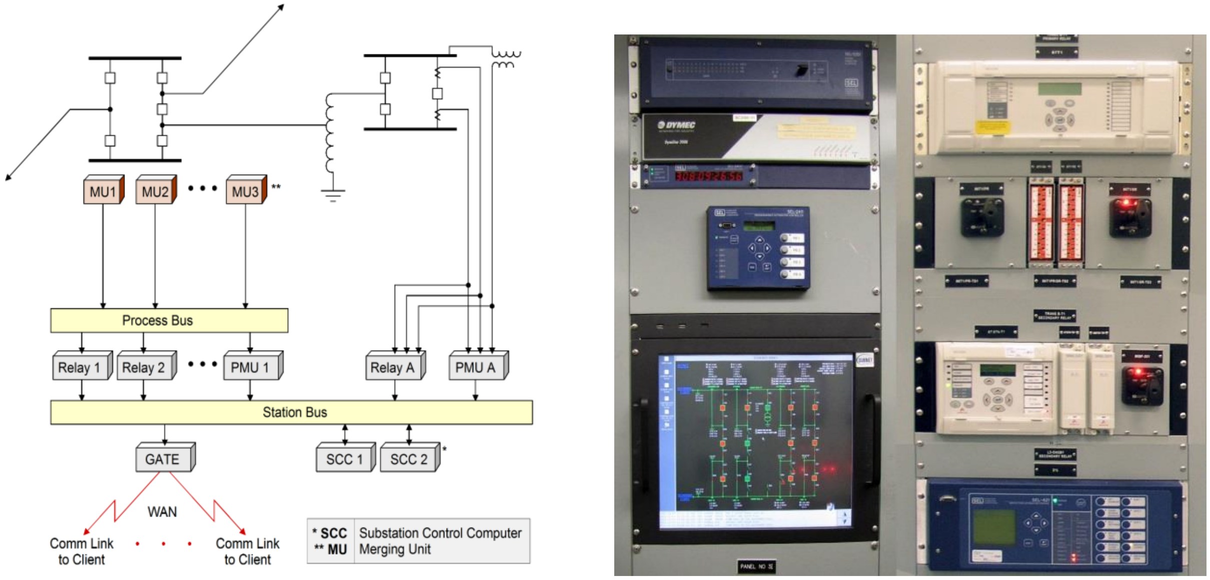


Figure 1.19c Evolution of Automation in Protection and Control
(Merging Units, Process Bus, Interoperable Relays, Communications, Man Machine Interface (circa 2009))

Another important issue is the fact that today the protective relays are also used for control and operation. Specifically, in the past the electric power system was instrumented with a dedicated SCADA system. SCADA stands for **S**upervisory **C**ontrol **A**nd **D**ata **A**cquisition system. Since numerical relays can provide the measurements, supervisory control and tripping capability, then it is natural to use the numerical relays for this purpose and eliminate the need for a separate system. Numerical relays can do so without affecting their main function of protection. For this reason, new substations have an integrated SCADA system based on numerical relays and automation hardware and software such as those shown in Figure 1.19c.

From a historic point of view the present status and standards evolved from an EPRI project under the name UCA (Utility Communication Architecture). This project developed the general framework for communications among digital relays and in general IEDs. The concepts were picked up by standards developing entities and resulted in many different standards that address various aspects of automation. This activity is very important as it addresses standardization of how the information is collected, packaged, transmitted, interpreted and used by other devices and used by applications. It should be understood as the digital automation technology is evolving many standard protocols for handling the transmission and utilization of data and information were developed, some of them proprietary to specific entities, some of them public standards. The end result is that presently we have many communication protocols, some of them proprietary. This makes the task of integrating IEDs from different manufacturers into an operational automation system very difficult. The effort to develop the IEC 61850 standard has as objective to provide a unified standard for physical communication layers and protocols as well as packaging the information in a way that can be correctly interpreted and used by any device in the system. More details will be provide in subsequent chapters.

10.0 Power System Relaying: Research Issues

The performance of protective relays is of paramount important to the reliability and safety of any power system. As technology evolves, protective relaying schemes improve. In the last few decades we have seen amazing technology that benefited the field of protection. Yet we do not have a perfect 100% reliable protective system for power systems. This is exemplified by the fact that we have a number of protection gaps, i.e. abnormal conditions in a system that we cannot detect and properly isolate from the system. It is expected that new technologies and new research and approaches to protection will provide reliable solutions for all protection problems. This section provides few thoughts on current research issues in the area of power system relaying, the underlying technology and trends.

The capabilities of numerical relays have increased with time as more powerful microprocessors are used. The protective algorithms have become more sophisticated and its application and coordination with other protective devices have become very complex. Complexity of relaying functions and settings creates concerns regarding training requirements for new protective relaying engineers and possibility of human errors. Another concern is the fact that despite the recent advances we still have protection gaps: for example we do not have reliable protection schemes for high impedance faults. For both of these concerns, i.e. complexity and protection gaps, the role of automation becomes even more critical. Automation can help by providing the infrastructure to automate many checks and balances that are of concern as the complexity of protective schemes increases. Here we discuss some of them and offer comments how research can provide good solutions to these concerns.

Field validation of calibration and settings of relays: Traditionally, when new relays are installed, a commissioning process takes place to validate the field installation. This process involves testing the correct values of the instrument transformers, the connections to the relay (polarity, phase sequence, etc.), the setting of the relay, etc. For each type of relay, commissioning procedures have been developed. As new installations become more complex with a typical substation having multiple numerical relays and each relay is a multifunctional relay the complexity of field testing increases drastically. Automating this process provides benefits in terms of reduced manpower in performing these tests and avoidance of possible human errors. Research into more automated field testing procedures will continue.

Hidden Failures: The protective relaying system comprises many components that are subject to failure. A failure of one component may occur in such a way that could not be recognized (hidden failure) until a fault occurs in which case it is possible that the hidden failure may affect the security of the relaying scheme, i.e. an insecure operation (operation for a condition that is tolerable) or worse a no-operation for a condition that should be cleared. A hidden failure can be also as simple as a wiring to the wrong tap of an instrument transformer, or wrong wiring of the zero sequence current circuit, etc. There are efforts to develop on-line real time systems for continuously monitoring the performance of the protective relaying scheme and possibly identifying any hidden failures in real time. Such an effort is based on the substation based dynamic state estimator. Specifically, the data from the relays are continuously compared to the model of the system. Any discrepancies will indicate an abnormality which can be identified and corrected before it is too late. Communications – automation – standards play a crucial role in the smooth implementation of these approaches.

Instrumentation and its reliability – saturation, transients and effects: Instrumentation and the accuracy and integrity of the instrumentation is critical to protection applications and control functions. Instrument transformers do occasionally fail as well and many times these failures may remain undetected, such as an open delta, etc. There is also new technology in instrument transformers, such as the optical VT/CT which provide directly a digital output with communications such as interfaces to connect to a LAN or to communicate via Bluetooth, etc.

In 1989, the federal government initiated the development of the GPS satellite system by placing on orbit a number of satellites each year - the system was completed in 1991. This system provides a time reference anywhere on the globe with precision better than one microsecond. Then, by providing a GPS receiver to the numerical relays, synchronizing the digitization of the waveforms, and using hardware that do not introduce timing errors greater than a microsecond, we obtain measurements that are synchronized with very high accuracy (see section 6). This approach opens up many possibilities as we can now take measurements at distant locations and these measurements will be synchronized. Most relay manufacturers offer this capability today. This technology allows direct monitoring of wide areas as well as improved special protection systems (SPS). Few comments on these applications follow.

The term **W**ide **A**rea **M**onitoring **S**ystem (WAMS) was introduced to facilitate the dynamic monitoring of a wide area of a power system with the specific applications of system protection. The first WAMS installation on the Eastern Interconnection was on the NYPA system (1993) for the purpose of monitoring dynamic disturbances and geomagnetic disturbances through harmonics. The western systems' WAMS installations were dedicated to event monitoring and disturbance analysis. The term monitoring implies that it is a system of data acquisition and a communications infrastructure that brings the data into a central location. Since the intended application is system protection, the data collection to a central location should be fast enough to facilitate system protection. This implies that time latencies must be at the sub-cycle region for electrical events. Traditionally WAMS systems use data concentrators and fast dedicated communication links to achieve the necessary performance.

It is recognized that WAMS have other applications than system protection and control. One important target application is stability monitoring of the system. Another one is situational awareness. As a matter of fact, one of the leading drivers for grid modernization is the improvement of situational awareness capabilities for managing the bulk power transmission system electrically and thermally. Wide area time domain GPS synchronized sampling systems (WATSS) and Dynamic Line Ratings are both recognized by many electric utilities, government, and research entities as a key technology for situational awareness and the Smart Grid. WATSS has the potential to provide timely and reliable system information in phasor form which constitutes the cornerstone for control and protection of the electric power system (short time), and in conjunction with Dynamic Line Ratings to manage the system and facilitate markets (longer time). As a result any wide area monitoring system may serve many clients with different requirements in terms of frequency and time latencies in the data.

It is recognized that GPS synchronized data acquisition systems are the key technology to achieve the objectives of wide area monitoring. With the introduction of the GPS synchronized measurements, the WAMS technology has made some evolutionary steps. Presently we can use this advanced technology to achieve: (a) data validation at the local level and (b) data compactions to minimize communication latencies thus achieving the objectives of WAMS. These advanced technologies coupled with the knowledge of transmission line transfer capacity in real time, taking into account actual weather conditions between substations and across regions, enhance our ability to extract a validated model of the system in real time. These technologies provide the infrastructure to perform grid control functions with precision and speed not possible with other technologies. A list of possible control applications and functions is:

- Distributed State Estimation
- System Protection (electrical and thermal)
- Visualization / situational awareness / Alarming
- System stability
- Voltage control
- Frequency control
- Post mortem analysis / Play back capability
- Parameter estimation / Model validation
- Predictive analysis / Look ahead
- Oscillation monitoring
- Islanding monitoring / Controlled islanding / Restoration
- Control of Renewable Resources
- System Optimization
- Load control
- Dynamic Line Thermal Monitoring (Dynamic Line Ratings)
- Voltage Security Monitoring

The author was involved with the development of a white paper to identify issues and establish performance targets, identify needs in standards (gaps) and provide a roadmap towards achieving these goals for wide area monitoring systems (WAMS) in order to enhance wide area situational awareness. The material here is from the white paper to which the author has substantially contributed and developed the illustrations. The work of all members of the group is acknowledged.

A broad definition for any wide area monitoring system is: a system that is capable of providing accurate data (both numerical values and time tags) at a central location of a wide area and with a rate that is appropriate for the intended applications. A visual of the WAMS definition is shown in Figure 1.20. The figure shows the substation control computer (or data concentrator) that collects data and transports the data to a central location CC (control center or any other facility).

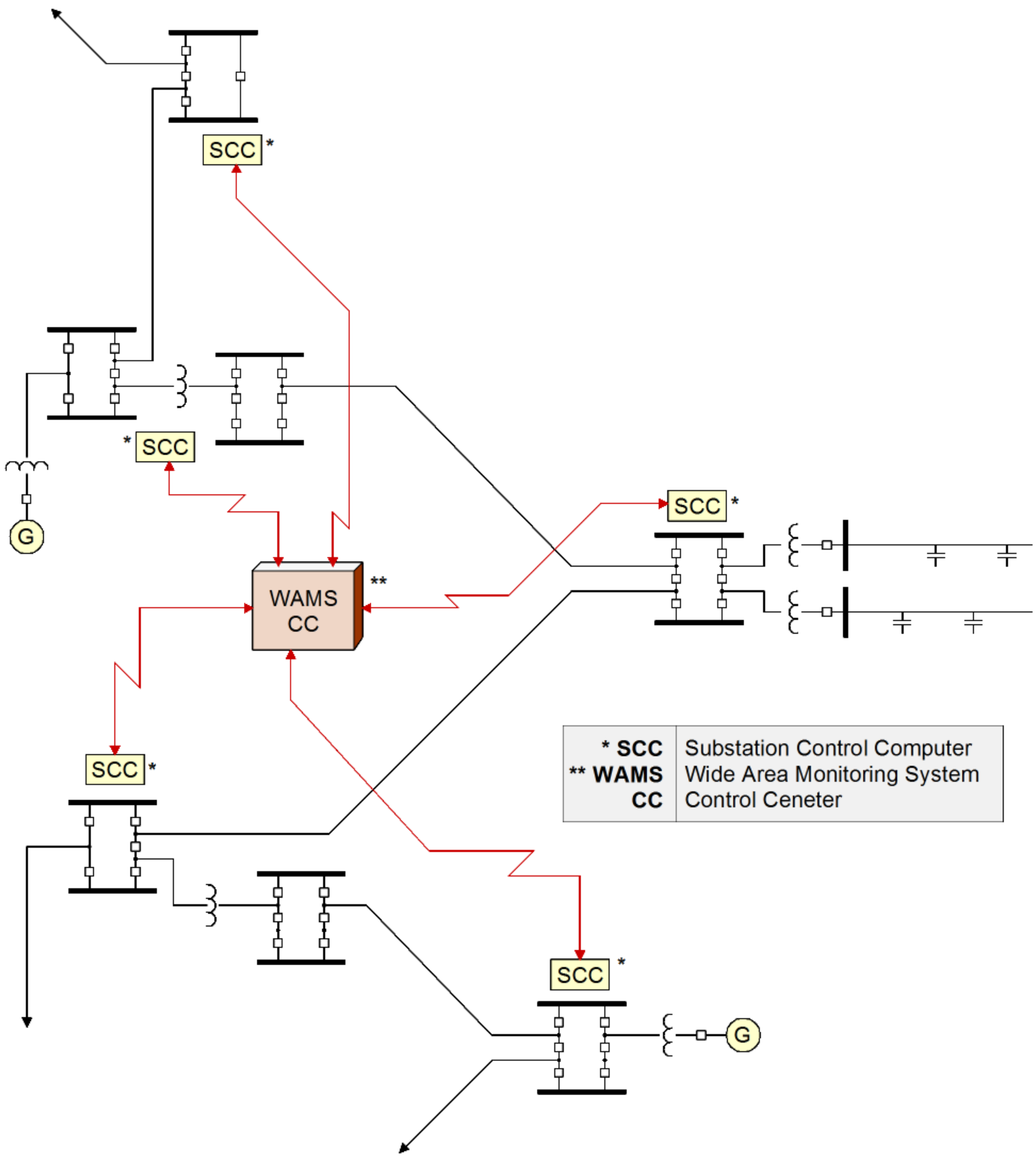


Figure 1.20 Pictorial of a Wide Area Monitoring System

The technology for the substation control computer or data concentrator and data collecting devices (IEDs and PMUs) has evolved. Figure 1.21 shows a modern mixed system. Note that merging units may be collecting data directly at the instrument transformers, where data is digitized, time tagged and then transmitted to the substation process bus. Older systems may have wire communications from the instrument transformers to various IEDs as shown on the right side of Figure 1.21. The IEDs are connected to the station bus. A data concentrator (substation control computer) is also connected to the substation bus as shown in the figure. Communications are enabled via gates connected to the station bus.

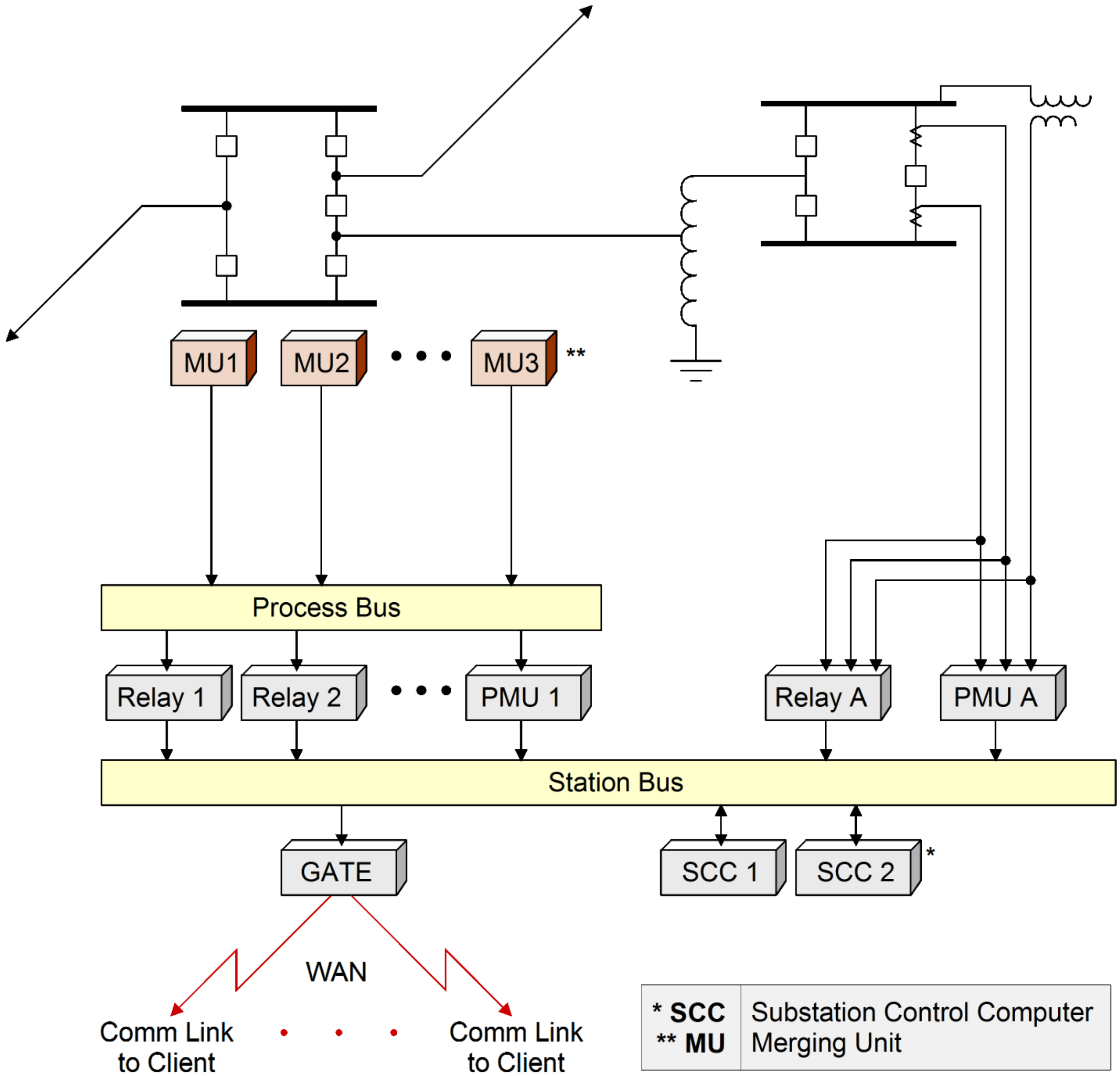


Figure 1.21: Data Collection for WAMS at a Substation

While Figures 1.20 and 1.21 illustrate what is possible with today's technology and certainly some recently constructed stations do have the indicated capability, there are many older substations that are not as automated as Figures 1.20 and 1.21 suggest. In addition there are many gaps in the technology and challenges that need to be addressed. Some of the general issues in this space are described in greater detail in the next section.

The standards available today to address the systems depicted in Figures 1.20 and 1.21 determine (a) the interoperability of the merging units, IEDs, Process bus, and station bus, (b) the exchange of data – streaming data, and (c) storing and retrieving data. A partial list of these standards is provided below.

- IEEE C37.118 Synchrophasor Streaming Data
- IEC 61850 Protocols, Configuration, Information Models
- IEEE C37.1 SCADA and Automation Systems
- Distributed Network Protocol (DNP3)
- Modbus
- IEEE C37.111-1999 – COMTRADE
- IEEE C37.2 Device Function Numbers and Contact Designations
- IEEE 1588 Precision Time Protocol
- IEC 60870-6 Inter-Control Center Protocol (ICCP)
- IEEE 1613 Substation Hardening for Gateways
- IEEE 1379 Data Communications between IED's & RTU's in a Substation
- IEEE 1525 Standard for Substation Integration Communications
- IEEE 1711 Trial Use Standard for a Cryptographic Protocol for Cyber Security of substation Serial Links
- IEEE 1686 Substation Intelligent electronic Devices (IEDs) Cyber Security Standards Key Interoperability Barriers

It is important to note that the above mentioned standards do not address all the needs of the system depicted in Figures 1.20 and 1.21. For example synchronizing the data collected at the merging units is an area that is not well defined. There is activity to identify the gaps in these standards and update standards or create new standards to address all the needs. This effort is sort of endless as the technology advances coupled with new applications that generate new paradigms in this area. A number of key issues have been identified as the industry is moving towards the deployment of these technologies.

Key Issue 1: In a modern substation the amount of data collected is relatively large. Considering the number of substations in the power grid the overall amount of data is overwhelming. Transferring this amount of data through communication links at the speeds required by some applications is at best problematic even with the best communication technologies. Yet we need to recognize that the data represent redundant measurements by duplicate systems (relays, PMUs, fault recorders, meters, etc.). Extraction of the basic information included in this data will result in reduced amount of information points that need to be communicated.

Key Issue 2: The various IEDs connected to the process bus or the station bus must be interoperable in the sense that the substation control computers (data concentrators) should be able to collect the data from each IED with minimal latencies. Available standards and gap analysis of standards is provided in the literature [???].

Key Issue 3: Data Validation. It is important that the data be validated and characterized in terms of accuracy and timeliness before used by applications. Again because of the large amount of data, distributed validation and characterization of the data is very important.

Key Issue 4: Various applications require data at different rates, accuracy and timeliness. It is important to recognize the savings that can be accomplished by designing a WAMS to provide data to the most demanding applications (for example system protection or stability monitoring) and to be able to also provide data to other less demanding applications. A well designed WAMS can decimate data and provide data to any application at the rate, accuracy and timeliness required by the specific application. The industry approach is to create data concentrators (DC) to facilitate traffic and conditioning of the data. Much work is being presently done to design data concentrators specializing in synchrophasor data, i.e. PDCs (Phasor Data Concentrators).

Key Issue 5: Certain targeted applications for WAMS require data at fast rates, accurately synchronized and with very small time latency. Because the power grid is a geographically dispersed system spanning large distances, latencies cannot be reduced below travel times in the communication circuit (for example the travel time for a 150 mile long line using fiber optic communications is approximately 2 milliseconds one way). The challenge will be to develop distributed WAMS and applications that can use data in the vicinity of the application to avoid long latencies. The design of PDCs tries to address these issues with the objective of minimizing the latency in the data.

11.0 Power System Relaying: The Protection System of the Future

Considering the present state of technology in monitoring, protection and control of electric power systems, it is easy to realize that the possibilities towards a fully automated, self-organized, reliable and secure protection and control system are enormous. The technology also creates the possibility of substantial savings in the overall design implementation and operation of such a system. To achieve these goals much research effort must be expanded to develop new approaches to protection and control that fully utilize the capabilities of the technologies. Here we discuss a couple of approaches that have the capability to revolutionize protection and control.

The Substation of the Future: The first far looking approach is to recognize that the presently fragmented approach to protection and control can be easily integrated with present day technology. One approach towards this goal is illustrated in Figures 1.22 and 1.23. The ideas presented in these two figures are relatively simple to explain. First in Figure 1.22, it is shown that the data acquisition in any complex substation can be performed with GPS synchronized merging units, indicated as UGPSSM in the figure, that are placed next to the instrument transformers. The scheme integrates the data acquisition system into a single integrated subsystem that connects all the merging units to a process bus in the substation control house. The process bus could be a single device, or multiple devices. This system allows for redundancy, for example there may be more than one VT or CT connected to the same part of the system. At the process bus all the collected data are available. The connections and the data traffic are shown in Figure 1.23. Note that multiple IEDs (relays, PMUs, recorders, etc.) can be connected to the process bus. These devices process the data and create the "processed data", i.e. they compute quantities such as rms values, phasors (magnitude and phase), real power, reactive power, harmonics, etc. The processed data are available to the "station bus". The IEDs, relays, PMUs may also perform their own functions, i.e. perform a specific protective function and when appropriate issue a trip command to a breaker. The trip command will be communicated via the fiber optic to the merging unit in the field and the actual execution of the trip command.

Figure 1.23 also shows the flow of data and the types of data at each level of the architecture illustrated in Figure 1.22. While the figure shows multiple IEDs, relays, PMUs, etc. it is possible that one single PC, with the proper computing power to perform the processing of the data and the execution of various protective and control functions. In general, however, redundancy may be necessary to increase the reliability of the system.

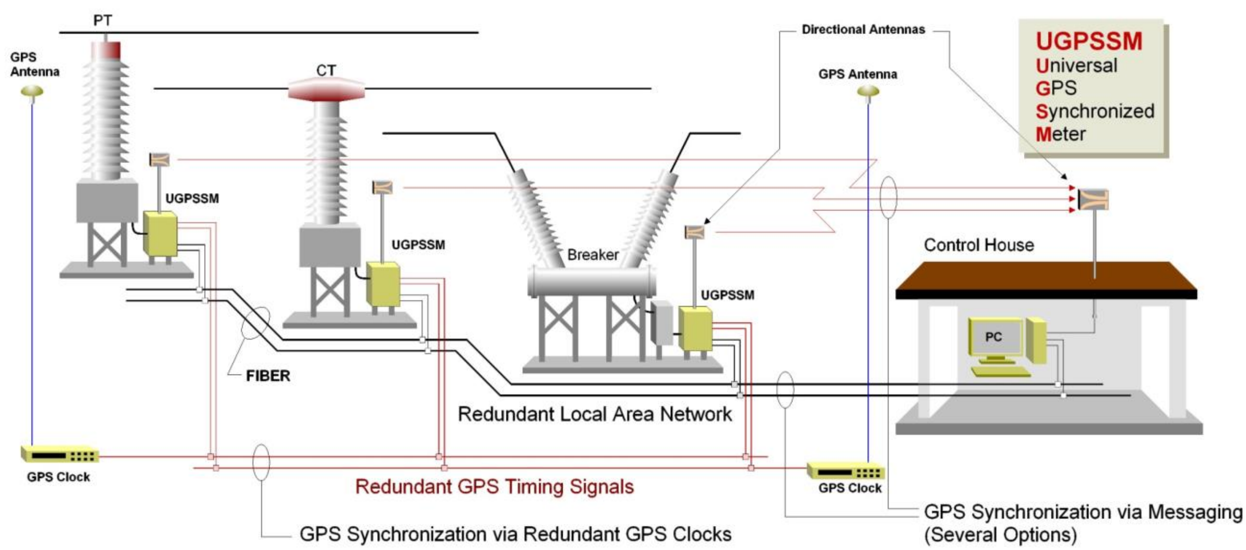


Figure 1.22: Possible Approach Towards Integration of Data Acquisition, Protection, Control and Communications

The proposed scheme allows for an integrated approach to protection. Consider the fact that present day protective relays rely on a small number of inputs to recognize the operating conditions of the device (zone) that they protect in order to take a decision. While smart engineers have design remarkable algorithms to identify intolerable conditions from a limited number of inputs, there are gray areas where these algorithms cannot provide the answer with certainty or with the required speed. More information will provide more certainty in these algorithms. With the proposed scheme it is possible to design a relay algorithm that uses all the data available at the process bus to identify intolerable conditions and the cause of the condition. In this case the protective decision (which breakers to operate and which not) is a rather simpler task. It is emphasized that the approach allows the connection of present day multifunctional relays to the process bus and operation of the protective functions as they are performed today but also allows the integration of all protective functions in one device (one IED).

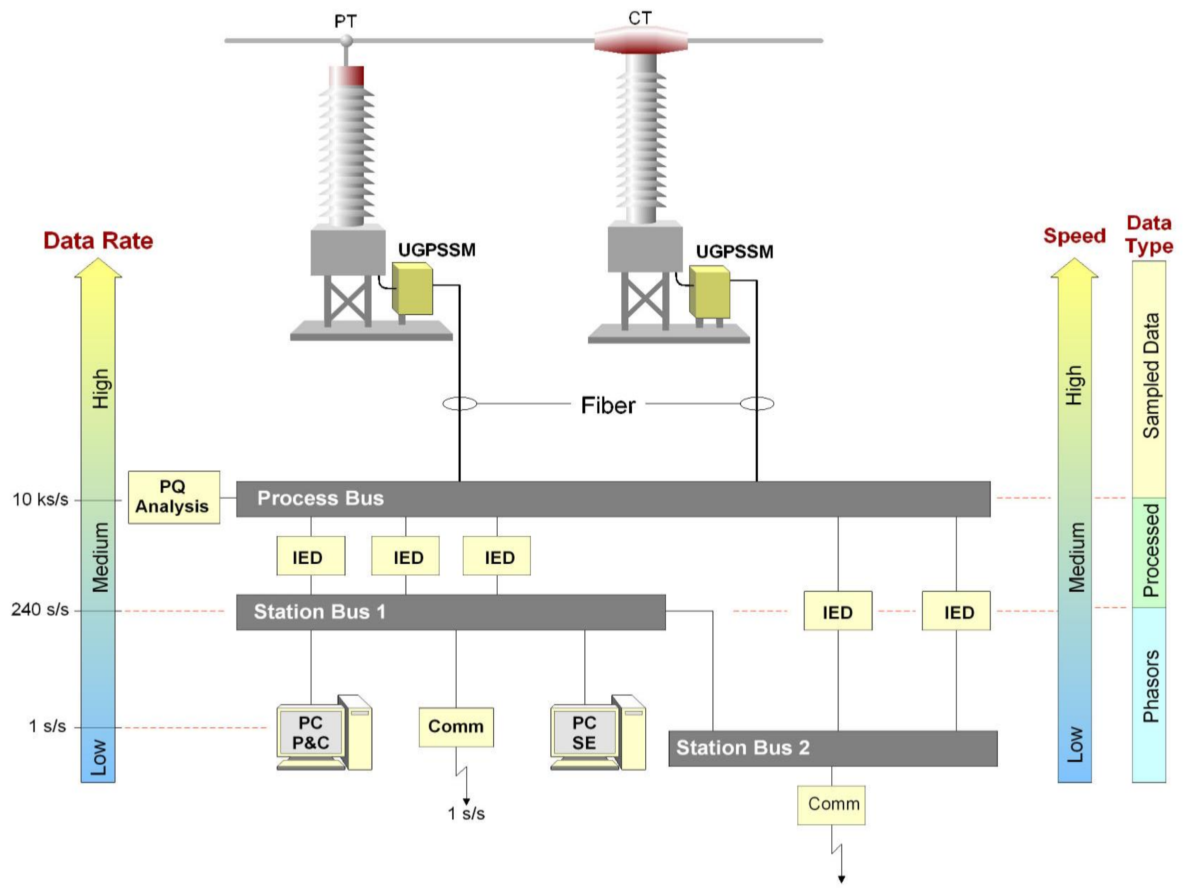


Figure 1.23: Possible Approach Towards Integration of Data Acquisition, Protection, Control and Communications

Dynamic State Estimation Based Protection: Another approach has recently emerged. This approach has been initiated at Georgia Tech with EPRI funding under the grid transformation initiative of EPRI. The drivers for the new approach are: (a) to minimize the complexity of protective relaying and if possible to evolve to setting-less protection schemes, (b) to provide secure and reliable protection of power components such as a generator, line, transformer, etc., and (c) to provide an infrastructure that will validate the model of the power system components.

The proposed method uses dynamic state estimation [???], [???], based on the dynamic model of the component, which accurately reflects the nonlinear characteristics of the component as well as the loading and thermal state of the component. The approach is briefly illustrated in Figure 1.24. The method requires a monitoring system of the component under protection that continuously measures terminal data (such as the terminal voltage magnitude and angle, the frequency, and the rate of frequency change) and component status data (such as tap setting (if transformer) and temperature). The dynamic state estimation processes these measurement data with the dynamic model of the component yielding the operating conditions of the component.

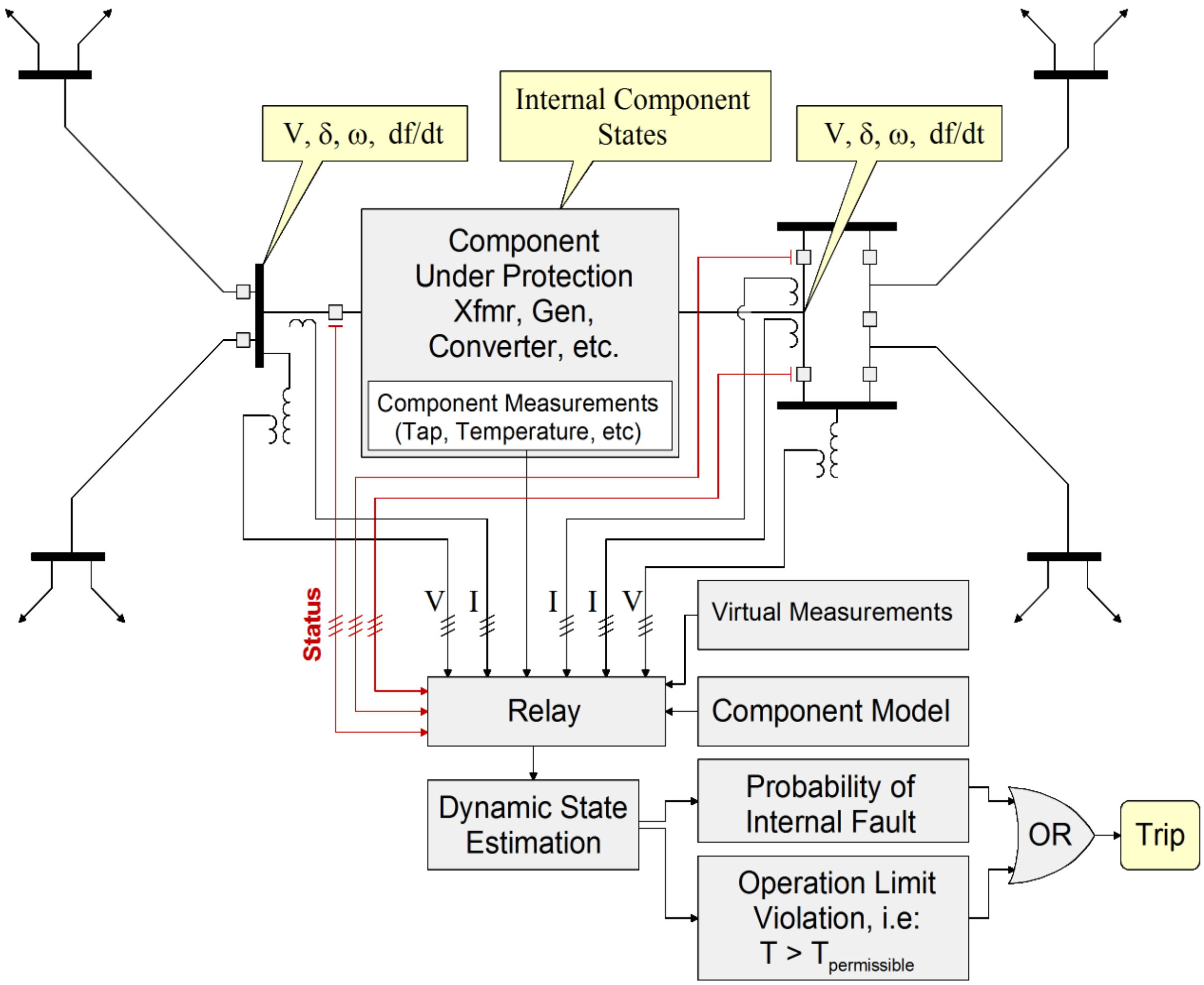


Figure 1.24 Protective Relaying Approach via Dynamic State Estimation

The operating condition can be compared to the operating limits of the component to develop the protection action. The logic for the protection action is illustrated in the figure below.

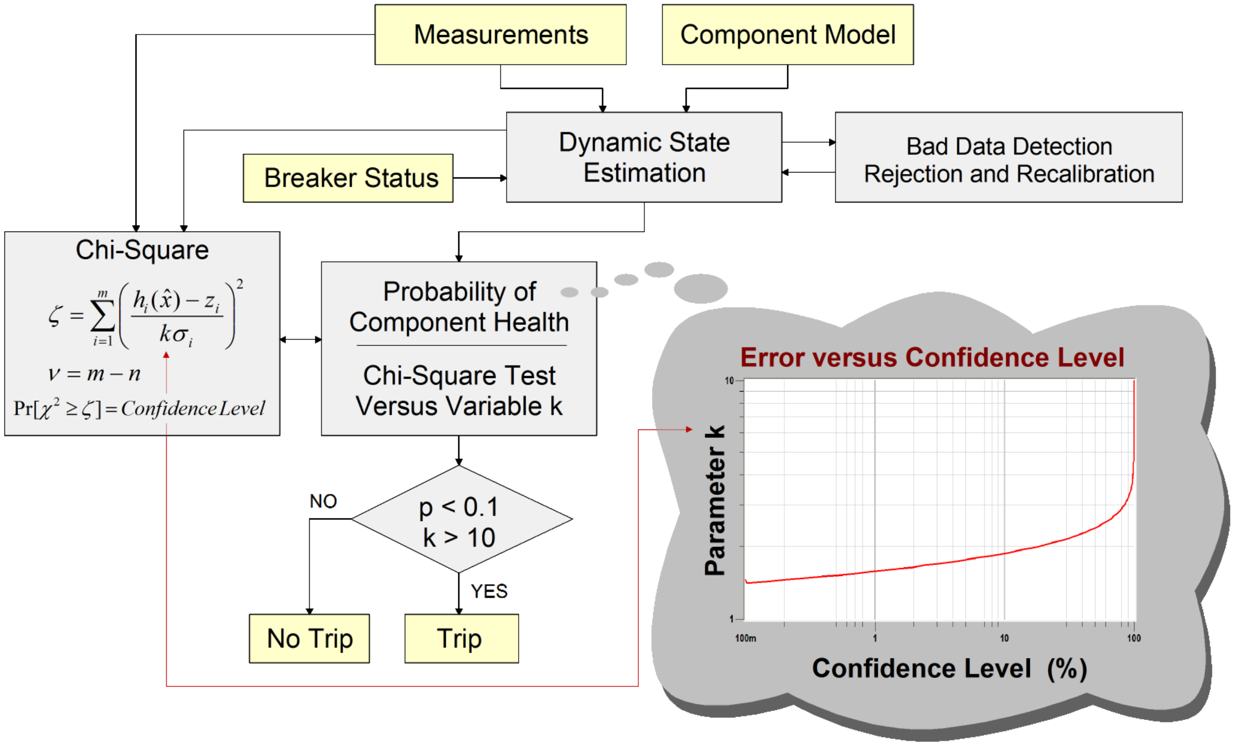


Figure 1.25 Data Flow and Logic of the Dynamic State Estimation Based Protective Relaying Approach

This approach faces some challenges which can be overcome with present technology. A partial list of the challenges is given below:

1. Ability to perform the dynamic state estimation in real time (each sampling period which can be very low, for example 250 microseconds).
2. Initialization issues.
3. Communications in case of a geographically extended component (i.e. lines).
4. New modeling approaches for components - for example the magnetic saturation characteristics of transformers may be modeled, converters in wind turbine systems must be accurately modelled and other.
5. Requirement for GPS synchronized measurements in case of multiple independent data acquisition systems, such as data acquisition systems at the two ends of a line, and other.
6. other

The modeling issue is fundamental in this approach. For success, the model must be high fidelity so that the component state estimator will reliably determine the operating status (health) of the component. For example consider a transformer during energization. The transformer will experience high in-rush current that represent a tolerable operating condition and therefore no relay action should occur. The component dynamic state estimator should be able to "track" the in-rush current and determine that they represent a normal (tolerable) operating condition. This requires a transformer model that accurately models saturation and in-rush current in the transformer. We can foresee the possibility that a high fidelity model used for protective relaying can be used as the main depository of the model which can provide the appropriate model for other applications. For example for the EMS applications, a positive sequence model can be computed from the high fidelity model and send to the EMS data base. The advantage of this approach will be that the EMS model will come from a field validated model (the utilization of the model by the relay in real time provide the validation of the model).

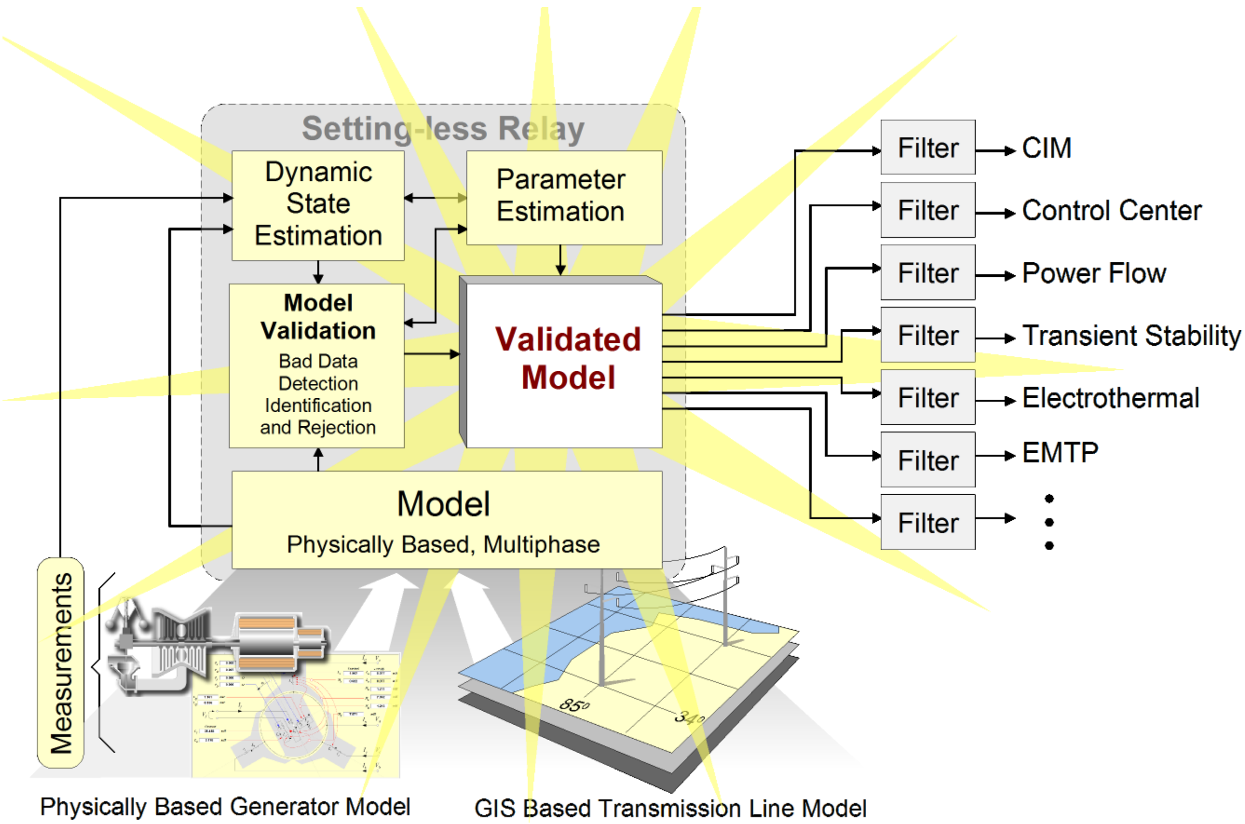


Figure 1.26 An Approach to State Estimation Based Relaying with Model Monitoring

The approach shown in Figure 1.24 provides additional benefits. First, the dynamic state estimation can be utilized to determine whether the overall protection system is consistent, the VT and CT ratios are the correct ones, the connections are consistent, etc. Any discrepancy will show up in the results of the dynamic state estimation. In addition, any faults in the instrumentation, i.e. VT failure, etc., will be immediately recognized by the state estimation. Therefore it provides a tool for detecting in an efficient way hidden failures. Another advantage is that the system can be utilized to estimate the parameters of the component that monitors from the redundant data. This will result in obtaining the correct and validated model of the component in real time. Thus the relay now can serve as the "creator and keeper" of the validated model of the component. The model can be also submitted to other applications or send to a central depository for the company. Filters can provide the model to specific application and in the proper form as it is illustrated in Figure 1.26.

12.0 Power System Relaying Test Bed

Illustrative examples are always an excellent pedagogical tool. In this book, we will use many such examples. Furthermore, we will organize these examples in such a way that they will be always familiar to the reader. For this reason, we introduce here a generic electric power system test bed that will serve for obtaining specific examples throughout the book. The electric power system test bed is illustrated in Figure 1.27. Note that the system comprises two generating substations, one transmission substation, one distribution substation and several distribution feeders. The test bed electric power system is a breaker oriented model and it will provide many illustrative examples for application of the protective relaying concepts discussed in the textbook. The model is available in WinIGS format. The program WinIGS will be used as the computational tool to perform a variety of protective relaying applications.

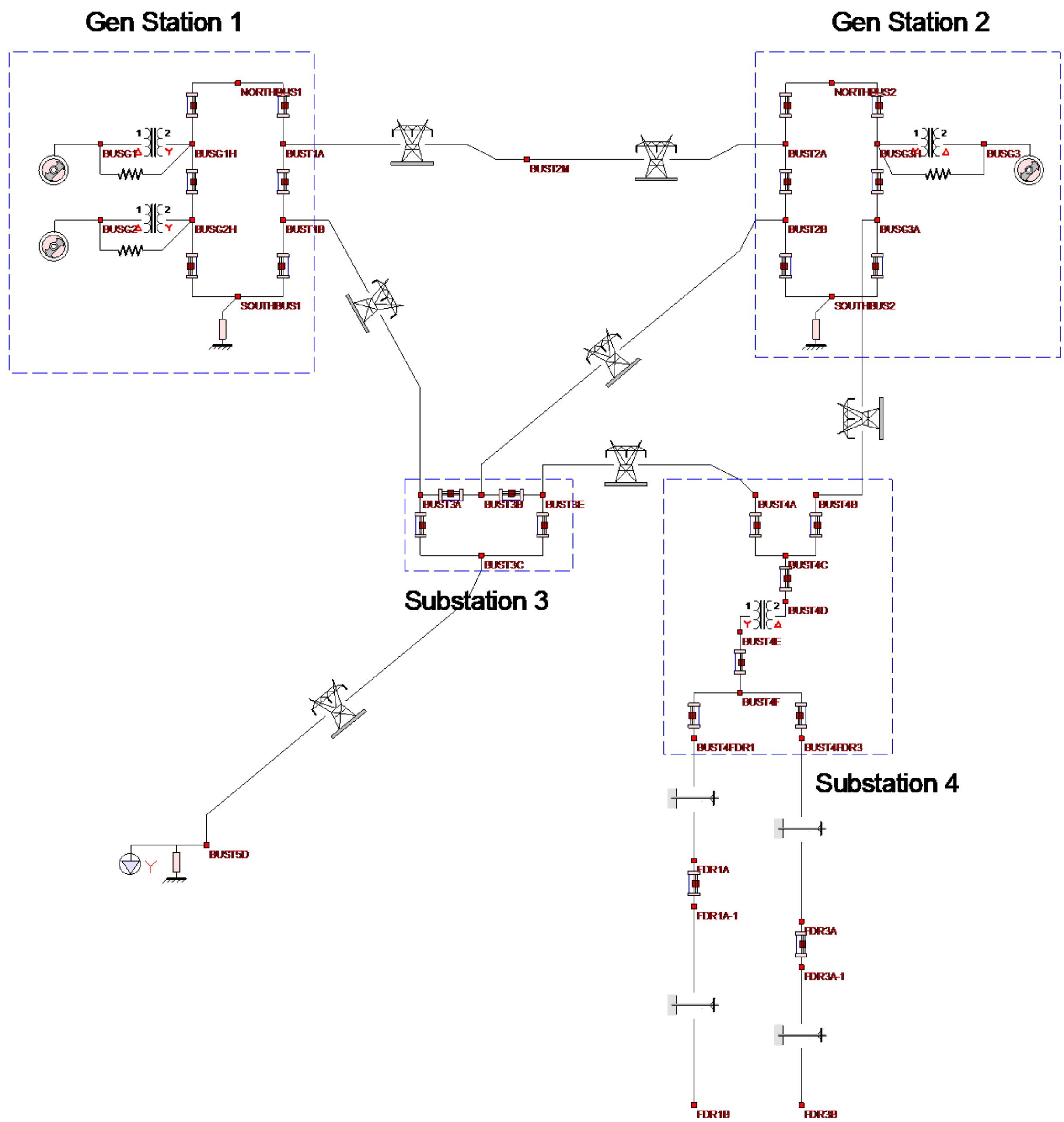


Figure 1.27. Electric Power System Test Bed

Comments for improvement:

1. Make Gen Station 1 a two kV level plant (230 kV, 115 kV).
2. Make the transmission substation a two kV level substation.

13.0 Problems

Problem P1.1: Obtain a copy of the IEEE Std C37.2-1996, "IEEE Standard Electrical Power System Device Function Numbers and Contact Designations".

Provide the functional description of the following devices (a couple of sentences will suffice – you are encouraged to avoid copying the standard and to use your own words):

21
24
27
32
40
46
47
50
51
59
81
87

Problem P1.2: Consider the electric power system of Figure P1.2. Select protection zones for this system. Be specific as to the boundaries of the protection zones. Place CTs and PTs as required.

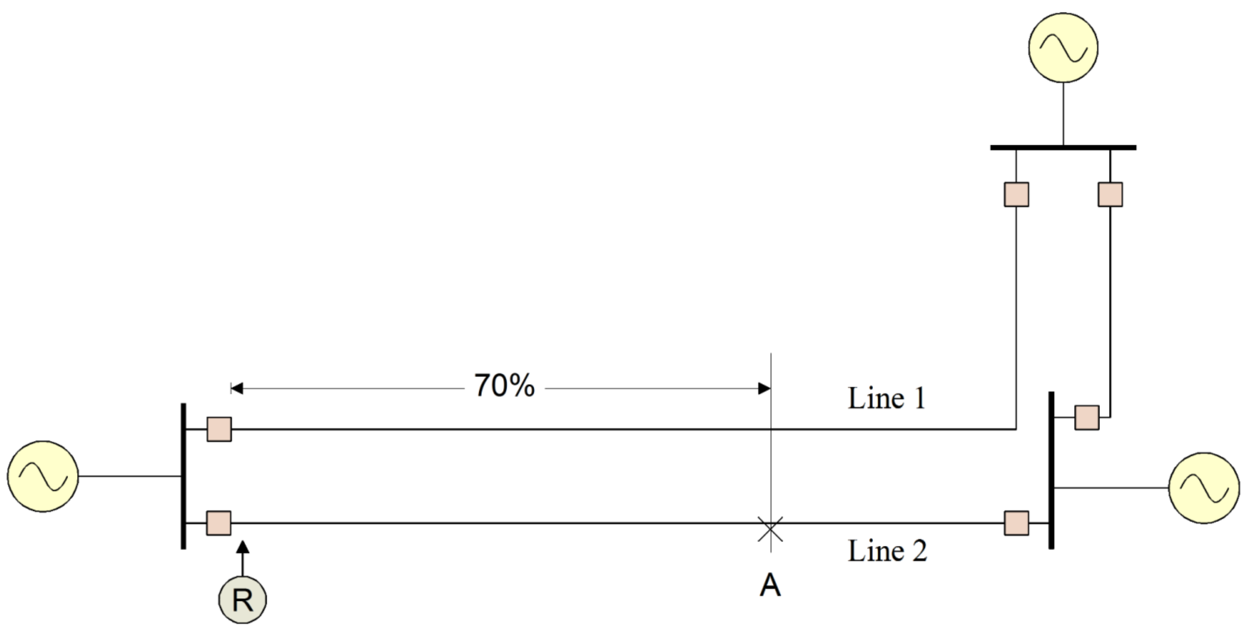


Figure P1.2

Problem P1.3: A non-utility generator is protected with the protection scheme shown in Figure P1.3. Describe with two sentences the function of each of the indicated relays.

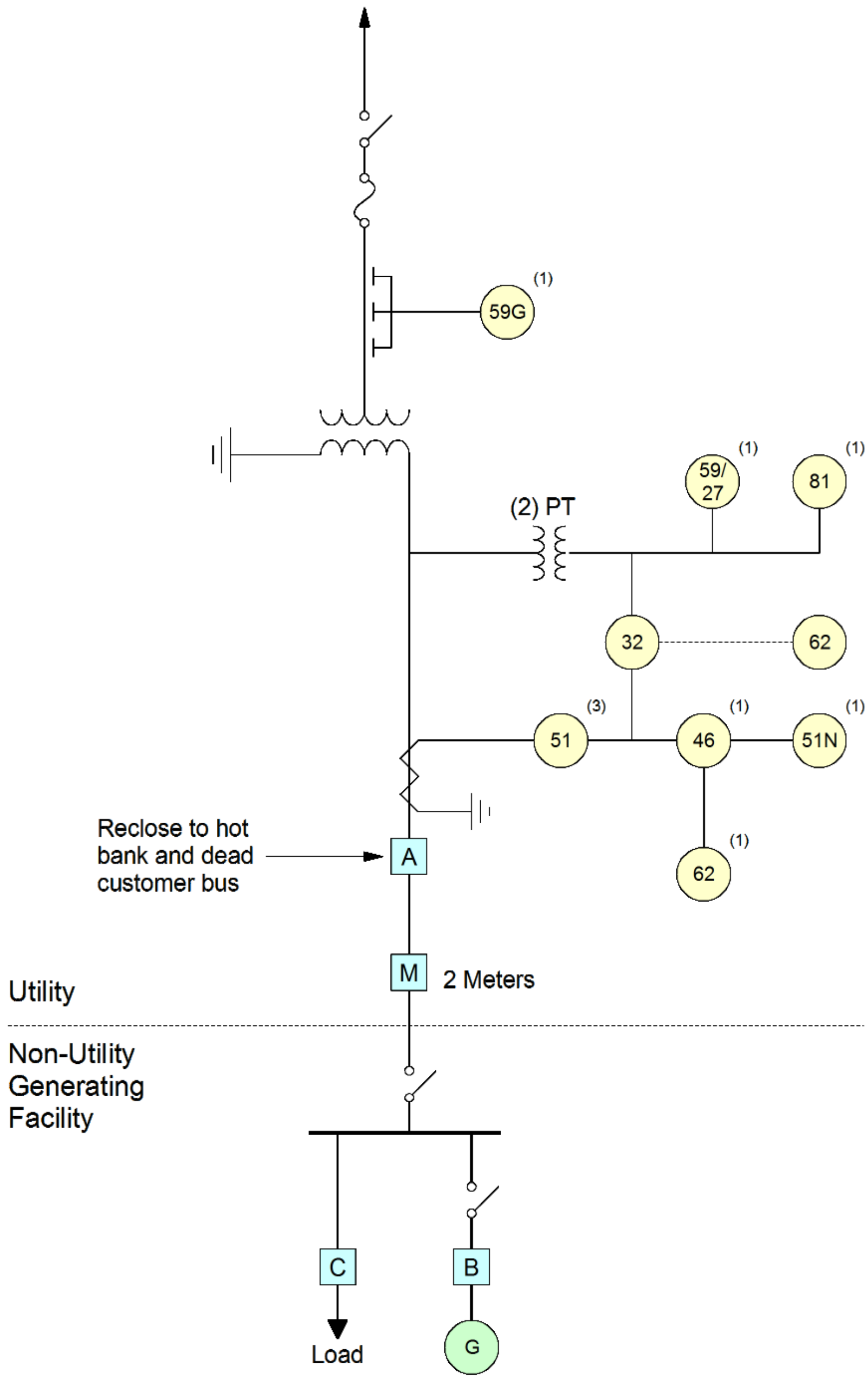


Figure P1.3

Solution: The functions of the indicated relays are:
59G: Ground overvoltage relay.
59/27: over and under voltage relay.
81: Frequency relay (normally over- and under- frequency).
32: Reverse power relay.
62: Delay timer.
46: Reverse phase relay.
51: Overcurrent relay.
51N: Neutral overcurrent relay.

Problem P1.4: Consider the electric power system of Figure P1.4. Select protection zones for this system. Be specific as to the boundaries of the protection zones. Make sure that the entire system is covered, i.e. there is no part of the system that does not belong to at least one protection zone.

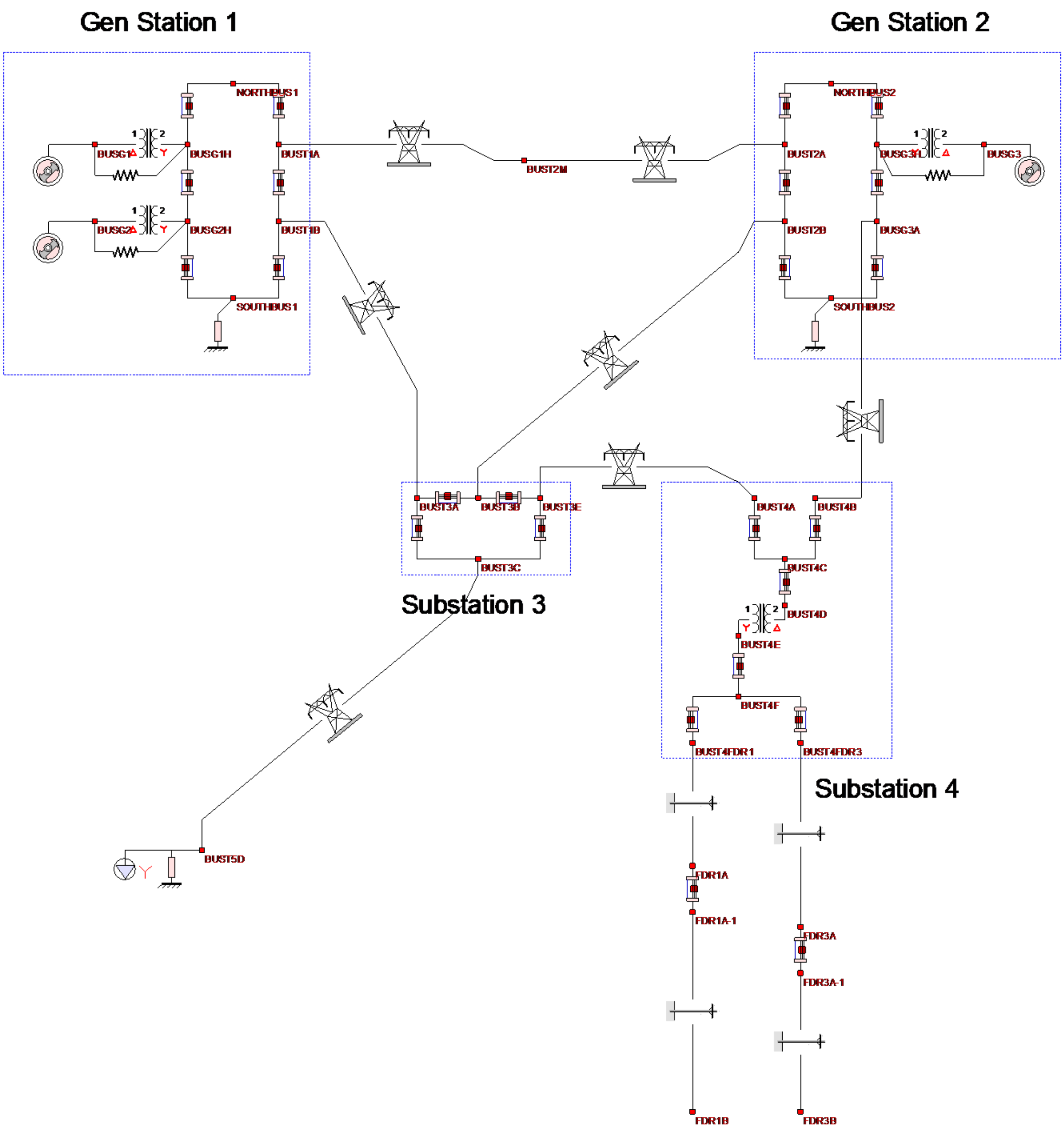

Gen Station 1
Gen Station 2
NORTHBUS1
BUSG1
BUSG1H
BUST1A
BUSG2
BUSG2H
BUST1B
SOUTHBUS1
BUST2M
NORTHBUS2
BUST2A
BUSG3H
BUSG3
BUST2B
BUSG3A
SOUTHBUS2
BUST3A
BUST3B
BUST3E
BUST3C
Substation 3
BUST4A
BUST4B
BUST4C
BUST4D
BUST4E
BUST4F
BUST4FDR1
BUST4FDR3
Substation 4
BUST5D
FDR1A
FDR1A-1
FDR3A
FDR3A-1
FDR1B
FDR3B

Figure P1.4

DRAFT and INCOMPLETE

Table of Contents

from

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Chapter 2
Basic Concepts

2.1 Introduction

Protective relaying practice requires understanding and analysis of power system performance under a variety of conditions. This can be achieved with a good knowledge of the basic concepts of power system operation and response to disturbances. The basic tools towards this understanding are modeling of power system components and analysis methods to determine the behavior of electric power systems. We devote three chapters to address these issues. The present chapter (Chapter 2) is devoted in the overview of the basic concepts. Chapter 3 is devoted to modeling of major power system components. Chapter 4 is devoted to analysis of power systems under faulty conditions. In addition, certain disturbances affect wide areas of the power system creating the need for a system wide approach to disturbance analysis and protection against these conditions. These issues will be addressed in Chapter 14.

2.2 Sinusoidal Steady State and Phasors

Under steady state operation, electric power system voltages and currents are purely sinusoidal waveforms. These waveforms can be expressed by the following equations:

$$i(t) = I_m \cos(\omega t + \varphi)$$
$$v(t) = V_m \cos(\omega t + \theta)$$

where I_m is the maximum value of the electric current
V_m is the maximum value of the voltage
ω is the angular frequency
φ is the phase of the electric current
θ is the phase of the voltage

This operating condition is referred to as Sinusoidal Steady State Condition (SSSC).

An alternative representation of sinusoidal waveforms is by means of **phasors**. The use of phasors greatly simplifies the manipulation of these signals, and many fundamental relationships can be easily expressed using phasor notation. An overview of these relationships is presented in this section.

First note that the root mean square (RMS) value of a sinusoidal voltage waveform is:

$$V_{rms} \equiv \sqrt{\frac{1}{T}\int_0^T v^2(\tau)d\tau} = \frac{V_m}{\sqrt{2}}$$

Similarly for a sinusoidal current waveform:

$$I_{rms} \equiv \sqrt{\frac{1}{T}\int_0^T i^2(\tau)d\tau} = \frac{I_m}{\sqrt{2}}$$

The above relations tell us that under SSSC, the root mean square value of the electric current or voltage is equal to the maximum value divided by the square root of 2. Thus we can write:

$$i(t) = \sqrt{2}I_{rms}\cos(\omega t + \varphi)$$
$$v(t) = \sqrt{2}V_{rms}\cos(\omega t + \theta)$$

Another interesting property can be obtained by writing the above equations in the following equivalent form:

$$i(t) = \mathrm{Re}\left(I_m e^{j(\omega t+\varphi)}\right) = \mathrm{Re}\left(\sqrt{2}I_{rms}e^{j(\omega t+\varphi)}\right)$$
$$v(t) = \mathrm{Re}\left(V_m e^{j(\omega t+\theta)}\right) = \mathrm{Re}\left(\sqrt{2}V_{rms}e^{j(\omega t+\theta)}\right)$$

Where $\mathrm{Re}(*)$ is the real part of the argument *. The above relationships are identities, which can be shown using Euler's identity

$$e^{ja} = \cos(a) + j\sin(a)$$

By rearranging

$$i(t) = \mathrm{Re}\left(\sqrt{2}I_{rms}e^{j\varphi}e^{j\omega t}\right) = \mathrm{Re}\left(\sqrt{2}\tilde{I}e^{j\omega t}\right)$$
$$v(t) = \mathrm{Re}\left(\sqrt{2}V_{rms}e^{j\theta}e^{j\omega t}\right) = \mathrm{Re}\left(\sqrt{2}\tilde{V}e^{j\omega t}\right)$$

where:

$$\tilde{I} = I_{rms}e^{j\varphi}$$
$$\tilde{V} = V_{rms}e^{j\theta}$$

Let's examine a useful geometric interpretation of the above equation. Consider Figure 2.5. The complex quantity $\tilde{I}e^{j\omega t}$ is a vector in the complex plane. Note that the angle between this vector and the real axis is ωt+φ, i.e. it is changing with time. The projection of this vector on the real axis is the instantaneous value of the electric current, i(t). As time progresses, the vector $\tilde{I}e^{j\omega t}$ rotates with angular speed ω, and as a result its projection on the real axis varies sinusoidally. The quantity $\tilde{I}e^{j\omega t}$ is called the rotating phasor of the electric current. Since the magnitude of the current phasor is defined as the RMS value of the current, it would be more precise to call it the *root mean square rotating phasor* of the electric current. We refer to $\tilde{I}$ as the phasor current or the complex current. Note that the value of the phasors and in particular the value of the

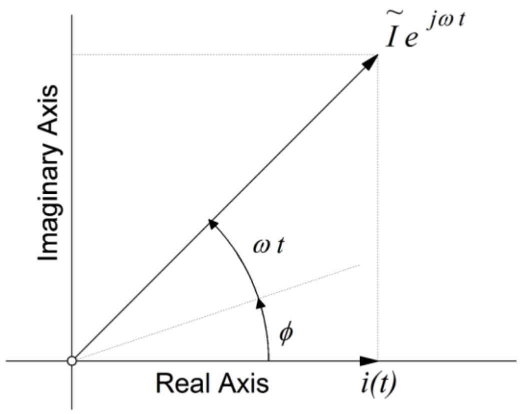


Figure 2.1 Geometric Interpretation of the Phasor Equation

phase angle depends on the time reference, for example if the time t is shifted by a certain value then the phase value will change by an amount equal to the angular frequency times the time shift.

A similar construction and discussion applies to the voltage quantity.

Electric power systems operate very close to sinusoidal steady state condition if there are no switching or nonlinear devices that may cause deviations from the sinusoidal steady state operation.

2.3 Symmetrical Components

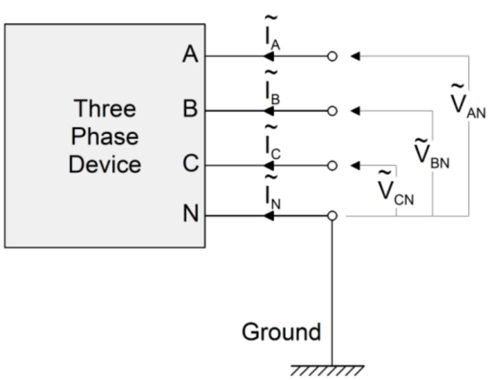


Figure 2.2: A Three Phase Device

Modern power systems are generating, transmitting and distributing electric power mostly through a three-phase electric power system. One reason for the proliferation of three phase AC power systems is that more power can be transmitted via three-phase power transmission than single phase power transmission for a given total conductor weight and insulation level. A second important reason is that three phase synchronous generators can efficiently generate three phase voltages and currents driven by a constant mechanical torque. Similarly, three-phase motors are compact; they are naturally self-starting, and can generate constant torque. These systems provide smooth and efficient operation.

The majority of power systems comprise three phase arrangements, which consist of the interconnection of three phase generators, three phase lines, three phase transformers, and other supporting equipment. Three phase transmission systems are configured as three wire (phase wires only), four wire (three phase wires plus neutral or shield (ground) wire(s)) or five wire systems (three phases, neutral, and shield (ground) wire(s)). A typical three-phase device with neutral is illustrated in Figure 2.2.

A three-phase device, such as a transformer or a motor may consist of single-phase elements connected in a three-phase arrangement. The elements may be connected in a *delta* or a *wye* configuration or any combination of these two. The delta and wye connections are illustrated in Figure 2.3a and 2.3b respectively.

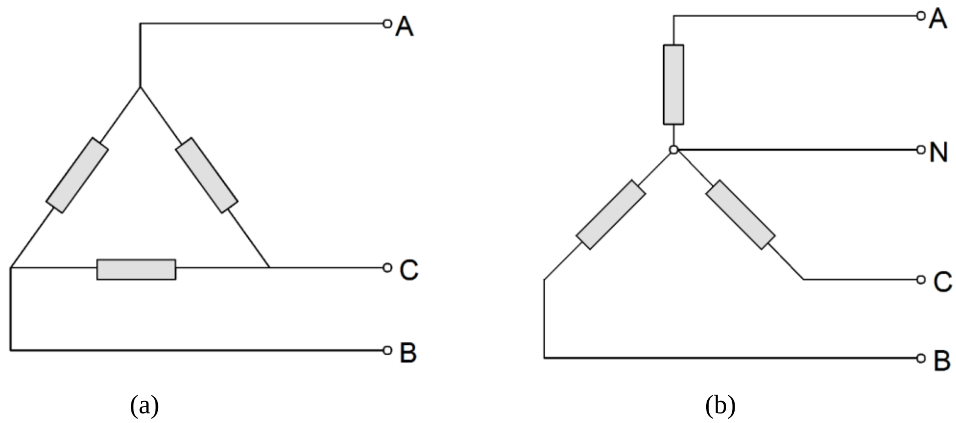


Figure 2.3: Three-Phase Devices Consisting of Three Single-Phase Elements (a) Delta Connection, (b) Wye Connection

In three phase system we have three or more voltages and three or more electric currents. It is desirable that the voltages and currents exhibit some type of symmetry. To characterize the three phase voltages and currents as well as the three phase power system, we introduce the following terminology that describes the operating conditions of the system as well the internal construction and characteristics of the three phase system.

Balanced Set of Three-Phase Voltages: A set of three-phase voltages, $v_a(t)$, $v_b(t)$, $v_c(t)$, is called balanced if and only if:

- The voltages vary sinusoidally with time.
- The amplitudes of the voltages are equal.
- There is a 120^o phase difference between any two voltages.

For example, the following set of three-phase voltages is balanced:

$$v_a(t) = \sqrt{2}V\cos(\omega t + \phi)$$
$$v_b(t) = \sqrt{2}V\cos(\omega t - 120^o + \phi)$$
$$v_c(t) = \sqrt{2}V\cos(\omega t - 240^o + \phi)$$

In the above equations, the phase angle of v_a leads the phase angle of v_b by 120^o. Similarly, the phase angle of v_b leads the phase angle of v_c by 120^o. This phase angle relationship among the three phases is called *positive phase sequence*. (Other phase sequences are discussed later in this chapter). Note that an analogous *Balanced Set* definition applies to a set of three-phase currents.

It is apparent that a set of balanced three-phase voltages can be completely specified by the following information:

- The voltage magnitude V,

- The angular frequency ω,
- The phase angle ϕ, and
- The phase sequence.

An alternative specification consists of the following:

- The phase A voltage phasor, $\tilde{V}_a = Ve^{j\phi}$
- The phase sequence

Throughout this text, when the sequence is not specified, it will be assumed to be the positive sequence.

An ideal three-phase source generates a set of balanced three-phase voltages, typically with positive phase sequence. Three-phase sources can be constructed by combining three single-phase sources of appropriate parameters. Two such configurations forming ideal three-phase voltage sources are illustrated in Figure 2.4.

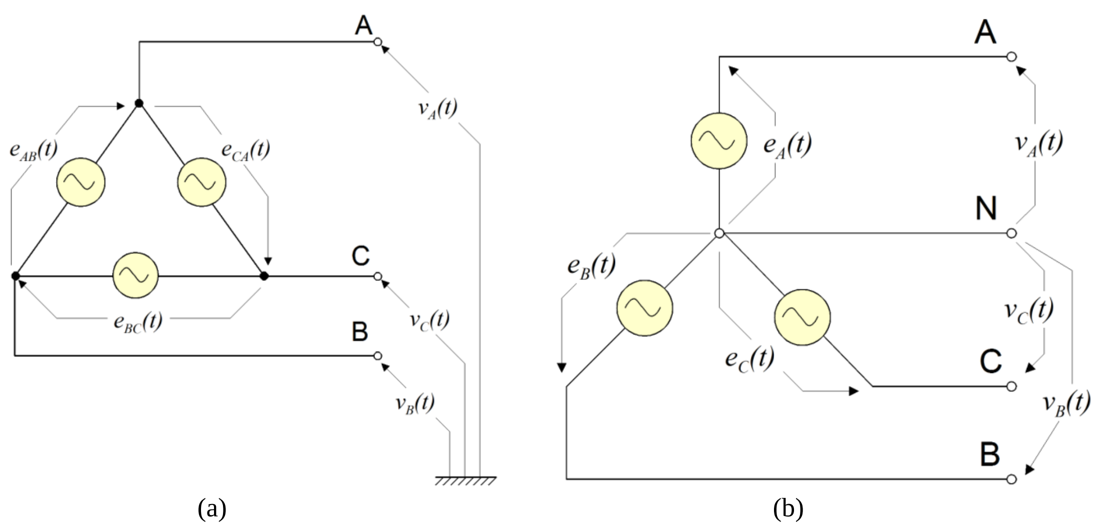
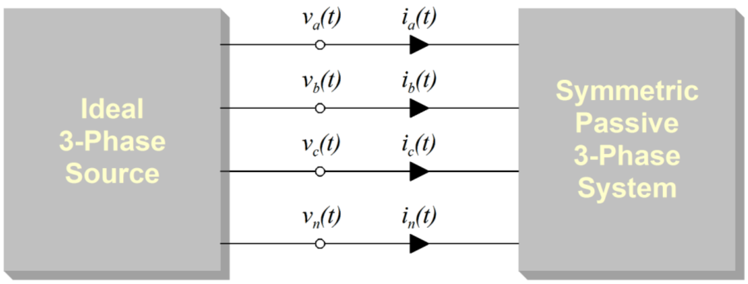


Figure 2.4 Ideal Three-Phase Voltage Source
[(a) Delta Connected, (b) Wye Connected]

Symmetric Three-Phase System: A three-phase passive system is called symmetric if and only if the following two statements are true:

- It is a linear system.
- When fed by a balanced set of three-phase voltages a balanced set of three-phase currents flows into it. Alternatively, when a set of three phase balanced currents are injected into the system the resulting three phase voltages are balanced.



$$\begin{pmatrix} Balanced Voltages \\ v_a(t), v_b(t), v_c(t) \end{pmatrix} \Leftrightarrow \begin{pmatrix} Balanced Currents \\ i_a(t), i_b(t), i_c(t) \end{pmatrix}$$

Figure 2.5: Definition of a Symmetric Three-Phase System

The definition of a symmetric three-phase system is illustrated in Figure 2.5. Practical three-phase systems comprise three-phase components that are symmetric or nearly symmetric. For example, three-phase transformers are symmetric three-phase devices, three-phase synchronous generators are nearly symmetric devices, overhead transmission lines are nearly symmetric, etc.

Traditional power system analysis techniques (i.e. load flow, fault analysis, transient stability analysis, etc.) have been developed on the assumption of symmetric three-phase systems. Since most practical three-phase power system elements are nearly symmetric, this assumption generates a small error. In most applications, this error is acceptable.

Most of the time three-phase power systems operate near balanced conditions. In this case we make the assumption that the system operates under balanced conditions, the system is symmetric and proceed on this basis. Whenever the balanced operation is disturbed, the analysis of the system can be performed in two ways:

(a) Direct analysis of the entire three phase system (exact model).
(b) Analysis by the method of *Symmetrical Components* (approximate model).

The Method of Symmetric Components

The method of symmetrical components was introduced by Charles L. Fortescue in a paper published in the AIEE transactions in 1918. It is based on a linear transformation of the system voltages and currents. This transformation converts an unbalanced three-phase system model into three uncoupled balanced three phase system models. We refer to these balanced three phase models as the (a) positive sequence, (b) negative sequence and (c) zero sequence. The analysis of each of the three balanced three-phase systems can be performed by considering one phase only. This procedure reduces the problem of analysis of an unbalanced three phase system into the analysis of three uncoupled single-phase system problems. This approach typically represents a reduction of computational effort by a factor of three.

The symmetrical component transformation is presented next. Consider a set of three phase voltages and electric currents $(\tilde{V}_a, \tilde{V}_b, \tilde{V}_c)$, and $(\tilde{I}_a, \tilde{I}_b, \tilde{I}_c)$, respectively. These sets are transformed by a transformation matrix T into a new set of voltages and electric currents $(\tilde{V}_1, \tilde{V}_2, \tilde{V}_0)$, and $(\tilde{I}_1, \tilde{I}_2, \tilde{I}_0)$ as follows:

$$\tilde{V}_{abc} = T\tilde{V}_{120} \qquad \text{(xxx)}$$
$$\tilde{I}_{abc} = T\tilde{I}_{120}$$

where:

$$\tilde{V}_{abc} = \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix}, \quad \tilde{V}_{120} = \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix}, \quad \tilde{I}_{abc} = \begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix}, \quad \tilde{I}_{120} = \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_0 \end{bmatrix}, \text{ and } T = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}, \quad a = e^{j120^o}$$

The inverse transformation is:

$$\tilde{V}_{120} = T^{-1}\tilde{V}_{abc} \qquad \text{(xxx)}$$
$$\tilde{I}_{120} = T^{-1}\tilde{I}_{abc}$$

where:

$$T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}, \qquad a = e^{j120^o}$$

Geometric Interpretation. The existence of three vectors that satisfy the equation $\tilde{V}_{abc} = T\tilde{V}_{120}$ is guaranteed by the fact that the matrix T is invertible. In fact this equation can be visualized geometrically on the complex plane as illustrated in Figure 2.6. Note that the Figure contains three vectors A, B and C (shown in blue color) of arbitrary magnitude and direction that are the sums of three sets of sets of vectors, each of which consists of three equal magnitude vectors (shown in red, black, and gray color respectively). Note that the red set is a positive sequence (phase A is followed by phase B, then by phase C assuming a counterclockwise rotation), the black set is the negative sequence, and the gray set is the zero sequence (three gray vectors are parallel, i.e. in phase).

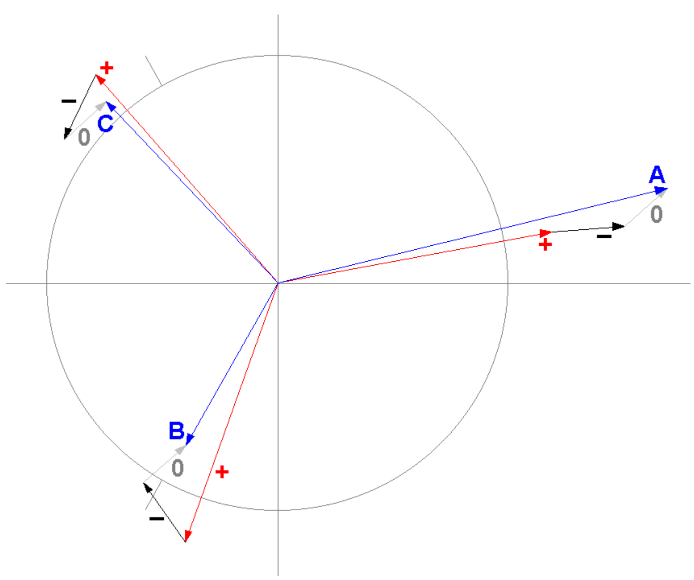


Figure 2.6: Decomposition of Three Vectors into Symmetrical Components

The importance of this transformation relies on the fact that it can transform the model of any symmetric three-phase device, into three ***uncoupled*** single-phase device models. This property of the transformation is illustrated with an example next.

Example E2.3: Consider a simple model of a three-phase generator, which is illustrated in Figure E2.3. The generator consists of three ideal voltage sources E_A, E_B, and E_C and three magnetically coupled coils. Each coil has a self-inductance L_s and two mutual inductances with the other two coils both equal to L_m. Write the model equations of the generator in terms of the phase voltages and electric currents and then transform these equations using the symmetrical transformation. Then compute the complex power delivered by the generator in terms of the symmetrical components.

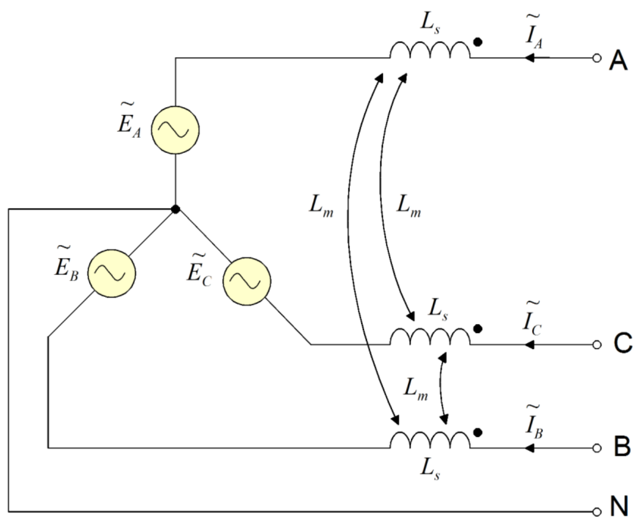


Figure E2.3: A Simplified Three Phase Source.

Solution: The three phase model equations for the simplified generator are:

$$\begin{aligned}
\tilde{V}_{AN} &= j\omega L_s \tilde{I}_A + j\omega L_m(\tilde{I}_B + \tilde{I}_C) + \tilde{E}_A \\
\tilde{V}_{BN} &= j\omega L_s \tilde{I}_B + j\omega L_m(\tilde{I}_A + \tilde{I}_C) + \tilde{E}_B \\
\tilde{V}_{CN} &= j\omega L_s \tilde{I}_C + j\omega L_m(\tilde{I}_A + \tilde{I}_B) + \tilde{E}_C
\end{aligned}$$

In compact matrix notation, above equations are written as follows:

$$\tilde{V}_{abc} = Z\tilde{I}_{abc} + \tilde{E}_{abc}$$

where:

$$\tilde{V}_{abc} = \begin{bmatrix} \tilde{V}_{AN} \\ \tilde{V}_{BN} \\ \tilde{V}_{CN} \end{bmatrix}, \quad \tilde{I}_{abc} = \begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix}, \text{ and } \quad Z = \begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix}$$

Note that above equations are *coupled*, i.e. each phase voltage is a function of all three phase currents. Now let's apply the symmetrical transformation.

$$\begin{aligned}
\tilde{V}_{abc} &= T\tilde{V}_{120} \\
\tilde{I}_{abc} &= T\tilde{I}_{120} \\
\tilde{E}_{abc} &= T\tilde{E}_{120}
\end{aligned}$$

Upon substitution into the matrix equation:

$$T\tilde{V}_{120} = ZT\tilde{I}_{120} + T\tilde{E}_{120}$$

Upon pre-multiplication of above equation by the matrix T:

$$\tilde{V}_{120} = T^{-1}ZT\tilde{I}_{120} + \tilde{E}_{120}$$

The key property of the symmetric component transformation is that the matrix product TZT^{-1} is a diagonal matrix, namely:

$$T^{-1}ZT = \begin{bmatrix} L_s - L_m & 0 & 0 \\ 0 & L_s - L_m & 0 \\ 0 & 0 & L_s + 2L_m \end{bmatrix}$$

This property holds provided that the system consists of symmetric devices, which implies that impedance matrices have all off-diagonal terms equal and all diagonal terms equal. This condition is valid or approximately valid for most 3-phase power system devices, such as transformers, generators, transmission lines etc. Writing the equations explicitly we obtain:

$$\tilde{V}_1 = j\omega(L_s - L_m)\tilde{I}_1 + \tilde{E}_1$$
$$\tilde{V}_2 = j\omega(L_s - L_m)\tilde{I}_2 + \tilde{E}_2$$
$$\tilde{V}_0 = j\omega(L_s + 2L_m)\tilde{I}_0 + \tilde{E}_0$$

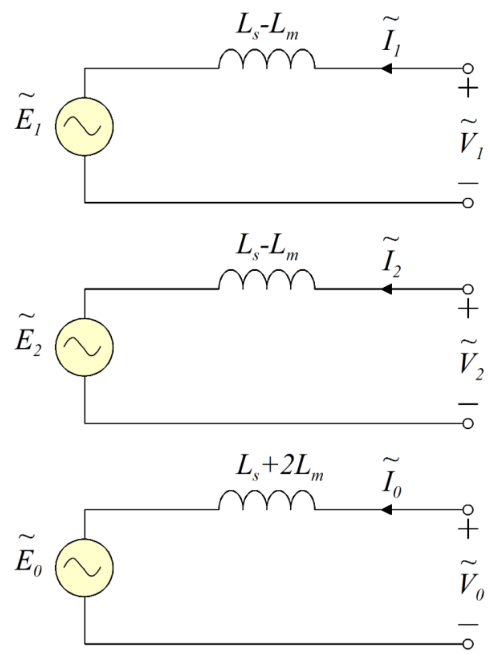


Figure E2.3a: The Sequence Networks of the Simplified Three Phase Source of Figure E2.3

Note that above equations are decoupled, i.e. the variables appearing in any one equation do not appear in any other equation. Consequently, these equations represent three independent single-phase networks, as shown in Figure E2.3a. It should be also noted that in case that the source is balanced, $\tilde{E}_2 = \tilde{E}_0 = 0$. The three networks of Figure E2.3a are referred to as the *positive sequence* model, the *negative sequence* model and the *zero sequence* model, respectively.

The complex power delivered by the generator is:

$$S = \tilde{V}_{an}\tilde{I}^*_a + \tilde{V}_{bn}\tilde{I}^*_b + \tilde{V}_{cn}\tilde{I}^*_c = \tilde{\mathrm{V}}^{\mathrm{T}}_{\mathrm{abc}}\tilde{I}^*_{abc}$$

Application of the symmetrical component transformation to the above equation yields:

$$S = (\mathrm{T}\tilde{\mathrm{V}}_{120})^T (T\tilde{I}_{120})^* = \tilde{\mathrm{V}}_{120}{}^{\mathrm{T}}\mathrm{T}^T T^* \tilde{I}_{120}{}^* =$$
$$= 3\tilde{\mathrm{V}}_1 \tilde{I}_1^* + 3\tilde{V}_2 \tilde{I}_2^* + 3\tilde{V}_0 \tilde{I}_0^*$$

The above equality can be proven by observing that

$$\mathrm{T}^T T^* = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Example E2.4: At a certain location of a three phase system, an engineer measures the following phase currents and phase to neutral voltages:

$$\tilde{I}_a = 200Ae^{j10^\circ} \qquad \tilde{V}_a = 15e^{j4^\circ}\, kV$$
$$\tilde{I}_b = 150Ae^{-j110^\circ} \qquad \tilde{V}_b = 14.5e^{-j120^\circ}\, kV$$
$$\tilde{I}_c = 160Ae^{-j240^\circ} \qquad \tilde{V}_c = 14.8e^{-j235^\circ}\, kV$$

Compute the symmetrical components: I_1, I_2, I_0 and V_1, V_2, V_0.

Solution: By direct computation:

$$\tilde{V}_{120} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = \begin{bmatrix} 14.7563e^{j3.02^0} \\ 0.4904e^{j7.43^0} \\ 0.3294e^{j141.70^0} \end{bmatrix}$$

$$\tilde{I}_{120} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix} = \begin{bmatrix} 169.44e^{j6.87^0} \\ 7.77e^{j28.32^0} \\ 24.39e^{j26.22^0} \end{bmatrix}$$

2.3.1 Real and Reactive Power - SSSC

The power flow in a system operating under sinusoidal steady state conditions can be described with relatively simple equations. Consider a two terminal, one port device as it is illustrated in Figure 2.6. Assume that the voltage and electric current at the port of the device are v(t) and i(t) respectively, where:

$$v(t) = \sqrt{2}V_{rms}\cos(\omega t + \theta)$$
$$i(t) = \sqrt{2}I_{rms}\cos(\omega t + \varphi)$$

The instantaneous power flowing into the device is

$$p(t) = v(t)i(t) = 2V_{rms}I_{rms}\cos(\omega t + \theta)\cos(\omega t + \varphi)$$

Using appropriate trigonometric identities, the above expression becomes:

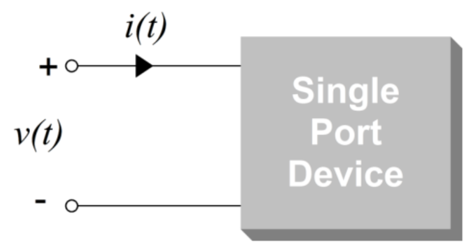


Figure 2.6 Electric Power Flow into a Single Port Device

$$p(t) = V_{rms}I_{rms}\cos(\theta - \varphi) + V_{rms}I_{rms}\cos(2\omega t + \theta + \varphi)$$

Note that the instantaneous power equation consists of two terms: one that is independent of time and another term that is a sinusoidal function of time. Note that when the instantaneous power is integrated over time, in order to evaluate the average power, the time varying sinusoidal term integrates to zero. Only the constant term provides a non-zero contribution. Specifically, the average power flowing into the device is:

$$P = \frac{1}{T}\int_0^T p(\tau)d\tau = V_{rms}I_{rms}\cos(\theta - \varphi)$$

Note that the average power is equal to the product of the RMS values of voltage and current multiplied by the cosine of the angle (θ - φ). The last term, namely $\xi = \cos(\theta - \varphi)$, is known as the *power factor*.

The time varying term of the instantaneous power represents a pulsating power, i.e. power that flows into and out of the device with a net flow of zero. Note that the angular frequency of this term is twice the power frequency. This power was named the *reactive power* many decades ago. In 1932, Fryze provided a theory, which simplified the representation of reactive power. Specifically, Fryze postulated that the 'apparent' power absorbed by a device is simply the product of the voltage across its terminals times the current through it:

$$S_a = V_{rms}\, I_{rms}$$

Then he postulated that the reactive power Q is related to the real power P (he postulated that P and Q are orthogonal) and the apparent power S as follows:

$$Q^2 = S_a^2 - P^2 = V_{rms}^2 I_{rms}^2 (1 - \cos^2(\theta - \varphi)) = V_{rms}^2 I_{rms}^2 \sin^2(\theta - \varphi)$$

or

$$Q = \pm V_{rms}I_{rms}\sin(\theta - \varphi)$$

He adopted the convention of using the + sign resulting to the definition:

$$Q = V_{rms}I_{rms}\sin(\theta - \varphi)$$

This theory, in essence, postulates that the real and reactive powers are projections of the apparent power on the real and imaginary axes of the complex plane. Algebraically, this can be stated as follows:

$$S = P + jQ = V_{rms} I_{rms} \cos(\theta - \varphi) + jV_{rms} I_{rms} \sin(\theta - \varphi) = \tilde{V}_{rms} \tilde{I}^*_{rms}$$

The above relationship involves four quantities, S, S_a, P, and Q. All of them express power but they are all different physical quantities. To distinguish them, the following nomenclature has been adopted many years ago and it is used in power engineering:

Quantity	Name	Units
S	Complex Power	VA (Volt Ampere)
S_a	Apparent Power	VA (Volt Ampere)
P	Real Power	W (Watt)
Q	Reactive Power	VAr (Volt Ampere reactive)

Now let's examine the term $Q = S_a \sin(\theta - \varphi)$. If the phase of the electric current, φ, is less than θ, then $\sin(\theta - \varphi) > 0$ and $Q > 0$. In this case we say that the electric current ***lags*** the voltage because if the voltage and electric current phasors are depicted on the same complex plane as it is illustrated in Figure 2.5, the voltage phasor appears to be *ahead* of the electric current phasor as both phasors rotate with speed ω in the counterclockwise direction. (Note that the positive direction for phase angle is assumed to be the counterclockwise direction. See also Figure 2.7). On the other hand, if $\theta < \varphi$, then $Q < 0$. Again, by observing the voltage and current phasors we conclude that in this case, the current phasor ***leads*** the voltage phasor. When specifying power consumption in power engineering, it is customary to state the value of the real power and the power factor (i.e the value of the term $\cos(\theta - \varphi)$). Since $\cos(\theta - \varphi) = \cos(\varphi - \theta)$, we do not know whether $\theta > \varphi$ or $\theta < \varphi$. This information is given, by also stating whether the electric current is *lagging* or *leading* the voltage. For example, a power factor of 0.8 *lagging* means that $\cos(\theta - \varphi) = 0.8$ and the electric current phasor *lags* the voltage phasor, i.e. $\theta > \varphi$.

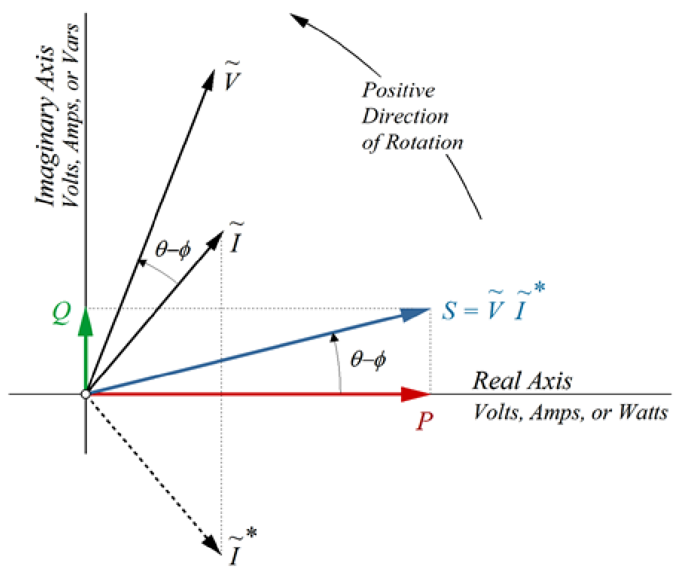


Figure 2.7: Phasor Representation of Voltage, Electric Current, Real Power, and Reactive Power

The power relationships for sinusoidal steady state are summarized in Table 2.2

Table 2.2 Summary of Power Relationships - Sinusoidal Steady State

Quantity	**Equation**
Instantaneous Values	$i(t) = I_m \cos(\omega t + \varphi)$, $v(t) = V_m \cos(\omega t + \theta)$
Phasors	$\tilde{I} = I_{rms} e^{j\varphi}$, $\tilde{V} = V_{rms} e^{j\theta}$
Instantaneous Power	$p(t) = v(t)i(t) = 2V_{rms} I_{rms} \cos(\omega t + \theta)\cos(\omega t + \varphi)$
Real Power	$P = \frac{1}{T}\int_0^T p(\tau)d\tau = V_{rms} I_{rms} \cos(\theta - \varphi)$
Power Factor	$\xi = \cos(\theta - \varphi)$, (leading if $\phi > \theta$, lagging otherwise)
Apparent Power	$S_a = V_{rms}\, I_{rms}$
Reactive Power	$Q = V_{rms} I_{rms} \sin(\theta - \varphi)$
Complex Power	$S = \tilde{V}_{rms} \tilde{I}^*_{rms} = V_{rms} I_{rms} \cos(\theta - \varphi) + jV_{rms} I_{rms} \sin(\theta - \varphi) = P + jQ$

2.3.2 Real, Reactive and Distortion Power - PSSC

In modern power systems we have many devices that include power electronics that switch on and off periodically during each cycle of the power frequency. These devices distort the sinusoidal operation of the system. Nonlinear devices can also distort the sinusoidal operation of the system. The distortion is periodic, i.e. it is repeated each cycle of the operation of the system in exactly the same way. We refer to these operating conditions as the Periodic Steady State Conditions (PSSC). In this section we consider the definitions of power when the system operates under periodic steady state conditions (PSSC). The voltage and current waveforms are not sinusoidal but periodic. Consider again the single port device as in Figure 2.6. If v(t) and i(t) are not pure sinusoids but periodic, they can be expanded into a Fourier series, i.e.

$$v(t) = \sum_n a_n \cos(n\omega t + \theta_n)$$
$$i(t) = \sum_n b_n \cos(n\omega t + \varphi_n)$$

The real power is computed as the average power flow, i.e.

$$P = \frac{1}{T}\int_0^T v(\tau)i(\tau)d\tau$$

Using above Fourier expansion, and after a few manipulations, the real power is

$$P = \frac{1}{2}\sum_n a_n b_n \cos(\theta_n - \varphi_n)$$

Note that the real power can be determined directly from the real power definition. However, for the reactive power there is no unique generalization of the sinusoidal case approach. In fact, for the periodic steady state operation, the reactive power can be defined in a variety of ways. The literature is full of proposals for the definition of reactive power under non-sinusoidal periodic conditions. In this book we have adopted the simple definition of reactive power of the fundamental frequency (as defined by Fryze). In addition, the concept of distortion power is introduced. Specifically, the distortion power D is defined from the equation

$$S^2 = P^2 + Q_B^2 + D^2$$

where:

S	is the apparent power defined as the product of the voltage and electric current rms values, i.e. $S = V_{rms} I_{rms}$
P	is the real power
Q_B	is the fundamental reactive power, i.e. $Q_B = V_1 I_1 \sin(\theta - \varphi)$
D	is the distortion power.

The distortion power, as defined above, provides a measure of the waveform distortion. Specifically, for the sinusoidal steady state case, the distortion power is exactly zero, and in the general non-sinusoidal case, the following relationship holds:

$$S^2 \geq P^2 + Q_B^2$$

Thus D^2 is never negative. The reader is encouraged to prove this property.

The power relationships for non-sinusoidal periodic steady state are summarized in Table 2.3.

Table 2.3 Power Relationships Summary - Periodic Steady State

Quantity	**Equation**
Instantaneous Values	$v(t) = \sum_n a_n \cos(n\omega t + \theta_n)$, $i(t) = \sum_n b_n \cos(n\omega t + \varphi_n)$
Phasors	$\tilde{I}_n = I_{n,rms} e^{j\varphi_n}$, $\tilde{V}_n = V_{n,rms} e^{j\theta_n}$, for $n = 0, 1, 2, 3, \ldots$
Instantaneous Power	$p(t) = v(t)i(t)$
Real Power	$P = \frac{1}{T}\int_0^T v(\tau)i(\tau)d\tau$, or $P = \frac{1}{2}\sum_n a_n b_n \cos(\theta_n - \varphi_n)$
Power Factor	$\delta = \cos(\theta_1 - \varphi_1)$, (leading if $\phi_I > \theta_I$, lagging otherwise)
Apparent Power	$S = V_{rms} I_{rms}$
Reactive Power	$Q_B = V_1 I_1 \sin(\theta - \varphi)$
Distortion Power	$D^2 = S^2 - P^2 - Q_B^2$

2.4 Instrument Transformers

Instrument transformers transform the level of power system voltages and currents to levels appropriate for relay inputs. Instrument transformers are classified into Voltage and Current transformers. These devices exist in a variety of sizes and implementation technologies. The design and characteristics of instruments transformers will be examined in Chapter 6. In this section we consider the ideal characteristics and representation of instrument transformers.

2.4.1 Transformer Polarity

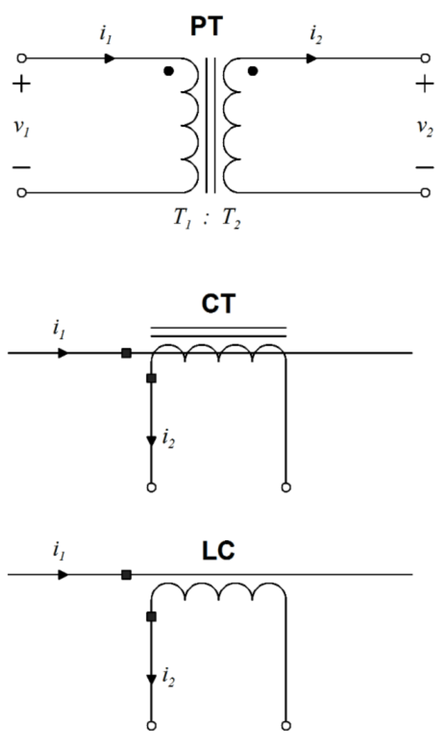


Figure 2.8: Instrument Transformer Polarity Marks

The polarity of instrument transformer windings is important in many relaying applications where the phase of the transformed waveforms must be preserved. The standard notation indicating winding polarity is illustrated in Figure 2.8. The dots or square blocks marks placed near one terminal of each winding have the following interpretation:

The voltages measured from the marked to the unmarked terminals of each winding are in phase. The electric current flowing into the marked terminal of the first winding is in phase with the electric current flowing out of the marked terminal of the second winding.

2.4.2 Phase Shifts in Transformers

Transformers may be connected in a variety of configurations, especially those with multiple windings. The connection configuration may introduce a phase shift between the various sides of the transformer. This phase shift should be accounted for in any application.

Figure 2.9 illustrates a two winding three phase transformer. The phase shift between the two sides of the transformer is computed by observing the basic relationship among the voltages of the individual single phase transformer.

$$\frac{\tilde{V}_A - \tilde{V}_B}{N_1} = \frac{\tilde{V}_a}{N_2} \qquad \rightarrow \qquad \frac{\tilde{V}_A}{\tilde{V}_a} = \frac{N_1}{N_2\sqrt{3}} e^{-j30^0}$$

Figure 2.10 illustrates a three-winding three-phase phase-shifting transformer. The phase shift between the two sides of the transformer is computed by observing the basic relationship among the voltages of the individual single phase transformer. Specifically:

$$\tilde{V}_{a'} = k_1\tilde{V}_a + k_3 k_2 (\tilde{V}_c - \tilde{V}_b)$$

Under steady state balanced conditions:

$$\frac{\tilde{V}_{a'}}{\tilde{V}_a} = k_1 + k_3 k_2 \sqrt{3} e^{j90^0}$$

Note that by changing the transformer tap the phase shift between primary and secondary changes.

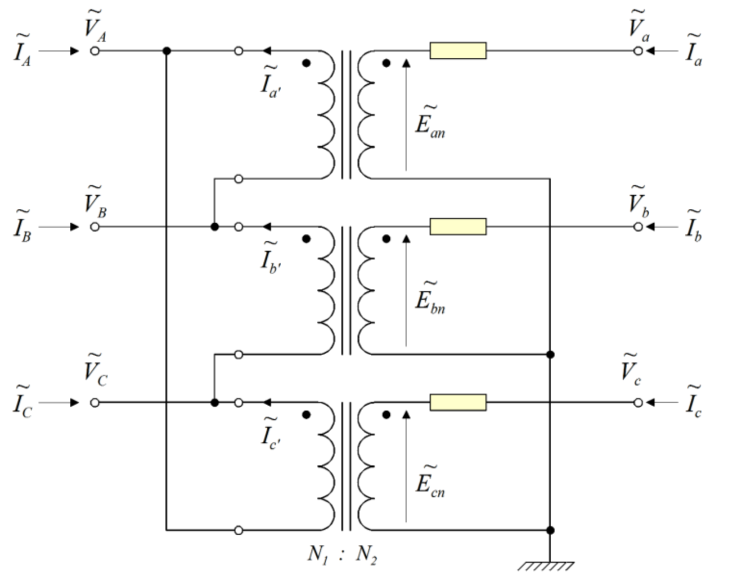


Figure 2.9 Two-Winding Three-Phase Transformer – Delta/Wye Connected

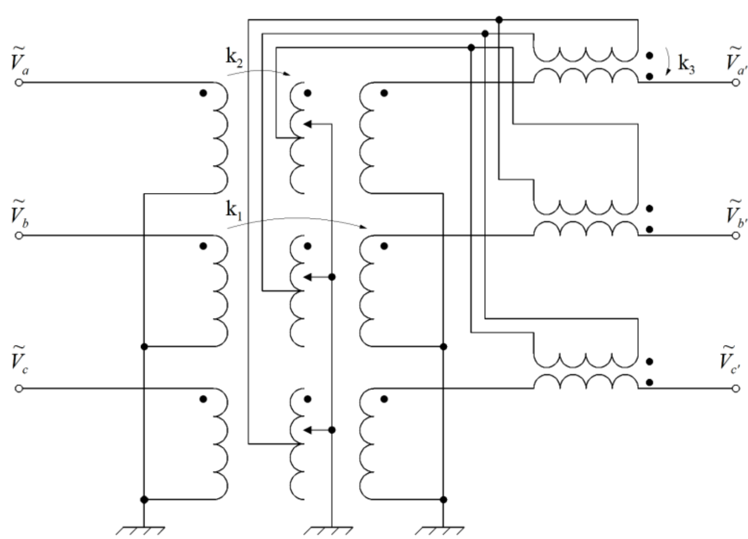


Figure 2.10 Three-Winding Three-Phase Phase-Shifting Transformer

Historical Note:

Electromechanical relays use transformers for creating phase shifts and extracting specific quantities (such as positive sequence, etc.) for the logical operations of the relay. In numerical relays this is not necessary anymore since any transformation can be achieved by numerically processing the digitized data of the voltage and current waveforms. The following section provides additional information.

2.5 Waveform Synthesis

In many relaying applications, there is a need to synthesize or extract specific information from the voltages and currents of a three phase power system. Examples are: the rms value of a voltage from the waveform of the same voltage, the positive sequence phasor from a set of three phase voltages, the zero sequence rms value of a set of three phase voltages, etc. For this purpose specific filters are constructed. For electromechanical relays, these filters are analog devices (comprising resistors, inductors, capacitors and transformers), while for numerical relays these filters are simply digital filters. The output of the filters may be used in a variety of applications, for example as the polarizing voltage in a directional relay, etc. In this section we describe the construction of some of the most common filters.

2.5.1 Analog Zero Sequence Waveform Synthesis

Figure 2.11 illustrates an analog circuit that generates an output that is proportional to the zero sequence current of the three phase system.

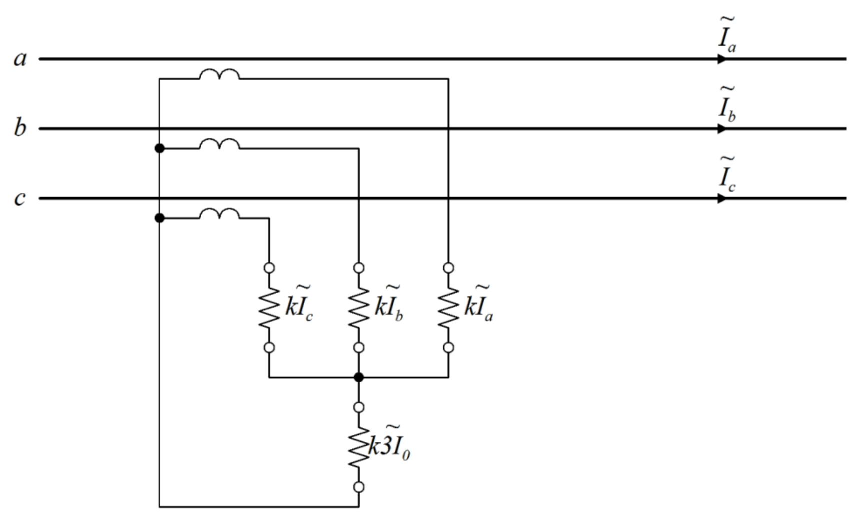


Figure 2.11 Synthesis of the Zero Sequence Current

The zero sequence voltage can be generated by an open delta connected transformer. Figure 2.12 illustrates the application of the zero sequence synthesis circuits for both voltage and current for a directional ground relay.

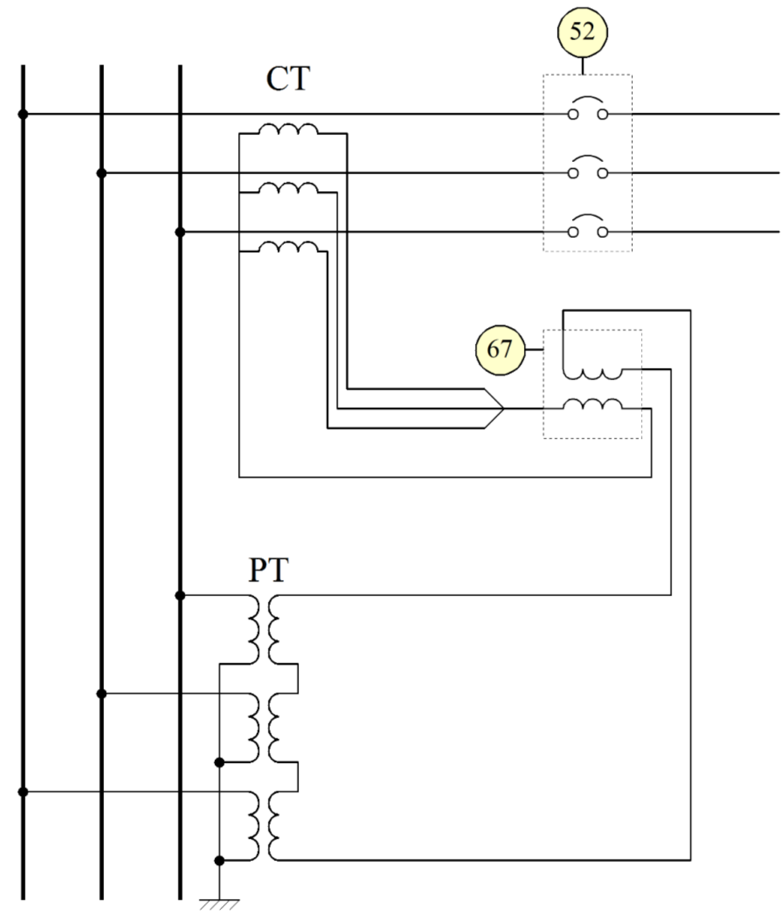


Figure 2.12 Synthesis of the Zero Sequence Voltage and Current for a Directional Ground Relay

2.5.2 Analog Positive and Negative Sequence Waveform Synthesis

Voltages proportional to the positive and negative sequence components are generated using the circuit illustrated in Figure 2.13. The circuit consists of three transformers and a phase shifting network consisting of resistors (R) and capacitors (C). It can be shown that selecting the R and C values so that $R = X_C$, and assuming negligible circuit loading, yields a 90 degree phase shift between the voltage across the primary of transformer T_3 and the voltage developed between the RC interconnection point and the center tap of transformer T_3 secondary winding. The circuit output can be expressed as follows:

$$\tilde{V}_{out} = k_1(\tilde{V}_a - \tilde{V}_b) + k_2(\tilde{V}_a - \tilde{V}_c) + jk_3(\tilde{V}_b - \tilde{V}_c)$$

Rearranging the above equation yields:

$$\tilde{V}_{out} = (k_1 + k_2)\tilde{V}_a + (-k_1 + jk_3)\tilde{V}_b + (-k_2 - jk_3)\tilde{V}_c$$

For the above voltage to be positive sequence:

$$\begin{gathered}(k_1 + k_2) = 1/3 \\ (-k_1 + jk_3) = 1/3(-0.5 + j0.8660254) \\ (-k_2 - jk_3) = 1/3(-0.5 - j0.8660254)\end{gathered}$$

The above equations are satisfied for:

$$k_1 = k_2 = 0.16666 \text{and} \quad k_3 = 0.288675$$

Note that reversing the R and C positions changes the phase shift from +90 to -90 degrees. Thus adding a second branch with R and C positions reversed yields a second output providing the negative sequence voltage. This can be easily shown by substituting j by –j in the above equations.

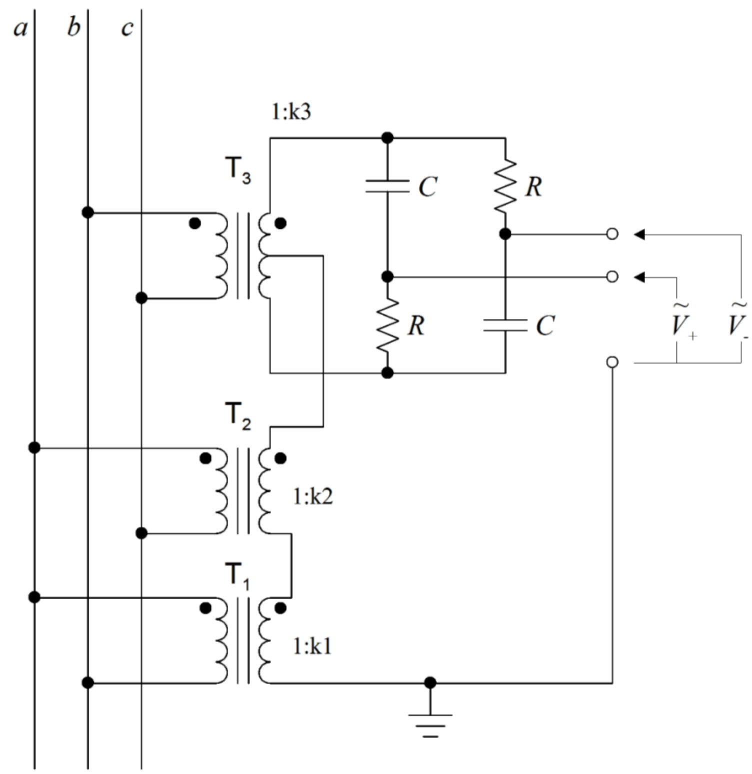


Figure 2.13: Positive/Negative Sequence Component Synthesis Circuit

Example E2.5: It is desired to design a filter circuit that will provide a voltage output that will be proportional to the negative sequence current of a three phase system. For this purpose consider the circuit of Figure E2.5. Can you select the values of the resistors R_1 and R_2, and the reactances x_s and x_m such that the voltage output v_{out} is proportional to the positive sequence current in the three phase system? The three indicated CTs are identical. The output is connected to a high input impedance relay.

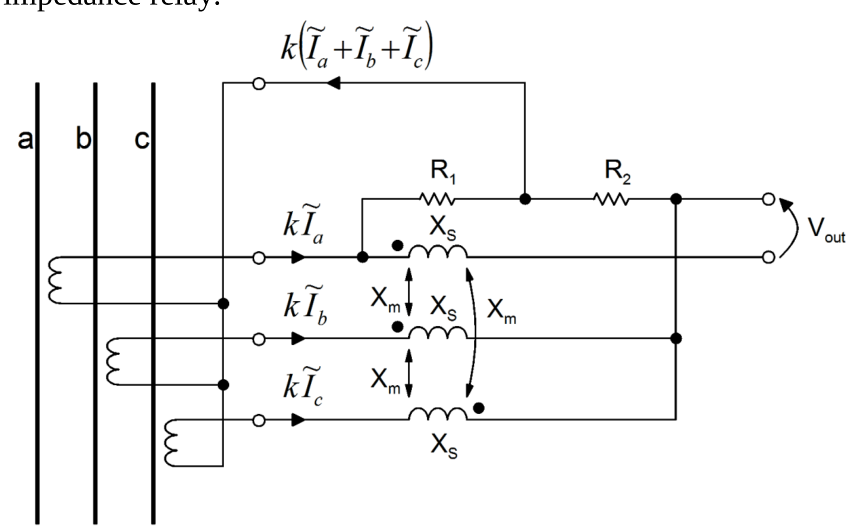


Figure E2.5

Solution: Considering the circuit, the following holds:

$$\tilde{V}_{out} = k(\tilde{I}_b + \tilde{I}_c)R_2 - k\tilde{I}_a R_1 + k\tilde{I}_b jx_m - k\tilde{I}_c jx_m, \text{ or}$$

$$\tilde{V}_{out} = -kR_1\left(\tilde{I}_a + \left(-\frac{R_2}{R_1} - j\frac{x_m}{R_1}\right)\tilde{I}_b + \left(-\frac{R_2}{R_1} + j\frac{x_m}{R_1}\right)\tilde{I}_c\right)$$

It is desired that: $\tilde{V}_{out} \sim \tilde{I}_a + a^2\tilde{I}_b + a\tilde{I}_c$

Thus:

$$\left.\begin{aligned} -\frac{R_2}{R_1} - j\frac{x_m}{R_1} &= a^2 \\ -\frac{R_2}{R_1} + j\frac{x_m}{R_1} &= a \end{aligned}\right\} \quad \Rightarrow \quad \begin{aligned} \frac{R_2}{R_1} &= 0.5 \\ \frac{x_m}{R_1} &= 0.866 \end{aligned}$$

2.5.3 Digital Zero Sequence Waveform Synthesis

In microprocessor based implementations, the zero sequence component is numerically computed by simply adding the three phase samples at each sampling time instant. The concept is illustrated graphically in Figure 2.14.

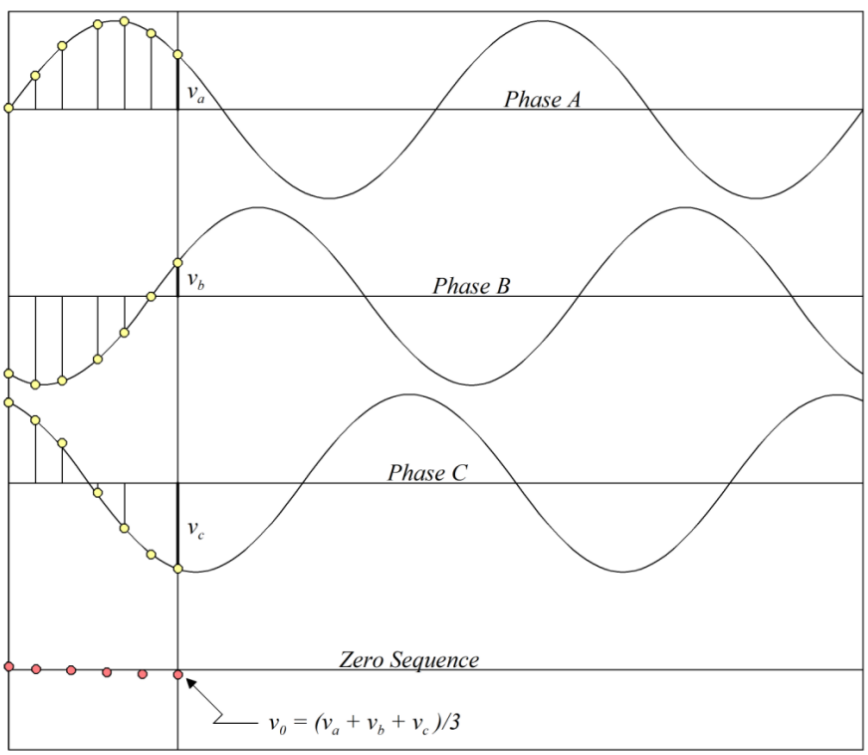


Figure 2.14: Synthesis of the Zero Sequence Component in Numerical Relays

2.5.4 Digital Positive and Negative Sequence Waveform Synthesis

Positive and negative sequence components are computed by first computing the phase A, B and C phasors, and subsequently applying the symmetrical component transformation:

$$v_a(t) = \sqrt{2}\left(A_1 \cos\omega t + A_2 \sin\omega t\right) \Rightarrow V_a e^{i\phi_a}$$

Note:

$$V_a = \sqrt{A_1^2 + A_2^2}$$

$$\phi_a = \arctan\left(-\frac{A_2}{A_1}\right)$$

2.5.5 Phasor Extraction - Magnitude and Phase Computation

A periodic waveform can be expanded in terms of it Fourier series. Specifically, a voltage waveform $v_a(t)$ can be written as:

$$v_a(t) = a_1 \cos\omega t + a_2 \sin\omega t + harmonics$$

Assume that the above waveform is uniformly sampled at a rate of N samples per period. Assuming that the fundamental frequency ω is known, the parameters a_1 and a_2 can be computed by computing the following two sums:

$$A = \sum_{i=1}^{N} v_a(t_i)\cos\omega t_i = a_1 \sum_{i=1}^{N} \cos^2 \omega t_i + a_2 \sum_{i=1}^{N} \cos\omega t_i \sin\omega t_i$$

$$B = \sum_{i=1}^{N} v_a(t_i)\sin\omega t_i = a_1 \sum_{i=1}^{N} \sin\omega t_i \cos\omega t_i + a_2 \sum_{i=1}^{N} \sin^2 \omega t_i$$

Note that the right hand side sums evaluate to:

$$\sum_{i=1}^{N} \cos^2 \omega t_i = N/2,$$

$$\sum_{i=1}^{N} \sin^2 \omega t_i = N/2, \text{ and}$$

$$\sum_{i=1}^{N} \cos\omega t_i \sin\omega t_i = 0$$

Thus:

$$a_1 = 2A/N, \text{ and } a_2 = 2B/N$$

Given the parameters a_1 and a_2, the phasor magnitude and phase are computed as follows:

$$v_a(t) = \sqrt{2}\,V_a \cos(\omega t + \varphi_a) \qquad \Leftrightarrow \qquad V_a e^{j\varphi_a}$$

$$V_a = \sqrt{\frac{a_1^2 + a_2^2}{2}}$$

$$\varphi_a = \arctan\left(-\frac{a_2}{a_1}\right)$$

The above equations assume that the exact power system frequency is known. Considerable error is introduced if the value ω used in the computations does not match the actual power system frequency. A parametric analysis that quantifies this error is presented next.

Figure 2.15 illustrates the computed phasor magnitude error (in %) if the power system frequency varies from 59.5 to 60.5, while the term ω in the above equations is set to the nominal power system frequency (2π60), and for varying levels of 3d harmonic. Note that the error increases with the frequency deviation from its nominal value. It also increases further with the level of harmonic distortion. The error is nearly independent of the sampling rate, assuming no aliasing has occurred.

Figure 2.16 illustrates the computed phasor phase angle error (in degrees) if the power system frequency varies from 59.5 to 60.5, while the term ω in the above equations is set to the nominal power system frequency (2π60). Again, the error increases with the frequency deviation from its nominal value. However, it is nearly independent of harmonic distortion as well as the sampling rate, assuming no aliasing has occurred.

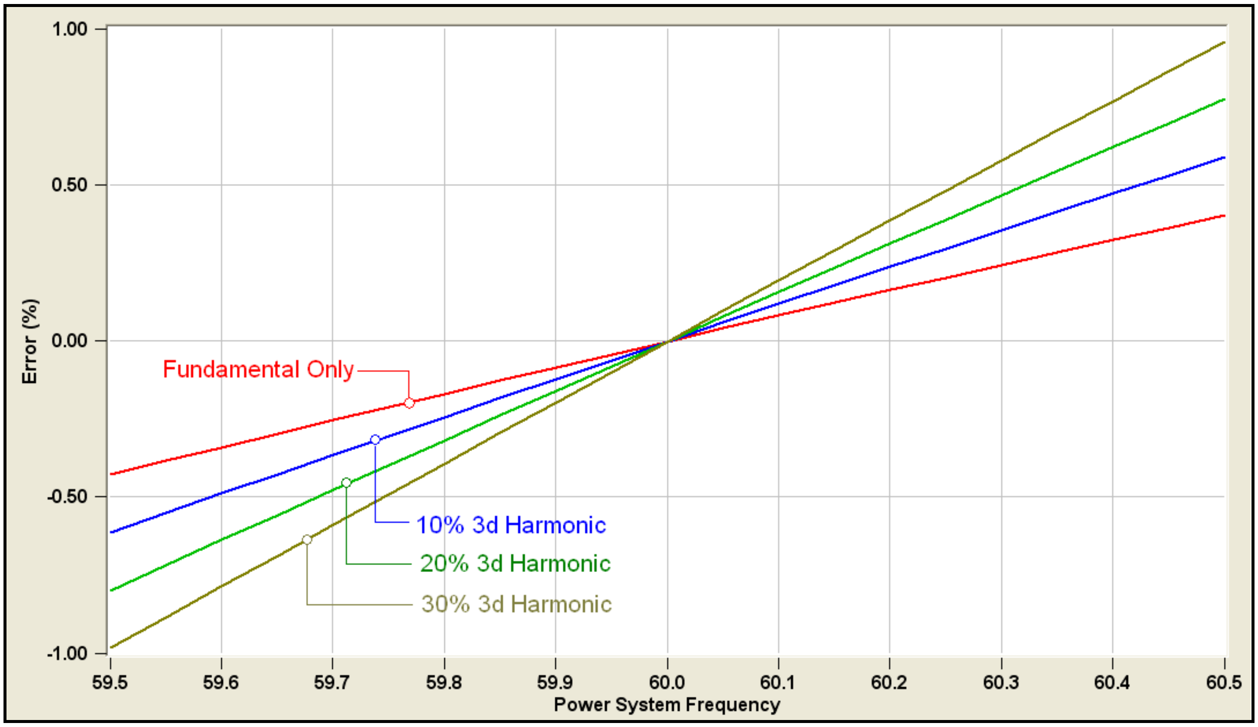


Figure 2.15: Phasor Magnitude Computation Error versus Power System Frequency and 3d Harmonic Level.

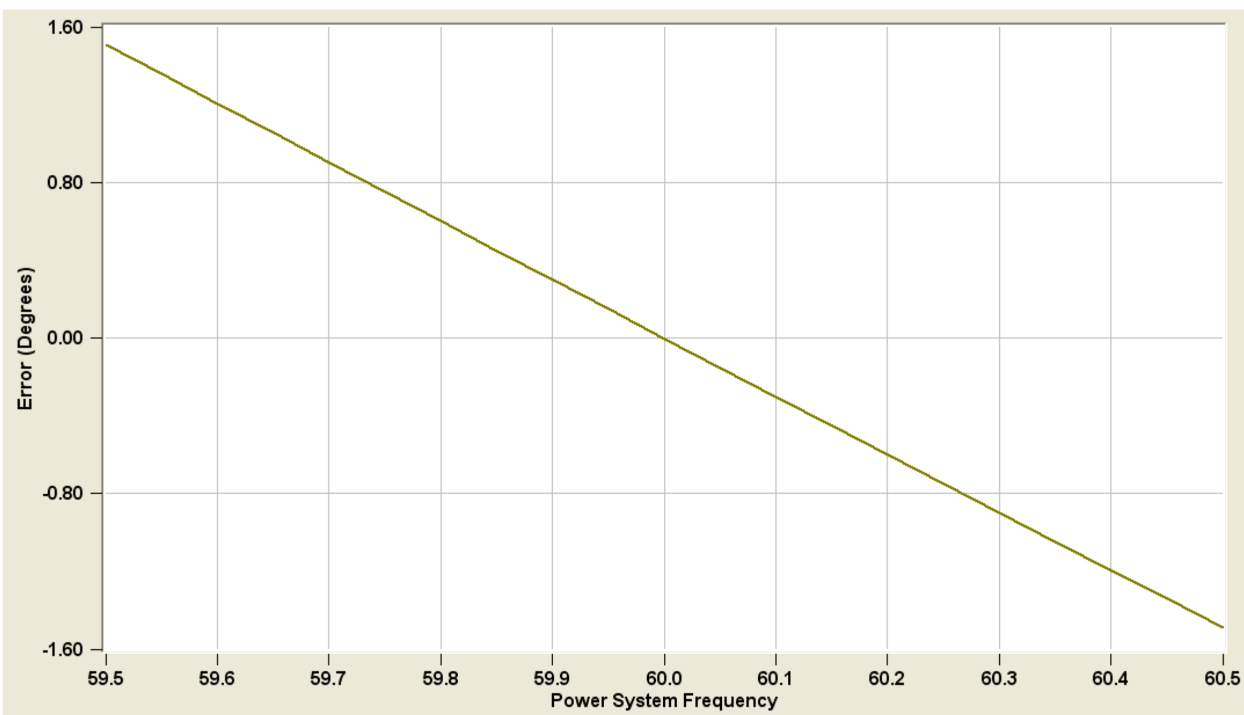


Figure 2.16: Phasor Phase Computation Error versus Power System Frequency

2.5.6 Phasor Extraction - Fundamental Frequency Computation

Many relaying applications require accurate determination if the instantaneous value of the power system fundamental frequency. For example, the accuracy of the waveform magnitude and phase computation presented in the previous section depends on the accuracy of the assumed value for the power system fundamental frequency. One practical method for performing this task is presented in this section.

The presented method is based on the computation of the rate of change of a phasor phase angle. A computationally efficient recursive implementation is described next.

The following two sums are recursively updated every time a new waveform sample becomes available:

$$V_1(k) = \sum_{i=k}^{k+N-1} x(i)\cos(\omega_0 Ti)$$

$$V_2(k) = \sum_{i=k}^{k+N-1} x(i)\sin(\omega_0 Ti)$$

where:

x(i) input waveform sample sequence (voltage or current)
ω_o power base frequency (= 2 π f_0)
T sampling period
N Number of samples in one period = $\dfrac{2\pi}{\omega_0 T}$

At every time instant the two sums are updated, the phasor angle ϕ is computed as follows:

$$\varphi(k) = \arctan\left(-V_2(k), V_1(k)\right)$$

The frequency is computed from the rate of change of the above value as follows:

$$f = f_0 + \frac{\Delta\varphi_k}{2\pi T}$$

where:

$$\Delta\varphi_k = \varphi(k) - \varphi(k-1) \text{ and,}$$

T sampling period.
f_0 nominal power frequency.

Note that the value of $\Delta\varphi_k$ must be corrected whenever φ_k crosses from π to -π, or from -π to π by adding or subtracting 2π.

2.5.7 Phasor Extraction - Circular Array Implementation

The phasor extraction computations can be implemented with high computational efficiency using a circular array based algorithm. Computations such as moving averages and Discrete Fourier transforms can be implemented in this way. Consider for example the values $V_1(k)$ and $V_2(k)$ used in the frequency tracking algorithm presented in section 2.5.6:

$$V_1(k) = \sum_{i=k}^{k+N-1} x(i)\cos(\omega_0 Ti)$$
$$V_2(k) = \sum_{i=k}^{k+N-1} x(i)\sin(\omega_0 Ti)$$

The circular array implementation of this computation is as follows. Two circular arrays are setup with N entries each. All entries are initialized to zero. Also two accumulators which will contain the present values of $V_1(k)$ and $V_2(k)$ are initialized to zero. As each sample $x(i)$ becomes available the following steps are performed:

1 Compute the values $y(i) = x(i)\cos(\omega_0 Ti)$ and $z(i) = x(i)\sin(\omega_0 Ti)$

2 Update the two accumulators as follows:

$$V_1(k) = V_1(k-1) + y(i) - y(i-N)$$
$$V_2(k) = V_2(k-1) + z(i) - z(i-N)$$

Note that $y(i-N)$ and $z(i-N)$ are the oldest values stored in the circular arrays.

3 Store the values $y(i)$ and $z(i)$ in the circular arrays. This causes the values stored N steps ago, $y(i\text{-}N)$ and $z(i\text{-}N)$ to be overwritten.

4 Compute the phasor value, as follows:

$$\tilde{V}(k) = \frac{\sqrt{2}}{N}\left(V_1(k) + jV_2(k)\right)$$

The above algorithm is illustrated graphically in Figure 2.17.

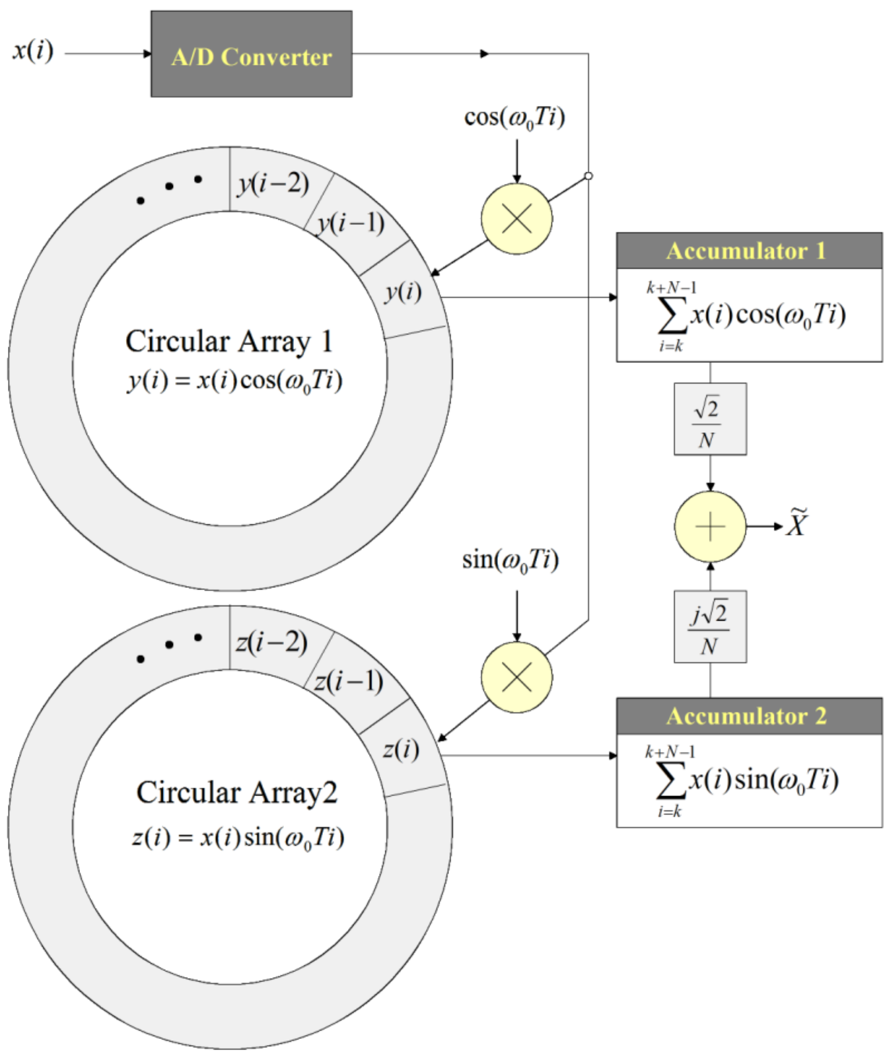


Figure 2.17 Illustration of Circular Array Implementation for Phasor Computations

2.5.8 Comparison Between Analog and Digital Computer Implementations

Figure 2.18 illustrates the general block diagram of a typical digital implementation of a protective relay. It consists of a bank of analog to digital converters that continuously sample a number of voltage and current channels. The sampled data are processed by a microprocessor which uses algorithms designed for the specific relay application. The microprocessor controls a number of contactors which can be used to control breakers or trigger external alarm circuits based on the results of the executing algorithm. A display, keyboard and other i/o options (such as RS232, ethernet ports, etc.) are typically also included which allow the user to view the relay status as well as view and modify the relay settings.

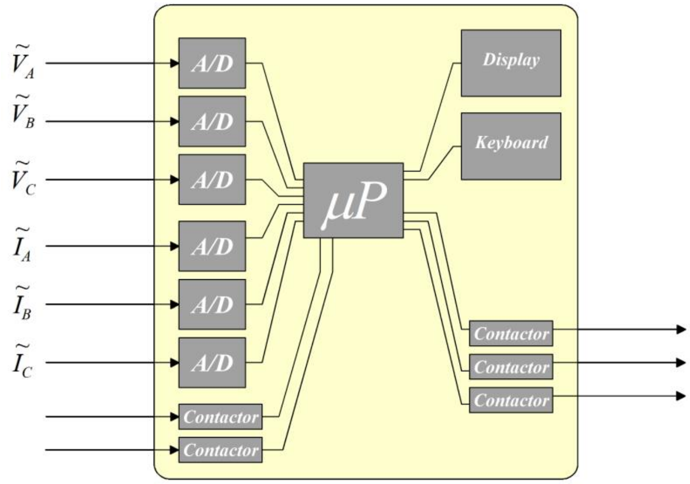


Figure 2.18: Typical Digital Relay Implementation – Block Diagram

The above implementation allows complete flexibility in implementing of a variety of relay functions. As a matter of fact, the same relay hardware can be used to implement any desirable relay function, such as over-current, distance, directional, differential protection etc. Digital relays can be programmed to simulate the operation of electromechanical relays, or perform new original functions. The only component that must be modified in order to change the relay function is the software. Conversely, analog (electromechanical) relay implementations can perform only the single function they are originally designed for.

An additional benefit arising from the digital computer implementation is that relays can also provide the function of *digital event recording.*

2.6 Synchronized Sampling

In many applications it is desirable to compute the phase angles of voltage and current phasors with respect to a common time reference. This capability is useful in fault locating applications where phasor measurements are available from the two ends of a transmission line. Another application where absolute phase reference is essential is system state measurements based on direct phasor measurements across the entire power system. A practical method to obtain a common time reference is the use of the Global Positioning System (GPS). GPS based clocks can provide time reference signals with accuracy better that 1 microsecond. Using this technology, data acquisition systems located at different locations can sample synchronously within the accuracy of the GPS timing signals. Note that a timing error of Δt in measuring a periodic signal of period T results in phase error of:

$$\textit{Time Error}: \Delta t \quad \Leftrightarrow \quad \textit{Phase Error}: 360 \cdot \frac{\Delta t}{T} \text{(degrees)}$$

For a 60 Hz signal and a timing error of 1 microsecond the resulting error is 0.0216 degrees. This error level is adequate for most power frequency phasor calculations. Consider for example the computation of power flow through a transmission line. Assuming that the major component of the series impedance is inductive, the power flow is approximately given by:

$$P \cong \frac{V^2 \delta}{X}$$

Where V is the phase voltage, X is the series reactance, and δ is the phase difference between the voltages at the two terminals of the line. If δ is 2 degrees then, the Power error for 0.02 degrees phase error is approximately 1%. Thus a 1 microsecond timing accuracy is a reasonable requirement for such applications.

The IRIG-B Time Code

In order to reduce equipment cost, many relays and other data acquisition systems capable of synchronous sampling do not include a built in a clock source such as a GPS receiver, but accept a timing signal from an external clock. Thus, a single GPS receiver can be installed in a substation and provide timing signals to any number of data acquisition equipment. The timing signal standard most commonly used for this purpose is the IRIG time code format. The IRIG time code standard was developed by the Telecommunications Working Group of Inter-Range Instrumentation Group (IRIG), which is the standards part of Range Commanders Council (RCC). Work on this standard started in 1956, while the latest version is IRIG Standard 200-04, published in September 2004. The standard contains several different time-codes identified by alphabetic designations A, B, D, E, G, and H. The most commonly used code is IRIG-B, which is briefly described next

The IRIG-B code is a serially transmitted binary signal consisting of a sequence of "frames". Each frame contains time and date information plus some additional "Control Function" bits that are reserved for special applications. A frame is transmitted every second and consists of binary signal containing 74 bits, and each bit is represented by a variable width pulse occurring within a 10 millisecond interval. The information in the 74 bits of a frame is listed in Table 2.4. Note that the first 6 numbers forming the time stamp are encoded in Binary Coded Decimal (BCD). The Control bits are not defined by the standard as they are for internal use by the Range Commanders Council. The seconds of day are in *straight* binary form, and represent the number of seconds since midnight.

Table 2.4: IRIG-B Frame Information Encoding

Number of Bits	Encoding	Information
7	BCD	Seconds of Minute (0-59)
7	BCD	Minutes of Hour (0-59)
6	BCD	Hours of Day (0-24)
10	BCD	Days of Year (0-366)
9	BCD	Year (last two digits)
18	Binary	Control Bits
17	Binary	Seconds of Day (0-86399)

IRIG-B signal may be transmitted in three modes:

- Unmodulated
- Amplitude Modulated
- Modified Manchester Modulated

A snapshot of an unmodulated and an amplitude modulated IRIG signal are illustrated in Figure 2.19. The modulated carrier signal frequency is specified at 1 kHz. Therefore, each 10ms period during which a bit is transmitted contains 10 carrier waveform cycles. An advantage of the modulated IRIG-B transmission is that it contains no DC component and thus it can be easily transmitted through galvanic isolation circuits such as transformers or capacitive coupled stages. A major disadvantage is that the time resolution is considerably inferior to the unmodulated transmission. Specifically, the unmodulated signal time resolution is typically in the order of 100 nanoseconds. This is facilitated by the short rise time of the pulses comprising the unmodulated signal. On the other hand, the modulated signal rise time is determined by the 1 kHz carrier frequency, resulting in time resolution in the order of 100 microseconds. A minor disadvantage of the unmodulated IRIG signal is that it contains a significant DC component which is a function of the transmitted data. Thus passing this signal through DC blocking galvanic isolation circuits may cause signal deterioration and data errors. The Manchester Modulated transmission mode combines the advantages of the other two modes. Detailed descriptions of Manchester Modulated scheme are described in the IRIG standard.

Figure 2.20 shows an example of a complete unmodulated IRIG-B frame. Note that the frame starts with two 8 millisecond pulses which indicate the beginning of the frame (P0). Binary ones are represented by 5 millisecond pulses while binary zeroes are represented by 2 millisecond pulses. Additional single 8 millisecond pulses separate the sequential data fields (seconds, minutes, hours, etc.).

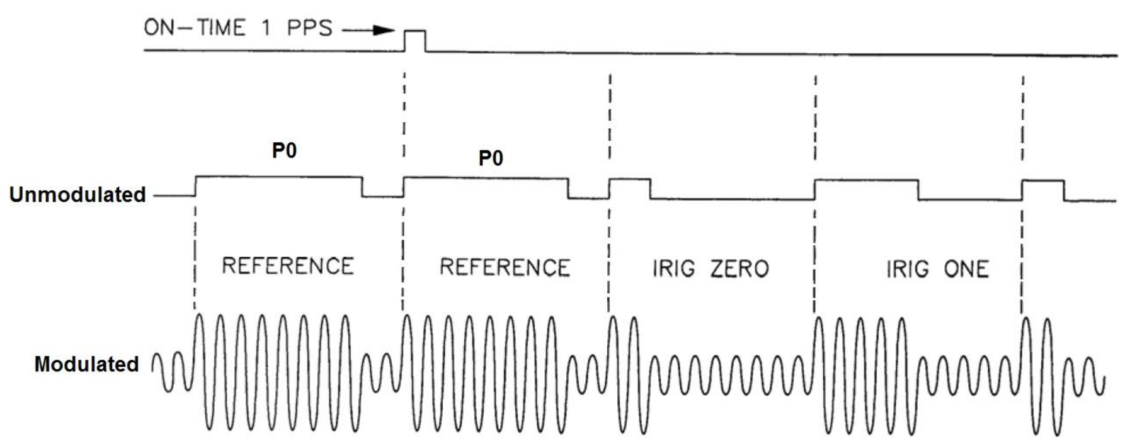


Figure 2.19: A Snapshot of an Unmodulated and a Modulated IRIG-B Signal

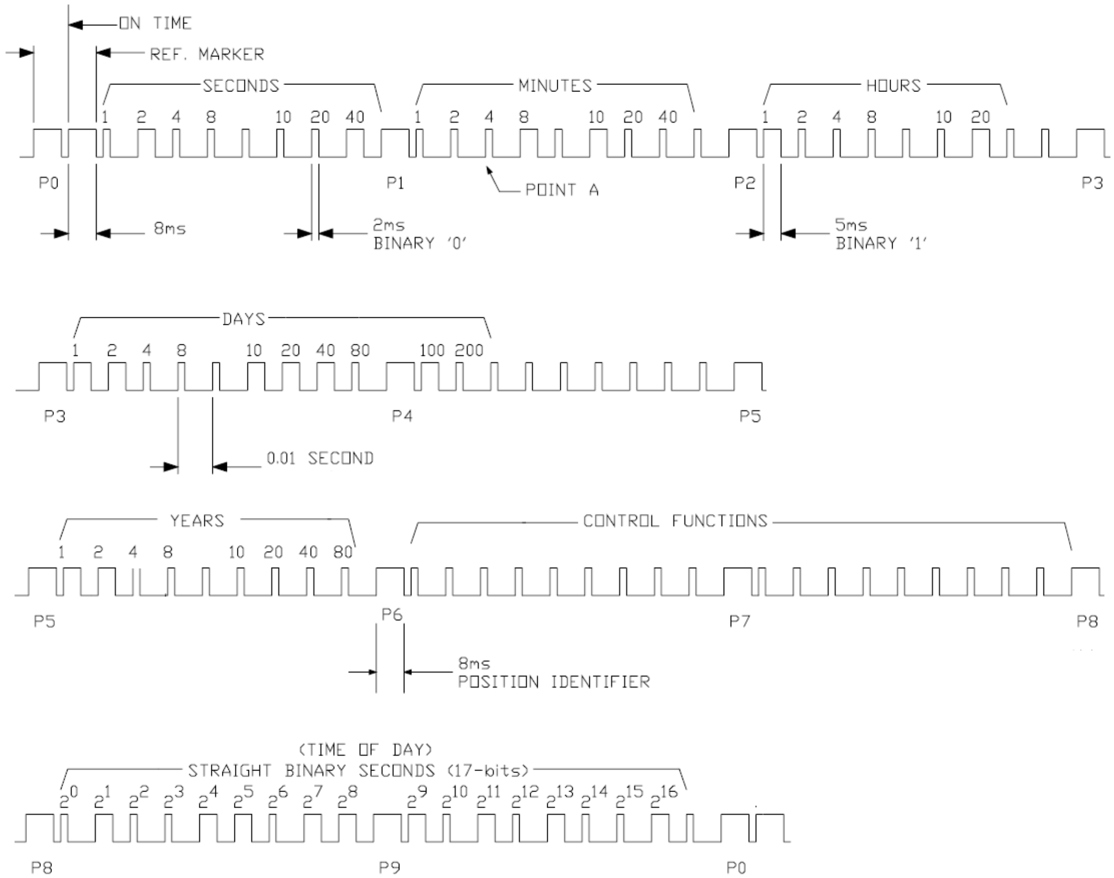


Figure 2.20: Example of IRIG-B Unmodulated Frame

The IEEE Synchrophasor Standard

A standard defining both the measurement procedures and transmission of phasor measurements has been created by IEEE. The original standard was created in 1995, updated in 2005 and in 2011. The 2011 version has been split into two documents, Std-C37.118.1 and Std-C37.118.2. The first document covers measurement techniques while the second document covers communications. A brief description of the concepts defined in the standard follows.

The standard introduces the term “Synchrophasor” defined as a phasor with magnitude equal to the RMS value of a monitored voltage or current waveform, and phase angle defined using a cosine function, at the nominal system frequency, referenced to “Universal Coordinated Time” (UTC). Specifically, the magnitude X_m and phase φ angle are defined by the equation:

$$x(t) = \sqrt{2}X_m \cos(\omega t + \varphi)$$

where x(t) is the monitored voltage or current waveform and ω is the nominal system base frequency. From this definition follows that a synchrophasor with 0 degrees phase angle corresponds to a sinusoidal function which reaches its maximum value at the UTC second rollover, while a sinusoidal function with a positive slope zero crossing occurring at the UTC second has a phase angle of -90 degrees.

Note that since the synchrophasors are always referenced to the nominal system base frequency, the phase angle of a synchrophasor with frequency lower than the nominal frequency will decrease with time. This phenomenon is illustrated graphically in Figure 2.21.

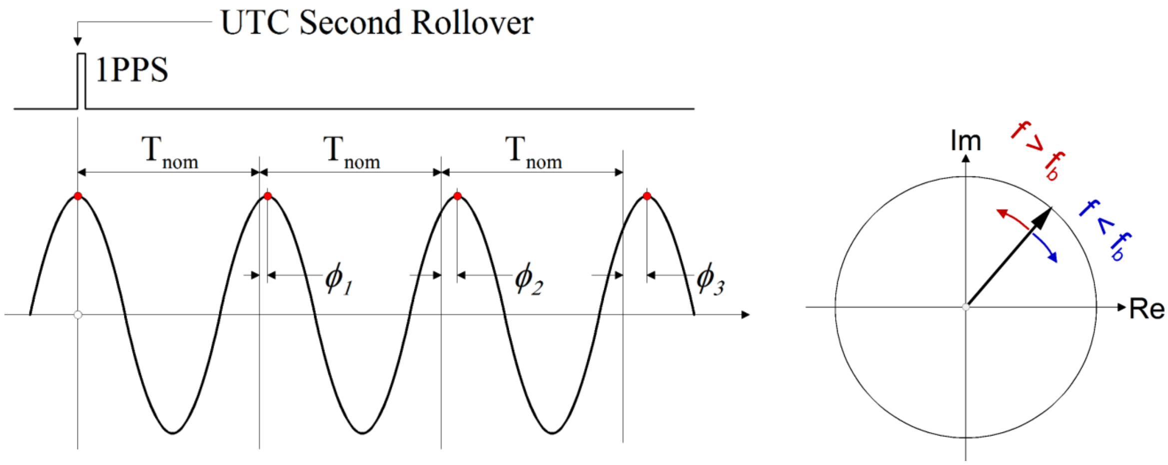


Figure 2.21: Synchrophasor Phase Angle Variation at Off-Nominal Frequencies

The first cycle in this Figure reaches its maximum value on the UTC second rollover time instant, while successive cycles reach their maximum values at time instants which are increasingly delayed from the start of the corresponding nominal period window. Plotting this synchrophasor on a complex plane it will appear to rotate clockwise at a rate equal to the difference between the nominal and actual frequencies. Conversely, if the actual frequency is higher than the nominal frequency the synchrophasor will rotate counterclockwise.

Note that the UTC second rollover time instant corresponds to the rising edge of a timing signal referred to as a “One Pulse per Second” signal (1PPS) (See Figure 2.21). This signal a common output of GPS based clocks. It is a signal that provides synchronization accuracy with typical resolution of less than a microsecond. This signal is typically transmitted along with an AM modulated IRIG signal in order to achieve high resolution synchronization. It is unnecessary if the IRIG signal is unmodulated, or “Modified Manchester” modulated.

The IEEE synchrophasor standard requires that a device that measures phasors, referred to as Phasor Measurement Unit (PMU) generates a data stream containing synchrophasors of the monitored voltage and current waveforms, as well as the system frequency and the rate of change of frequency (ROCOF) derived from the monitored waveforms. These data are to be written into “Data Frames” and transmitted typically via a wide area network to other devices. The standard defines a minimum set of data frame transmission rates that a PMU must be capable of generating. For 60 Hz systems PMUs must be able to generate synchrophasor data frames at the rates of 10, 12, 15, 20, 30, and 60 synchrophasors per second. At all rates, there should always be a phasor corresponding on the UTC hour rollover, and the remaining samples taken at uniformly spaced time intervals.

The IEEE synchrophasor standard specifies limits on the synchrophasor measurement error, as well as frequency and rate of change of frequency measurement errors. The synchrophasor error is expressed in terms of the "Total Vector Error". The total vector error (TVE) is illustrated in Figure 2.22 and is defined as the magnitude of the difference between the measured phasor and the theoretical exact phasor. A disadvantage of the TVE is that it does not differentiate between magnitude and phase errors. For certain applications for which the phase error is more critical than the magnitude error (for example real power flow computations), a separate magnitude and phase error specification is preferable.

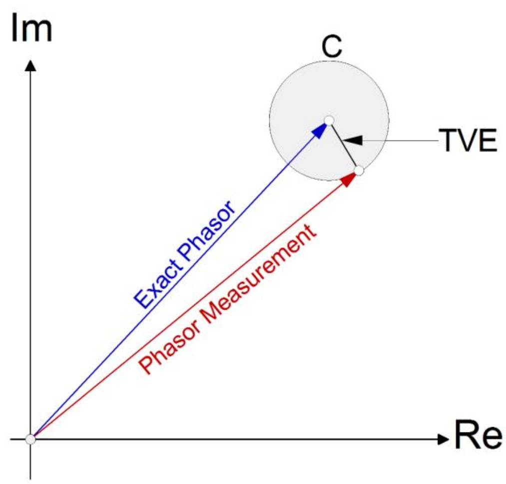


Figure 2.22: Total Vector Error

Note that the synchrophasor measurement error may vary considerably depending on the conditions of the measured signals. Depending on the algorithm used to extract phasors from sampled waveform data, the error may be considerable higher during transients (such as magnitude or frequency variation), and waveform harmonic distortion. For example, in several PMU implementations averaging filters are used in order to reduce errors during steady state conditions. However, such filters actually increase the computed phasor errors during transient conditions. Thus it is important to test phasor computation algorithms against a variety of signal conditions.

The IEEE synchrophasor standard includes a detailed specification of the format of the transmitted data. Specifically, two types of synchrophasor frames are defined: (a) configuration frames and (b) data frames. Configuration frames contain PMU setup information such as number of input channels, channel names, station name, numeric encoding (floating point or integer) etc. The information in configuration frames changes very infrequently, and thus configuration frames are transmitted only after a request from a device receiving the synchrophasor stream. Data frames contain the frame time stamp, clock quality indicators, the measured phasor values, the measured system frequency, and the ROCOF. The standard also allows the transmission of a number of analog and discrete values. These values are typically used to transmit station status data such as transformer tap settings and breaker status. All values in data frames are encoded in binary form, resulting in a very compact message format.

Time Tagging Versus True Synchronous Sampling

PMUs use a UTC synchronized reference clock to provide a sequence of phasor measurements with phase angle referenced to UTC time using two approaches: (a) Time tagging and (b) synchronized sampling. In the **time tagging** approach the sampling clock of the A/D converters is free running (i.e. not synchronized to UTC in any way) and each sample is assigned a time tag by reading the reference clock at the time each sample is taken. From the time tagged data samples at the desired time instants are estimated by interpolation.

In the **synchronized sampling** approach, the A/D converter clock is synchronized to UTC so that a sample is always taken within a microsecond or less from the at the UTC second rollover. This approach ensures that phasors can be directly calculated at the desired time intervals without requiring interpolation. The obvious advantage of the synchronized sampling approach is that it avoids interpolation errors, which may be significant during transients. Of course, the hardware required for the implementation of synchronized sampling is more complex as it requires a dedicated A/D converter for every channel, driven by a common sampling clock "disciplined" to the UTC second rollover. (Phase-locked-loop technology is a common approach in achieving the required clock synchronization). Figure 2.23 illustrates the hardware organization required for synchronized sampling.

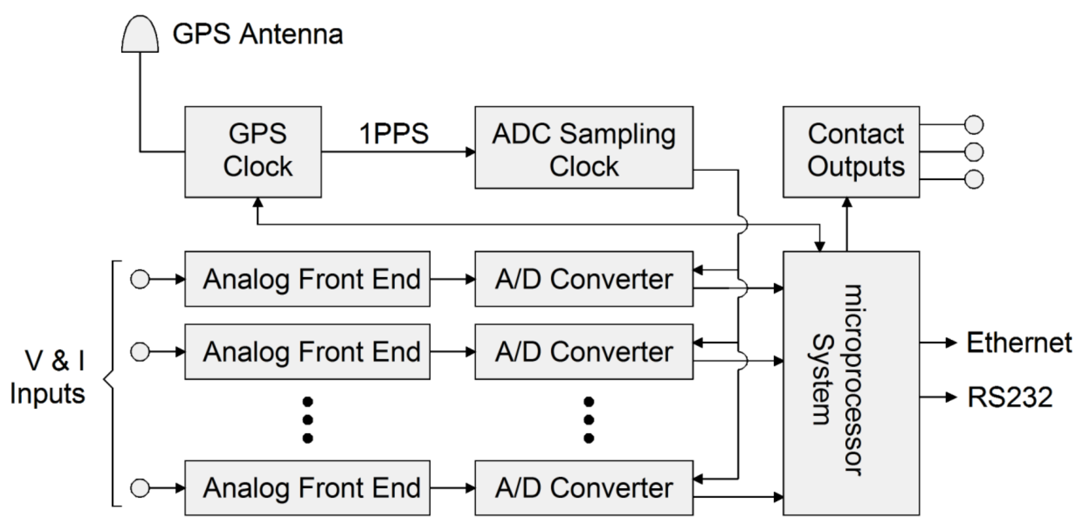


Figure 2.23: Example Block Diagram of Data Acquisition System with Synchronized Sampling Capability

2.7 Interoperability

Interoperability is a property referring to the ability of diverse systems to work together. In general, it can be defined as the "property of a product or system, whose interfaces are completely understood, to work with other products or systems, present or future, without any restricted access or implementation". Note it applies to any device or system that is typically an embedded system that may interface to a cyber system as well as to a physical system exchanging information with the rest of the world. In the context of power system protection, interoperability applies to any relay, merging unit, meter, fault recorder, monitoring device, routers, automation systems and other.

TO BE COMPLETED

2.8 System Grounding

System grounding plays an important role in the performance of a power system during asymmetrical ground faults. As a result relay operation is affected by grounding. During ground faults, the potential of the system neutrals and grounds may be elevated to a substantial voltage with respect to the "Remote Earth" voltage. This potential is commonly referred to as the Ground Potential Rise, or GPR. Furthermore, the voltages on un-faulted phases with respect to the neutral voltage may be elevated to values higher than the corresponding nominal voltages. Figure 2.24 shows the single line diagram of a simple power system consisting of a 3 phase source (at bus 10), a transmission line, and a load (bus 15). A single line to ground fault occurring at bus 15 is simulated using a high fidelity model which includes explicit representation of transmission line grounding, its phase and shield conductors, as well as the grounding models at both ends of the line. Figure 2.25 illustrates the resulting voltages and currents occurring during the ground fault. Note that the un-faulted line voltage (phase C, bottom curve) assumes a higher voltage during the fault than before the fault initiation or after the fault is cleared. Both of these phenomena are obviously undesirable since they can impact human safety and cause equipment damage. Both of these phenomena can be mitigated by a high quality (low impedance) grounding system.

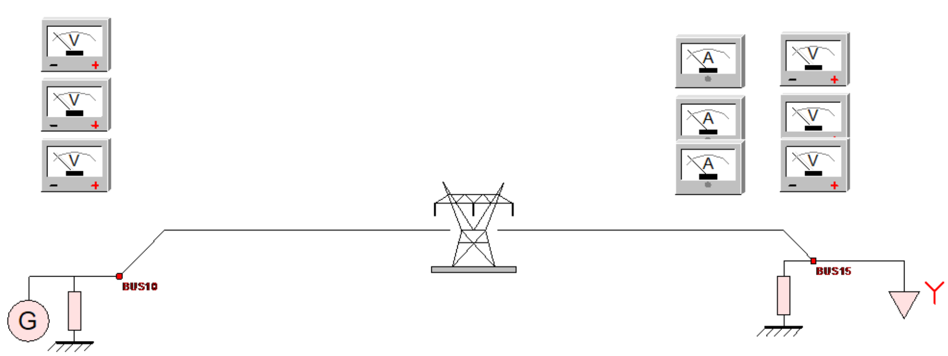


Figure 2.24: Single Line Diagram of Test System for Ground Potential Rise Evaluation

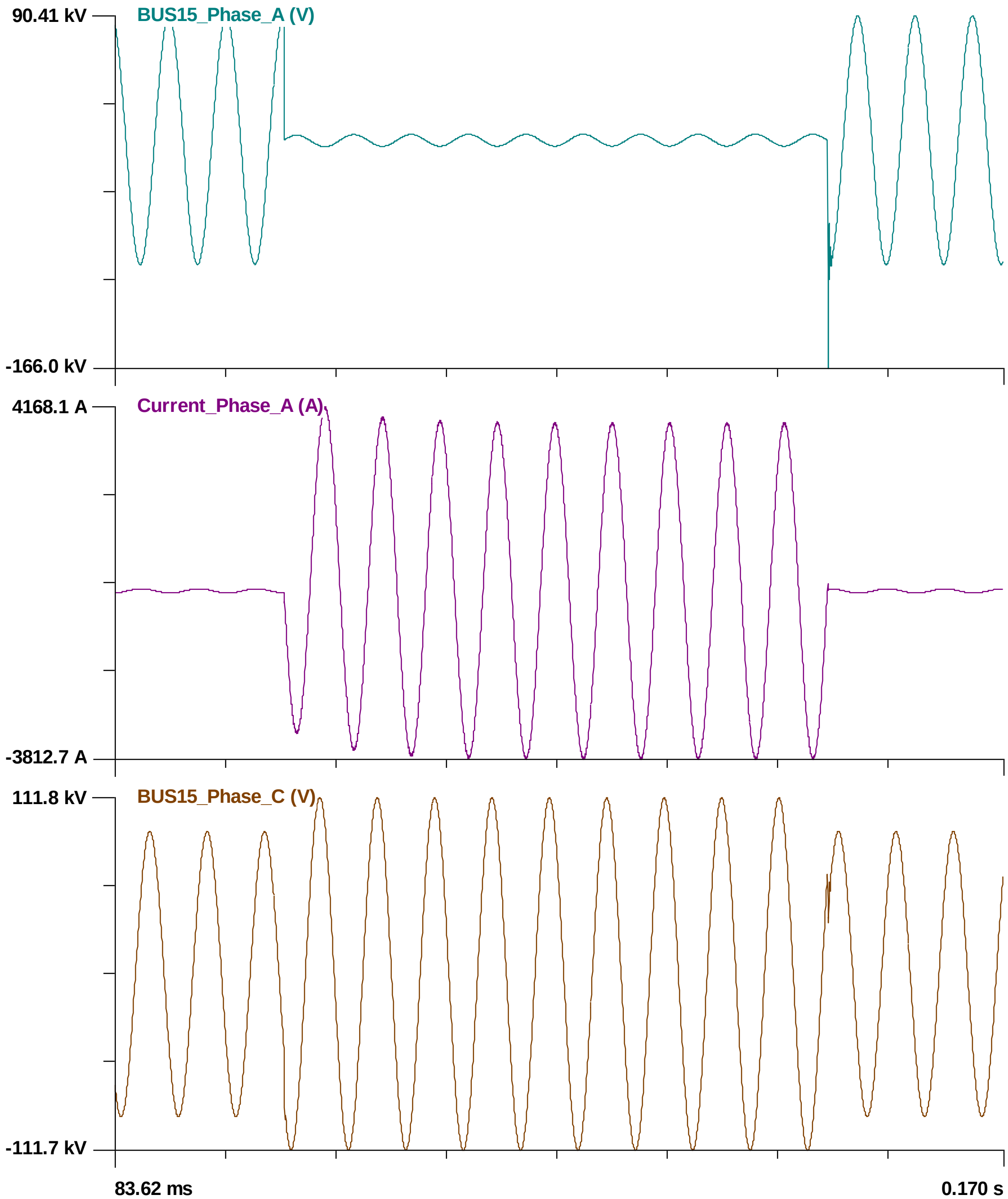


Figure 2.25: Typical Voltages and Currents during a Single Line to Ground Fault

Top Trace:	**Faulted Phase Voltage (Phase A)**
Middle Trace:	**Fault Current (Phase A)**
Bottom Trace:	**Un-faulted Phase Voltage (Phase C)**

Furthermore, excessive un-faulted phase over-voltages and ground potential rise can also result in protective relay miss-operation. Thus it is desirable to mitigate these phenomena by ensuring that the power system is "effectively grounded". In fact the term "Effectively Grounded" system is defined as a system which during an unsymmetrical ground fault the un-faulted phase voltages do not exceed 80% of their nominal line to line voltage, or 138% of the nominal phase to ground voltage. A related metric of the grounding system effectiveness is the "Coefficient of Grounding" (COG) which is defined as the ratio of the maximum un-faulted phase voltages over the nominal phase voltages. Figure 2.26 provides a quick way to compute the coefficient of grounding given the sequence components of a 3-phase power system impedance at the fault location.

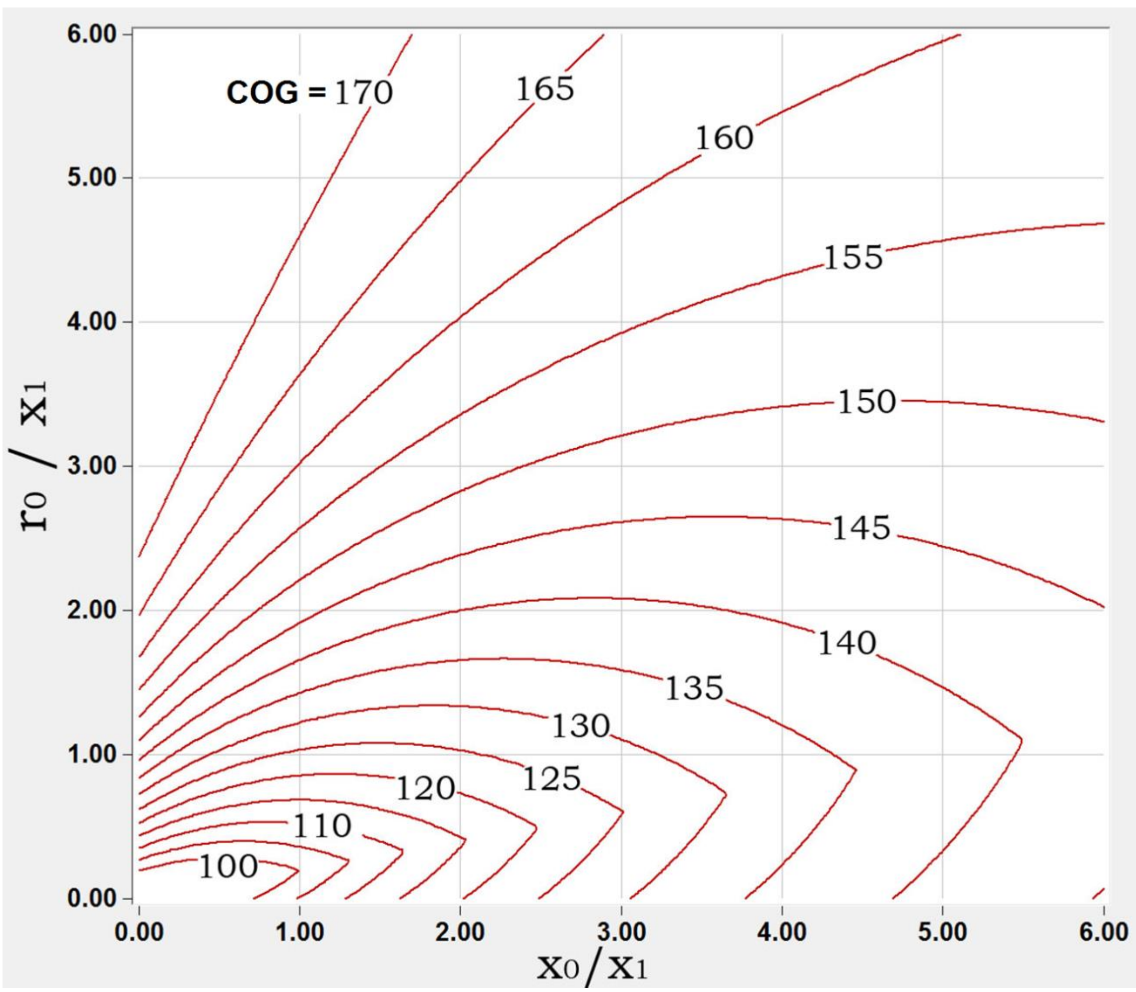


Figure 2.26: Coefficient of Grounding versus Impedance Sequence Components at the Fault Location. (Curves Generated Assuming $r_1 = r_2 = 0.2 \cdot x_1$)

From Figure 2.26 it can be concluded that if the following conditions are met then a system is effectively grounded:

$$x_0 / x_1 < 3 \quad \text{and} \quad r_0 / x_1 < 1$$

2.9 Network Solutions

In this section we discuss techniques for the solution of network problems. The network solutions pertinent for relay applications may involve faulted power systems (symmetric faults or asymmetric faults) or systems under transient conditions. Most network solutions today are based on nodal methods. We present the basis of this method in the next section.

2.9.1 Nodal Analysis

Nodal analysis is a powerful method for circuit analysis. It applies to any circuit consisting of passive elements and current sources.

Example E2.6: Figure E2.6 illustrates a three-phase power system which consists of a wye connected source, a delta connected motor, a delta connected load, and two transmission lines. For simplicity the internal impedance of both source and motor is assumed to be zero. The system is symmetric and operates under balanced conditions. The following have been measured:

$$\tilde{E}_a = 7.2\, e^{j0^o} \quad \text{kV}$$
$$\tilde{E}_{AB} = 12.0\, e^{j5^o} \quad \text{kV}$$

a) Compute the electric current $\tilde{I}_a$ and $\tilde{I}_{AB}$ shown in the Figure. Provide both magnitude and phase.

b) What is the total real power absorbed by the motor?

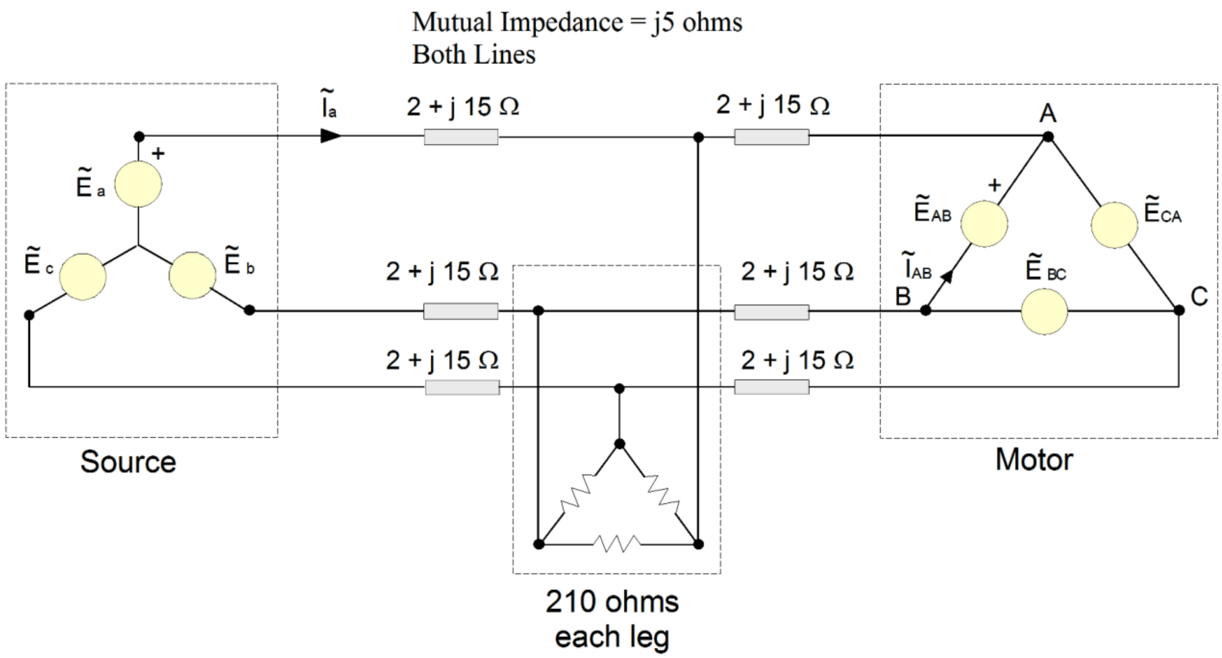
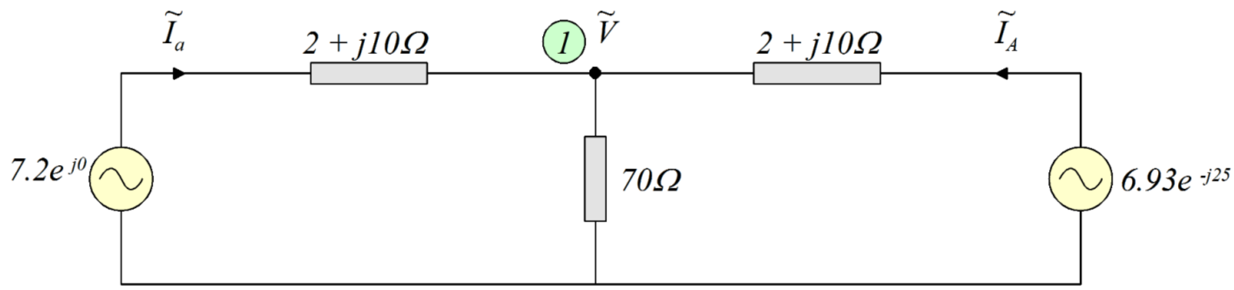


Figure E2.6

Solution: The per phase equivalent of the above three phase circuit is shown below.



Applying Kirchoff's current law at Node 1 yields:

$$\frac{7.2 - \tilde{V}}{2 + j10} - \frac{\tilde{V}}{70} + \frac{6.93e^{-j25^0} - \tilde{V}}{2 + j10} = 0$$

solving the above equation for the voltage phasor $\tilde{V}$ yields:

a) $\tilde{V} = 6.7826e^{-j16.29} kV$

$$\tilde{I}_a = \frac{7.2 - \tilde{V}}{2 + j10} = 0.1985e^{-j8.64^0} kA$$

$$\tilde{I}_A = \frac{6.93e^{-j25^0} - \tilde{V}}{2 + j10} = 0.1031e^{-j178.71^0} kA$$

$$\tilde{I}_{AB} = \frac{\tilde{I}_A}{\sqrt{3}} e^{j30^0} = 0.0595e^{j208.71^0} kA$$

b) $S = 3 \times 6.93e^{-j25^0} (-0.1031e^{-j178.71^0}) \; MVA$

$$S = 2.1434e^{-j23.71^0} \; MVA$$

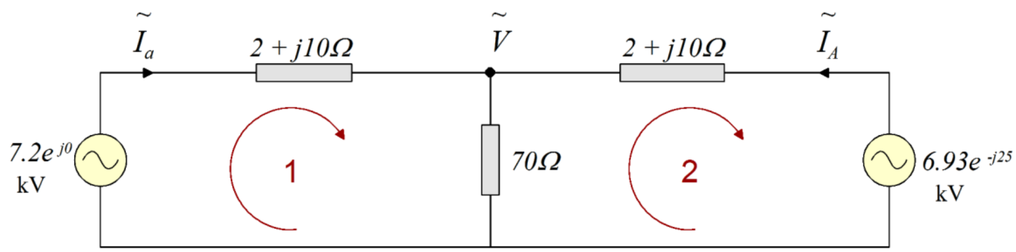
$$S = 1.962 MW - j0.862 MVA$$

2.9.2 Loop Analysis

Loop analysis is also a powerful method for circuit analysis. It applies to any circuit consisting of passive elements and voltage sources.

Example E2.7: Consider the example problem of Example E2.6. Compute the same quantities as in Example E2.6 by using loop analysis

Solution: The equivalent circuit is:



Loop Equation 1

$$7.2e^{j0^0} = (2 + j10)\tilde{I}_a + 70(\tilde{I}_a + \tilde{I}_A)$$

Loop Equation 2

$$-6.93e^{-j25^0} = -70(\tilde{I}_a + \tilde{I}_A) - (2 + j10)\tilde{I}_A$$

Solution

$$\tilde{I}_a = 0.1985e^{-j8.64^0} kA$$
$$\tilde{I}_A = 0.1031e^{j178.71^0} kA$$
$$\tilde{V} = 7.2e^{j0^0} - (2 + j10)\tilde{I}_A = 6.7826e^{j16.29^0} kV$$

2.10 Summary and Discussion

In this chapter we have presented basic concepts for typical analysis problems encountered in protective relaying applications.

2.11 Problems

Problem P2.1: At a certain location of a three phase system, an engineer measures the following phase currents:

$$\tilde{I}_a = 12.78 e^{j9.8^o} \ kA$$

$$\tilde{I}_b = 1.98 e^{-j79.8^o} \ kA$$

$$\tilde{I}_c = 2.47 e^{j69.8^o} \ kA$$

Compute the positive sequence component: $\tilde{I}_1$.

Solution: The symmetrical components are directly computed with:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 12.78\ e^{j9.8^0} \\ 1.98\ e^{-j79.8^0} \\ 2.47\ e^{j69.8^0} \end{bmatrix} = \begin{bmatrix} 5.2546\ e^{j5.66^0} \\ 2.8813\ e^{j16.30^0} \\ 4.6766\ e^{j10.45^0} \end{bmatrix}$$

Problem P2.2: A protection engineer would like to design an analog system that will provide the positive sequence voltage at a specific location of a three-phase system. For this purpose, he uses three transformers as it is illustrated in Figure P2.2. Help the protection engineer to select the parameters of the transformers (transformation ratio) and the values of the resistor and capacitor so that the output will be the positive sequence voltage.

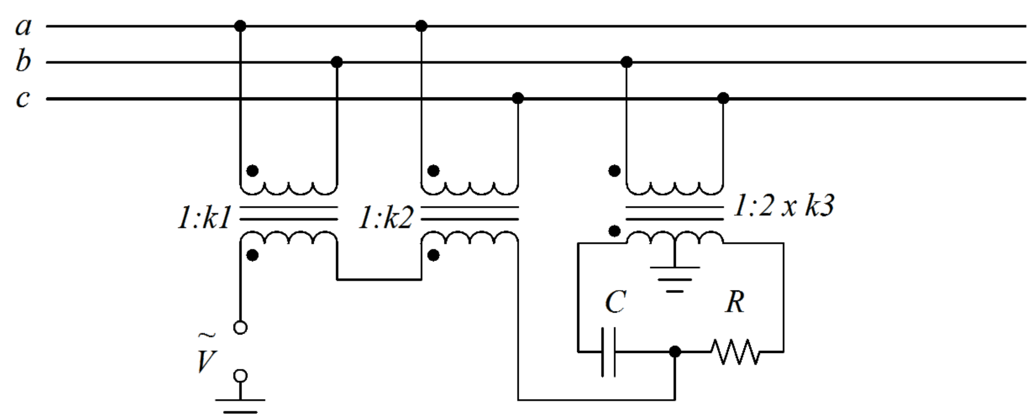


Figure P2.2

Solution: The output voltage is:

$$\tilde{V} = k_1\left(\tilde{V}_a - \tilde{V}_b\right) + k_2\left(\tilde{V}_a - \tilde{V}_c\right) + k_3\left(\tilde{V}_b - \tilde{V}_c\right)\frac{R + jX_C}{R - jX_C}$$

$$\text{Select: } R = X_C \quad \Rightarrow \quad \frac{R + jX_C}{R - jX_C} = \frac{1+j}{1-j} = j$$

$$\tilde{V} = (k_1 + k_2)\tilde{V}_a + (-k_1 + jk_3)\tilde{V}_b + (-k_2 - jk_3)\tilde{V}_c$$

For the above voltage to be positive sequence:

$$(k_1 + k_2) = 1/3$$
$$(-k_1 + jk_3) = 1/3(0.5 + j0.8660254)$$
$$(-k_2 - jk_3) = 1/3(0.5 - j0.8660254)$$

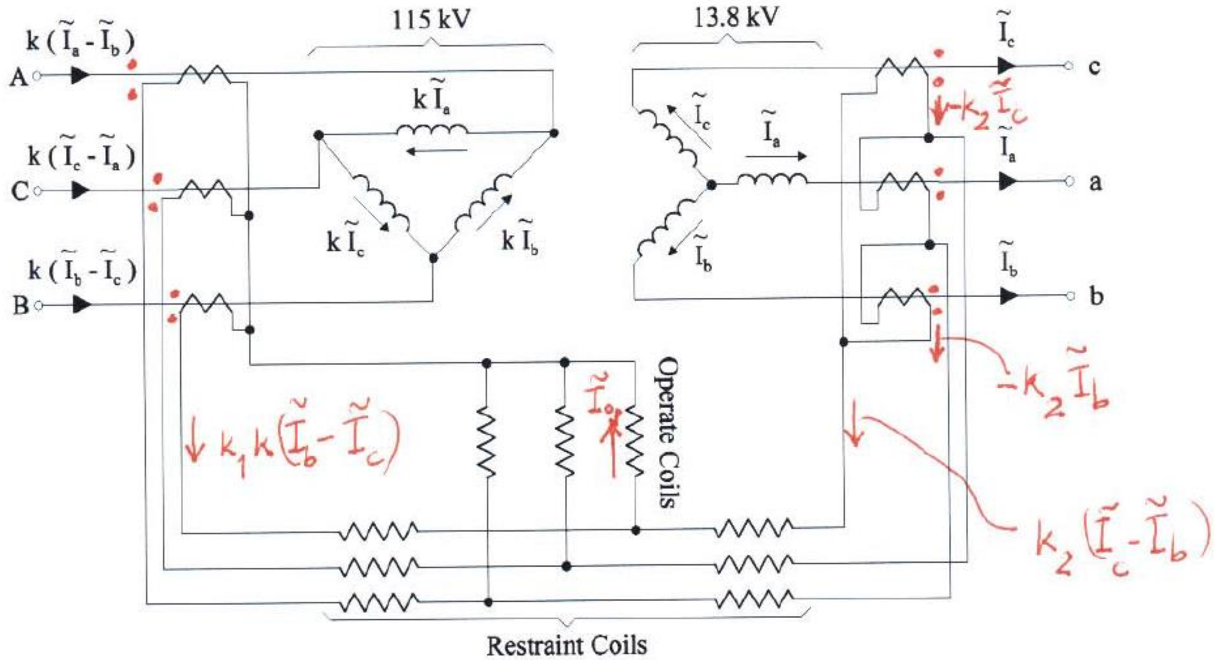
The above equations are satisfied for:

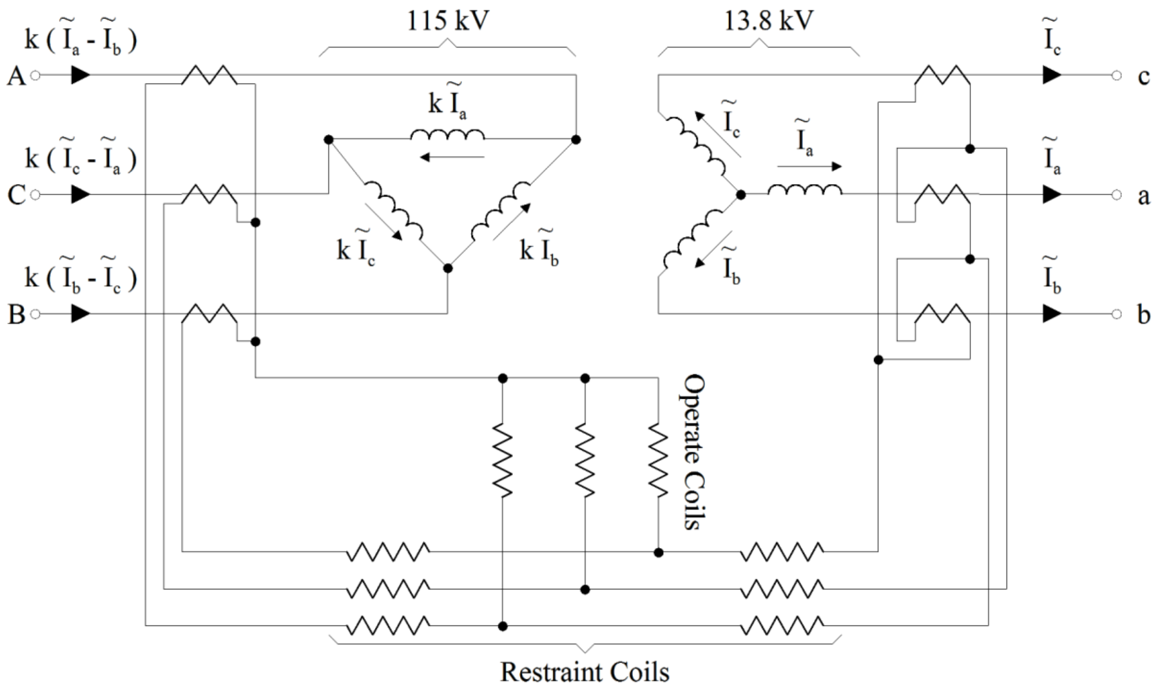
$$k_1 = k_2 = 0.16666$$
$$k_3 = 0.288675$$

Problem P2.3: A protection engineer would like to design an analog system that will provide the negative sequence voltage at a specific location of a three-phase system. For this purpose, he uses three transformers as it is illustrated in the figure. Help the protection engineer to select the parameters of the transformers (transformation ratio) and the values of the resistor and capacitor so that the output will be the negative sequence voltage.

Problem P2.4: A three phase transformer is rated 36 MVA, 115kV/13.8kV, delta-wye connected. It is desired to generate a circuit that will have an output as close as possible to the net current flowing into the transformer. For this purpose, CTs of the following transformation ratios are available: (a) 1200:5, (b) 1500:5, (c) 1600:5, (d) 150:5, and (e) 200:5. Select the proper CTs for this application and compute the operating current when the transformer load at the secondary is nominal, at nominal voltage and power factor 1.0.

Solution: Consult connectivity diagram of three phase transformer.





The operating coil current is:

$$\tilde{I}_{op} = k_1 k\left(\tilde{I}_b - \tilde{I}_a\right) + k_2\left(\tilde{I}_a - \tilde{I}_b\right), \quad k = \frac{13.8}{115\sqrt{3}} = 0.0693$$

$$\tilde{I}_{op} = \left(0.0693 k_1 - k_2\right)\left(\tilde{I}_b - \tilde{I}_a\right)$$

Need to select k_1 and k_2 so that $\tilde{I}_{op}$ is minimum. Using only the given transformation ratios, select:

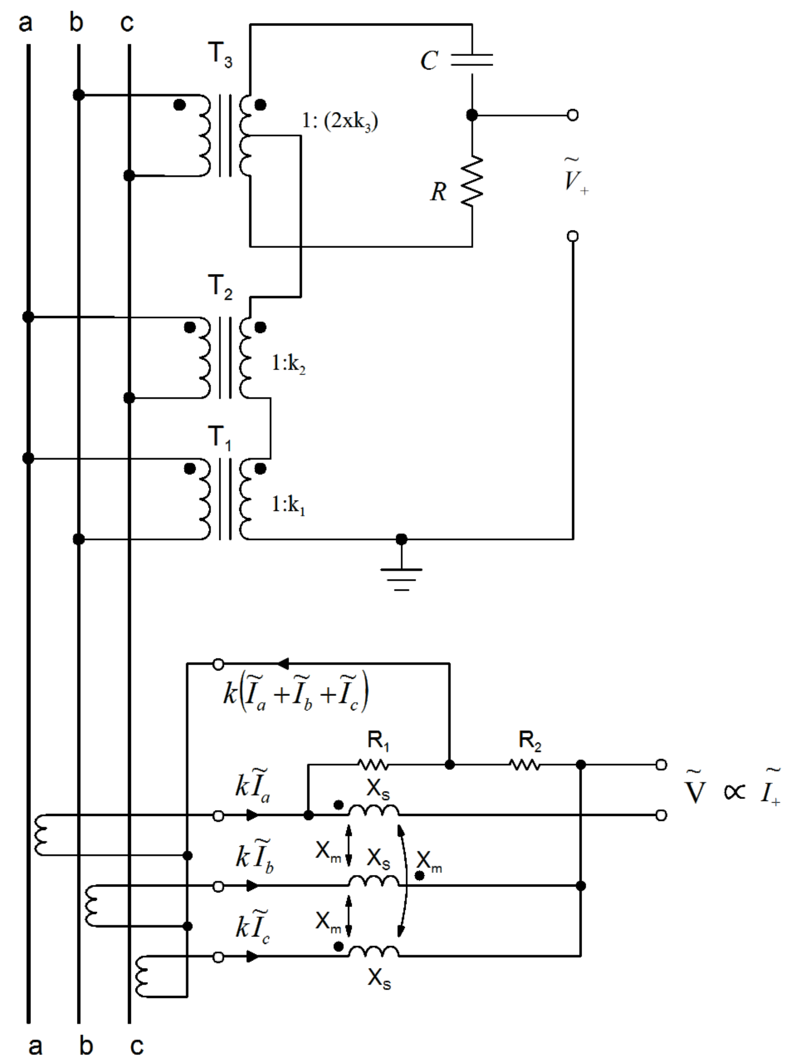
$$k_1 = \frac{5}{150} \quad and \quad k_2 = \frac{5}{1600}, \text{ then}$$

$$\tilde{I}_{op} = -0.0008156\left(\tilde{I}_b - \tilde{I}_a\right)$$

Problem P2.5: A protection engineer would like to design a directional relay that works with the positive sequence voltages and currents at the point of application. Help the protection engineer to design an analog system that will generate the positive sequence voltage and current into the voltage and current coils of the relay.

(a) Draw the entire analog circuit that will accomplish this task.
(b) Describe advantages/disadvantages of using positive sequence current and voltage for direction detection in a relay.

Solution: The circuit is shown in the figure below.



The output of the first circuit is:

$$\tilde{V} = k_1(\tilde{V}_a - \tilde{V}_b) + k_2\left(\tilde{V}_a - \tilde{V}_c\right) + k_3\left(\tilde{V}_c - \tilde{V}_b\right)\frac{R + jX_C}{R - jX_C}$$

Select: $R = X_C \quad \Rightarrow \quad \dfrac{R + jX_C}{R - jX_C} = \dfrac{1 + j}{1 - j} = j$

$$\tilde{V} = (k_1 + k_2)\tilde{V}_a + (-k_1 + jk_3)\tilde{V}_b + (-k_2 - jk_3)\tilde{V}_c$$

For the above voltage to be positive sequence:

$$(k_1 + k_2) = 1/3$$
$$(-k_1 + jk_3) = 1/3(0.5 + j0.8660254)$$

$$(-k_2 - jk_3) = 1/3(0.5 - j0.8660254)$$

The above equations are satisfied for:

$$k_1 = k_2 = 0.16666$$
$$k_3 = 0.288675$$

The output of the second circuit is:

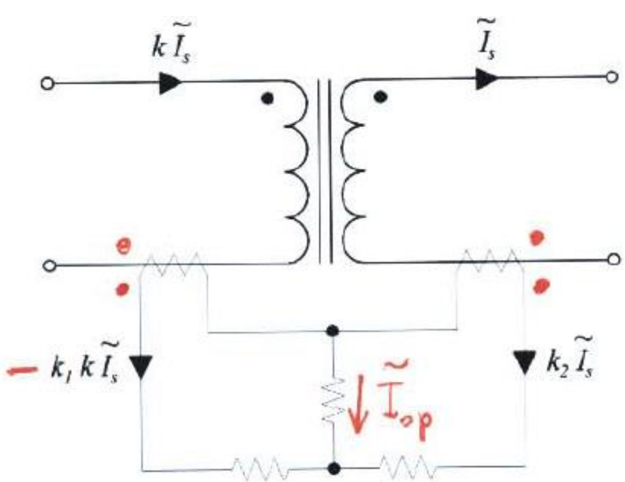
$$\tilde{V} = kR_2\left(\tilde{I}_b + \tilde{I}_c\right) - kR_1\tilde{I}_a - kx_m\tilde{I}_b + kx_m\tilde{I}_c = -kR_1\left[\tilde{I}_a + \left(-\frac{R_2}{R_1} + \frac{x_m}{R_1}\right)\tilde{I}_b + \left(-\frac{R_2}{R_1} - \frac{x_m}{R_1}\right)\tilde{I}_c\right]$$

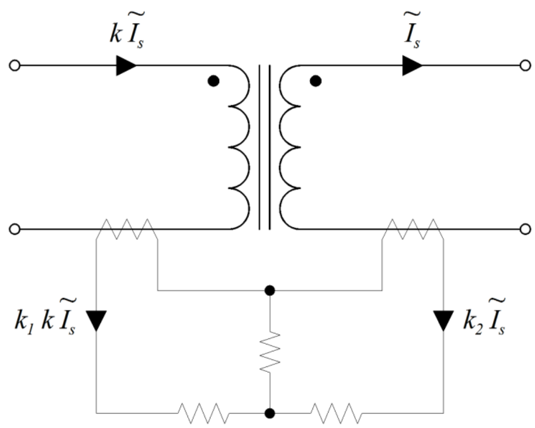
Select:

$$\frac{R_2}{R_1} = 0.5, \qquad \frac{x_m}{R_1} = 0.866$$

Problem P2.6: A single phase transformer is rated 1.248 MVA, 13.8kV/480V. It is desired to generate a circuit that will have an output as close as possible to the net current flowing into the transformer. For this purpose, CTs of the following transformation ratios are available: (a) 3000:5, (b) 2500:5, (c) 1500:5, (d) 600:5, (e) 200:5, and (e) 100:5. Select the proper CTs for this application and compute the operating current when the transformer loaded at the secondary is nominal, at nominal voltage and power factor 1.0.

Solution: Consult the connectivity diagram.





The operating coil current is:

$$\tilde{I}_{op} = k_1 k\tilde{I}_S - k_2 \tilde{I}_S, \quad k = \frac{480}{13{,}800} = 0.0348$$

$$\tilde{I}_{op} = (0.0348k_1 - k_2)\tilde{I}_S$$

Need to select k1 and k2 so that Iop is minimum. Using only the given transformation ratios, select:

$k_1 = \dfrac{5}{100} \quad and \quad k_2 = \dfrac{5}{3000}$, then

$$\tilde{I}_{op} = 0.0000725\tilde{I}_S$$

At nominal loading:

$$\tilde{I}_{op} = 0.0000725\tilde{I}_S = 0.0000725\frac{1{,}250{,}000}{480} = 0.1885\ Amperes$$

Problem P2.7: At a certain location of a three phase system, an engineer measures the following phase currents and phase to neutral voltages:

$$\tilde{I}_a = 400Ae^{j10^\circ} \qquad \tilde{V}_a = 15e^{j4^\circ}\,kV$$
$$\tilde{I}_b = 350Ae^{-j110^\circ} \qquad \tilde{V}_b = 14.5e^{-j120^\circ}\,kV$$
$$\tilde{I}_c = 360Ae^{-j240^\circ} \qquad \tilde{V}_c = 14.8e^{-j235^\circ}\,kV$$

Compute the symmetrical components: $\tilde{I}_1, \tilde{I}_2, \tilde{I}_0, \quad and \quad \tilde{V}_1, \tilde{V}_2, \tilde{V}_0$.

Solution: Use the symmetrical transformation directly on the provided data.

$$\tilde{V}_{120} = T\tilde{V}_{abc}, \quad \tilde{I}_{120} = T\tilde{I}_{abc}$$

$$T = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}, \qquad \tilde{V}_{120} = \begin{bmatrix} 14.75e^{j3.02^0} \\ 0.49e^{j7.43^0} \\ 0.33e^{j141.70^0} \end{bmatrix} kV$$

$$\tilde{I}_{120} = \begin{bmatrix} 368.77e^{j6.76^0} \\ 9.36e^{j113.18^0} \\ 35.92e^{j29.05^0} \end{bmatrix} Amperes$$

Problem P2.8: At a certain location of a three phase system, an engineer measures the following phase currents and phase to neutral voltages:

$$\tilde{I}_a = 200Ae^{j10^\circ} \qquad \tilde{V}_{an} = 15e^{j4^\circ} kV$$
$$\tilde{I}_b = 160Ae^{-j110^\circ} \qquad \tilde{V}_{bn} = 13.5e^{-j120^\circ} kV$$
$$\tilde{I}_c = 160Ae^{-j240^\circ} \qquad \tilde{V}_{cn} = 14.8e^{-j235^\circ} kV$$

Compute the symmetrical components: $\tilde{I}_1, \tilde{I}_2, \tilde{I}_0, \quad and \quad \tilde{V}_1, \tilde{V}_2, \tilde{V}_0$.

Solution: Use the symmetrical transformation directly on the provided data.

$$\tilde{V}_{120} = T\tilde{V}_{abc}, \quad \tilde{I}_{120} = T\tilde{I}_{abc}$$

$$T = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}, \quad \tilde{V}_{120} = \begin{bmatrix} 14.42e^{j3.09^0} \\ 0.69e^{-j18.98^0} \\ 0.50e^{j100.55^0} \end{bmatrix} kV$$

$$\tilde{I}_{120} = \begin{bmatrix} 172.77e^{j6.93^0} \\ 7.817e^{j53.0^0} \\ 22.11e^{j20.24^0} \end{bmatrix} Amperes$$

Problem P2.9: Consider the simplified electric power system of Figure P2.9 consisting of a balanced generator, a symmetric transmission line and a symmetric electric load. Each phase of the symmetric line has a self-impedance of j9 ohms. The mutual impedance between any two phases is j4 ohms. Other system parameters are indicated in the figure.

(a) Compute and graph the positive sequence model of this system.

(b) Compute the real power absorbed by the electric load using the positive sequence model from part (a)..

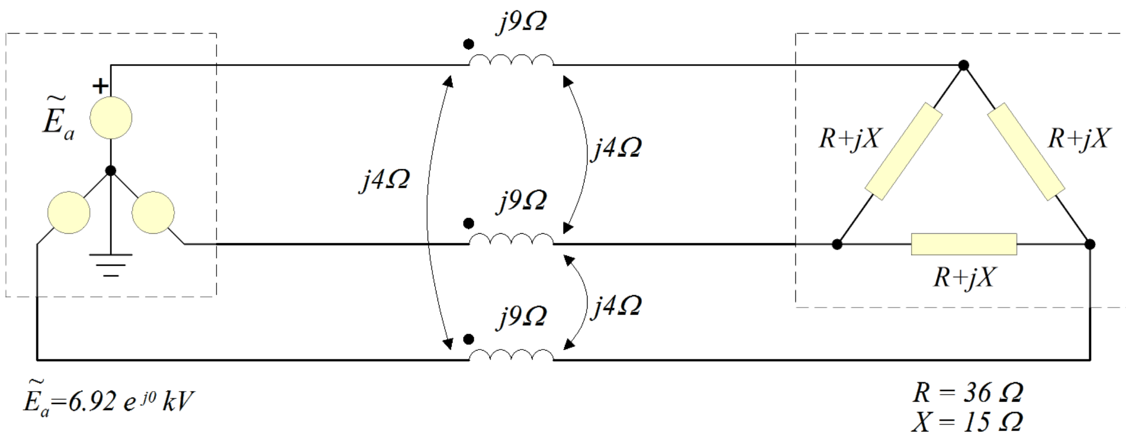


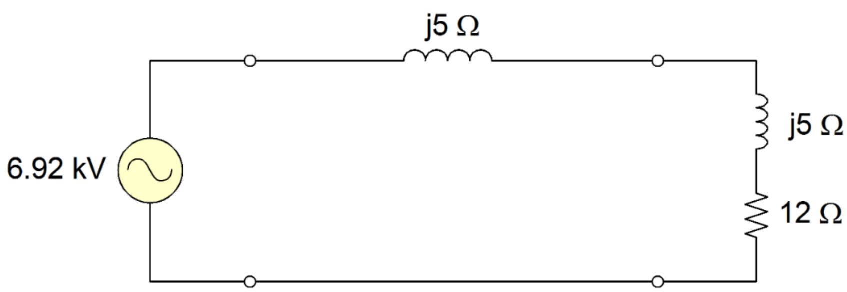
Figure P2.9

Solution: (a) The positive sequence model of the load is computed by simply applying the symmetrical transformation on the load equations. Let y be the inverse of the load impedance. Then:

$$\tilde{I}_{abc} = y\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\tilde{V}_{abc} \rightarrow \quad T^{-1}\tilde{I}_{abc} = yT^{-1}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}TT^{-1}\tilde{V}_{abc} \rightarrow$$

$$\tilde{I}_{120} = y\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\tilde{V}_{120}$$

Above equation states that the positive sequence circuit of the load consists of an admittance equal to 3y. Thus the positive sequence circuit is (by simply replacing each component with its positive sequence model):



(b) The positive sequence current is:

$$\tilde{I}_1 = \frac{6.92}{j5 + 12 + j5} = 0.443e^{-j39.805^0} \quad kAmperes$$

The real power is:

$$P = 3\,\mathrm{Re}\left(\tilde{E}_a \tilde{I}_1^*\right) = 3\,\mathrm{Re}\left\{(6.92\,kV)\left(0.443e^{j39.805^0}\ kA\right)\right\} = 7.065\,MW$$

Problem P2.10: A relay has recorded the three phase voltages and currents at a certain location of a power system. The captured data have been stored in COMTRADE format in the files PSR_Chapter01_Ex01.cfg and PSR_Chapter01_Ex01.dat.

1. Write a computer program to read the data of Phase A to Neutral voltage. For this purpose, consult the IEEE Std C37-111-1999 that describes the COMTRADE format.
2. Compute and graph the frequency of the Phase A to Neutral voltage over the entire data record. Use two cycle time frame.
3. Compute and graph the phase angle of the Phase A to Neutral voltage over the entire data record. Use two cycle time frame.
4. Compute and graph the magnitude of the Phase A to Neutral voltage over the entire data record. Use two cycle time frame.

DRAFT and INCOMPLETE

Table of Contents

from

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Know your physics and the rest will follow (R. K. Kalman)

Chapter 3
Modeling for Power System Relaying Analysis

3.1 Introduction

The power system comprises generators, step-up/step-down transformers, autotransformers, transmission lines (overhead or underground operating at various kV levels), reactors, capacitors, distribution lines, end-use equipment (customers), motors, etc. As examples, Figure 3.1 illustrates an artistic visualization of power systems and Figure 3.2 illustrates typical distribution systems (utility) and end use equipment (customers). The illustrated power system and the medium voltage distribution systems are typical designs of US utilities to supply electric power to commercial, residential and industrial customers.

Any power system analysis method must be able to model and analyze systems similar to the ones illustrated in Figures 3.1 and 3.2. The phenomena to be analyzed on these systems are numerous, i.e. power frequency, harmonics, dynamic transients, switching transients, lightning transients, in-rush current transients, etc. Each of these phenomena may include different frequency spectra. The models to be used should reproduce the response of the system to these phenomena with high fidelity. In this chapter we examine modeling techniques for various power system components that provide this capability.

The various electric power system components that must be modeled are:

- Transmission Lines
- Transformers
- Generators
- Induction Machines
- Capacitors
- Reactors
- Converters
- Adjustable speed drives
- Power supplies
- etc.

Some of the power system components are linear, i.e. they do not distort the applied voltage and current while others are distorting, i.e. they introduce distortion of the waveform, such as converters, adjustable speed drives, power supplies, transformers, etc.

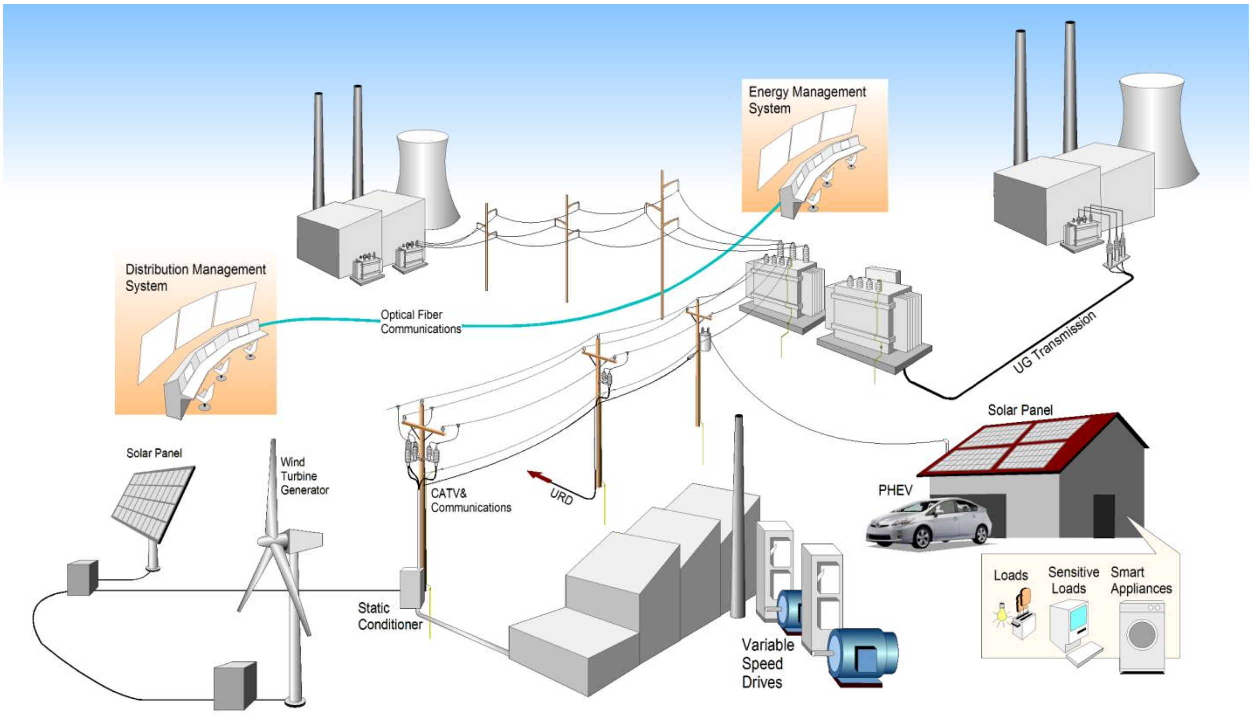


Figure 3.1: A Power System Comprising Generation, Transmission and Distribution - Overhead and Underground

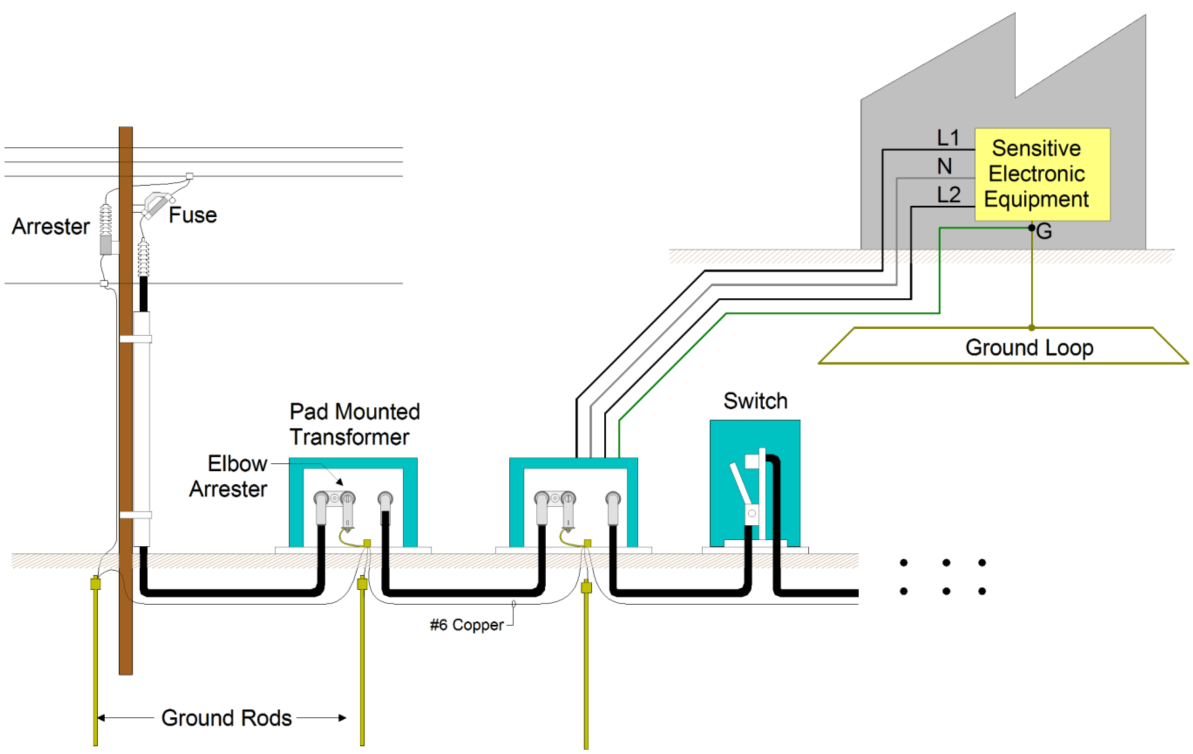


Figure 3.2: Typical Overhead and Underground Distribution Systems

We alluded to the fact that the component models should reproduce the response of the components to specific inputs and phenomena with high fidelity. It is important to recognize that the selection of the appropriate model depends on the phenomena to be studied. For example, to study power frequency phenomena in a transformer, a simple model will suffice. If, however, high frequency phenomena are to be studied, then a totally different transformer model will be necessary. Similarly, if transformer in-rush currents are to be computed, a totally different model must be utilized, specifically one that captures the nonlinearity of the transformer core and properly represent the dependence of the magnetizing current on the magnetic flux of the transformer core. The model selection also depends on the time period of concern. For example, if the steady state of the system is to be analyzed, appropriate steady state models should be used. If, however, inrush current phenomena in transformers are to be studied, another set of models must be employed. Therefore one should realize that the phenomena under study and the time period of concern will determine the selection of the proper model. The most usual phenomena under study and time periods of concern are listed below.

Phenomena Under Study

- Power Frequency
- Line Switching
- Capacitor Bank Switching
- Transient Recovery Voltage
- Lightning

Period of Concern

- Steady State
- Short Term (seconds)
- Milliseconds
- Microseconds

In the rest of this chapter, models of the most usual power system components will be introduced with comments about their applicability to specific phenomena under study. It should be understood, that most of the phenomena that affect power quality are typically of relatively low frequency.

3.2 Transmission and Distribution Line Modeling

Transmission and distribution lines can be of many varieties: overhead three phase, single phase, underground three phase cables, underground single phase cables, etc. The distinction between transmission and distribution depends on the intended purpose of the power circuit. Specifically if the intended use is to supply customers (residential, commercial and industrial) then we refer to this line as distribution. In general distribution lines operate at medium voltage (a few kVs to about 35 kV) and most times they operate radially. Power circuits operating at higher voltages are typically classified as transmission circuits. Mathematically, the methods for modeling transmission and distribution circuits are identical. We present some typical transmission and distribution lines and then we address the modeling of these components.

The components of overhead transmission lines and distribution lines are illustrated in Figure 3.3. A three-phase overhead line consists of three phase conductors HA, HB, and HC, which are suspended with insulators from towers. Most designs include an overhead ground wire (OHGW (**O**ver**H**ead **G**round **W**ire) or shield wire) to provide protection against lightning. Many OHGW also include a tube with optical fibers for communications. The OHGW is typically connected to the neutral of the system and may be grounded at each tower. The tower grounding system may consist of counterpoise (illustrated in Figure 3.3), rings, ground rods, etc. A typical overhead transmission line terminates to two substations. The OHGW is typically connected to the grounding system of the substations. Figure 3.3 illustrates the termination of the OHGW to the substation ground mat. A three-phase overhead distribution line is also illustrated in Figure 3.3. It consists of three phase conductors, indicated as LA, LB, and LC, and a multiply grounded neutral conductor. The neutral conductor is typically bonded to the substation ground mat and to the grounds of the distribution poles.

Overhead power lines are suspended on towers or poles. The design of transmission towers depends on the operating voltage of the line and other mechanical strength considerations. Three example tower/pole designs are illustrated in Figures 3.4, 3.5, and 3.6 for 230-kV, 115-kV, and 12-kV lines, respectively. Note that the 12-kV line, which is typically used in distribution circuits, does not have an OHGW. Instead, it has a fourth conductor, the neutral, which is suspended below the phase conductors. While electrically the OHGW and neutral are similar, the naming difference reflects the fact that the OHGW is not intended to carry electric current under normal operating conditions while the neutral is intended to carry the return current under normal operating conditions. The size of the neutral conductor is comparable to that of the phase conductors and it is intended to carry potentially the full load current. The reason for this practice is the fact that distribution circuits may supply single phase loads connected between a phase and the neutral conductor. This practice generates unbalanced conditions and the neutral conductor may carry a substantial electrical current.

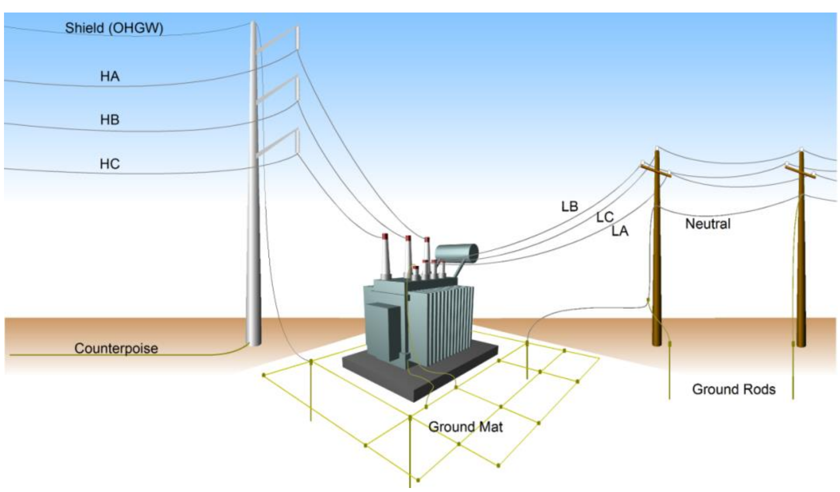


Figure 3.3: Overhead Power Lines: a Transmission and a Distribution Line Connected via a Power Transformer

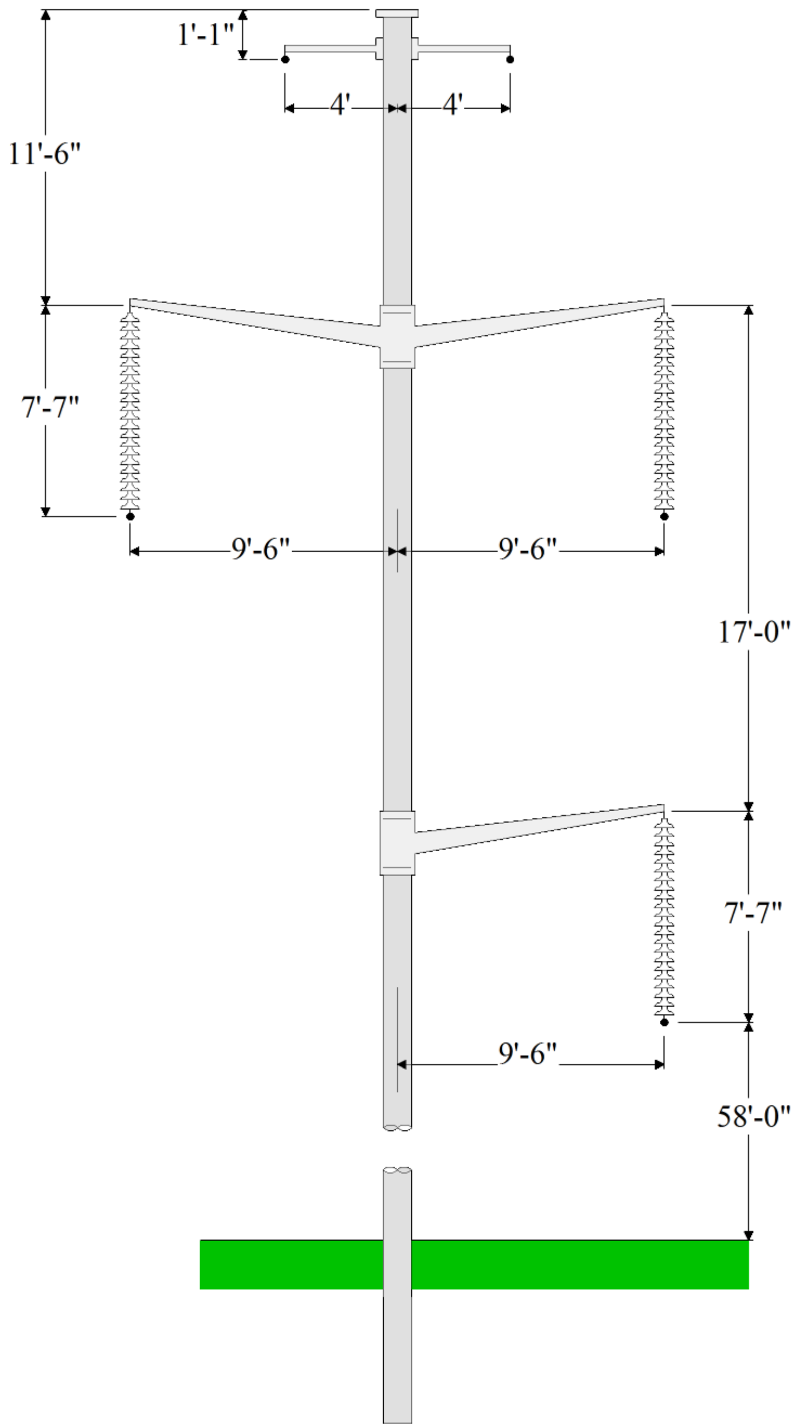


Figure 3.4: Design of a 230-kV H-frame Transmission Tower
(Courtesy of Georgia Power Co.)

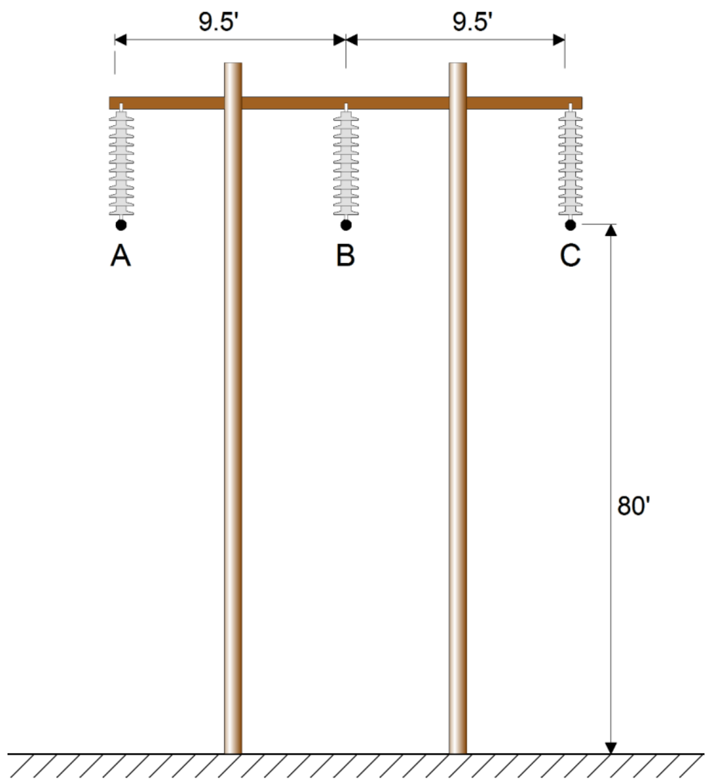


Figure 3.5: Design of a 115-kV H-Frame Transmission Tower

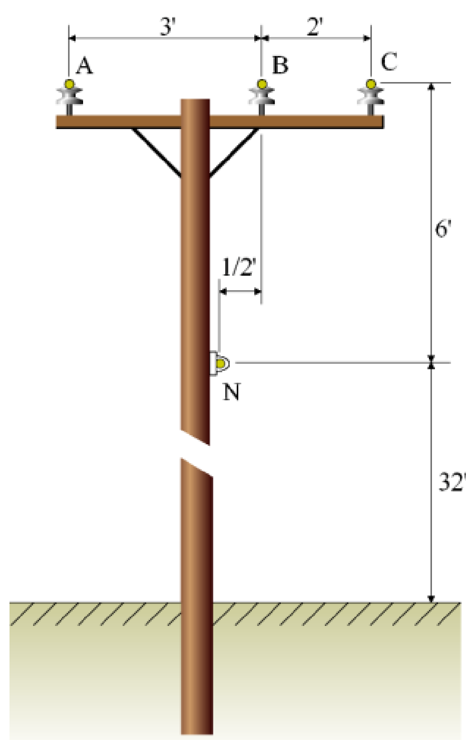


Figure 3.6: Design of a 12-kV Single-Pole Distribution

Recent advances in technology have made DC transmission an economically attractive alternative over long distances. A typical DC transmission line is illustrated in Figure 3.7. It consists of two bundle conductors, the positive and negative poles, and an overhead ground conductor.

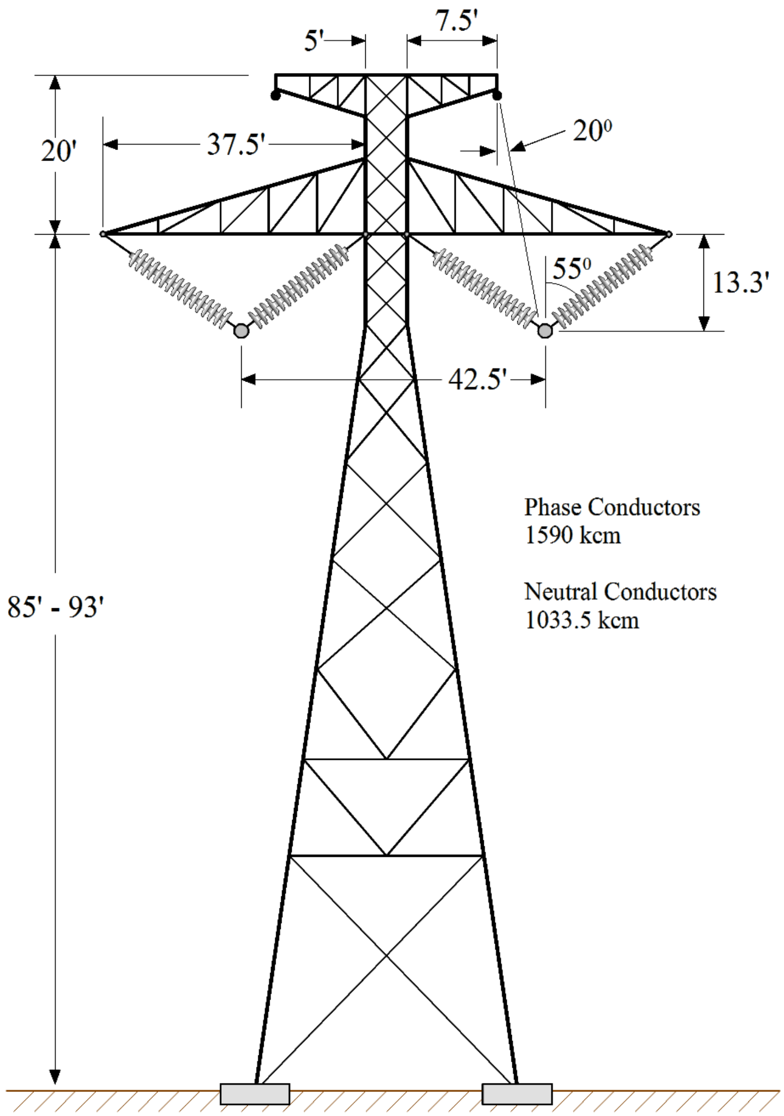
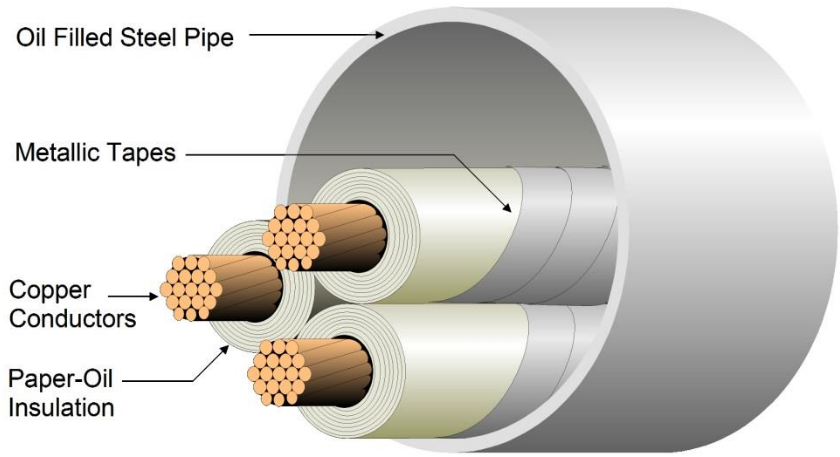


Figure 3.7: Design of a ±400-kV HVDC Tower
(Courtesy of the Electric Power Research Institute)

Power lines can be also constructed from power cables. Cables may be three phase, or single phase cables connected in a three phase arrangement. A typical three phase construction with three single phase power cables is illustrated in Figure 3.8a and a typical three phase power cable construction is illustrated in Figure 3.8b.

(a)



(b)

Figure 3.8: Typical Power Cables: (a) 3-Single Phase Solid Dielectric, (b) Three Phase Oil Filled

A distribution system comprises power lines and voltage-step-down equipment for electric service at industrial, commercial, and residential sites. A distribution system may comprise three-phase transmission lines, with typical operating voltages of 12 to 35 kV line to line, and three-phase, two phase, or single phase tapped lines. The construction of these lines may be overhead or underground. These possibilities are illustrated in Figure 3.2. Figure 3.2 suggests that distribution systems may operate (and in fact they do operate) under unbalanced conditions. Some of this imbalance may transmit to the transmission system. This means that distribution systems present some unique analysis problems. In addition, recent advances in end-use equipment technology have resulted in electric loads that may be interacting with the system dynamically. For example, solid-state motor controllers, rectifiers, and so on, inject harmonics into the distribution system. Analysis and understanding of these phenomena require that the distribution system be modeled and understood not only for the power frequency (60-Hz in the United States, 50 Hz in Europe) but also for other frequencies, such as the harmonics of 60 Hz.

For several technical and safety reasons, electric power installations must be grounded. Grounding of power systems is achieved by embedding metallic structures (conductors) into earth and electrically connecting these conductors to the neutral of the power system. In this way a low impedance is provided between the power system neutral and the vast conducting soil, which guarantees that the voltage of the neutral, with respect to earth, will be low under all conditions. Grounding is necessary for several reasons: (a) to assure correct operation of electrical devices, (b) to provide safety during normal or fault conditions, (c) to stabilize the voltage during transient conditions, and (d) to dissipate lightning strokes. An example of the physical construction of a substation with the underlying grounding system is illustrated in Figure 3.9.

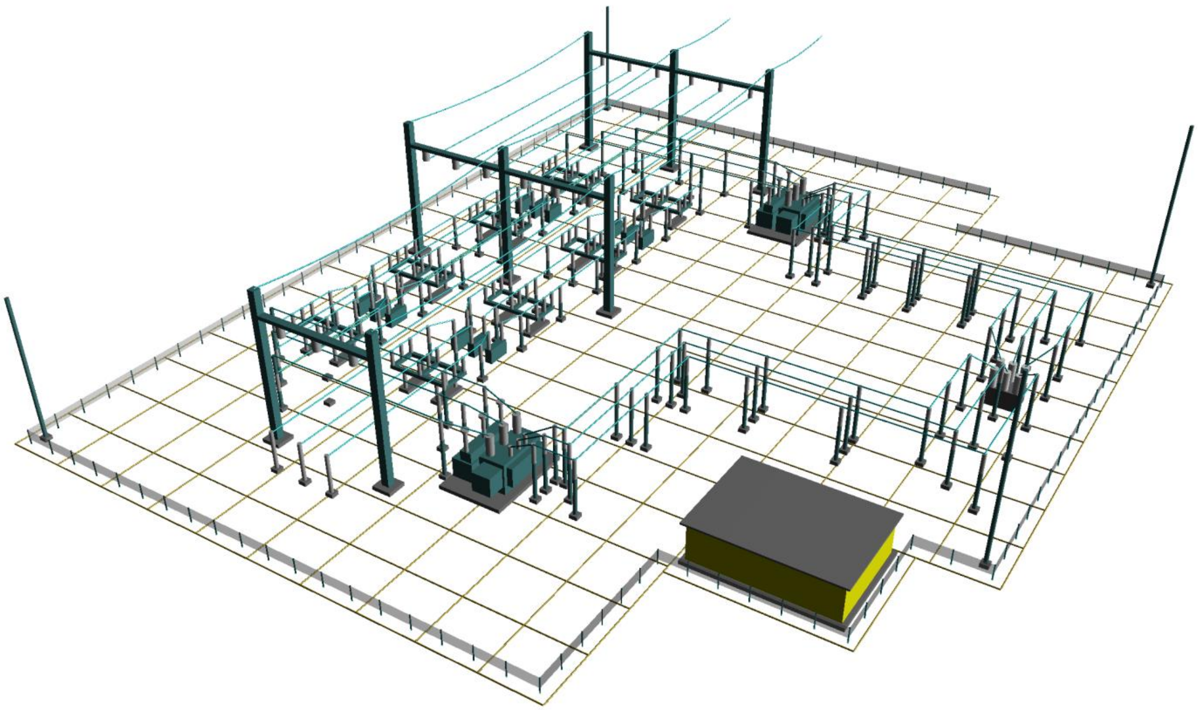


Figure 3.9: Example of the Physical Arrangement of a Substation Illustrating the Grounding, Fences and Electrical Equipment

The described physical structures are typically modeled with proper mathematical models. The presentation of line modeling will be done in several steps. First, we shall examine the per unit length parameters of a power line. These parameters are: resistance, inductance, and capacitance.

Next, analysis procedures will be introduced by which equivalent circuits of power lines will be developed. Depending on the objectives of the analysis the mathematical models may be different for the same physical structure. As an example for analysis of a power lines under steady state 60 Hz sinusoidal operation, a π-equivalent circuit completely captures the behavior of the line. However for the same line, this equivalent circuit is inadequate to describe transients on the line. In general, the following models of transmission and distribution lines and relative applications may be encounter:

1. Three-phase power lines can be approximated in terms of their sequence equivalent circuits (positive, negative and zero sequence). These models represent an approximation of the actual behavior of a line. They are extensively used for power flow studies, short-circuit analysis, and stability studies.

2. Power lines can be also modeled with explicit representation of transmission tower, neutral wires or ground wires, grounding systems and substation grounding systems. These models are applicable for ground potential rise computations, safety analysis and for design of grounding systems [???].
3. Distributed parameter models of power lines can be also developed. These models are applicable for fast electrical transient analysis, (such as switching transients, lightning transients) and the design of overvoltage protection. These models will not be considered in this book.

In this section, the basic equations of a transmission line model are presented for low frequencies. We focus on the derivation of the resistance, inductance and capacitance of the line and subsequent extraction of appropriate equivalent circuits.

3.2.1 Resistance

The resistance of power conductors is dependent upon the frequency of the electric current. For example the DC resistance (r_{dc}, f=0 Hertz) can be directly computed from the conductor material resistivity:

$$r_{dc} = \rho \frac{1}{A}\, ohms/meter$$

where ρ is the resistivity of the conductor material and A is the cross section of the conductor.

The computation of the AC resistance, r_{ac}, of a power conductor can be quite complicated, depending on the geometry (cross section) of the conductor. For cylindrical conductors, the AC resistance of the conductor is given in terms of Bessel functions:

$$r_{ac} = r_{dc} \frac{ka}{2} \frac{M_0(ka)}{M_1(ka)} \sin\left(\theta_1(ka) - \theta_0(ka) - \frac{\pi}{4} \right)\ ohms/meter$$

where: $k = \sqrt{\omega\mu\sigma}$, $\omega = 2\pi f$, a is the radius of the conductor

Note ka is a pure number (dimensionless)

$M_0(ka),\ \ \theta_0(ka)$: are the magnitude and phase respectively of the modified Bessel function, order zero and argument ka.

$M_1(ka),\ \ \theta_1(ka)$: are the magnitude and phase respectively of the modified Bessel function, order one, argument ka.

Tabulation of these functions can be found in the references. For convenience, the values of these functions for the argument value up to 10 are provided in Table 3.1. Derivation of above equations for the ac resistance of cylindrical conductors can be found in [???].

For other conductor cross section geometries, the reader is encouraged to consult the references.

Table 3.1: Modulus and Phase of Modified Bessel Functions

z	$M_0(z)$	$\theta_0(z)$	$M_1(z)$	$\theta_1(z)$	z	$M_0(z)$	$\theta_0(z)$	$M_1(z)$	$\theta_1(z)$
0.000	1.0000	0.00	0.0000	135.00	1.300	1.0438	23.75	0.6548	147.07
0.025	1.0000	0.01	0.0125	135.00	1.350	1.0508	25.54	0.6808	148.02
0.050	1.0000	0.04	0.0250	135.02	1.400	1.0586	27.37	0.7070	148.99
0.075	1.0000	0.08	0.0375	135.04	1.450	1.0672	29.26	0.7333	150.00
0.100	1.0000	0.14	0.0500	135.07	1.500	1.0767	31.19	0.7598	151.04
0.125	1.0000	0.22	0.0625	135.11	1.550	1.0871	33.16	0.7866	152.12
0.150	1.0000	0.32	0.0750	135.16	1.600	1.0984	35.17	0.8136	153.23
0.175	1.0000	0.44	0.0875	135.22	1.650	1.1108	37.22	0.8408	154.38
0.200	1.0000	0.57	0.1000	135.29	1.700	1.1242	39.30	0.8684	155.55
0.225	1.0000	0.73	0.1125	135.36	1.750	1.1387	41.41	0.8962	156.76
0.250	1.0001	0.90	0.1250	135.45	1.800	1.1544	43.54	0.9244	158.00
0.275	1.0001	1.08	0.1375	135.54	1.850	1.1712	45.70	0.9530	159.27
0.300	1.0001	1.29	0.1500	135.64	1.900	1.1892	47.88	0.9819	160.57
0.325	1.0002	1.51	0.1625	135.76	1.950	1.2085	50.08	1.0113	161.90
0.350	1.0002	1.75	0.1750	135.88	2.000	1.2290	52.29	1.0412	163.27
0.375	1.0003	2.01	0.1875	136.01	2.050	1.2509	54.51	1.0715	164.66
0.400	1.0004	2.29	0.2000	136.15	2.100	1.2741	56.74	1.1024	166.08
0.425	1.0005	2.59	0.2125	136.29	2.150	1.2986	58.98	1.1339	167.53
0.450	1.0006	2.90	0.2250	136.45	2.200	1.3246	61.22	1.1659	169.00
0.475	1.0008	3.23	0.2375	136.62	2.250	1.3520	63.46	1.1987	170.50
0.500	1.0010	3.58	0.2500	136.79	2.300	1.3808	65.71	1.2321	172.03
0.525	1.0012	3.95	0.2626	136.97	2.350	1.4111	67.95	1.2663	173.58
0.550	1.0014	4.33	0.2751	137.17	2.400	1.4429	70.19	1.3012	175.16
0.575	1.0017	4.73	0.2876	137.37	2.500	1.5111	74.65	1.3736	178.39
0.600	1.0020	5.15	0.3001	137.58	2.600	1.5855	79.09	1.4498	181.70
0.625	1.0024	5.59	0.3126	137.80	2.700	1.6665	83.50	1.5300	185.10
0.650	1.0028	6.04	0.3252	138.03	2.800	1.7541	87.87	1.6148	188.57
0.675	1.0032	6.52	0.3377	138.26	2.900	1.8486	92.21	1.7046	192.11
0.700	1.0037	7.01	0.3502	138.51	3.000	1.9502	96.52	1.7999	195.71
0.725	1.0043	7.51	0.3628	138.76	3.100	2.0593	100.79	1.9011	199.37
0.750	1.0049	8.04	0.3753	139.03	3.200	2.1760	105.03	2.0088	203.08
0.775	1.0056	8.58	0.3879	139.30	3.300	2.3009	109.25	2.1236	206.83
0.800	1.0064	9.14	0.4004	139.58	3.400	2.4342	113.43	2.2458	210.62
0.825	1.0072	9.72	0.4130	139.87	3.500	2.5764	117.60	2.3763	214.44
0.850	1.0081	10.31	0.4256	140.17	3.600	2.7280	121.75	2.5155	218.30
0.875	1.0091	10.92	0.4382	140.48	3.700	2.8894	125.87	2.6640	222.17
0.900	1.0102	11.55	0.4508	140.80	3.800	3.0613	129.99	2.8227	226.07
0.925	1.0114	12.19	0.4634	141.12	3.900	3.2443	134.10	2.9920	229.98
0.950	1.0127	12.86	0.4760	141.46	4.000	3.4391	138.19	3.1729	233.90
0.975	1.0140	13.53	0.4886	141.80	4.500	4.6179	158.59	4.2783	253.67
1.000	1.0155	14.23	0.5013	142.16	5.000	6.2312	178.93	5.8091	273.55
1.025	1.0171	14.94	0.5140	142.52	5.500	8.4473	199.28	7.9253	293.48
1.050	1.0188	15.66	0.5267	142.89	6.000	11.5008	219.62	10.8502	313.45
1.075	1.0207	16.40	0.5394	143.27	6.500	15.7170	239.96	14.8961	333.46
1.100	1.0227	17.16	0.5521	143.66	7.000	21.5479	260.29	20.5003	353.51
1.125	1.0248	17.93	0.5648	144.05	7.500	29.6223	280.61	28.2737	373.59
1.150	1.0270	18.72	0.5776	144.46	8.000	40.8176	300.92	39.0697	393.69
1.175	1.0294	19.52	0.5904	144.87	8.500	56.3586	321.22	54.0807	413.82
1.200	1.0320	20.34	0.6032	145.29	9.000	77.9565	341.52	74.9740	433.96
1.225	1.0347	21.17	0.6161	145.73	9.500	108.0039	361.81	104.0822	454.11
1.250	1.0376	22.02	0.6290	146.17	10.000	149.8476	382.10	144.6705	474.28

3.2.2 Inductance

We examine the modeling methods for computing the inductance of transmission circuits. First the fundamentals are presented, followed by more practical analysis methods.

3.2.2.1 Basic Magnetic Field Equation around a Conductor

Conceptually, the phenomena to be studied can be explained through the simple two-conductor line illustrated in Figure 3.10. Assume that electric current i(t), which is time dependent, flows through one conductor and returns through the other conductor. The current flow generates a magnetic field that is time dependent, i.e. it follows the time variation of the electric current. Consider an infinitesimal length dx of conductor. Let $d\lambda(t)$ be the magnetic flux linking the electric current i(t) flowing in the infinitesimal length dx of the conductor. By definition, the inductance of the length dx of the conductor is dL, where

$$dL = \frac{d\lambda(t)}{i(t)} \tag{3.1}$$

Since the magnetic flux linkage is time varying, a voltage dv(t) will be induced along length dx of the conductor:

$$dv(t) = \frac{d\lambda(t)}{dt} = dL\frac{di(t)}{dt}$$

Now assume that the inductance of the conductor is L henries per meter; then

$$dL = L\,dx$$

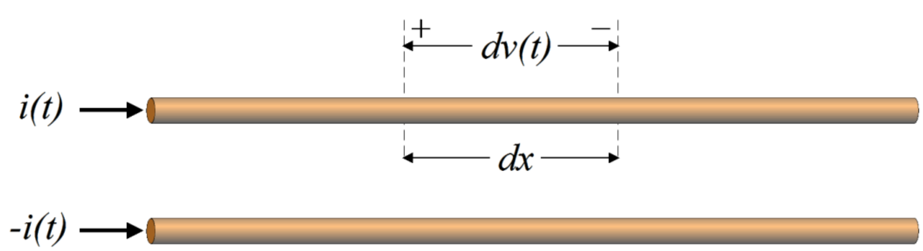


Figure 3.10: A Simple Two Conductor Line

Upon substitution in the equations above and subsequent solution for L, we have

$$L = \frac{\frac{dv(t)}{dx}}{\frac{di(t)}{dt}} \quad \text{H/m (Henries/meter)} \tag{3.2}$$

Equation (3.1) or (3.2) defines the inductance of a conductor. Specifically, Eq. (3.1) states that the inductance equals the magnetic flux linkage divided by the electric current. Alternatively, Equation (3.2) states that the inductance equals the induced voltage per unit length divided by the time derivative of the electric current.

A transmission line is a complicated structure, comprising two or more conductors. Our objective in this chapter is to characterize each conductor with its inductance and also any pair of conductors with a mutual inductance.

We introduce the basic concepts by considering the magnetic field of an infinity long conductor of circular cross section. For simplicity, assume that the conductor material is nonmagnetic. In other words, the permeability of the conductor material is μ_0. A cross section of the conductor is shown in Figure 3.11a. The radius of the conductor is a. Further assume that the conductor carries an electric current i(t), which is uniformly distributed in the cross section of the conductor (i.e. constant current density). Under these assumptions, it is relatively easy to compute the magnetic field of the configuration and subsequently the inductance of the line.

Because of the existing cylindrical symmetry, the magnetic field intensity $\underline{H}$ at a point A, illustrated in Figure 3.11a, will be perpendicular to the radial direction and the magnitude will be constant on the circular contour with center O and radius r. In other words, the magnitude of the magnetic field intensity, H, is a function of the radius r only [i.e. H(r)]. H(r) is computed with a direct application of Ampere's law on the described configuration. There are two cases.

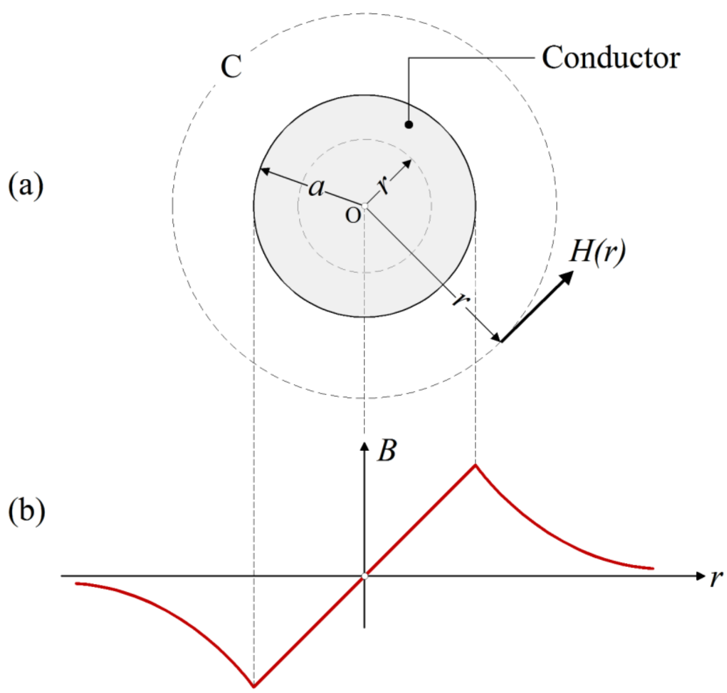


Figure 3.11: Infinitely Long Circular Conductor
[(a) Cross Section, (b) Magnetic Flux Density Along a Radial Direction]

Case a. The point A is located outside the conductor:

$$r \geq a$$

Application of Ampere's law yields

$$i(t) = \int_C \underline{H}(r) \cdot d\underline{\ell} = 2\pi r H(r)$$

Upon solution for H(r), we obtain:

$$H(r) = \frac{i(t)}{2\pi r}, \quad r > a \tag{3.3}$$

The magnetic flux density is given by

$$B(r) = \mu_0 H(r) = \mu_0 \frac{i(t)}{2\pi r}, \quad r > a \tag{3.4}$$

Case b. The point A is located inside the conductor:

$$r \leq a$$

Application of Ampere's law yields:

The electric current inside C_2: $\quad i_{C_2} = \int_{C_2} \underline{H}(r) \cdot d\underline{\ell} = 2\pi r H(r)$

In general the computation of the current inside the curve C_2 may be quite complicated. For simplicity and for low frequencies, we introduce the simplifying assumption that the electric current density is constant inside the conductor In this case:

The electric current inside C_2: $\quad i_{C_2} = \dfrac{\pi r^2}{\pi a^2} i(t) = \left(\dfrac{r}{a}\right)^2 i(t), \quad r \leq a$

Substitution and subsequent solution for H(r) yields

$$H(r) = \frac{1}{2\pi a}\left(\frac{r}{a}\right) i(t), \quad r \leq a \tag{3.5}$$

and

$$B(r) = \mu_0 H(r) = \frac{\mu_0}{2\pi a}\left(\frac{r}{a}\right) i(t), \quad r \leq a \tag{3.6}$$

The results are summarized in Figure 3.11b, where the magnetic flux density B(r) is plotted as a function of r along a radial direction.

From the magnetic flux density B, the magnetic flux Φ crossing any surface S is computed from the integral

$$\Phi = \int_S B \bullet ds$$

If the surface S crosses the conductor and since the electric current is distributed inside the conductor, the magnetic flux will link variable portions of the electric current. In this case the use of the concept of magnetic flux linkage is expedient. The magnetic flux linkage is defined by

$$\lambda = \int_S wB \bullet ds$$

where w is the portion of electric current linked with the infinitesimal magnetic flux $B \bullet ds$.

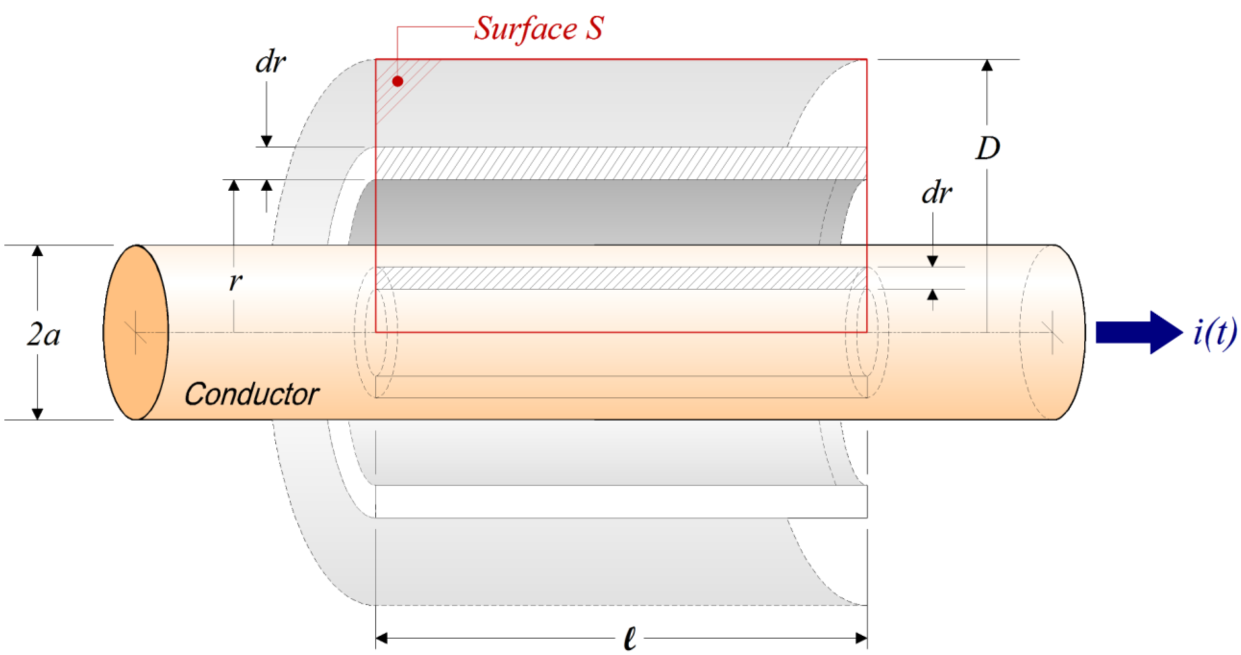


Figure 3.12: Geometry of surface S

Given the magnetic flux linkage though a surface S, the induced voltage v(t) along the perimeter of the surface is computed by

$$v(t) = \frac{d\lambda(t)}{dt}$$

As an example, consider a rectangular surface S, of dimensions ℓ and D, located on a plane passing through the axis of the conductor. The surface S is defined in Figure 3.12. Consider the two illustrated infinitesimal strips of the area ℓdr located on the surface S and parallel to the axis of the conductor. One infinitesimal strip is located inside the conductor at a distance $r_1 \le a$ from the axis. The magnetic flux through the infinitesimal strip $dS_1 = \ell dr \ at \ r = r_1 \le a$ (inside the conductor), links $\frac{\pi r^2}{\pi a^2}$ portion of the electric current. Thus the magnetic flux linkage $d\lambda_{\text{int}}(t)$ is

$$d\lambda_{\text{int}}(t) = \frac{\pi r^2}{\pi a^2} B(r)\ell = \frac{\mu_0 r^3 i(t)}{2\pi a^4} \ell dr$$

The magnetic flux linkage of the second infinitesimal strip $dS_2 = \ell dr \ at \ r = r_2 \ge a$ (outside the conductor), links the entire electric current through the conductor. The magnetic flux linkage of this infinitesimal strip $d\lambda_{ext}(t)$ is

$$d\lambda_{ext}(t) = \frac{\mu_0 i(t)}{2\pi r} \ell dr$$

The total magnetic flux linkage through the surface S is

$$\lambda(t) = \int_{r=0}^{a} \frac{\mu_0 r^3 i(t)}{2\pi a^4} \ell dr + \int_{r=a}^{D} \frac{\mu_0 i(t)}{2\pi r} \ell dr$$

Evaluation of the integrals provides the following result:

$$\lambda(t) = \frac{\mu_0 \ell i(t)}{2\pi} \left(\frac{1}{4} + \ln\left(\frac{D}{a}\right)\right) \tag{3.7}$$

Equation (3.7) is usually written in the following compact form:

$$\lambda(t) = \frac{\mu_0 \ell i(t)}{2\pi} \ln\left(\frac{D}{d}\right), \quad d = ae^{-\frac{1}{4}} \tag{3.8}$$

The quantity d is known as the geometric mean radius of the conductor. The physical meaning of the geometric mean radius is that a thin hollow conductor of radius equal to the geometric mean radius and carrying the same electric current i(t), produces the same magnetic flux linkage as the conductor under consideration. This interpretation will be illustrated by the following example.

Example E3.1: An infinitely long hollow conductor of average radius d and infinitesimal thickness carries as electric current i(t). The conductor is illustrated in Figure E3.1a. For clarity, it is shown with finite thickness.

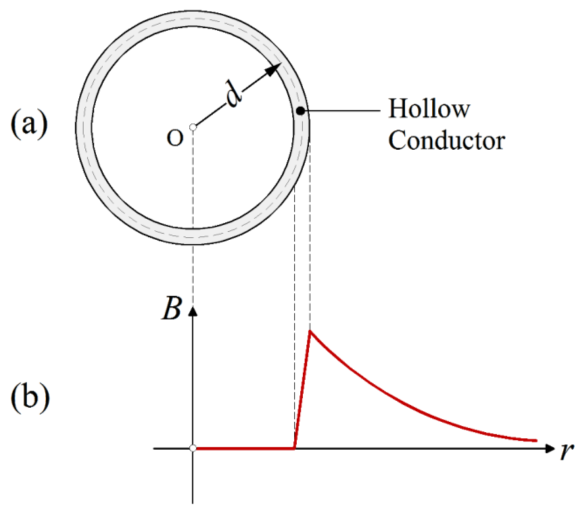


Figure E3.1 Magnetic Field around a Hollow Conductor Carrying Electric Current

Show that the magnetic flux linking a rectangular surface of dimensions ℓ and D, with one ℓ-long side located on the axis of the conductor, is

$$\lambda(t) = \frac{\mu_0 \ell i(t)}{2\pi} \ln\left(\frac{D}{d}\right)$$

Solution: The magnetic field density around this configuration is illustrated in Fig. E3.1b. Specifically, the magnetic field density is

$$B(r) = \begin{cases} 0, & r \le d \\ \dfrac{\mu_0 i(t)}{2\pi r}, & r \ge d \end{cases}$$

The magnetic flux linkage is

$$\lambda(t) = \int_{r=d}^{D} \frac{\mu_0 i(t)}{2\pi r} \ell dr = \frac{\mu_0 \ell i(t)}{2\pi} \ln\left(\frac{D}{d}\right)$$

This completes the proof.

The induced voltage across the conductor due to the magnetic flux is readily computed from

$$v(t) = \frac{d\lambda(t)}{dt} = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{D}{d}\right) \frac{di(t)}{dt}$$

By definition, the inductance of the conductor is

$$L_t = \frac{\lambda(t)}{i(t)} = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{D}{d}\right)$$

On a per unit length basis, the inductance is

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{d}\right) \tag{3.10}$$

One should observe that the inductance of the conductor is dependent on the width D of the selected surface S. Since the width D can be selected arbitrarily, the result above does not have any physical meaning. This peculiarity occurs because the path of return of the electric current i(t) has been neglected. It is apparent that in order to compute the inductance of the conductor in a unique and meaningful way, it is necessary to consider the entire circuit which includes the path of return of the electric current. In any practical situation, all conductors or objects carrying electric current will be located in a finite area. In this case, as we shall see in subsequent sections, the inductance of the conductors can be uniquely defined. Despite the lack of realism of the configuration being considered, the results obtained are fundamental for the computation of the inductances of realistic transmission line configurations, as we shall see in subsequent sections.

In summary we have derived expressions for the magnetic field density and magnetic flux linkage of a current carrying conductor. We will use these results for the analysis of practical transmission lines.

3.2.2.2 Inductive Equations of a Multi-Conductor Line

In general a line configuration involves multiple conductors. Each conductor carries a certain electric current. Because of physical considerations (conservation of charge) the sum of the electric currents must be equal to zero. Such an arrangement is shown in Figure 3.13. The current of each conductor will establish a magnetic field around it which will link all other conductors. The net result will be an induced voltage on each conductor. Considering conductor j, the induced voltage will be along the conductor as it is shown in Figure 3.13. For computing this voltage one must determine the magnetic flux linkage per unit length of the conductor.

Consider a rectangular frame with one side of the frame located on the axis of conductor j. The frame extends to a distance x from the axis of the conductor and its length is l. The flux linkage through this frame with respect to the current through conductor j, i.e. the flux linkage of conductor j will be

$$\lambda_{jx}(t) = \lambda_{jjx}(t) + \sum_k \lambda_{jkx}(t)$$

where $\lambda_{jkx}(t)$ is the contribution of conductor k to the flux linkage of conductor j.

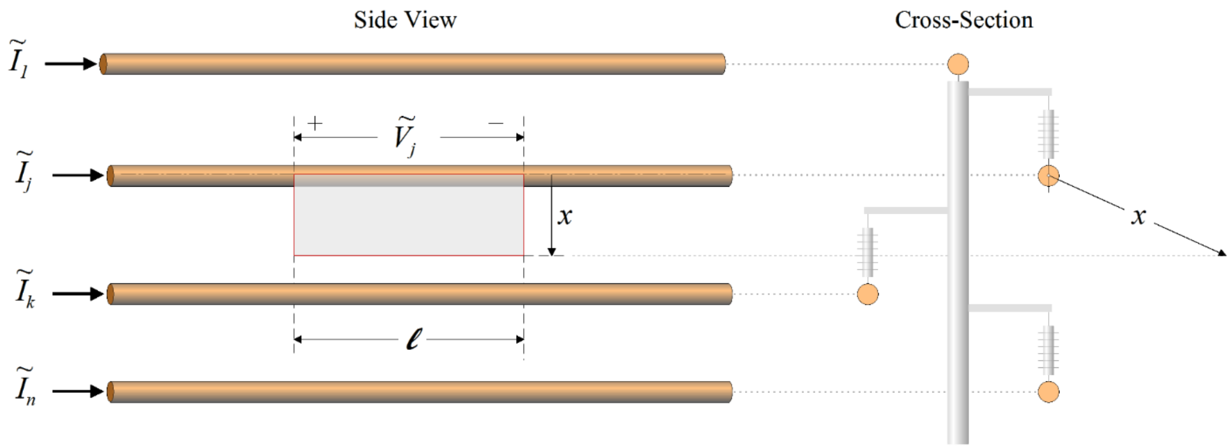


Figure 3.13: Illustration of Induced Voltage

To compute this term consider Figure 3.14, which illustrates the cross section of the system of conductors (only conductors j and k are shown) and the frame jx. We would like to determine the flux linkage through the frame jx defined with the axis of conductor j and a line parallel to conductor j passing through point x. Note that the contribution to the magnetic flux linkage from the current of conductor j is:

$$\lambda_{jjx}(t) = \frac{\mu_0 \ell i_j(t)}{2\pi} \ln\left(\frac{D_{jx}}{d_j}\right)$$

Also note that the contribution to the magnetic flux linkage of conductor j from the electric current of conductor k is the magnetic flux linkage through the surface defined with the line d_{jk}. This magnetic flux equals the flux linkage through the line mx which is given by

$$\lambda_{jkx}(t) = \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{D_{kx}}{d_{jk}}\right)$$

Note that the distance d_{km} is the same as the distance d_{jk}. The total magnetic flux linkage through the frame jx can be formed from the contribution to the flux from all conductors, i.e.:

$$\lambda_{jx}(t) = \frac{\mu_0 \ell i_j(t)}{2\pi} \ln\left(\frac{D_{jx}}{d_j}\right) + \sum_k \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{D_{kx}}{d_{jk}}\right)$$

The above equation can be written in compact form as follows:

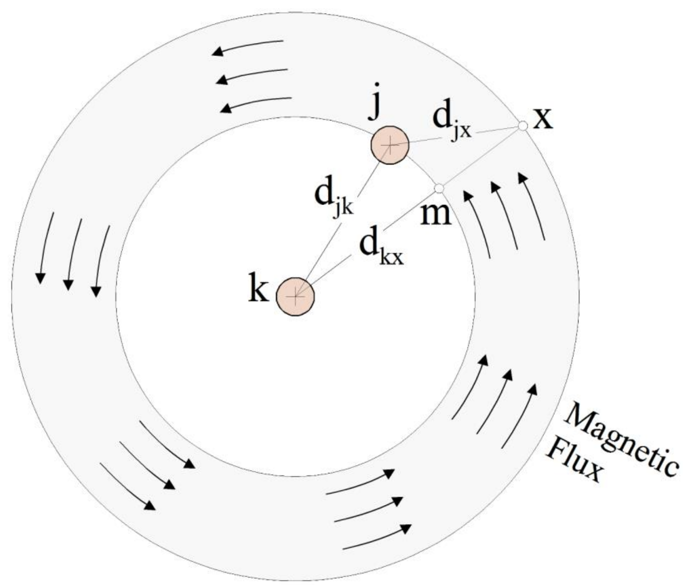


Figure 3.14: Illustration of Magnetic Flux Through Plane d_{jx} due to Electric Current i_k (t)

$$\lambda_{jx}(t) = \sum_{k=1}^{n} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{D_{kx}}{d_{jk}}\right)$$

where n is the number of conductors, d_{jk} is the distance between conductors j, k if $j \neq k$, and d_{jj} is the geometric mean radius of conductor j.

It is easy to prove that under the observation that $\sum_{k=1}^{n} i_k(t) = 0$ and as the point x goes to infinity:

$$\lambda_{jx}(t) \to \lambda_j(t) = \sum_{k=1}^{n} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{1}{d_{jk}}\right)$$

Proof: Since the sum of all currents equals zero, then the current of the last conductor n can be written as the negative sum of all other currents:

$$i_n(t) = -\sum_{k=1}^{n-1} i_k(t)$$

Upon substitution in the expression for the magnetic flux linkage:

$$\lambda_{jx}(t) = \sum_{k=1}^{n-1} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{d_{kx}}{d_{jk}}\right) - \sum_{k=1}^{n-1} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{d_{nx}}{d_{jn}}\right)$$

The above expression can be rewritten in the following form:

$$\lambda_{jx}(t) = \sum_{k=1}^{n-1} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{1}{d_{jk}}\right) - \sum_{k=1}^{n-1} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{1}{d_{jn}}\right) + \sum_{k=1}^{n-1} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{d_{kx}}{d_{nx}}\right)$$

Note that the last sum will vanish as the point x goes to infinity because each term will become zero (logarithm of 1.0). The second sum can be expressed in terms of the current in conductor n. Thus:

$$\lambda_j(t) = \sum_{k=1}^{n} \frac{\mu_0 \ell i_k(t)}{2\pi} \ln\left(\frac{1}{d_{jk}}\right)$$

This concludes the proof.

The induced voltage along the conductor is computed as the time derivative of the magnetic flux linkage of the conductor.

$$v_j(t) = \frac{d\lambda_j(t)}{dt} = \sum_{k=1}^{n} \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{1}{d_{jk}}\right) \frac{di_k(t)}{dt}$$

Assuming sinusoidal steady state conditions,

$$v_j(t) = \mathrm{Re}\left[\sqrt{2}\tilde{V}_j e^{j\omega t}\right]$$

$$i_k(t) = \mathrm{Re}\left[\sqrt{2}\tilde{I}_k e^{j\omega t}\right]$$

Upon substitution with manipulations

$$\tilde{V}_j = \sum_{k=1}^{n} \frac{j\omega\mu_0 \ell}{2\pi} \ln\left(\frac{1}{d_{jk}}\right) \tilde{I}_k = \sum_{k=1}^{n} x_{jk} \tilde{I}_k$$

where

$$x_{jk} = \frac{j\omega\mu_0 \ell}{2\pi} \ln\left(\frac{1}{d_{jk}}\right)$$

The previous results can be directly used to determine the induced per unit length of any line.

3.2.2.3 Inductive Equations of a Multi-Conductor Line Above Earth

Overhead or underground power transmission lines are characterized by the fact that earth is one of the paths for the flow of electric current. Electric current can flow into the soil through the grounding system (substation ground mat, pole grounds, etc.) and vice versa can flow into the shield/neutral wires from the soil. During normal operating conditions, some electric current flows in the conductive earth soil. This current it is generally generated by a combination of inductive, conductive and capacitive phenomena. In general, the magnitude of the earth current during normal operating conditions is comparatively low. During abnormal operating conditions (faults), a substantial amount of electric current may flow through earth. In any case, the earth current induces a voltage along the transmission line, thus affecting the performance of the power line. As a matter of fact, most three-phase overhead transmission circuits are designed in such a way that during ground faults the majority of the fault current flows through the earth.

The distribution of the current in the earth follows a complex, non-uniform pattern. As a result, the computation of the inductive reactance of the earth path and the mutual inductance between the earth path and overhead conductors is very complex. In this section we present the expressions for the series impedances, derived by Carson [???] and Rudenberg [???]. The results have been converted into the metric system of units (or English system) and adapted to the two-conductor system above earth as it illustrated in Figure 3.15. Specifically, consider the simplest configuration of two overhead conductors, j and k respectively, parallel to the surface of the earth and carrying electric currents $\tilde{I}_j$ *and* $\tilde{I}_k$, respectively. The configuration is illustrated in Figure 3.15a and 3.15b. Assume that there are not any other conductors in the vicinity. Then the current through the soil path, i.e. the earth current $\tilde{I}_e$, is $\tilde{I}_e = -\tilde{I}_j - \tilde{I}_k$. Carson [???] has given a solution to this problem in terms of a complex infinite series. A converted version of Carson's result (converted into the metric system of units) is provided below (only the first few terms of the infinite series are retained). Specifically, the induced voltage along the conductor a is:

$$\tilde{V}_a = \left[r_a + j\frac{\omega\mu}{2\pi}\ln\frac{D_{aa'}}{d} + \frac{\omega\mu}{\pi}(P_{aa} + jQ_{aa}) \right]\tilde{I}_a + \left[j\frac{\omega\mu}{2\pi}\ln\frac{D_{ab'}}{d_{ab}} + \frac{\omega\mu}{\pi}(P_{ab} + jQ_{ab}) \right]\tilde{I}_b$$

Where:

r_a is the conductor AC resistance at frequency ω computed as follows:

$$r_a = r_{dc}\,\frac{ka}{2}\,\frac{M_0(ka)}{M_1(ka)}\sin\left(\theta_1(ka) - \theta_0(ka) - \frac{\pi}{4}\right) ohms/meter$$

$k = \sqrt{\omega\mu\sigma}$, σ is the conductor conductivity, and ω is angular frequency of the electric current.

d is the geometric mean radius of the overhead conductor ***a***, which is calculated in terms of the conductor actual radius a as follows:

$$d = ae^{-\frac{\xi}{4}}$$

where :
$$\xi = \frac{4}{ka}\frac{M_0(ka)}{M_1(ka)}\sin\left(\theta_0(ka) - \theta_1(ka) + \frac{3\pi}{4}\right)$$

$\tilde{I}_a$ is the current through the overhead conductor ***a***, and $\tilde{I}_b$ is the current through the overhead conductor ***b***. The terms P_{aa}, Q_{aa}, P_{ab}, Q_{ab} are computed in terms of infinite series, the first few terms of which are given below:

$$P_{aa} = \frac{\pi}{8} - \frac{x}{3\sqrt{2}} + \frac{x^2}{16}(0.6728 + \ln\frac{2}{x}) + \frac{x^3}{45\sqrt{2}} - \frac{\pi x^4}{1536} + ...$$

$$Q_{aa} = -0.0386 + \frac{1}{2}\ln\frac{2}{x} + \frac{x}{3\sqrt{2}} - \frac{\pi x^2}{64} + \frac{x^3}{45\sqrt{2}} - \frac{x^4}{384} - \frac{x^4}{384}\left(\ln\frac{2}{x} + 1.0895\right) + ...$$

$$P_{ab} = \frac{\pi}{8} - \frac{y}{3\sqrt{2}}\cos\theta + \frac{y^2}{16}\cos 2\theta(0.6728 + \ln\frac{2}{y}) + \frac{y^2}{16}\theta\sin 2\theta + \frac{y^3}{45\sqrt{2}}\cos 3\theta - \frac{\pi y^4}{1536}\cos 4\theta + ...$$

$$Q_{ab} = -0.0386 + \frac{1}{2}\ln\frac{2}{y} + \frac{y}{3\sqrt{2}}\cos\theta - \frac{\pi y^2}{64}\cos 2\theta + \frac{y^3}{45\sqrt{2}}\cos 3\theta - \frac{y^4}{384}\theta\sin 4\theta - \frac{y^4}{384}\cos 4\theta(\ln\frac{2}{y} + 1.0895) + ...$$

where:

$$x = k_s D_{aa'} = 2k_s h_a$$

$$y = k_s D_{ab'}, \quad \theta = \sin^{-1}\left(\frac{d_{ab}}{D_{ab'}}\right)$$

$k_s = \sqrt{\omega\mu / \rho}$, where ρ is the soil resistivity

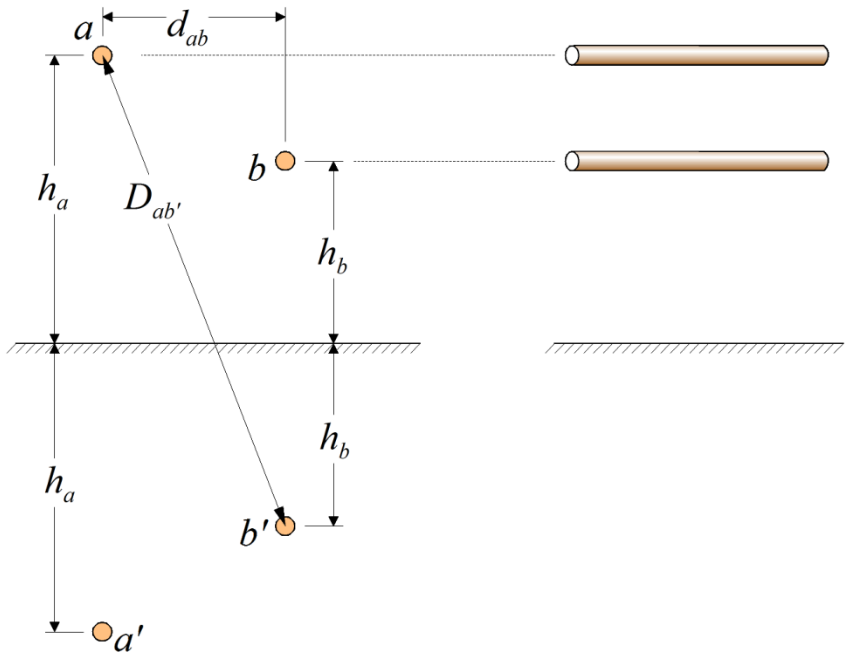


Figure 3.15: Two Parallel Power Conductors Above Soil

Note that the above equation provides the self and mutual series impedances of any conductor or any pair of conductors respectively. Specifically, the self-series impedance of conductor a is:

$$z_{a,series} = r_a + j\frac{\omega\mu}{2\pi}\ln\frac{D_{aa'}}{d} + \frac{\omega\mu}{\pi}(P_{aa} + jQ_{aa})$$

The mutual series impedance between conductors ***a*** and ***b*** is:

$$z_{ab,series} = j\frac{\omega\mu}{2\pi}\ln\frac{D_{ab'}}{d_{ab}} + \frac{\omega\mu}{\pi}(P_{ab} + jQ_{ab})$$

The above impedances are given in per unit length. Note that these equations can be repeated for any conductor and any pair of conductors of any complex arrangement of n conductors.

Equivalent Depth of Return Method: This method is obtained from the general solution (Carson [???]) presented earlier if only the first term of the infinite series is retained. The basic equations of this model can be stated with the aid of Figure 3.15 which illustrates two horizontal conductors above earth. The two conductors may be the two phases of a line, a phase conductor and a shield conductor, etc. The induced voltage on conductor ***a*** is expressed in terms of the equivalent depth of return, D_e, defined by

$$D_e = 2160\sqrt{\frac{\rho}{f}}\,(feet) = 658.368\sqrt{\frac{\rho}{f}}\,(meters)$$

where ρ is the soil resistivity in Ohm-meters, and f is the electric current frequency in Hz. The induced voltage per unit length of conductor a is

$$\tilde{V}_a = \left(r_a + r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{d_a} \right)\tilde{I}_a + \left(r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{D_{ab}} \right)\tilde{I}_b$$

$$r_a = r_{dc}\frac{ka}{2}\frac{M_0(ka)}{M_1(ka)}\sin\left(\theta_1(ka) - \theta_0(ka) - \frac{\pi}{4} \right) ohms/meter$$

$$r_e = \frac{\omega\mu}{8},\ k = \sqrt{\omega\mu\sigma}\text{, and } d_a = ae^{-\frac{\xi}{4}}$$

$$\text{with}\quad \xi = \frac{4}{ka}\frac{M_0(ka)}{M_1(ka)}\sin\left(\theta_0(ka) - \theta_1(ka) + \frac{3\pi}{4} \right)$$

a = radius of conductor ***a***
ω = angular frequency
μ = permeability of free space ($4\pi x10^{-7}$ H/m)
σ = conductivity of the conductor
M_0, θ_0 = modulus and phase of the modified Bessel function of first kind and zero order
M_1, θ_1 = modulus and phase of the modified Bessel function of first kind and first order.

Above equation provides the self and mutual series impedance of any conductor or any pair of conductors respectively. Specifically, the series self-series impedance of conductor a is:

$$z_{a,series} = r_a + r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{d_a}$$

The mutual series impedance between conductors ***a*** and ***b*** is:

$$z_{ab,series} = r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{D_{ab}}$$

The above method is called the equivalent depth of return method. It is also many times referred to as Carson’s equation. This simplified formula is valid only for usual soil resistivities (20 to 500 Ohm.m) and for low frequencies such as the power frequency (50 or 60 Hz), and for usual overhead line configurations.

Interpretation of the Equivalent Depth of Return Method: A physical interpretation of the equivalent depth of return method can be provided as follows. The equivalent depth, D_e, defines the cross section of the soil under the line where the majority of the electric current returns. For example, consider the simple case of one conductor above earth carrying an electric current and the electric current returns through the earth. The return current is spread into the soil and most of the current returns to the source through a semi-circle with radius equal to the equivalent depth of return. The higher the frequency the smaller the radius will be. This interpretation applies to any electric current under a multi-conductor line in which case the return current through the earth will be the negative sum of all currents in the conductors of the line. A visualization of this interpretation is provided in Figure 3.16.

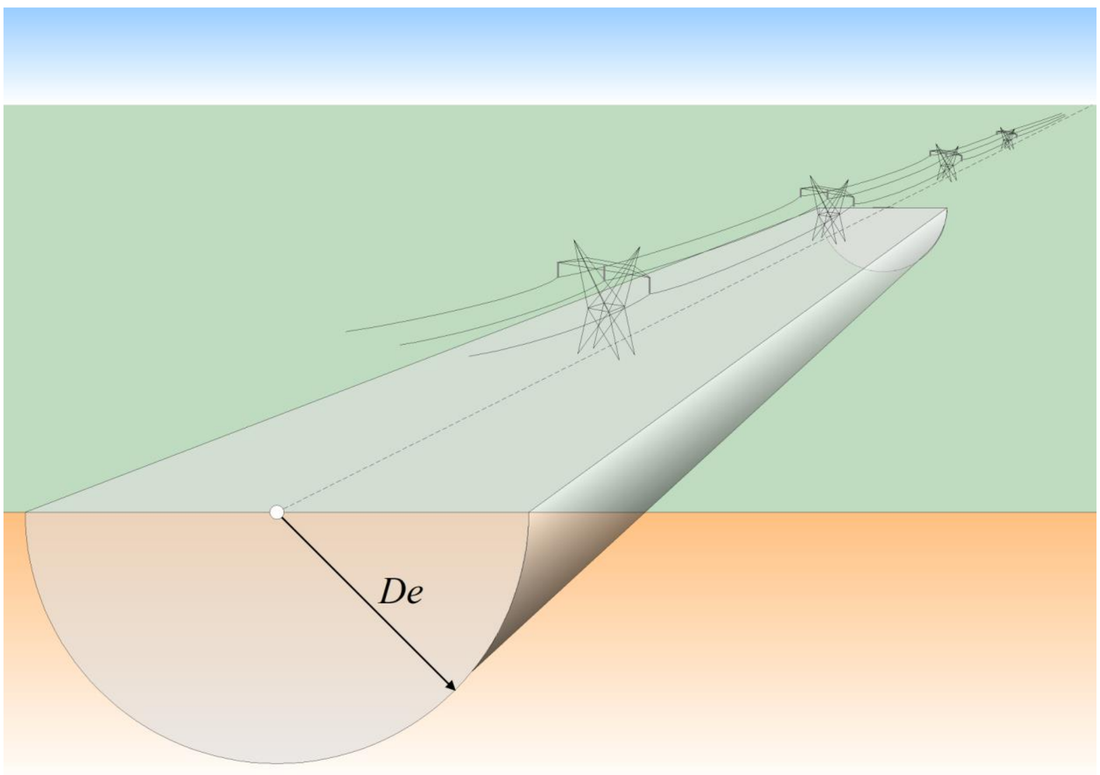


Figure 3.16: Interpretation of the Equivalent Depth of Return Method

Complex Depth of Return Method: Another method which is provided in closed form and it is remarkably accurate over a wide frequency range is the complex depth of return method. The basic equations of this model can be stated with the aid of Figure 3.15 which illustrates two horizontal conductors above earth. The two conductors may be the two phases of a line, a phase conductor and a shield conductor, etc. The induced voltage on conductor *a* is expressed in terms of the complex depth p [???] [???], defined by:

$$p = \frac{1.0}{\sqrt{j\omega\mu / \rho}}$$

where ρ is the soil resistivity. The induced voltage per unit length of conductor a is

$$\tilde{V}_a = \left(r_a + j\frac{\omega\mu}{2\pi}\ln\frac{2(h_a+p)}{d} \right)\tilde{I}_a + \left(j\frac{\omega\mu}{2\pi}\ln\frac{\sqrt{(h_a+h_b+2p)^2+d_{ab}^2}}{\sqrt{(h_a-h_b)^2+d_{ab}^2}} \right)\tilde{I}_b$$

$$r_a = r_{dc}\frac{ka}{2}\frac{M_0(ka)}{M_1(ka)}\sin\left(\theta_1(ka)-\theta_0(ka)-\frac{\pi}{4}\right) ohms/meter$$

$$k=\sqrt{\omega\mu\sigma}\text{ , and } d_a = ae^{-\frac{\xi}{4}}$$

$$\text{with}\quad \xi = \frac{4}{ka}\frac{M_0(ka)}{M_1(ka)}\sin\left(\theta_0(ka)-\theta_1(ka)+\frac{3\pi}{4}\right)$$

Where:

h_a, h_b = heights of conductors a and b above ground (meters)
d_{ab} = horizontal separation between conductors ***a*** and ***b*** (meters)
a = radius of conductor ***a*** (meters)
ω = angular frequency (rad/s)
μ = permeability of free space ($4\pi x10^{-7}$H/m)
σ = conductor conductivity (S/m)
ρ = soil resistivity (Ωm)
M_0, θ_0 = modulus and phase of the modified Bessel function of first kind and zero order.
M_1, θ_1 = modulus and phase of the modified Bessel function of first kind and first order.

The above equation provides the self and mutual series impedance of any conductor or any pair of conductors respectively. Specifically, the series self-series impedance of conductor ***a*** is:

$$z_{a,series} = r_a + r_e + j\frac{\omega\mu}{2\pi}\ln\frac{2(h_a+p)}{d}$$

The mutual series impedance between conductors ***a*** and ***b*** is:

$$z_{ab,series} = r_e + j\frac{\omega\mu}{2\pi}\ln\frac{\sqrt{(h_a+h_b+2p)^2+d_{ab}^2}}{\sqrt{(h_a-h_b)^2+d_{ab}^2}}$$

The above method is called the *complex depth of return method* which is a closed-form approximation to Carson's solution and was suggested by Semlyen and Deri [???]. This closed-form solution yields a remarkably close agreement with the exact Carson's solution in a wide range of frequencies (0 to 10 MHz) for typical overhead line configurations.

Summary of the Three Methods: Note that each one of the three presented methods provide the self and mutual impedance of two parallel conductors above earth. These results can be easily generalized to an n-conductor configuration above soil by considering two conductors at a time. Specifically, the series impedance of an n-conductor power line is provided by

$$Z = R + j\omega L = \begin{bmatrix} R_{11}+jX_{11} & R_{12}+jX_{12} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ R_{n1}+jX_{n1} & R_{n2}+jX_{n2} & \cdots & R_{nn}+jX_{nn} \end{bmatrix}$$

The elements of the above matrix can be computed with any of the three methods presented.

Example E3.2: Consider the three-phase electric power line of Figure E3.2. The phase conductors are ACSR, 556,500 cm, 26 strands. The line does not have an overhead ground wire. The soil resistivity ρ is 75 Ωm. Compute the resistance and inductance matrices of this line using: (a) The equivalent depth of return, (b) The complex depth of return method.

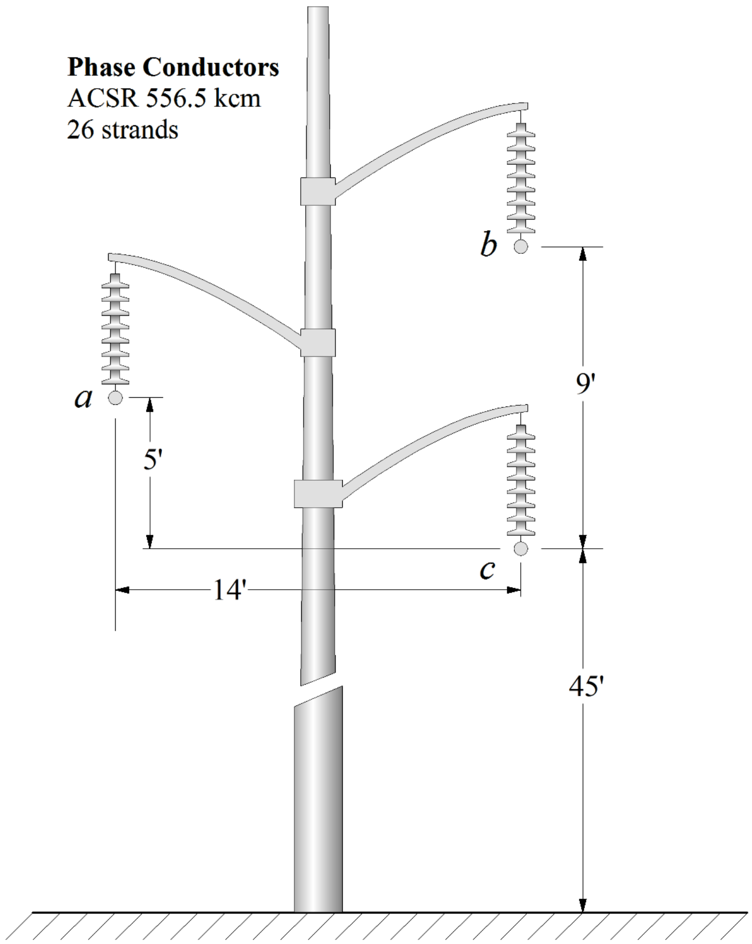


Figure E3.2

Solution: (a) The series impedance of the line using the equivalent depth of return method is:

$$R + j\omega L = \begin{bmatrix} r + r_e & r_e & r_e \\ r_e & r + r_e & r_e \\ r_e & r_e & r + r_e \end{bmatrix} + j\frac{\omega\mu}{2\pi}\begin{bmatrix} \ln\frac{D_e}{d_{aa}} & \ln\frac{D_e}{d_{ab}} & \ln\frac{D_e}{d_{ac}} \\ \ln\frac{D_e}{d_{ba}} & \ln\frac{D_e}{d_{bb}} & \ln\frac{D_e}{d_{bc}} \\ \ln\frac{D_e}{d_{ca}} & \ln\frac{D_e}{d_{cb}} & \ln\frac{D_e}{d_{cc}} \end{bmatrix}$$

From ACSR conductor tables we obtain the following conductor parameters:

- Conductor Resistance at 60 Hz, r = 0.1611 Ω/mile = 0.0001 Ω/m.
- Conductor Geometric Mean Radius: 0.0315 feet.

Furthermore,

$$D_e = 2{,}160\sqrt{\frac{75}{60}} = 2{,}415\ ft$$
$$d_{ab} = d_{ba} = 14.56\ ft$$
$$d_{ac} = d_{ca} = 14.87\ ft$$
$$d_{bc} = d_{cb} = 9.0\ ft$$
$$d_{aa} = d_{bb} = d_{cc} = 0.0315\ ft$$

Upon substitution into the above impedance matrix formula:

$$R + j\omega L = 10^{-3}\begin{bmatrix} 0.159 & 0.059 & 0.059 \\ 0.059 & 0.159 & 0.059 \\ 0.059 & 0.059 & 0.159 \end{bmatrix} + j10^{-3}\begin{bmatrix} 0.8478 & 0.3853 & 0.3838 \\ 0.3853 & 0.8478 & 0.4215 \\ 0.3838 & 0.4215 & 0.8478 \end{bmatrix} \quad Ohms/meter$$

(b) The series impedance of the line using the complex depth of return method is:

$$R + j\omega L = \begin{bmatrix} z_{a,series} & z_{ab,series} & z_{ac,series} \\ z_{ba,series} & z_{\mathrm{b},series} & z_{bc,series} \\ z_{ca,series} & z_{cb,series} & z_{\mathrm{c},series} \end{bmatrix}$$

Where:

$$z_{\mathrm{i},series} = r_i + j\frac{\omega\mu}{2\pi}\ln\frac{2(h_i + p)}{d},\ i = a, b, c \text{ , and}$$

$$z_{ik,series} = j\frac{\omega\mu}{2\pi}\ln\frac{\sqrt{(h_i + h_k + 2p)^2 + d_{ik}^2}}{\sqrt{(h_i - h_k)^2 + d_{ik}^2}}, \quad ik = ab, bc, ca$$

$$\text{where:} \quad p = \frac{1.0}{\sqrt{j\omega\mu / \rho}}$$

Upon substitution one obtains:

$$p = 281.349 - j281.349\, m$$

and:

$$R + j\omega L = 10^{-3}\begin{bmatrix} 0.157 & 0.057 & 0.057 \\ 0.057 & 0.157 & 0.057 \\ 0.057 & 0.057 & 0.158 \end{bmatrix} + j10^{-3}\begin{bmatrix} 0.8559 & 0.3906 & 0.3876 \\ 0.3906 & 0.8561 & 0.4034 \\ 0.3876 & 0.4034 & 0.8557 \end{bmatrix} \quad Ohms / meter$$

Comparing the results of (a) and (b) it appears that they are remarkably close.

3.2.3 Capacitance

In this section we discuss methods by which the capacitance of a transmission line can be computed. For this purpose we employ an approach analogous to the one for computing the inductive reactance of a transmission line. Recall that for the computation of the inductive reactance, the magnetic field around the transmission line was examined. For the computation of the line capacitance, the electric field around the line will be examined. The source of this electric field is electric charge, which is deposited on the surface of the line conductors. The analysis of the electric field results in a model relating the electric charge and the conductor voltage. The time derivative of the total electric charge on the surface of the conductors is by definition the capacitive current (or the charging current) of the line. Utilizing this definition, the model can be transformed into a relationship between the line voltage and the capacitive current. The line capacitance can be extracted from this model.

This general approach will be utilized to introduce the analysis of capacitive phenomena in lines in a step-by-step procedure. Specifically, first the simplest case of a single circular conductor will be examined to establish the basic equations. Then the analysis will be extended to two parallel conductors and the general n-conductor line configuration.

3.2.3.1 Basic Electric Field Equations around a Conductor

Consider the simple case of one circular infinitely long conductor. We shall assume that the conductor is electrically charged and we shall seek the relationship between the electric charge and the conductor voltage. Specifically, assume that the conductor is charged with electric charge q (coulombs per meter). Because of symmetry, the electric charge will be uniformly distributed on the conductor surface. The electric charge generates an electric field around the conductor. Because of symmetry, the electric field intensity E will be radially directed and the magnitude will depend only on the distance of the point of observation from the axis of the conductor, as illustrated in Figure 3.17:

$$\vec{E} = E(r)\vec{a}_r \tag{3.13}$$

Where $\vec{a}_r$ is a unit vector in the radial direction r.

Consider a cylinder of length ℓ and circular bases of radius r. The axis of the cylinder is coincident with the axis of the conductor, as it is illustrated Figure 3.17. Let S be the surface of the cylinder and V its volume. Application of Gauss's law yields:

$$\iiint_V \rho dv = \iint_S \vec{D}.d\vec{s} = \iint_S \varepsilon \vec{E}.d\vec{s} \tag{3.14}$$

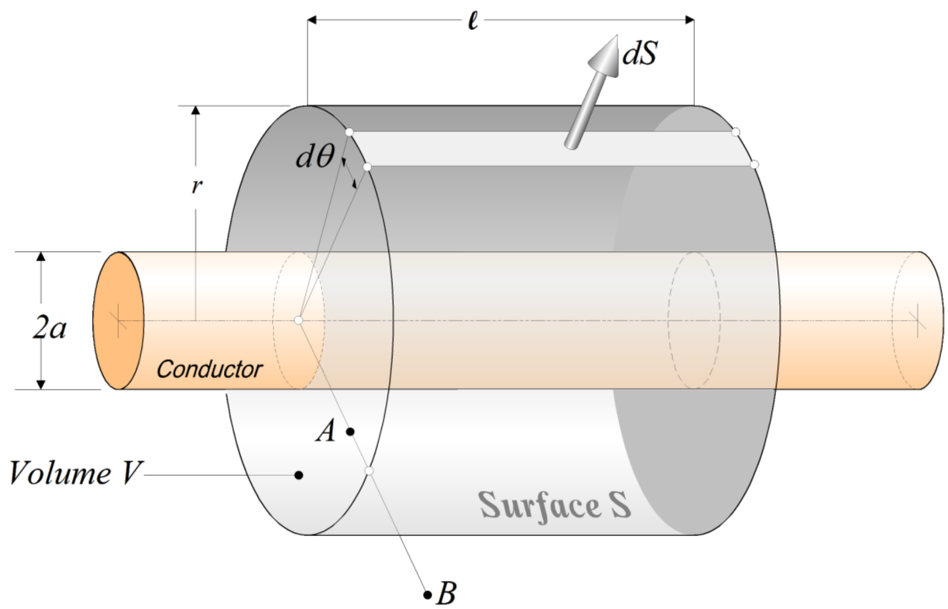


Figure 3.17: An Infinitely Long Circular Conductor

where

ρ = electric charge density, C/m^3
$\vec{E}$ = electric field intensity
$\vec{D}$ = electric field density
dv = infinitesimal volume
$d\vec{s}$ = infinitesimal surface area vector

The volume integral of the electric charge density inside the volume of the cylinder equals the total electric charge enclosed in the volume. It can be immediately computed by observing that electric charge exists only on the conductor surface at a density of q coulombs per meter. Thus

$$\iiint_V \rho dv = q\ell$$

The surface integral on the right-hand side of Equation (3.14) is computed as follows:

$$\iint_S \vec{D}.d\vec{s} = \iint_{S_1} \vec{D}.d\vec{s} + \iint_{S_2} \vec{D}.d\vec{s} + \iint_{S_3} \vec{D}.d\vec{s}$$

where S_1, S_2 are the bases of the cylinder and S_3 is the side surface of the cylinder. Note that because the electric field is radially directed, the contributions of the bases of the cylinder will vanish, that is,

$$\iint_{S_1} \vec{D}.d\vec{s} = \iint_{S_2} \vec{D}.d\vec{s} = 0.0$$

As has been discussed, the magnitude of the electric field intensity $\vec{E}$ and therefore $\vec{D}$ is a function of the radial distance r only. Thus on the surface S_3, the magnitude of the electric field density, D(r), is constant. In addition, the vector $\vec{D}$ is perpendicular to the surface S_3 and thus parallel to $d\vec{s}$. Thus

$$\iint_{S_3} \vec{D}.d\vec{s} = 2\pi r\ell D(r)$$

Substitution into Eq. (4.2) yields

$$q\ell = 2\pi r\ell D(r) = 2\pi r\ell \varepsilon E(r)$$

In above equation we used the constitutive relationship: D(r) = εE(r). Solution of above equation for E(r) yields:

$$E(r) = \frac{q}{2\pi\varepsilon r} \qquad (3.15)$$

The electric field inside the conductor is zero.

The computed electric field intensity provides the basis for computation of the potential difference between any two points A and B. This difference is the voltage V_{AB} between point A and B, defined by:

$$V_{AB} = \Phi(A) - \Phi(B) = \int_{C_{A\to B}} \vec{E}(r).d\vec{\ell}$$

The value of above integral depends only on points A and B (the reader is encouraged to prove it). Evaluation of the integral yields:

$$V_{AB} = \int_{C_{A \to B}} \vec{E}(r).d\vec{\ell} = \frac{q}{2\pi\varepsilon} \ln \frac{d_B}{d_A} \qquad (3.16)$$

where: d_A *and* d_B are the distances of points A and B respectively from the axis of the conductor.

Equation (3.16) relates the electric charge on the conductor to the potential difference between two points located at radial distances d_A *and* d_B, respectively, from the axis of the conductor. Equation (3.16) is the basic equation utilized in the analysis of transmission line capacitance.

3.2.3.2. Capacitive Equations of a Multi-Conductor Line

Consider a configuration of n conductors which are parallel and infinitely long. The conductor cross section is circular. Figure 3.18 shows a cross section of the configuration. Assume that electric charge $q_i(t)$ per unit length has been accumulated on the surface of conductor i which is uniformly distributed over the surface of the conductor. As a first step, we consider the potential of conductor i with respect to an arbitrarily selected point of reference X which is illustrated in Figure 3.18. For this purpose the principle of superposition and the results of section 3.2.3.2 are employed to yield

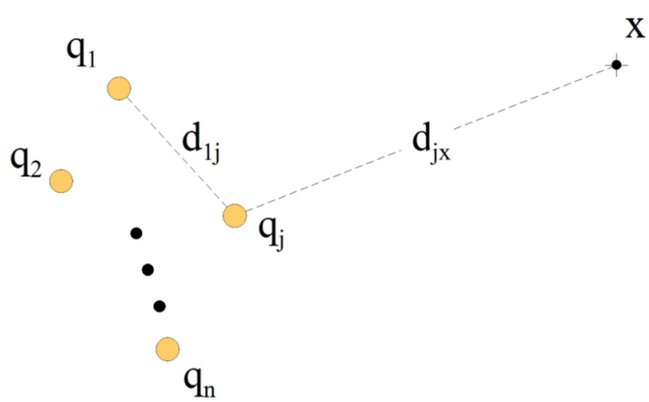


Figure 3.18: General Configuration of n-Parallel Conductors

$$V_{ix}(t) = \Phi_i(t) - \Phi_x(t) = \frac{1}{2\pi\varepsilon} \sum_{j=1}^{n} q_j(t) \ln \frac{d_{jx}}{d_{ij}} \qquad (3.17)$$

where

d_{ij} = distance between the axes of conductors i and j

d_{jx} = distance between the axis of conductor j and point X

$\Phi_i(t)$ = potential of conductor ***i*** at time t

$\Phi_x(t)$ = potential of point X at time t

Note that $d_{ii} = a_i$, the radius of conductor ***i***.

Equation (3.17) expresses the potential difference between conductor ***i*** and an arbitrarily selected point x. If point x is taken to infinity, the voltage V_{ix} will become the absolute voltage of conductor i, V_i. To derive the absolute voltage of conductor i, the general expression for V_i is rewritten as:

$$V_{ix}(t) = \frac{1}{2\pi\varepsilon}\sum_{j=1}^{n} q_j(t)\ln\frac{1}{d_{ij}} - \frac{1}{2\pi\varepsilon}\sum_{j=1}^{n} q_j(t)\ln\frac{1}{d_{jx}}$$

Now observe that if the n conductors are the only objects with electric charge, the sum of the electric charges, $q_1(t),\ldots\ldots, q_n(t)$, must equal zero, that is,

$$\sum_{j=1}^{n} q_j(t) = 0 \qquad (3.18)$$

In this case it can be shown that (the reader is encouraged to prove it):

$$\lim_{x\to\infty}\frac{1}{2\pi\varepsilon}\sum_{j=1}^{n} q_j(t)\ln\frac{1}{d_{jx}} = 0 \qquad (3.19)$$

Then the absolute voltage of conductor i is

$$V_i(t) = \frac{1}{2\pi\varepsilon}\sum_{j=1}^{n} q_j(t)\ln\frac{1}{d_{ij}} \qquad (3.20)$$

The proof of the limit of Eq. (3.19) follows.

Proof: Equation (3.18) is solved for $q_n(t)$:

$$q_n(t) = -\sum_{j=1}^{n-1} q_j(t)$$

Then substitute above in the quantity of equation (3.19) and rearrange to obtain.

$$\frac{1}{2\pi\varepsilon}\sum_{j=1}^{n} q_j(t)\ln\frac{1}{d_{jx}} = \frac{1}{2\pi\varepsilon}\sum_{j=1}^{n-1} q_j(t)\ln\frac{1}{d_{jx}} + \frac{1}{2\pi\varepsilon}q_n(t)\ln\frac{1}{d_{nx}} = \frac{1}{2\pi\varepsilon}\sum_{j=1}^{n-1} q_j(t)\ln\frac{d_{nx}}{d_{jx}}$$

Note that as $x \to \infty$, the ratio $\frac{d_{nx}}{d_{jx}} \to 1.0$. The logarithm of this ratio goes to zero. Thus equation (3.19).

It is expedient to repeat the assumptions under which Eq. (3.20) has been obtained:

Assumption 1: The sum of all charges equals zero, i.e. $\sum_{j=1}^{n} q_j(t) = 0$, and **Assumption 2**: The electric charge is uniformly distributed on the surface of the conductors. This assumption is equivalent to: $d_{ij} \gg a_i, \quad i \neq j$.

The first assumption is valid for any transmission line configuration, assuming that all conductors have been accounted for. For overhead lines, since the conducting soil represents one of the conductors, this means that the earth must be also accounted for. This issue is addressed in the next section. The second assumption is always valid for overhead circuits. For circuits with bundled conductors, three phase cables, etc. the assumption may not result in accurate results. More sophisticated computational methods must be employed in these cases.

Equation (3.20) can be transformed into an equation relating the conductor capacitive current to the conductor voltage. For this purpose, Eq. (3.20) is differentiated with respect to time, yielding.

$$\frac{dv_i(t)}{dt} = \sum_{j=1}^{n} \frac{1}{2\pi\varepsilon} \ln\left(\frac{1}{d_{ij}}\right) \frac{dq_j(t)}{dt}$$

By definition, the time derivative of the conductor electric charge is the capacitive current of the conductor (or charging current):

$$\frac{dq_j(t)}{dt} = i_j^{'}(t) \quad \text{capacitive current of conductor j}$$

Upon substitution, we have

$$\frac{dv_i(t)}{dt} = \sum_{j=1}^{n} \frac{1}{2\pi\varepsilon} \ln\left(\frac{1}{d_{ij}}\right) i_j^{'}(t) \tag{3.21}$$

Equation (3.21) is the basic equation for modeling the capacitive effects of a multiconductor power line. For sinusoidal steady-state analysis, Eq. (3.21) is converted into an algebraic equation. For this purpose, recall that under sinusoidal steady-state conditions, the voltage and currents will have the following general time variation:

$$v_i(t) = \mathrm{Re}\left[\sqrt{2}\tilde{V}_i e^{j\omega t}\right]$$

$$i_i^{'}(t) = \mathrm{Re}\left[\sqrt{2}\tilde{I}_i^{'} e^{j\omega t}\right]$$

where $\tilde{V}_i$ *and* $\tilde{I}_i^{'}$ are complex numbers representing the phasors of the voltage and the capacitive current. Substitution in Eq. (3.21) and solution for $\tilde{V}_i$ gives us

$$\tilde{V}_i = \sum_{j=1}^{n} \frac{1}{j\omega 2\pi\varepsilon} \ln\left(\frac{1}{d_{ij}}\right) \tilde{I}_j^{'}, \quad i = 1, 2, ..., n \tag{3.22}$$

where $d_{ii} = a_i$, the radius of the conductor i.

It is expedient to define the following quantities:

$$x'_{ij} = \frac{1}{j\omega 2\pi\varepsilon} \ln\frac{1}{d_{ij}} \quad \mathrm{i} \neq \mathrm{j} \ \ \text{ohm·meters} \tag{3.23a}$$

$$x'_{ii} = \frac{1}{j\omega 2\pi\varepsilon} \ln\frac{1}{a_i} \ \ \text{ohm·meters} \tag{3.23b}$$

which will be called the <u>separation component of the capacitive reactance</u> and the <u>conductor component of the capacitive reactance</u> respectively. These quantities depend on the geometry and material of the components of the capacitive reactance. Using these variables, Equation (3.22) takes the following simple form:

$$\tilde{V}_i = \sum_{j=1}^{n} x_{ij} \tilde{I}_i^{'} \tag{3.24}$$

It is noted that the components of capacitive reactance for all commercially available conductors have been tabulated. As in the case of the components of inductive reactance, note that the mathematically rigorous reader will be offended by the expressions for x'_{ii} and x'_{ij} since they involve the terms $\ln\frac{1}{a_i}$ and $\ln\frac{1}{d_{ij}}$. It should be observed that if the quantities a_i and d_{ij} are expressed in the same units, the final result will be correct. For this reason it has been accepted that a_i and d_{ij} will be expressed in feet under the understanding that each quantity x'_{ii}, x'_{ij} is meaningless if considered individually.

In summary, the capacitive effects of a power line are represented with Eq. (3.21). Specifically, for each conductor in a power line, one equation can be written relating the capacitive current of the conductors and the time derivative of the conductor voltage. For sinusoidal steady-state analysis, these equations are converted into a set of algebraic equations [Equation (3.24)] relating the phasors of the conductor capacitive currents to the phasor of the conductor voltage.

3.2.3.3 Capacitive Equations of a Multi-Conductor Line Above Earth

Most overhead transmission lines have ground wires to protect them against lightning. Overhead distribution lines have neutral conductors for unbalanced current return. All overhead power lines are suspended above earth. Neutral/ground wires and the earth are conducting media in the vicinity of the line which may be charged with electric charge due to the electric field of the line. Alternatively, these conducting media alter the electric field of the line and affect the capacitance of the line. In this section we examine methods by which the effects of earth and neutral or overhead ground wires on line capacitance can be quantified.

The effect of neutral/ground wires can be computed in a straightforward way by treating these wires in the same way as the phase conductors. It should be observed that the voltage of the neutral/ground wires will be much different from the voltage of the phase conductors. Actually, the voltage of neutral or ground wires is approximately zero at normal operating conditions. For usual applications it is assumed that the voltage of neutral or ground wires is exactly zero.

Computation of the effect of earth on the capacitive reactance of a line, in general, is a difficult problem. To simplify the analysis, it is assumed that the earth is a semi-infinite perfectly conducting medium. In this case the theory of images is applied directly, yielding a rather simple analysis procedure. Specifically, the problem of a transmission line located above earth is replaced with another equivalent problem which does not include the earth, but includes the images of the conductors with respect to the surface of the earth.

Consider a multi-conductor line above earth. The space around the line consists of two media: a non-conducting and a highly conducting medium. Assume the interface to be a plane, as illustrated in Figure 3.19. The conductors of the line are located in the non-conducting medium. Earth conductor is charged with electric charge. The charged conductors will establish an electric field in medium 1. The electric field in medium 2 will be zero since medium 2 is a perfect conductor. The theory of images [???] guarantees that the electric field in the space of medium 1 established by the charged conductor is identical to the electric field generated by two conductors, the original conductor located in the free space, and another which is the geometric image of the actual conductor with respect to the plane interface of the two media. If the electric charge on the conductor is q, the electric charge of its image is -q. This condition guarantees that the electric field intensity on the interface will be perpendicular to the plane interface. Thus the boundary conditions of the problem are matched. A consequence of this condition is that if the voltage of the conductor is V, the voltage of its image will be -V.

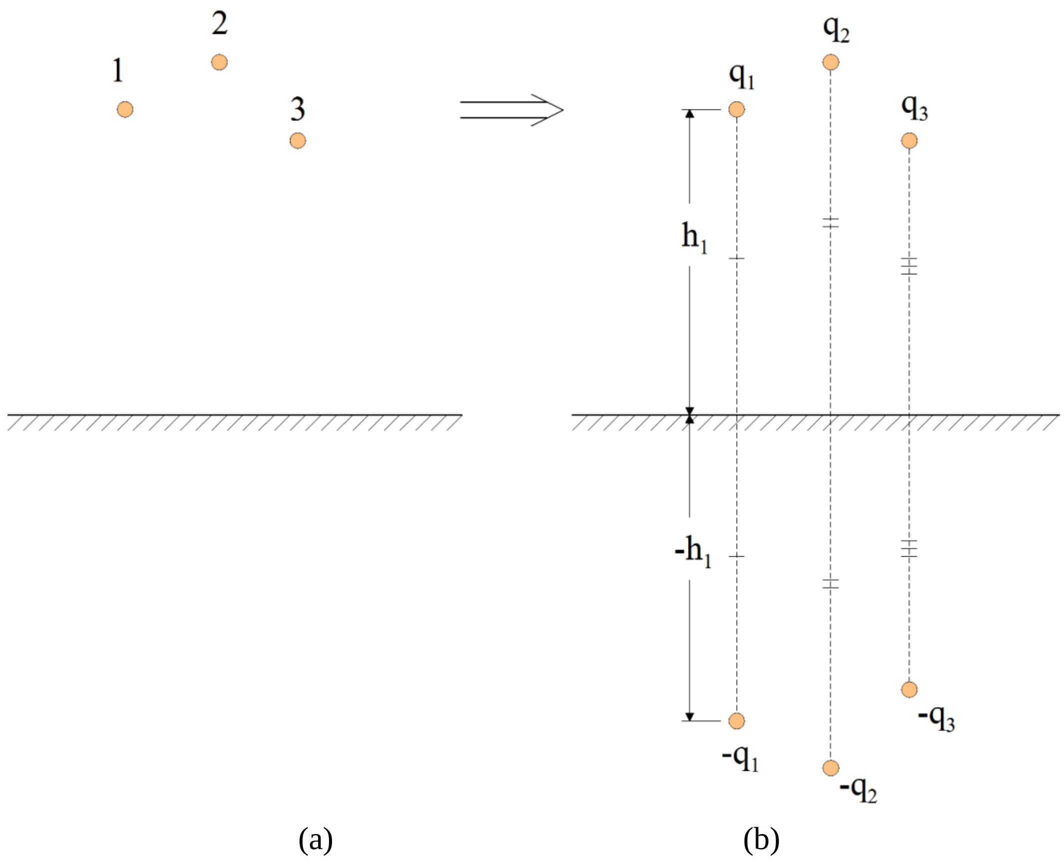


Figure 3.19: Multi-conductor Line Above Earth
[(a) Conductor Arrangement, (b) Conductor and Image Arrangement]

Consider the general transmission line suspended above earth, as illustrated in Figure 3.19a. Application of the theory of images results in the equivalent configuration of Figure 3.19b. Subsequently, the capacitive currents of the conductors are computed as follows: The voltages of the conductors, $\tilde{V}_1, \tilde{V}_2, ..., \tilde{V}_n$ are expressed in terms of the capacitive currents $\tilde{I}_1, \tilde{I}_2, ..., \tilde{I}_n$. In this analysis the capacitive currents of the images are also included. The voltage of conductor i will be:

$$\tilde{V}_i = \sum_{j=1}^{n} x'_{ij} \tilde{I}'_j - \sum_{j=1}^{n} x'_{ij'} \tilde{I}'_j, \quad i = 1, 2, ..., n \qquad (3.26)$$

where:

$$x'_{ij} = \frac{1}{j\omega 2\pi\varepsilon} \ln \frac{1}{d_{ij}}$$

$$x'_{ij'} = \frac{1}{j\omega 2\pi\varepsilon} \ln \frac{1}{d_{ij'}}$$

d_{ij} = distance between conductors i, j

$d_{ij'}$ = distance between conductors i, and the image of conductor j (which is the same as the distance between conductor j and the image of conductor i)

Equation (3.26) is rewritten by combining the terms with the same electric current, yielding the compact form:

$$\tilde{V}_i = \sum_{j=1}^{n} \frac{1}{j\omega 2\pi\varepsilon} \ln\left(\frac{d_{ij'}}{d_{ij}}\right) \tilde{I}_j^{'}, \quad i = 1, 2, ..., n \qquad (3.27)$$

Assuming that the voltages $\tilde{V}_1, \tilde{V}_2, ..., \tilde{V}_n$ are known, Eq. (3.27) is solved to provide the capacitive currents $\tilde{I}_1, \tilde{I}_2, ..., \tilde{I}_n$. The earth will also carry a capacitive currents, $\tilde{I}_e^{'}$, which is given by the equation

$$\tilde{I}_e^{'} = -\sum_{j=1}^{n} \tilde{I}_j^{'}$$

The procedure is illustrated with as example involving a three-phase line.

Example E3.3: Consider the three phase line of Example E3.2 illustrated in Figure E3.2. Compute the capacitance matrix of this line by (a) ignoring the earth effect, and (b) taking into account the earth effect. Compare the positive sequence capacitance with and without the earth effect.

Using result (b) compute the capacitive current of this transmission line assuming it is connected to a balanced 115 kV source and the line length is 10 miles.

(a) Solution Ignoring the Earth Effect

Using the formula for the capacitive reactance matrix terms:

$$x'_{ij} = \frac{1}{j\omega 2\pi\varepsilon} \ln\left(\frac{1}{d_{ij}}\right)$$

The capacitive reactance matrix X with d_{ij} expressed in feet is:

$$X = j\begin{bmatrix} 155.15 & -127.70 & -128.69 \\ -127.70 & 155.15 & -104.76 \\ -128.69 & -104.76 & 155.15 \end{bmatrix} \quad M\Omega \bullet m$$

The positive sequence reactance is computed as the difference between the self and the average of the mutual terms, as follows:

$$x_1 = x_s - x_m = \frac{1}{3}(X_{11} + X_{22} + X_{33}) - \frac{1}{3}(X_{12} + X_{13} + X_{23})$$
$$= 155.15 + \frac{1}{3}(127.70{+}128.69{+}104.76) M\Omega\, m = 275.53\, M\Omega\, m$$

and the positive sequence capacitance is obtained from the positive sequence capacitive reactance as:

$$c_1 = \frac{1}{\omega x_1} = \frac{1}{377 \times 275.53 \times 10^6} F / m = 9.6269\, pF / m$$

(b) Solution Including the Earth Effect

Using the formula for the capacitive reactance matrix terms:

$$x'_{ij} = \frac{1}{j\omega 2\pi\varepsilon} \ln\left(\frac{d_{ij'}}{d_{ij}}\right)$$

The first diagonal term is:

$$x'_{11} = \frac{1}{j\omega 2\pi\varepsilon} \ln\left(\frac{d_{11'}}{d_{11}}\right) = \frac{1}{j60 \times 4\pi^2 \times 8.854 \times 10^{-12}} \ln\left(\frac{38.48\, m}{0.0118\, m}\right) \Omega m = 374.72 M\Omega m$$

Computing the remaining terms in a similar manner, the complete capacitive reactance matrix X is as follows:

$$X = j\begin{bmatrix} 374.72 & 94.17 & 88.95 \\ 94.17 & 378.39 & 114.33 \\ 88.95 & 114.33 & 369.70 \end{bmatrix} \quad M\Omega\, m$$

As in part (a), the positive sequence reactance is computed as the difference between the average self and mutual terms, as follows:

$$x_1 = x_s - x_m = \frac{1}{3}(X_{11} + X_{22} + X_{33}) - \frac{1}{3}(X_{12} + X_{13} + X_{23})$$
$$= \frac{1}{3}(374.72{+}378.39{+}369.70 - 94.17 - 88.95 - 114.33) M\Omega\, m = 275.11 M\Omega\, m$$

and the positive sequence capacitance is obtained from the positive sequence capacitive reactance as:

$$c_1 = \frac{1}{\omega x_1} = \frac{1}{377 \times 275.11 \times 10^6} F / m = 9.6413\, pF / m$$

Note that as expected, the actual positive sequence capacitance is slightly higher than the value computed ignoring the earth effect. (The error in this case is only 0.15%, and generally decreases with line height).

In order to compute the charging current, we evaluate the capacitive susceptance matrix ***B** = j*ω***C*** as the inverse of ***X***:

$$B = j\omega C = j\begin{bmatrix} 2.938 & -0.5710 & -0.5303 \\ -0.5710 & 3.0621 & -0.7985 \\ -0.5303 & -0.7985 & 3.0794 \end{bmatrix} \quad nS/m$$

and therefore:

$$C = \begin{bmatrix} 7.7933 & -1.5145 & -1.4067 \\ -1.5145 & 8.0271 & -2.1180 \\ -1.4067 & -2.1180 & 8.1685 \end{bmatrix} \quad pF/m$$

and finally the capacitive current is computed by multiplying the susceptance matrix by the voltage vector corresponding to a balanced 115 kV line-to-line system, as follows:

$$I = j\omega CV = jBV = jB\begin{bmatrix} 66{,}390 / 0^0 \\ 66{,}390 / -120^0 \\ 66{,}390 / 120^0 \end{bmatrix} = \begin{bmatrix} 3.7268 / 90.578^0 \\ 3.9696 / -26.961^0 \\ 4.0069 / -153.550^0 \end{bmatrix} A$$

Note that even though the voltage is balanced the charging current is unbalanced, due to the asymmetry of the line capacitance.

3.2.4 Line Models for Sinusoidal Steady State

We consider the sinusoidal steady-state operating conditions. In this case the imposed voltages and currents on the transmission line vary sinusoidally with frequency f. Since the line is a linear system, the currents and voltages at any point, y, in the line will vary sinusoidally with time. Thus, in general,

$$i(y,t) = \mathrm{Re}\left[\sqrt{2}\tilde{I}(y)e^{j\omega t}\right] \tag{3.32a}$$

$$v(y,t) = \mathrm{Re}\left[\sqrt{2}\tilde{V}(y)e^{j\omega t}\right] \tag{3.32b}$$

where $\tilde{I}(y), \tilde{V}(y)$ are complex numbers (phasors) and ω = 2πf. The models of a single- or three-phase line under the conditions described are developed in the next sections.

3.2.4.1: Single-Phase Transmission Line

Let r, L and C be the resistance, inductance and capacitance all per unit length of a single phase line. If we consider an infinitesimal length *dy* of this line, then the lumped parameter model of the infinitesimal length is shown in Figure 3.20.

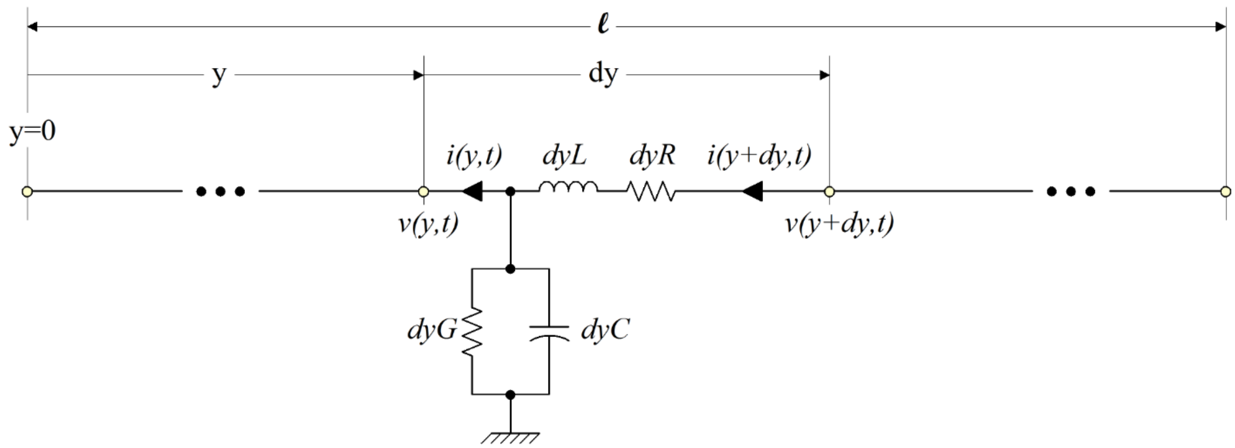


Figure 3.20: Transmission Line with Distributed Parameters

Applying Kirchoff's voltage and current law to this circuit:

$$v(y+dy,t)-v(y,t)=dyRi(y+dy,t)+dyL\frac{di(y+dy,t)}{dt}$$
$$i(y+dy,t)-i(y,t)=dyGv(y,t)+dy\,\mathrm{C}\frac{dv(y,t)}{dt}$$

Dividing both equations by *dy* and taking the limit as *dy* goes to zero, one obtains:

$$\frac{\partial v(y,t)}{\partial y}=Ri(y,t)+L\frac{\partial i(y,t)}{\partial t}$$
$$\frac{\partial i(y,t)}{\partial y}=Gv(y,t)+C\frac{\partial v(y,t)}{\partial t}$$

Differentiation of the first equation with respect to y, yields:

$$\frac{\partial^2 v(y,t)}{\partial y^2}=R\frac{\partial i(y,t)}{\partial y}+L\frac{\partial^2 i(y,t)}{\partial y\partial t}=RGv(y,t)+RC\frac{\partial v(y,t)}{\partial t}+L\frac{\partial^2 i(y,t)}{\partial y\partial t}$$

Differentiation of the second equation with respect to t and substituting the result in above equation one obtains:

$$\frac{\partial^2 v(y,t)}{\partial y^2} = RGv(y,t) + RC\frac{\partial v(y,t)}{\partial t} + L\left(G\frac{\partial v(y,t)}{\partial t} + C\frac{\partial^2 v(y,t)}{\partial t^2}\right)$$

$$= RGv(y,t) + (RC + LG)\frac{\partial v(y,t)}{\partial t} + LC\frac{\partial^2 v(y,t)}{\partial t^2}$$

In summary, the differential equations of a single phase line are:

$$\frac{\partial^2 v(y,t)}{\partial y^2} = RGv(y,t) + (RC + LG)\frac{\partial v(y,t)}{\partial t} + LC\frac{\partial^2 v(y,t)}{\partial t^2}$$

$$\frac{\partial v(y,t)}{\partial y} = Ri(y,t) + L\frac{\partial i(y,t)}{\partial t}$$

Upon substitution of Eqs. (3.32) into above equations, we obtain

$$\sqrt{2}\,\mathrm{Re}\left\{e^{j\omega t}\frac{d^2\tilde{V}(y)}{dy^2}\right\} = \sqrt{2}\,\mathrm{Re}\left\{\,[-\omega^2 LC\tilde{V}(y) + j\omega(GL+CR)\tilde{V}(y) + GR\tilde{V}(y)]e^{j\omega t}\,\right\}$$

$$\sqrt{2}\,\mathrm{Re}\left\{e^{j\omega t}\frac{d\tilde{V}(y)}{dy}\right\} = \sqrt{2}\,\mathrm{Re}\left\{\,e^{j\omega t}(R + j\omega L)\tilde{I}(y)\right\}$$

The equation above must be satisfied for any time t. Thus the coefficients of the time functions on the two sides of the equation must be identical, yielding.

$$\frac{d^2\tilde{V}(y)}{dy^2} = \left[-\omega^2 LC + j\omega(GL+CR) + GR\right]\tilde{V}(y)$$

$$\frac{d\tilde{V}(y)}{dy} = (R + j\omega L)\,\tilde{I}(y)$$

Upon factorization of the right-hand-side expression, we have

$$\frac{d^2\tilde{V}(y)}{dy^2} = (R + j\omega L)(G + j\omega C)\tilde{V}(y) \qquad (3.33a)$$

$$\frac{d\tilde{V}(y)}{dy} = (R + j\omega L)\tilde{I}(y) \qquad (3.33b)$$

Now let's define

$z_s = \dfrac{1}{y_s} = R + j\omega L$ = series impedance per unit length of the line at frequency ω

$y_{sh} = \dfrac{1}{z_{sh}} = G + j\omega C$ = shunt admittance per unit length of the line at frequency ω

With the new notation, Equation (3.33) become

$$\frac{d^2\tilde{V}(y)}{dy^2} = z_s y_{sh} \tilde{V}(y) \qquad (3.34a)$$

$$\frac{d\tilde{V}(y)}{dy} = z_s \tilde{I}(y) \qquad (3.34b)$$

Equations (3.34) represent the single-phase line model at sinusoidal steady state. The general solution of Eq. (3.34a) is

$$\tilde{V}(y) = ae^{py} + be^{-py} \qquad (3.35)$$

where a, b are constants dependent on the boundary conditions of the line, and

$$p = \sqrt{z_s y_{sh}} = \sqrt{-\omega^2 LC + j\omega(GL + RC) + GR} \qquad (3.36)$$

Note that p is dependent on the angular frequency. The dimensions of the constant p are the inverse of length. The constant p characterizes the propagation of voltage through the transmission line. For this reason it is called the propagation constant. The real and imaginary parts of the propagation constant will be called the attenuation and phase constant, respectively. That is, p = κ + jη, where κ is the attenuation constant and η is the phase constant.

The general solution for the electric current phasor $\tilde{I}(y)$ is obtained by substituting Eq. (3.35) into Eq. (3.34b). The result is

$$\tilde{I}(y) = \frac{p}{z_s}(ae^{py} - be^{-py})$$

Observe that

$$\frac{p}{z_s} = \sqrt{\frac{y_{sh}}{z_s}}$$

Define

$$Z_0 = \frac{1}{Y_0} = \sqrt{\frac{z_s}{y_{sh}}} \tag{3.37}$$

Note that the quantity Z_0 has dimensions of impedance and it is characteristic of the transmission line under consideration. It will be called the characteristic impedance of the line. In terms of the characteristic impedance Z_0, the equation for the line current becomes

$$\tilde{I}(y) = \frac{a}{Z_0} e^{py} - \frac{b}{Z_0} e^{-py} = Y_0 a e^{py} - Y_0 b e^{-py} \tag{3.38}$$

In summary, the general solution for the voltage and current phasors at a location y of a single-phase line is given by Equations (3.35) and (3.38). The solution is expressed in terms of the propagation constant p, the characteristic impedance Z_0, and two constants a and b. The quantities p and Z_0 depend on the parameters of the line, while the constants a, b are dependent on the boundary conditions. If enough boundary conditions are given, for example the terminal voltage and current at an end of the line, the constants a and b can be expressed as a function of the boundary data.

As an example, we will assume that the voltage and current at the receiving end of the line of Figure 3.1 are known to be $\tilde{V}_R$ and $\tilde{I}_R$. Note that the receiving end of this line is characterized with y=0. Then

$$\tilde{V}(y=0) = \tilde{V}_R = a + b$$

$$\tilde{I}(y=0) = \tilde{I}_R = \frac{a}{Z_0} - \frac{b}{Z_0}$$

Upon solution of two equations above for the constants a and b we obtain

$$a = \frac{\tilde{V}_R + Z_0 \tilde{I}_R}{2}$$

$$b = \frac{\tilde{V}_R - Z_0 \tilde{I}_R}{2}$$

Substitution into Equations (3.8) and (3.9) gives us

$$\tilde{V}(y) = \tilde{V}_R \frac{e^{py} + e^{-py}}{2} + Z_0 \tilde{I}_R \frac{e^{py} - e^{-py}}{2} = \tilde{V}_R \cosh(py) + Z_0 \tilde{I}_R \sinh(py) \tag{3.39a}$$

$$\tilde{I}(y) = \frac{\tilde{V}_R}{Z_0} \frac{e^{py} - e^{-py}}{2} + \tilde{I}_R \frac{e^{py} + e^{-py}}{2} = Y_0 \tilde{V}_R \sinh(py) + \tilde{I}_R \cosh(py) \tag{3.39b}$$

Equations (3.39) provide the voltage and current phasors at any location y along the line in terms of the voltage and current at the receiving end of the line (y = 0). Of special interest are the voltage and current at the other end of the line (y = ℓ):

$$\tilde{V}_S = \tilde{V}\left(y=\ell\right) = \tilde{V}_R \cosh(p\ell) + Z_0 \tilde{I}_R \sinh(p\ell)$$
$$\tilde{I}_S = \tilde{I}(y=\ell) = Y_0 \tilde{V}_R \sinh(p\ell) + \tilde{I}_R \cosh(p\ell)$$

In compact matrix notation:

$$\begin{bmatrix} \tilde{V}_S \\ \tilde{I}_S \end{bmatrix} = \begin{bmatrix} \cosh(p\ell) & Z_0 \sinh(p\ell) \\ Y_0 \sinh(p\ell) & \cosh(p\ell) \end{bmatrix} \begin{bmatrix} \tilde{V}_R \\ \tilde{I}_R \end{bmatrix}$$

This equation states that sending-end voltage and current are a linear combination of the receiving-end voltage and current, and vice versa. Three parameters describe this model completely: (a) the characteristic impedance of the line Z_0; (b) the propagation constant of the line, p; and (c) the length of the line, ℓ. Note that the model depends only on the product $p\ell$ and the characteristic impedance Z_0. Alternatively, the following parameters completely describe the single-phase transmission line: (a) $\mathrm{A} = \cosh\left(p\ell\right)$, (b) $B = Z_0 \sinh\left(p\ell\right)$, and (c) $C = Y_0 \sinh\left(p\ell\right)$. In terms of the parameters A, B, C, the line equations (3.39) become

$$\tilde{V}_S = A\tilde{V}_R + B\tilde{I}_R \qquad (3.40a)$$
$$\tilde{I}_S = C\tilde{V}_R + A\tilde{I}_R \qquad (3.40b)$$

These parameters are known as the A, B, C constant of the line. Note that

$$A^2 - BC = \cosh^2\left(p\ell\right) - \sinh^2\left(p\ell\right) = 1.0$$

Thus the parameters A, B, and C are not independent. Knowledge of the two is enough to determine the third.

The above model of a single phase transmission lines can be represented with an equivalent circuit. The derivation of equivalent circuits is discussed in section 3.2.5.

3.2.4.2 Three-Phase Transmission Line

The same analysis can be applied to three-phase transmission lines. Assuming sinusoidal steady state, Equations (3.30a) and (3.31b) of the three-phase transmission line become

$$\frac{d^2\tilde{V}(y)}{dy^2} = (R + j\omega L)(G + j\omega C)\tilde{V}(y) \tag{3.41a}$$

$$\frac{d\tilde{V}(y)}{dy} = (R + j\omega L)\tilde{I}(y) \tag{3.41b}$$

Define the following matrices:

$$Z_s = R + j\omega L$$
$$Y_{sh} = G + j\omega C$$

Then

$$\frac{d^2\tilde{V}(y)}{dy^2} = Z_s Y_{sh}\tilde{V}(y) \tag{3.42a}$$

$$\frac{d\tilde{V}(y)}{dy} = Z_s \tilde{I}(y) \tag{3.42b}$$

The foregoing matrix differential equations in complex variables fully describe the performance of a general three-phase transmission line. Solution of these equations for specified boundary conditions will yield the electric currents and voltages of any phase at any location of the line. However, solution of the equations above is rather difficult. In the following section we discuss transformations that decompose the matrix equations (3.42) into scalar equations. In this way, the solution of the matrix equations (3.42) reduces to the solution of a set of scalar equations.

3.2.4.3 Modal Decomposition

The model of a three-phase transmission line under sinusoidal steady state condition is defined by Equations (3.42). Solution of these equations is in general complex because the matrices Z_s, Y_{sh} are full matrices resulting in a set of three coupled differential equations. To simplify the solution, observe that it is possible to find a transformation T of the voltage and current vector $\tilde{V}(y)$ and $\tilde{I}(y)$ as follows:

$$\tilde{V}(y) = T\tilde{V}^m(y) \quad \text{or} \quad \tilde{V}^m(y) = T^{-1}\tilde{V}(y) \tag{3.43a}$$

$$\tilde{I}(y) = T\tilde{I}^m(y) \quad \text{or} \quad \tilde{I}^m(y) = T^{-1}\tilde{I}(y) \tag{3.43b}$$

where T is a 3×3 matrix, $\tilde{V}^m(y)$ are the transformed voltages at location y of the line, and $\tilde{I}^m(y)$ are the transformed currents at location y of the line. Substitution of the transformation above into Equation (3.42) and subsequent pre-multiplication of the resulting equation by T results in

$$\frac{d^2\tilde{V}^{(m)}(y)}{dy^2} = T^{-1}Z_s Y_{sh} T\tilde{V}^{(m)}(y) = T^{-1}Z_s T T^{-1} Y_{sh} T\tilde{V}^{(m)}(y) \qquad (3.44a)$$

$$\frac{d\tilde{V}^{(m)}(y)}{dy} = T^{-1}Z_s T\tilde{I}^{(m)}(y) \qquad (3.44b)$$

Now assume that T has been selected in such a way that the matrices $T^{-1}Z_s T$ and $T^{-1}Y_{sh}T$ are diagonal matrices. In this case Equations (3.44) represent six uncoupled differential equations. The voltages $\tilde{V}^m(y)$ are called the modal voltages of the line and the transformation T is called a modal transformation matrix. Similarly, the currents $\tilde{I}^m(y)$ are called the modal currents. The procedures is called the modal decomposition. The advantages of modal decomposition are obvious. Solution of the decoupled equations (3.44) is identical to solution methods for single phase lines.

3.2.4.4 Sequence Models

A special case of the modal decomposition results in what is known as the sequence models of a three-phase line. Specifically, many transmission lines are transposed or their construction is such that the mutual parameters (inductance, capacitance) are approximately the same for any pair of phases and the phase self-parameters are also approximately the same for the three phases. For this reason it is justifiable to approximate a three-phase power line with a symmetric line. Mathematically, this is equivalent to assuming that the matrices Z and Y' have the following symmetric structure:

$$Z_s = \begin{bmatrix} z_{s,s} & z_{s,m} & z_{s,m} \\ z_{s,m} & z_{s,s} & z_{s,m} \\ z_{s,m} & z_{s,m} & z_{s,s} \end{bmatrix}$$

$$Y_{sh} = \begin{bmatrix} y_{sh,s} & y_{sh,m} & y_{sh,m} \\ y_{sh,m} & y_{sh,s} & y_{sh,m} \\ y_{sh,m} & y_{sh,m} & y_{sh,s} \end{bmatrix}$$

Note that if the matrices Z_s *and* Y_{sh} do not have this form, which is usually the case, they are put in this form using the following equations:

$$z_{s,s} = \frac{1}{3}\left(z_{aa} + z_{bb} + z_{cc}\right)$$

$$z_{s,\mathrm{m}} = \frac{1}{3}\left(z_{ab} + z_{bc} + z_{ca}\right)$$

$$y_{sh,s} = \frac{1}{3}\left(y_{aa} + y_{bb} + y_{cc}\right)$$

$$y_{sh,\mathrm{m}} = \frac{1}{3}\left(y_{ab} + y_{bc} + y_{ca}\right)$$

The product $Z_s Y_{sh}$ of the two matrices is computed to be

$$Z_s Y_{sh} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_2 \\ \alpha_2 & \alpha_1 & \alpha_2 \\ \alpha_2 & \alpha_2 & \alpha_1 \end{bmatrix}$$

where

$$\alpha_1 = z_{s,s} y_{sh,s} + 2 z_{s,m} y_{sh,m}$$

$$\alpha_2 = z_{s,m} y_{sh,m} + z_{s,s} y_{sh,m} + z_{s,m} y_{sh,s}$$

Now under the discussed assumption of symmetry, the modal transformation matrix T is defined as follows:

$T = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$, where $a = e^{j120^0}$. Note that the inverse of this matrix is:

$$T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}$$

The modal voltages $\tilde{V}^m(y)$ in this case will be denoted by

$$\tilde{V}_{120}(y) = \begin{bmatrix} \tilde{V}_1(y) \\ \tilde{V}_2(y) \\ \tilde{V}_0(y) \end{bmatrix}$$

and the modal currents will be denoted by

$$\tilde{I}_{120}(y) = \begin{bmatrix} \tilde{I}_1(y) \\ \tilde{I}_2(y) \\ \tilde{I}_0(y) \end{bmatrix}$$

Upon substitution of this modal transformation into Equation (3.44), we obtain

$$\frac{d^2\tilde{V}_{120}(y)}{dy^2} = M_{seq}\tilde{V}_{120}(y) \tag{3.45a}$$

$$\frac{d\tilde{V}_{120}(y)}{dy} = Z_{seq}\tilde{I}_{120}(y) \tag{3.45b}$$

where

$$M_{seq} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_0 \end{bmatrix}$$

$$Z_{seq} = \begin{bmatrix} z_1 & 0 & 0 \\ 0 & z_1 & 0 \\ 0 & 0 & z_0 \end{bmatrix}$$

$$m_1 = \alpha_1 - \alpha_2 = p_1^2$$
$$m_0 = \alpha_1 + 2\alpha_2 = p_0^2$$
$$z_1 = z_s - z_m$$
$$z_0 = z_s + 2z_m$$

The matrix equations (3.45) represent six scalar equations. It is expedient to write the six scalar equations explicitly and grouped them into three groups of two as follows:

$$\frac{d^2\tilde{V}_1(y)}{dy^2} = p_1^2\tilde{V}_1(y) \tag{3.46a}$$

$$\frac{d\tilde{V}_1(y)}{dy} = z_1\tilde{I}_1(y) \tag{3.46b}$$

$$\frac{d^2\tilde{V}_2(y)}{dy^2} = p_1^2\tilde{V}_2(y) \tag{3.47a}$$

$$\frac{d\tilde{V}_2(y)}{dy} = z_1\tilde{I}_2(y) \tag{3.47b}$$

$$\frac{d^2\tilde{V}_0(y)}{dy^2} = p_0^2\tilde{V}_0(y) \tag{3.48a}$$

$$\frac{d\tilde{V}_0(y)}{dy} = z_0\tilde{I}_0(y) \tag{3.48b}$$

It is now apparent that Equations (3.46), (3.47) and (3.48) represent three single-phase lines. We shall refer to Equation (3.46) as the positive sequence model of the line, Equations (3.47) as the negative sequence model, and Equations (3.48) as the zero sequence model of the line. Note that the parameters (p_1, z_1) of the negative sequence model are identical to those of the positive sequence model. Collectively, we shall refer to Equation (3.45) or equivalently. Equations (3.46), (3.47), and (3.48) as the sequence model of a three-phase line. The modal voltages and currents will be referred to as the symmetrical components of the currents and voltages. In addition, the parameters of the sequence models are defined as follows:

p_1, p_0 will be called the positive (or negative) and zero sequence propagation constants of the line.

z_1, z_0 will be called the per-unit length positive (or negative) and zero sequence series impedance of the line.

For the purpose of completing the discussion of the sequence model, recall that

$$M_{seq} = T^{-1}Z_sY_{sh}T$$

Consider the following:

$$M_{seq} = T^{-1}Z_sY_{sh}T = T^{-1}Z_sTT^{-1}Y_{sh}T = Z_{s,seq}Y_{sh,seq}$$

where:

$$Z_{s,seq} = T^{-1}Z_sT \qquad\qquad Y_{sh,seq} = T^{-1}Y_{sh}T$$

Upon evaluation of $Y_{sh,seq}$, we have

$$Y_{sh,seq} = \begin{bmatrix} y_{sh,1} & 0 & 0 \\ 0 & y_{sh,1} & 0 \\ 0 & 0 & y_{sh,0} \end{bmatrix}$$

where

$$y_{sh,1} = y_{sh,s} - y_{sh,m}$$
$$y_{sh,0} = y_{sh,s} + 2y_{sh,m}$$

Note that y'_1, y'_0 are the per-unit length positive (or negative) and zero sequence shunt admittance of the line.

In terms of the parameters y'_1, y'_0, the propagation constants p_1, p_2 and p_0 are

$$p_1 = p_2 = \sqrt{z_{s,1} y_{sh,1}} \tag{3.49a}$$

$$p_0 = \sqrt{z_{s,0} y_{sh,0}} \tag{3.49b}$$

and the characteristic impedances:

$$Z_0^1 = Z_0^2 = \sqrt{\frac{z_{s,1}}{y_{sh,1}}} \tag{3.50a}$$

$$Z_0^0 = \sqrt{\frac{z_{s,0}}{y_{sh,0}}} \tag{3.50b}$$

In summary, application of the symmetrical component transformation on the three-phase line equations results in the sequence models of the line (i.e., the positive, negative, and zero sequence models). Each model is identical to a single-phase line model. The parameters of the positive sequence models are equal to the parameters of the negative sequence model.

A physical interpretation of the sequence models of a three-phase transmission line is expedient. For this purpose, assume that only one symmetrical component of the voltage or current is present. As an example, assume that only the positive sequence current is present,
i.e. $\tilde{I}_1(y) \neq 0, \quad \tilde{I}_2(y) = 0, \quad and \quad \tilde{I}_0(y) = 0$
The actual phase currents $\tilde{I}_a(y)$, $\tilde{I}_b(y)$, $\tilde{I}_c(y)$ are obtained from the inverse transformation T^{-1}:

$$\tilde{I}_{abc}(y) = T \begin{bmatrix} \tilde{I}_1(y) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{I}_1(y) \\ \tilde{I}_1(y) e^{-j120^\circ} \\ \tilde{I}_1(y) e^{-j240^\circ} \end{bmatrix}$$

As is evident from the equation above, the three phase currents are balanced and of the positive sequence. The case is depicted in Figure 3.22a, which illustrates the three phase currents. Note that the electric current in the ground is zero.

Similarly, if we assume that only the negative sequence component is present $\left[i.e. \quad \tilde{I}_1(y) = 0, \quad \tilde{I}_2(y) \neq 0, \quad and \quad \tilde{I}_0(y) = 0 \right]$, the actual phase currents are

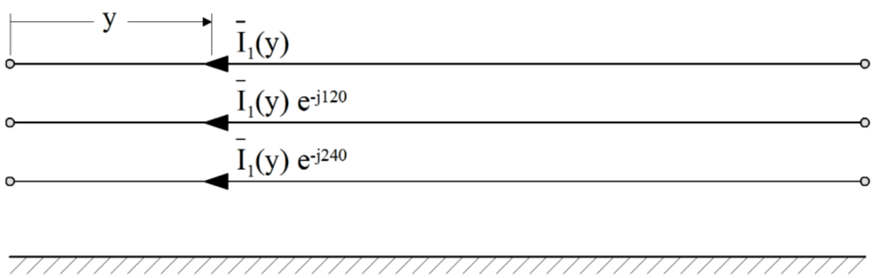
$$\tilde{I}_{abc}(y) = T\begin{bmatrix} 0 \\ \tilde{I}_2(y) \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{I}_2(y) \\ \tilde{I}_2(y)e^{j120^\circ} \\ \tilde{I}_2(y)e^{j240^\circ} \end{bmatrix}$$

Again, as is evident from equation above, the three phase currents are balanced but of the negative sequence. The case is depicted in Figure 3.22b, which illustrates the three phase currents. Note that the electric current in the earth is zero.

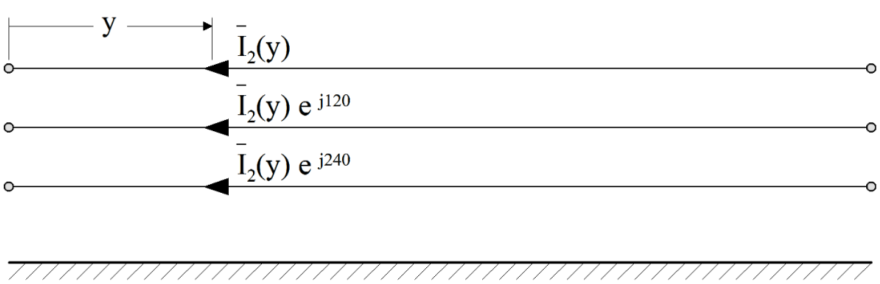
Finally, if we assume that only the zero sequence component is present, $\left[i.e.\ \ \tilde{I}_1(y)=0,\ \ \tilde{I}_2\left(y\right)=0,\ \ and\ \ \tilde{I}_0\left(y\right)\neq 0\right]$, the actual phase currents are

$$\tilde{I}_{abc}(y) = T\begin{bmatrix} 0 \\ 0 \\ \tilde{I}_0(y) \end{bmatrix} = \begin{bmatrix} \tilde{I}_0(y) \\ \tilde{I}_0(y) \\ \tilde{I}_0(y) \end{bmatrix}$$

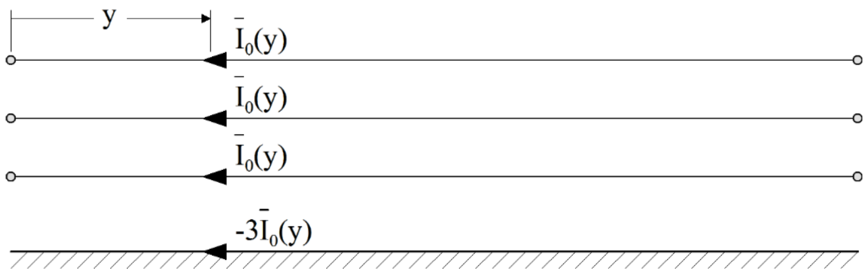
As is evident from the equation above, all three phase currents are identical. Sequence cannot be defined for these currents-thus the name "zero sequence". The earth current will be the negative of the sum of the phase currents [i.e., $-3\tilde{I}_0(y)$]. The case is depicted in Figure 3.22c.



(a)



(b)



(c)

Figure 3.20: Illustration of the Symmetrical Components on a Transmission Line

(a) Positive Sequence Components, (b) Negative Sequence Components,(c) Zero Sequence Components

Example E3.4: Consider the transmission line of Example E3.2 and Example E3.3. Compute the sequence parameters of the line.

Solution: The Z matrix of the line, computed at 60 Hz, and then symmetrized is:

$$Z = \begin{bmatrix} 0.2323 + j0.8901 & 0.0593 + j0.4330 & 0.0593 + j0.4330 \\ 0.0593 + j0.4330 & 0.2323 + j0.8901 & 0.0593 + j0.4330 \\ 0.0593 + j0.4330 & 0.0593 + j0.4330 & 0.2323 + j0.8901 \end{bmatrix} \times 10^{-3} \ ohms/meter$$

The Y' matrix of the line, computed at 60 Hz, and then symmetrized is:

$$Y' = \begin{bmatrix} j3.0331 & -j0.5556 & -j0.5556 \\ -j0.5556 & j3.0331 & -j0.5556 \\ -j0.5556 & -j0.5556 & j3.0331 \end{bmatrix} \times 10^{-9} \ S/m$$

The product ZY' is

$$ZY' = \begin{bmatrix} -2.2186 + jo.6387 & -0.5782 + j0.0178 & -0.5782 + j0.0178 \\ -0.5782 + j0.0178 & -2.2186 + jo.6387 & -0.5782 + j0.0178 \\ -0.5782 + j0.0178 & -0.5782 + j0.0178 & -2.2186 + jo.6387 \end{bmatrix} \times 10^{-12} \ \mathrm{m}^{-2}$$

Upon application of the transformation T, we have

$$z_1 = (0.173 + j0.4571) \times 10^{-3} \ \Omega/\ \mathrm{m}$$

$$z_0 = (0.3509 + j1.7561) \times 10^{-3} \ \Omega/\ \mathrm{m}$$

$$y'_1 = j3.5887 \times 10^{-9} \ \mathrm{S}/\ \mathrm{m}$$

$$y'_0 = j1.9219 \times 10^{-9} \ \mathrm{S}/\ \mathrm{m}$$

$$m_1 = (-1.6404 + j0.6209) \times 10^{-12} \ \mathrm{m}^{-2}$$

$$m_0 = (-3.375 + j0.6743) \times 10^{-12} \ \mathrm{m}^{-2}$$

The characteristic impedance and propagation constants of the sequence components are:

$$Z_0^1 = \sqrt{\frac{z_1}{y'_1}} = 369 e^{-j10.365^\circ} \ \Omega$$

$$p_1 = p_2 = \sqrt{z_1 y'_1} = 1.3244 \times 10^{-6} e^{j79.63^\circ} \ \mathrm{m}^{-1}$$

Zero sequence components:

$$Z_0^0 = \sqrt{\frac{z_0}{y'_0}} = 965 e^{-j5.65^\circ} \quad ohms$$

$$p_0 = \sqrt{z_0 y'_0} = 1.8552 \times 10^{-6} e^{j84.35^\circ} \text{ m}^{-1}$$

In summary of this section, the symmetrical component transformation provides a tool for the simplified solution of the equations of a multiphase line. It also yields the sequence models of a three-phase line. In this case the analysis of three-phase lines is reduced to the analysis of three single-phase transmission lines, the positive sequence, negative sequence and zero sequence line models.

3.2.5 Transmission Line Equivalent Circuits

In previous section we have developed models of single-phase, as well as three-phase lines, under steady-state conditions. The models are in terms of the A, B, C parameters or alternatively, in terms of the characteristic impedance, propagation constant, and line length. An alternative representation of the transmission lines under steady-state conditions is by means of equivalent circuits. This approach is more attractive because of the familiarity of engineers with circuits. This section presents the computation of equivalent circuits from the transmission line parameters. Only the single-phase line case will be demonstrated. Since a three-phase line can be reduced to three single-phase lines by means of the symmetrical component transformation, the extension to three-phase lines will be left to the reader as an exercise.

Consider Equations (3.40) of a single-phase line in terms of the terminal currents and voltages. From realization theory it is known that a two-port circuit can be found which is described with the same equations. Furthermore, this two-port circuit is not unique. From the multiplicity of equivalent circuits, one particular circuit has been popular among power engineers: the π equivalent. This circuit is introduced next.

To a transmission line with constants A, B, C, corresponds a π-equivalent circuit with elements Y_π, Y'_π as in Figure 3.21. The elements Y_π, Y'_π of the π-equivalent circuit are computed by first expressing the line terminal currents as a function of the line terminal voltages and subsequent application of circuit theory. Specifically, the line terminal currents as a function of line terminal voltages in terms of the parameters A, B and C, are:

$$\tilde{I}_1 = \frac{A}{B}\tilde{V}_1 - \frac{1}{B}\tilde{V}_2 \tag{3.51a}$$

$$\tilde{I}_2 = -\frac{1}{B}\tilde{V}_1 + \frac{A}{B}\tilde{V}_2 \tag{3.51b}$$

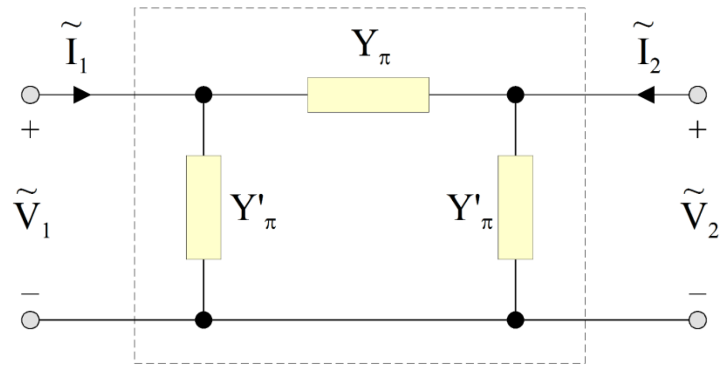


Figure 3.21: π -equivalent Circuit

On the other hand, the equation for the circuit of Figure 3.23 are:

$$\tilde{I}_1 = (Y_\pi + Y_\pi^{'})\tilde{V}_1 - Y_\pi \tilde{V}_2$$
$$\tilde{I}_1 = -Y_\pi \tilde{V}_1 + (Y_\pi^{'} + Y_\pi)\tilde{V}_2$$

For equivalence, the following must hold:

$$Y_\pi = \frac{1}{B} \tag{3.52a}$$

$$Y_\pi^{'} = \frac{A-1}{B} \tag{3.52b}$$

Equations (3.52) define the parameters of the π-equivalent circuit of a line. These parameters are the series admittance of the equivalent circuit, Y_π, and the shunt admittance of the equivalent circuit, Y'_π. The impedance parameters of the π equivalent circuit will be:

$$Z_\pi = \frac{1}{Y_\pi} = B = Z_0 \sinh(p\ell)$$

$$Z_\pi^{'} = \frac{1}{Y_\pi^{'}} = \frac{B}{A-1} = \frac{Z_0 \sinh(p\ell)}{\cosh(p\ell) - 1}$$

Nominal π-Equivalent Circuit: The nominal π-equivalent circuit of a transmission line is an approximation of the exact equivalents. In general, these approximations are valid for short lines; thus the name "short-line equivalent" is alternatively used. Consider the π-equivalent circuit described by the parameters Z_π, and Z'_π, as derived earlier. The nominal π-equivalent circuit is obtained by approximating the hyperbolic sine and cosine functions. Specifically, assuming that $p\ell << 1$, (this assumption is equivalent to the assumption of short line, i.e. ℓ is small), the functions are expanded into a series and then only the major terms are retained:

$$\sinh(p\ell) \cong p\ell$$

$$\cosh(p\ell) \cong 1 + \frac{(p\ell)^2}{2}$$

Substitution of the approximations above in the equations for the parameters Z_π, and Z'_π, yields:

$$Z_\pi \cong Z_0 p\ell = z\ell = (r + j\omega L)\ell \qquad (3.53a)$$

$$Z'_\pi \cong \frac{Z_0 p\ell}{1 + \frac{(p\ell)^2}{2} - 1} = 2\frac{z'}{\ell} = \frac{2}{(g + j\omega C)\ell} \qquad (3.53b)$$

Normally, the nominal pi-equivalent approximation can be made when the product pl is small. For example for 0.1% accuracy, it should be less than 0.025. For 60 Hz model, this means a line of about less than 12 miles.

The computation of equivalent circuits for a transmission line will be illustrated with an example.

Example E3.5: Consider the positive sequence model of the three-phase line of Example E3.4. The computed parameters are $Z_0 = 369e^{-j10.36°}$ ohms and $p = 1.3244\times10^{-6}\ e^{j79.63°}\ m^{-1}$. Assume that the line is 85 miles long and compute:

(a) The π-equivalent circuit.
(b) The nominal π-equivalent circuit.
(c) Compare the two models

Solution: First, the A, B, C parameters of the line are computed as follows:

$$p\ell = 0.18113e^{j79.63^\circ}$$

$$\cosh p\ell = 0.984659 + j0.005809 = 0.984676e^{j0.34^\circ}$$

$$\sinh p\ell = 0.032092 + j0.177323 = 0.180204e^{j79.74^\circ}$$

$$A = 0.984676e^{j0.34^\circ},\ B = 66.4953e^{j69.38^\circ},\ C = 0.000488e^{j90.1^\circ}$$

(a)

$$Z_\pi = B = (23.417 + j62.235)\ \Omega$$

$$Z'_\pi = \frac{B}{A-1} = (8.489 - j4053.59)\ \Omega$$

The π-equivalent circuit is illustrated in Fig. E6.1a.

(b) $$Z_\pi = z\ell = Z_0 p\ell = 66.837e^{j69.27^\circ}\ \Omega = (23.658 + j62.509)\ \Omega$$

$$Z'_\pi = \frac{2z'}{\ell} = \frac{2Z_0}{p\ell} = 4079.42e^{-j90^\circ}\ \Omega = (-j4074.4)\ \Omega$$

The nominal π-equivalent circuit is illustrated in Figure E3.6c.

(c) The two equivalent circuits are very close.

Example E3.6: A three phase transmission line has the following per unit length positive sequence parameters

Resistance: R = 0.08 ohms/mile
Inductance: L = 1.1×10^{-6} Henries/meter
Capacitance: C = 10.8×10^{-12} Farads/meter

The line is 200 miles long.

a) Compute the positive sequence π-equivalent circuit of the line.
b) Compute the positive sequence nominal π equivalent circuit of the line.

Solution:

a)
$$z = 0.08 + j0.667\ \Omega/\text{mi}$$
$$y = j6.551 \times 10^{-6}\ \text{S/mi}$$

$$Z_c = \sqrt{\frac{z}{y}} = 320.28\angle -3.418^\circ = 319.71 - j19.10\Omega$$

$$\gamma\ell = \ell\sqrt{zy} = 0.4196\angle 86.58^\circ = 0.0250 + j0.4189$$

$$Z' = Z_c \sinh \gamma\ell = 130.417\angle 83.36^\circ\ \Omega = 15.08 + j\ 129.64\ \Omega$$

$$\frac{y'}{2} = \frac{1}{Z_c}\tanh\left(\frac{\gamma\ell}{2}\right) = 1.19 \times 10^{-6} + j6.646 \times 10^{-4}\ S$$

$$\frac{2}{y'} = 2.682 - j\ 1504.2\ \Omega$$

b) The nominal equivalent circuit parameters are:

$$Z = zl = 16 + j133.4\ \ \Omega = 134.35\ \underline{/\ 83.16^0}\ \ \Omega$$

$$\frac{2}{Y} = \frac{2}{\ell y} = 1526.5\ \Omega$$

The results for this example are shown in Figure E3.9.

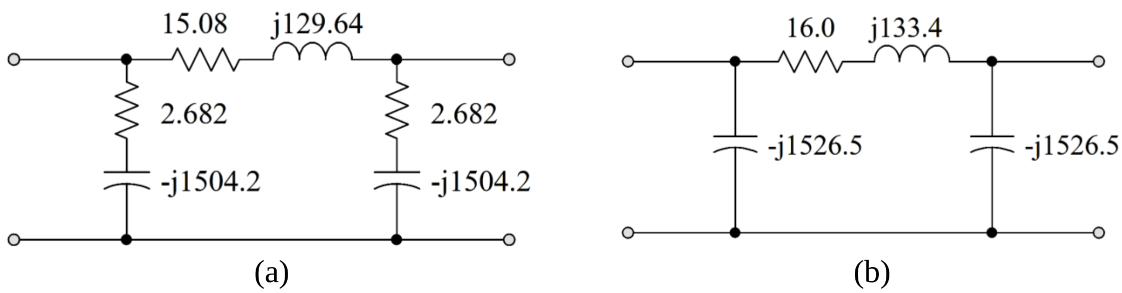


Figure E3.6: The Positive Sequence Equivalent Circuits
(a) pi-equivalent Circuit, (b) Nominal pi-equivalent Circuit
(All indicated values are in ohms)

The above computational methods for the parameters of lines and equivalent circuits of lines have been demonstrated for the fundamental power frequency. The same procedures can be applied for any frequency. As an example we apply these computational procedures for the line parameters and equivalent circuits for the 7th harmonic (420 Hz).

Example E3.6: Consider the 230 kV transmission line of Figure E3.6. For simplicity, assume that the phase conductors are aluminum one inch diameter of conductivity 40,000,000 S/m. The line is 57 miles long. Compute the positive, negative and zero sequence π-equivalent circuit of the line for the 7th harmonic. For simplicity, neglect the shield wires.

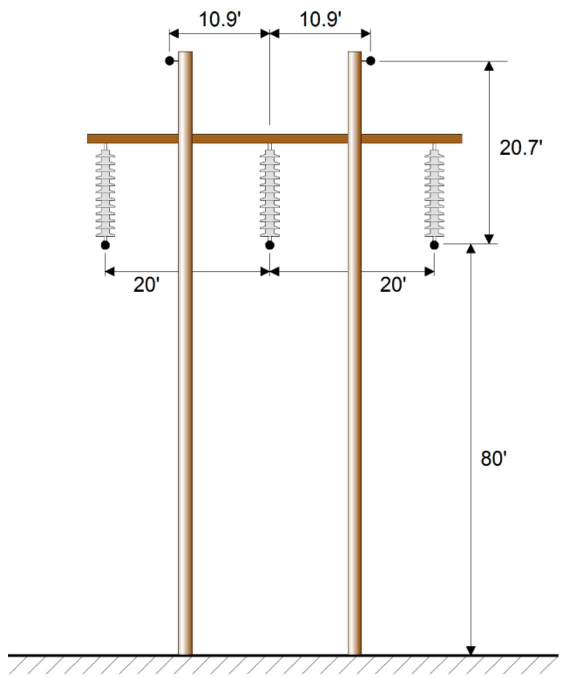


Figure E3.6

Solution: The resistance and inductance matrices are computed using Carsons equations. The results are:

$$k = \sqrt{k\omega\sigma} = 364.26\, m^{-1}$$
ka=4.625

$$M_0(ka) = 4.7$$
$$\theta_0(ka) = 162.0^0$$
$$M_1(ka) = 4.3$$
$$\theta_1(ka) = 260.0^0$$

$$r_{ac} = 9.96x10^{-5}\ ohms/m$$

$$\xi = 0.5689$$

$$d = ae^{-\frac{\xi}{4}} = 0.011016m = 0.03614\ ft$$

$$r_e = 0.00159f = 0.6678 ohms/mi = 41.5x10^{-5}\ ohms/m$$

$$D_e = 2160\sqrt{\frac{\rho}{f}} = 1{,}053.97\ ft = 321.25\, m$$

$$R = 10^{-5}\begin{bmatrix} 51.46 & 41.5 & 41.5 \\ 41.5 & 51.46 & 41.5 \\ 41.5 & 41.5 & 51.46 \end{bmatrix}\Omega / m$$

$$L = 10^{-6}\begin{bmatrix} 2.056 & 0.793 & 0.654 \\ 0.793 & 2.056 & 0.793 \\ 0.654 & 0.793 & 2.056 \end{bmatrix} H / m$$

$$C = 2\pi\varepsilon\begin{bmatrix} 8.253 & 2.087 & 1.416 \\ 2.087 & 8.253 & 2.087 \\ 1.416 & 2.087 & 8.253 \end{bmatrix}^{-1} = 2\pi\varepsilon\begin{bmatrix} 0.1312 & -0.0294 & -0.0151 \\ -0.0294 & 0.1360 & -0.0294 \\ -0.0151 & -0.0294 & 0.1312 \end{bmatrix}$$

From above matrices the following parameters are obtained:

Positive/Negative sequence:

$$r_1 = 99.6\,\mu\Omega / m,\quad L_1 = 1.3093\,\mu H / m,\quad C_1 = 8.758\,pF / m$$

Zero sequence:

$$r_0 = 1344.6\mu\Omega / m,\ L_0 = 3.5493\,\mu H / m,\quad C_0 = 4.651\,pF / m$$

The parameters of the equivalent circuits are:

Positive/Negative Sequence

3.3 Cable Modeling

Power cables are very common for medium and low voltage distribution systems. Recently, we have seen increased used of UG transmission cables 138 kV to 345 kV. There is a variety of cable designs. Figure 3.22 illustrates a concentric neutral medium voltage cable construction. Figure 3.23 illustrates a three phase cable construction (medium or high voltage). Figure 3.24 illustrates a three wire secondary voltage cable (2x120V).

Cable designs have rather complicated geometries and accurate analysis and computation of their parameters is very complex and for all practical purposes it is done by computer modeling. In this chapter, we present the basis of and computational procedures for the evaluation of the parameters of cables. The theory is followed by examples of cable parameters for usual cable geometries.

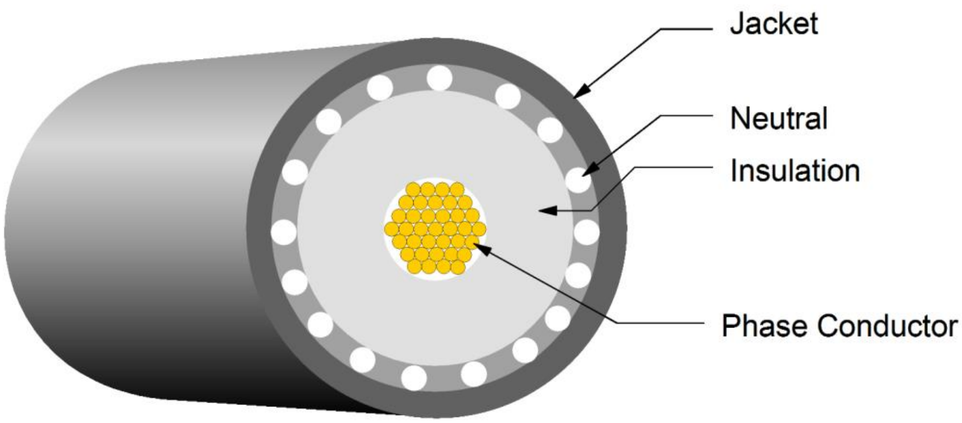


Figure 3.22: Concentric Neutral Medium Voltage Cable

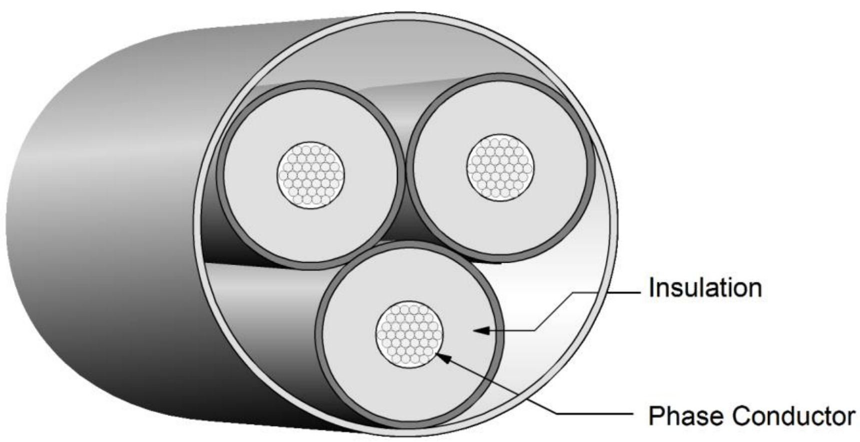


Figure 3.23: Three-Phase Medium Voltage Cable

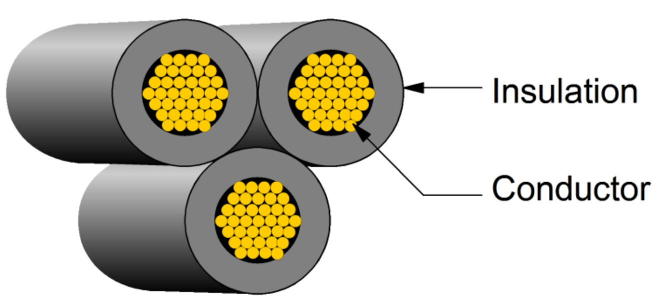


Figure 3.24: Secondary System 600V Class Cable

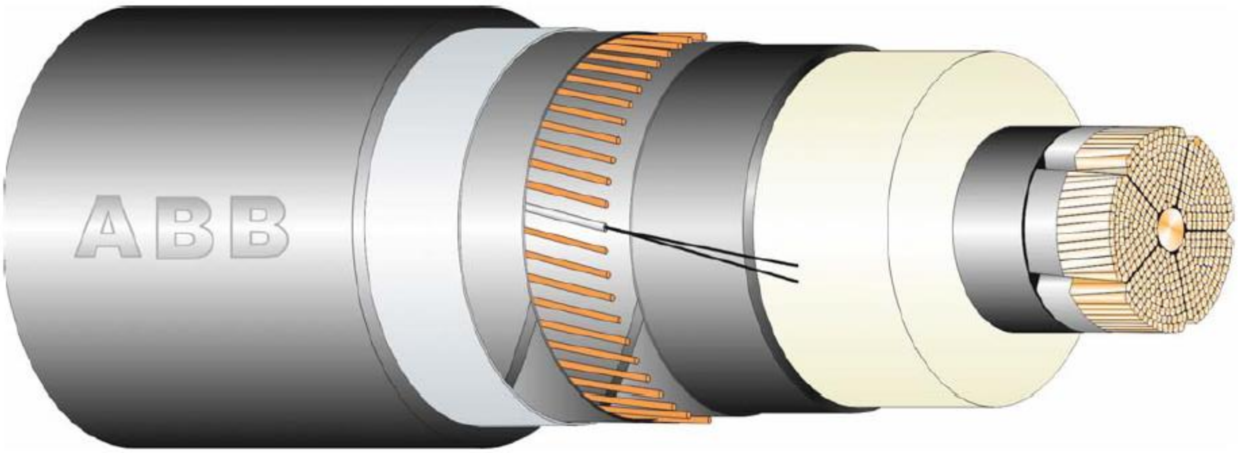


Figure 3.25: 135 kV, 3000 kcm Transmission Cable – Manufacturer: ABB
(note the fibers in a metal tube occupying the location of one copper shield wire)

3.3.1 Methodologies for Cable Parameter Computation

to be added

3.3.2 Typical Cable Parameters

The figures that follow present typical variations of concentric neutral cable parameters versus frequency. One should observe that while the cable reactance is rather insensitive to soil resistivity, the cable resistance is quite sensitive to soil resistivity, especially as the frequency increases. For all practical purposes, the parameters of cables are computed by computer programs.

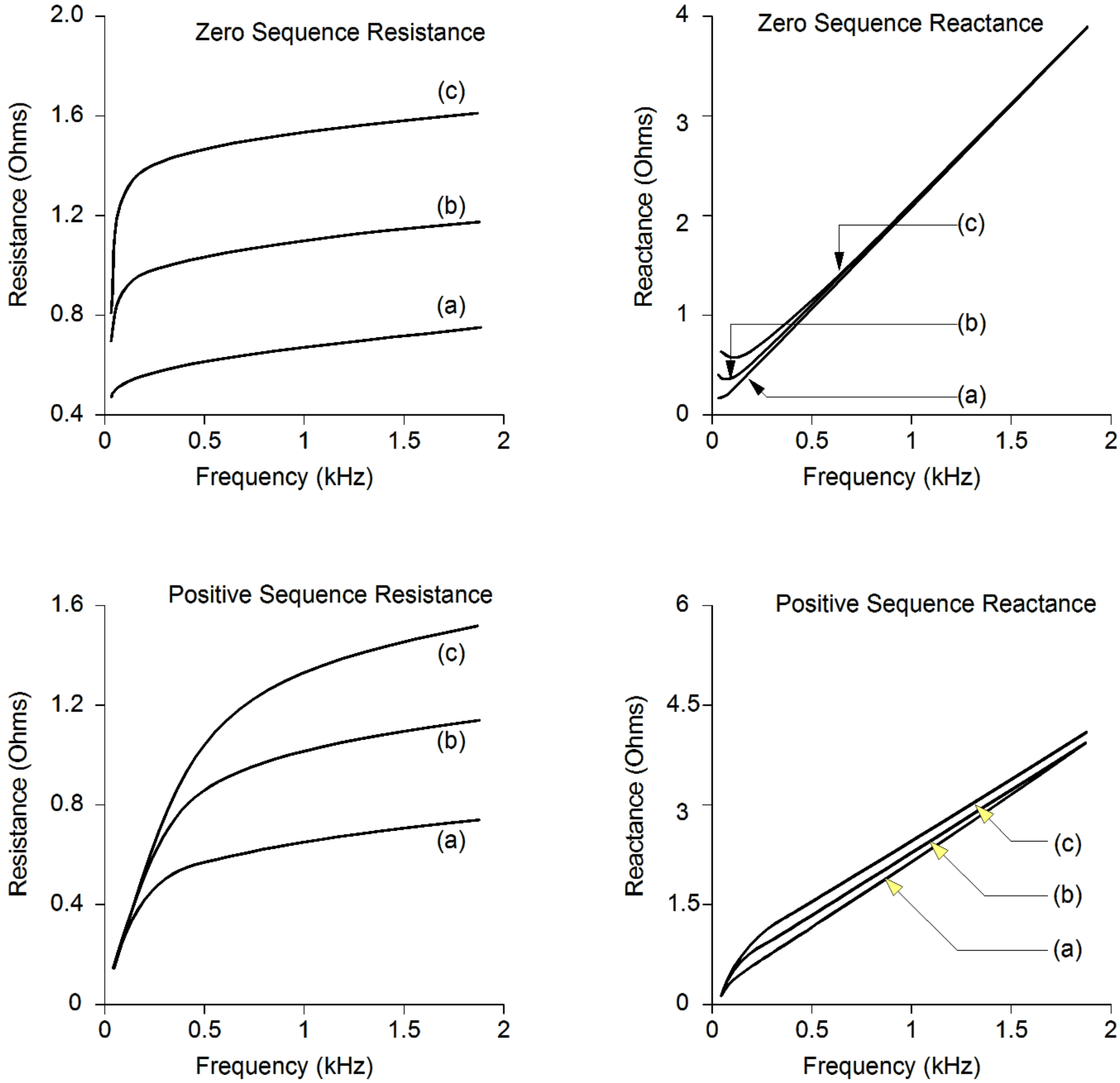


Figure 3.25: Parametric Analysis of 15 kV Concentric Neutral Cable Sequence Parameters
[(a) Soil Resistivity of 10 ohm.m, (b) Soil Resistivity of 100 ohm.m, (c) Soil Resistivity of 1000 ohm.m]

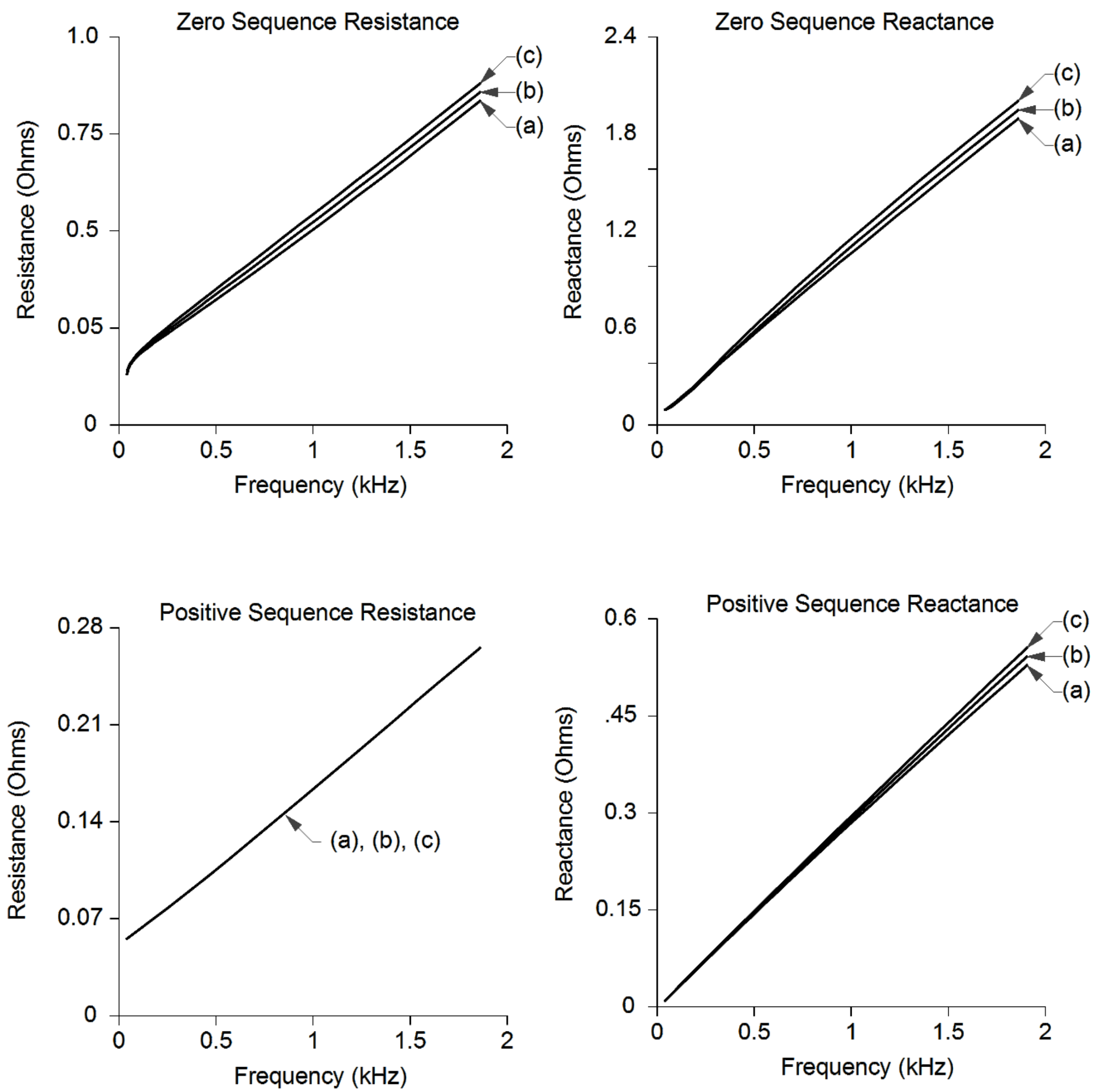
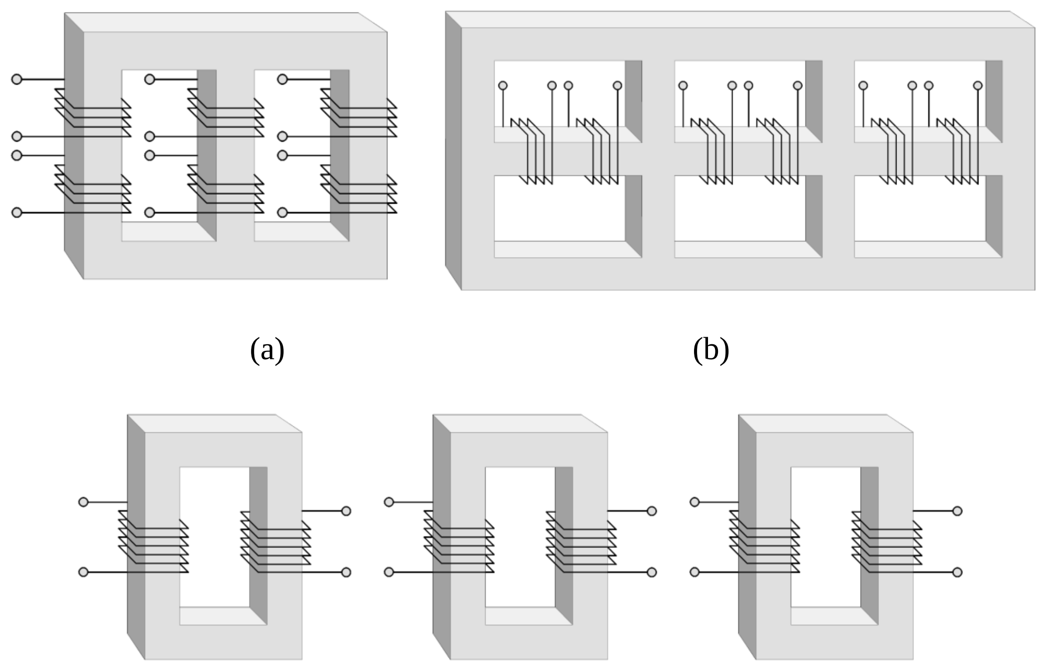


Figure 3.26: Parametric Analysis of 600 V Cable Sequence Parameters
[(a) Soil Resistivity of 10 ohm.m, (b) Soil Resistivity of 100 ohm.m, (c) Soil Resistivity of 1000 ohm.m]

3.4 Transformer Modeling

Transformers can be single phase or three-phase, two windings or multiple windings, and some windings may be center-tapped. In general, the coils of a transformer are electrically isolated from each other enabling the isolation of the circuits that may be connected to these coils. Three phase transformers can be constructed in a number of ways. Three of the most usual constructions are illustrated in Figure 3.27. Figure 3.27a illustrates a three phase core type transformer. The core has three legs, on each leg there are two windings for a total of six windings. Similarly, Figure 3.27b illustrates a shell type transformer which also has six windings. Figure 3.27c illustrates a “bank” of three single phase transformers. This arrangement also has six windings. The six windings of any configuration (a), (b), or (c) are grouped in two groups of three, the primary and the secondary. For example, in Figure 3.27a the primary may be the three windings located on the upper part of each leg and the secondary may be the other three winding. Both the primary and secondary windings may be connected in a delta or wye configurations leading to four possible arrangements of a three phase transformer: (a) delta-delta, (b) wye-delta, (c) delta-wye and (d) wye-wye. These arrangements are schematically represented in Figure 3.28. Note that from the circuit point of view, all three phase transformer constructions are similar, i.e. all have six winding grouped into three phases. However, the magnetic circuit of each one of these constructions is different. For example, the three phase transformer bank consists of three independent magnetic circuits. The shell and core type three phase transformers are characterized with coupled magnetic circuits of the three phases.



(a) (b)

(c)

Figure 3.27: Three Phase Transformers
(a) Core Type, (b) Shell Type, (c) Three Single Phase Transformer Bank

The model of a three phase transformer bank is the simplest since it consists of the interconnection of three single phase transformers. Replacing each one of the single phase transformers with its equivalent circuit, the equivalent circuit of the three phase transformer is obtained. This has been done in Figure 3.29 where the simplified equivalent circuit of a single phase transformer has been used. The Figure illustrates a delta-wye connection.

In subsequent paragraphs we will consider first the ideal three phase transformer model for the purpose of examining its basic characteristics. Then the non-ideal transformer model will be studied. The use of the symmetrical transformation to the three phase transformer model will result in the sequence models.

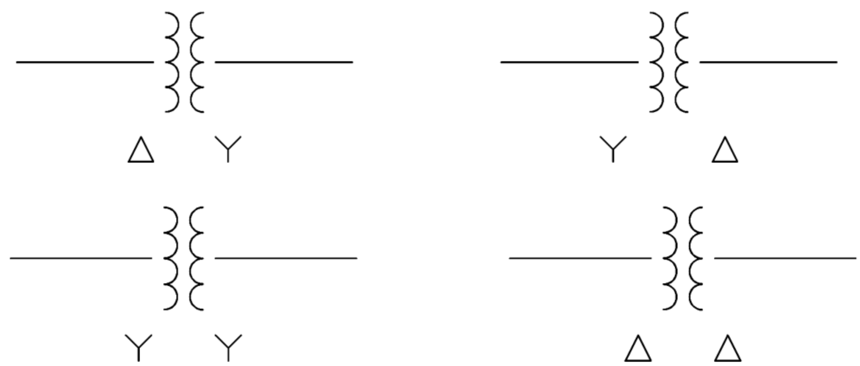


Figure 3.28: Schematic Representation of Three Phase Transformers

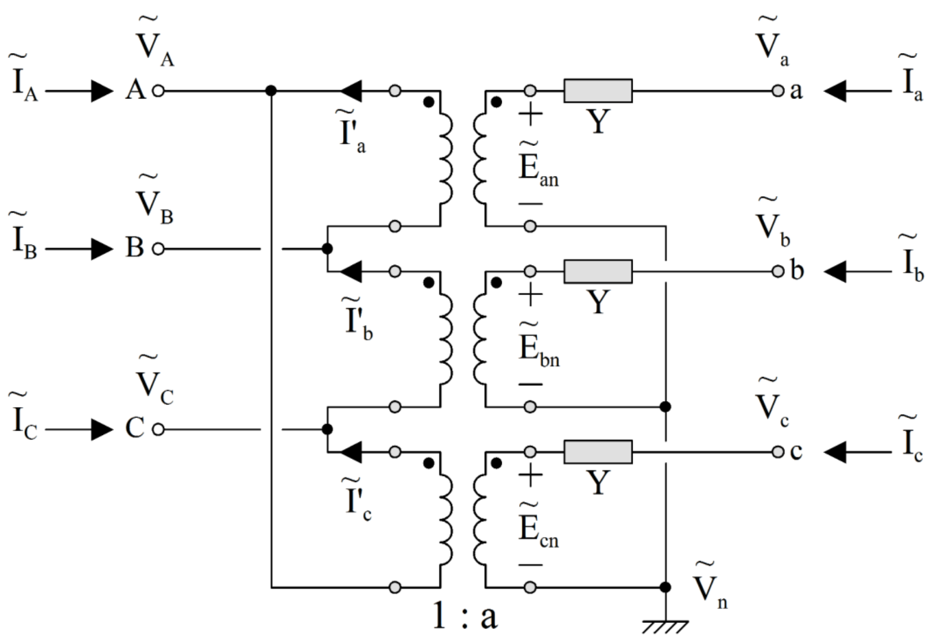


Figure 3.29: Delta-Wye Connected Three Phase Transformer Model

3.4.1 The Ideal Three Phase Transformer

An ideal three phase transformer consists of three ideal single phase transformers. The transformer of Figure 3.29 will be ideal if $\Upsilon = \infty$ (short circuit). The voltage relationships of an ideal three phase transformer are:

$$\tilde{V}_{AB} = a^{-1}\tilde{V}_{an}$$
$$\tilde{V}_{BC} = a^{-1}\tilde{V}_{bn}$$
$$\tilde{V}_{CA} = a^{-1}\tilde{V}_{cn}$$

Under balanced operating conditions, the voltages will be:

$$\tilde{V}_{Bn} = \tilde{V}_{An} e^{-j120^o}$$
$$\tilde{V}_{Cn} = \tilde{V}_{An} e^{-j240^o}$$
$$\tilde{V}_{bn} = \tilde{V}_{an} e^{-j120^o}$$
$$\tilde{V}_{cn} = \tilde{V}_{an} e^{-j240^o}$$

Note that:

$$\tilde{V}_{AB} = \tilde{V}_{An} - \tilde{V}_{Bn} = \tilde{V}_{An} - \tilde{V}_{An} e^{-j120^o} = \sqrt{3}\tilde{V}_{An} e^{j30^o}$$
$$\tilde{V}_{BC} = \tilde{V}_{Bn} - \tilde{V}_{Cn} = \tilde{V}_{An} e^{-j120^o} - \tilde{V}_{An} e^{+j120^o} = \sqrt{3}\tilde{V}_{An} e^{-j90^o} = \sqrt{3}\tilde{V}_{Bn} e^{j30^o}$$
$$\tilde{V}_{CA} = \tilde{V}_{Cn} - \tilde{V}_{An} = \tilde{V}_{An} e^{+j120^o} - \tilde{V}_{An} e^{-j120^o} = \sqrt{3}\tilde{V}_{An} e^{-j210^o}$$

Now the relationship between the primary and secondary voltages can be found.

$$\tilde{V}_{An} = \frac{e^{-j30^0}}{a\sqrt{3}}\tilde{V}_{an}$$
$$\tilde{V}_{Bn} = \frac{e^{-j30^0}}{a\sqrt{3}}\tilde{V}_{bn}$$
$$\tilde{V}_{Cn} = \frac{e^{-j30^0}}{a\sqrt{3}}\tilde{V}_{cn}$$

Above equations indicate that the per phase (positive sequence) equivalent model of a delta-wye connected ideal three phase transformer is a single phase ideal transformer with transformation ratio $n = a\sqrt{3}e^{j30^0}$.

3.4.2 Non-Ideal Three Phase Transformer Model

The non-ideal three phase transformer model can be derived from the proper interconnection of the non-ideal single phase transformers. For simplicity we assume that each single phase transformer is represented with its simplified non-ideal model. For the case of a delta-wye connected transformer, the result is illustrated in Figure 3.29.

For the circuit of Figure 3.29, the following relationships are valid:

$$\tilde{I}_a = (\tilde{V}_a - \tilde{E}_a)Y \qquad \tilde{I}_a^{'} = a\tilde{I}_a$$
$$\tilde{I}_b = (\tilde{V}_b - \tilde{E}_b)Y \qquad \tilde{I}_b^{'} = a\tilde{I}_b$$
$$\tilde{I}_c = (\tilde{V}_c - \tilde{E}_c)Y \qquad \tilde{I}_c^{'} = a\tilde{I}_c$$

$$\tilde{E}_a = a(\tilde{V}_A - \tilde{V}_B) \qquad \tilde{I}_A = \tilde{I}_c^{'} - \tilde{I}_a^{'}$$
$$\tilde{E}_b = a(\tilde{V}_B - \tilde{V}_C) \qquad \tilde{I}_B = \tilde{I}_a^{'} - \tilde{I}_b^{'}$$
$$\tilde{E}_c = a(\tilde{V}_C - \tilde{V}_A) \qquad \tilde{I}_C = \tilde{I}_b^{'} - \tilde{I}_c^{'}$$

Upon elimination of the variables $\tilde{E}_a, \tilde{E}_b, \tilde{E}_c$ and $\tilde{I}_a^{'}, \tilde{I}_b^{'}, \tilde{I}_c^{'}$ and expressing the remaining currents as a function of the voltages we obtain a set of six equations which, written in matrix notation, are:

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \\ \tilde{I}_A \\ \tilde{I}_B \\ \tilde{I}_C \end{bmatrix} = Y \begin{bmatrix} 1 & 0 & 0 & -a & a & 0 \\ 0 & 1 & 0 & 0 & -a & a \\ 0 & 0 & 1 & a & 0 & -a \\ -a & 0 & a & 2a^2 & -a^2 & -a^2 \\ a & -a & 0 & -a^2 & 2a^2 & -a^2 \\ 0 & a & -a & -a^2 & -a^2 & 2a^2 \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \\ \tilde{V}_A \\ \tilde{V}_B \\ \tilde{V}_C \end{bmatrix}$$

Note that above equation expresses the input/output relationship of the three phase transformer. In compact matrix form, above equation can be written as

$$\begin{bmatrix} \tilde{I}_{abc} \\ \tilde{I}_{ABC} \end{bmatrix} = Y \begin{bmatrix} I & -aE \\ -aE^T & a^2F \end{bmatrix} \begin{bmatrix} \tilde{V}_{abc} \\ \tilde{V}_{ABC} \end{bmatrix}$$

where I is the 3x3 identify matrix, and the matrices E and F are:

$$E = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Above equations represent the simplified model of a delta-wye connected three phase transformer. The same procedure can provide the models for other connections, i.e. delta-delta, wye-wye and wye-delta connections.

3.4.3 Sequence Circuits of Three Phase Transformers

Three phase transformers are inherently symmetric three phase elements. This means that by applying the symmetrical transformation, their model can be transformed to three equivalent circuits, namely the positive, negative and zero sequence equivalent circuits. The procedure will be illustrated on a delta-wye connected transformer model developed in the previous paragraph. It should be understood that the procedure equally applies to any other configuration.

The phase voltages and currents are substituted with their corresponding symmetrical components as follows:

$$\tilde{I}_{abc} = T\,\tilde{I}_{120}$$
$$\tilde{I}_{ABC} = T\,\tilde{I}_{120}^{'}$$
$$\tilde{V}_{abc} = T\,\tilde{V}_{120}$$
$$\tilde{V}_{ABC} = T\,\tilde{V}_{120}^{'}$$

Replacing the phase quantities with the symmetrical components, the equation for the three phase transformer becomes:

$$\begin{bmatrix} \tilde{I}_{120} \\ \tilde{I}_{120}^{'} \end{bmatrix} = Y \begin{bmatrix} T^{-1} I\, T & -a\, T^{-1} E\, T \\ -a\, T^{-1} E^T T & a^2 T^{-1} F\, T \end{bmatrix} \begin{bmatrix} \tilde{V}_{120} \\ \tilde{V}_{120}^{'} \end{bmatrix}$$

Note that by direct evaluation, the following apply:

$$T^{-1} I\, T = I$$

$$T^{-1}ET = \begin{bmatrix} \sqrt{3}e^{j30^o} & 0 & 0 \\ 0 & \sqrt{3}e^{-j30^o} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T^{-1}E^TT = \begin{bmatrix} \sqrt{3}e^{-j30^o} & 0 & 0 \\ 0 & \sqrt{3}e^{j30^o} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad T^{-1}FT = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Upon substitution and grouping the six equations into three groups of two we obtain:

$$\tilde{I}_1 = Y\tilde{V}_1 - \sqrt{3}aYe^{j30^o}\tilde{V}_1^{'}$$
$$\tilde{I}_1^{'} = -\sqrt{3}aYe^{-j30^o}\tilde{V}_1 + 3a^2Y\tilde{V}_1^{'}$$

$$\tilde{I}_2 = Y\tilde{V}_2 - \sqrt{3}aYe^{-j30^o}\tilde{V}_2^{'}$$
$$\tilde{I}_2^{'} = -\sqrt{3}aYe^{j30^o}\tilde{V}_2 + 3a^2Y\tilde{V}_2^{'}$$

$$\tilde{I}_0 = Y\,\tilde{V}_0,$$
$$\tilde{I}_0^{'} = 0$$

Note that above relations represent three independent set of equations corresponding to three equivalent circuits which are illustrated in Figure 3.30. An example will illustrate the procedure.

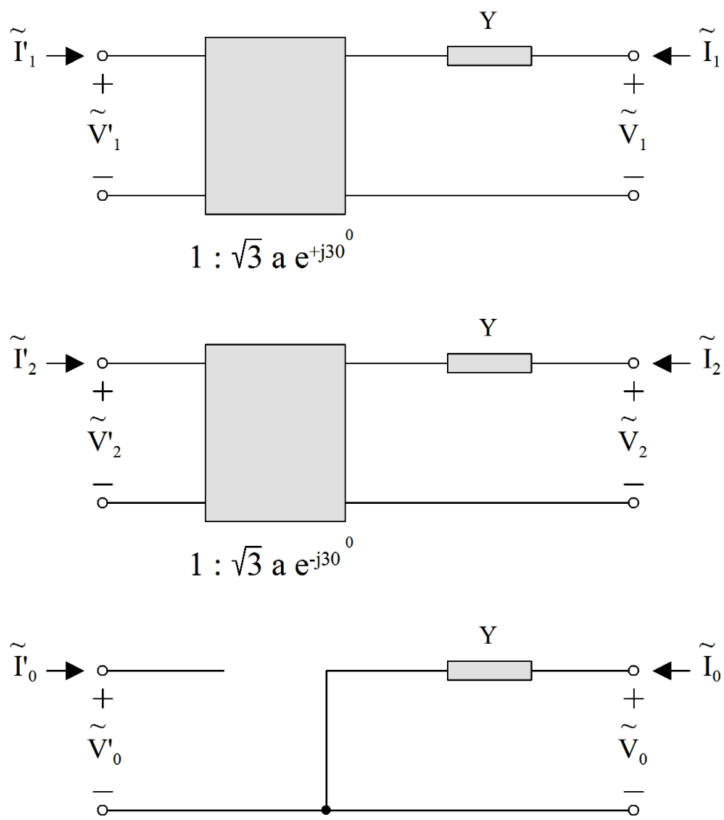
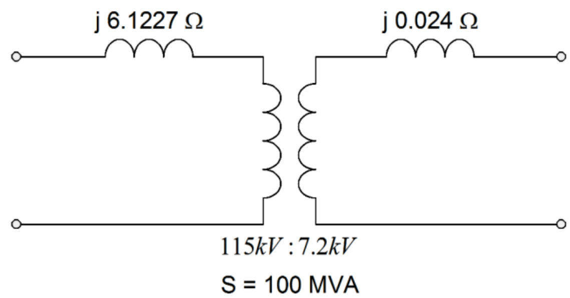


Figure 3.30: Sequence Equivalent of a Delta - Wye Connected Three Phase Transformer

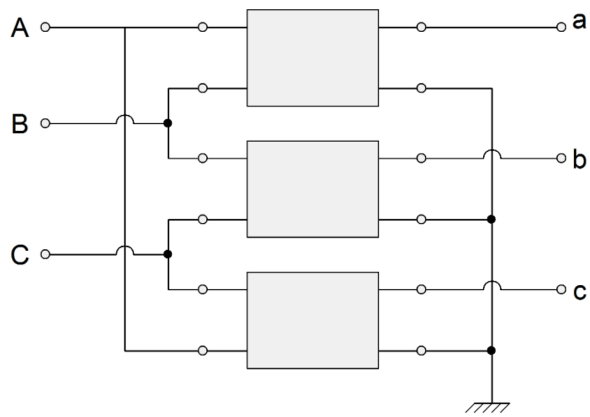
Example E3.7: A three phase transformer bank is made from three single-phase transformers. Each single phase transformer has the equivalent circuit of Figure E3.7a. The three phase connections are illustrated in Figure E3.7b.

a) Draw the positive sequence equivalent circuit of the three phase transformer bank with all impedances shown on the <u>right</u> hand side. The transformer ratio and the impedance values should be clearly marked in actual quantities, i.e. volts and ohms.
b) Draw the positive sequence equivalent circuit of the three phase transformer bank with all impedances shown on the <u>left</u> hand side. The transformer ratio and the impedance values should be clearly marked in actual quantities, i.e. volts and ohms.
c) Draw the per phase equivalent circuit <u>in per unit</u> of the three phase transformer bank using the following bases.

Left Hand Side:	S_B = 300 MVA (three phase) V_{B1} = 66.395 kV (line to neutral)
Right Hand Side:	S_B = 300 MVA (three phase) V_{B2} = 7.2 kV (line to neutral)



(a)



(b)

Figure E3.7: Construction of a Three Phase Transformer Bank
(a) Circuit Model of Single Phase Transformer, (b) Three Phase Connections. Each Block Represents the Single Phase Transformer of (a)

Solution: The equivalent circuit of Figure E3.7a can be modified by referring the j6.6125 ohm leakage impedance on the right hand side. By doing so, the three phase transformer model becomes identical to the circuit of Figure 3.11 with

$$Y = \frac{1}{j0.048\,ohms} = -j20.8333\,S \quad and \quad a = \frac{7.2}{115.0} = 0.0626$$

a) By utilizing the results of the previous section, the positive sequence equivalent circuit of the transformer is shown in Figure E3.7a.

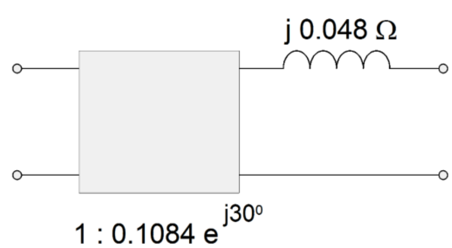


Figure E3.7a

b) By referring the impedance on the left hand side, the positive sequence equivalent circuit of Figure E3.7b is obtained.

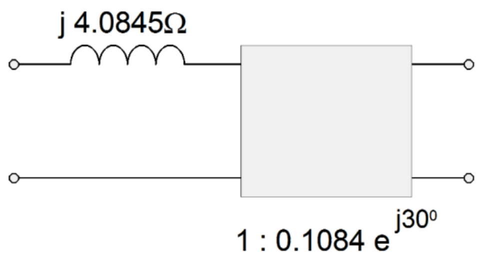


Figure E3.7b

c) The base impedance at the left hand side is:

$$Z_{b1} = \frac{V_{b1}^2}{S_{b1}} = \frac{115^2}{300} = 44.0833\,ohms$$

The per-unit value of the impedance is:

$$Z_{1u} = \frac{j4.0845}{44.0833} = j0.0927$$

The per unit transformation ratio is:

$$1:0.1084e^{j30^0} \rightarrow \quad 1/66.395:0.1084e^{j30^0}/7.2 \rightarrow \quad 1:1e^{j30^0}$$

Therefore the positive sequence per-unit equivalent circuit is shown in Figure E3.7c. Similarly the circuit of Figure E3.7d is developed.

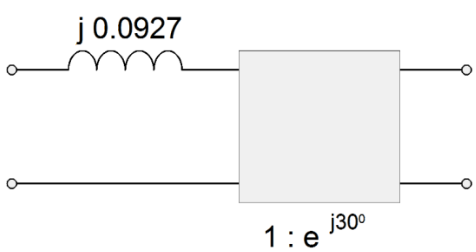


Figure E3.7c

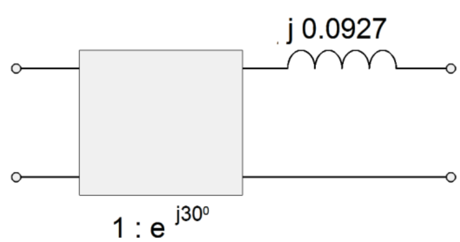


Figure E3.7d

3.4.4 Transformer Model for Inrush Current Computations

Transformers are normally made with iron core. Iron core transformers exhibit nonlinearities whenever the iron core is saturated. The iron core of a transformer can be represented with a nonlinear inductor and a nonlinear resistor.

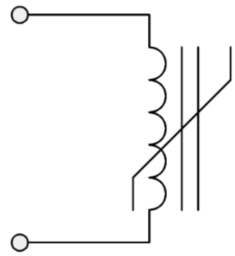


Figure 3.31: A Saturable Inductor Schematic Symbol

The equations describing a typical iron core represented with a saturable inductor are:

$$i_1(t) = i_0 \left| \frac{\lambda(t)}{\lambda_0} \right|^n sign(\lambda(t)) \tag{3.1}$$

$$i_2(t) = -i_0 \left| \frac{\lambda(t)}{\lambda_0} \right|^n sign(\lambda(t)) \tag{3.2}$$

$$0 = v_1(t) - v_2(t) - \frac{d\lambda(t)}{dt}$$

Above model provides the electric current required to support the magnetic flux of the transformer.

The core loss is a function of frequency and magnetic flux. Specifically, core loss consists of hysteresis loss and eddy current loss. Approximate equations for these losses are:

Hysteresis Loss:

$$P_h = a_h f B_{\max}^{\nu}, \quad \nu \approx 1.7$$

Eddy-Current Loss:

$$P_e = a_e f B_{\max}^2$$

where a_h *and* a_e are constants, $B_{\max}$ is the maximum magnetic flux density in the core.

3.4.5 Transformer Model for High Frequencies

At very high frequencies, a power transformer behaves quite differently. For example at high frequencies, the iron core of the transformer will not respond to the very fast changing field and it will not achieve full magnetization. In addition the core losses will be very high. Beyond a certain frequency, the response of the iron core will be non-existent to the point that the core can be neglected. In this case the transformer coil can be modeled as a distributed inductance and capacitance, the parameters of these components determined by the coil alone. Modeling of transformers at high frequencies is beyond the scope of this book.

3.5 Generator Modeling

Generators are very complex devices that exhibit a rather complex behavior. First generators do produce harmonics under normal operating conditions due to the construction of the windings. In addition, the generator exhibits different impedances along its direct and quadrature axes. This makes the interaction of the generator with the network rather complex when the waveforms are distorted.

Generator models for power frequency analysis can be classified into three categories depending on the time period of concern. For short duration phenomena (typically two cycles), the appropriate generator model is the so-called subtransient model. For longer duration periods (typically half second), the appropriate generator model is the transient model. Finally for long periods of time (or steady state), the appropriate model is the synchronous model. As an example Figure 3.32 illustrates the positive sequence transient model of the generator. Note that the generator exhibits a different value of impedance along the d-axis and a different along the q-axis.

A simplified way to model a generator in the presence of waveform distortion is to consider the generator impedances to the flow of specific harmonics. For this purpose it is expedient to analyze the electric currents into harmonics and each harmonic into sequence components.

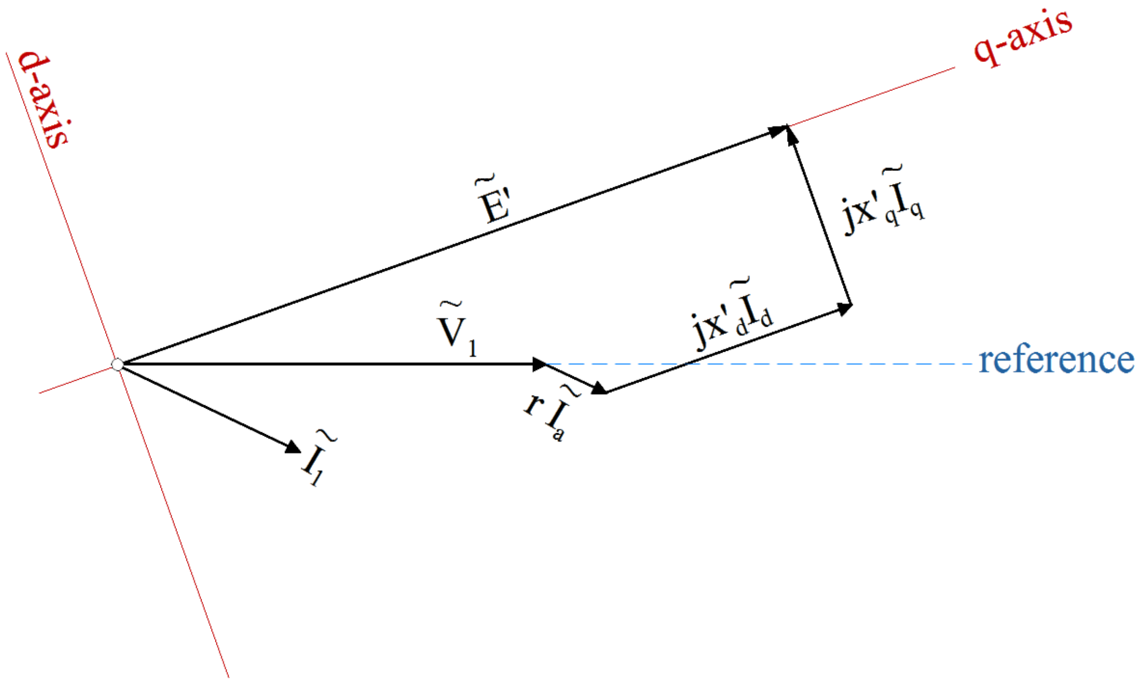
The generator sequence impedances are:

$$L_h^{+} \cong \frac{L_d^{''} + L_q^{''}}{2}$$

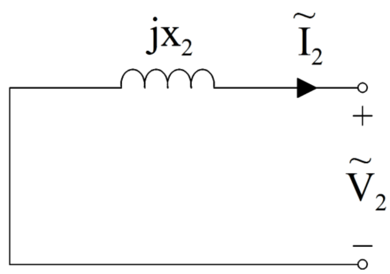
$$L_h^{-} \cong \frac{L_d^{''} + L_q^{''}}{2}$$

$$L_h^{0} \cong L^{0}$$

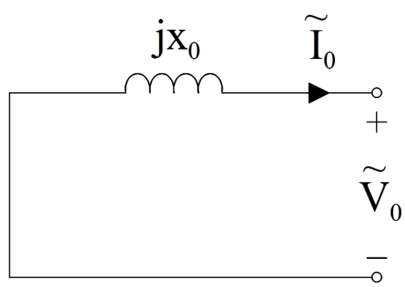
It is important to recognize that harmonic currents in the armature of a synchronous machine will generate a magnetic flux in the air-gap of the machine. For example a positive sequence 5th harmonic current will generate a rotating magnetic field in the air-gap that will rotate at a speed equal five times the synchronous speed in the direction of the rotation of the rotor. Similarly, a negative sequence 5th harmonic current will generate a rotating magnetic field with speed equal five times the synchronous speed but at a direction opposite the rotation of the rotor. Finally a zero sequence 5th harmonic current will generate a pulsating magnetic field. These magnetic fields will generate a torque by the same mechanism as in an induction motor. This torque many times is harmful to the life expectancy of synchronous machines.



(a)



(b)



(c)

Figure 3.32: Synchronous Generator Transient Model – Phasor Diagram
(a) Positive sequence phasor diagram, (b) Negative sequence equivalent circuit, (c) Zero sequence equivalent circuit

3.6 Inverter Interfaced Generation Models

In the last couple of decades, we have experience the penetration of generation that is interfaced to the power grid via inverters. The most well-known systems are wind turbine systems and PV plants. The models of these systems are quite complex as they involve rotating machines and power electronics. As an example the figures below illustrate the most common wind turbine systems, referred to as type 3 and type 4. Figure 3.33 shows a wind turbine system that interfaces to the power grid through a rectifier that rectifies the power into DC and then it converts into AC of the proper frequency before connecting to the grid. The system has complete rectification and inversion of the full power generated by the wind turbine/generator. Figure 3.34 illustrates another system that has the wind system generator connected directly to the power grid and the frequency is controlled with a set of rectifier/inverter connected to the rotor winding of a double fed induction machine. As the speed of rotation of the wind turbine/generator system changes, the frequency of the rotor windings is changing in such a way that the armature currents and voltages are of the grip frequency.

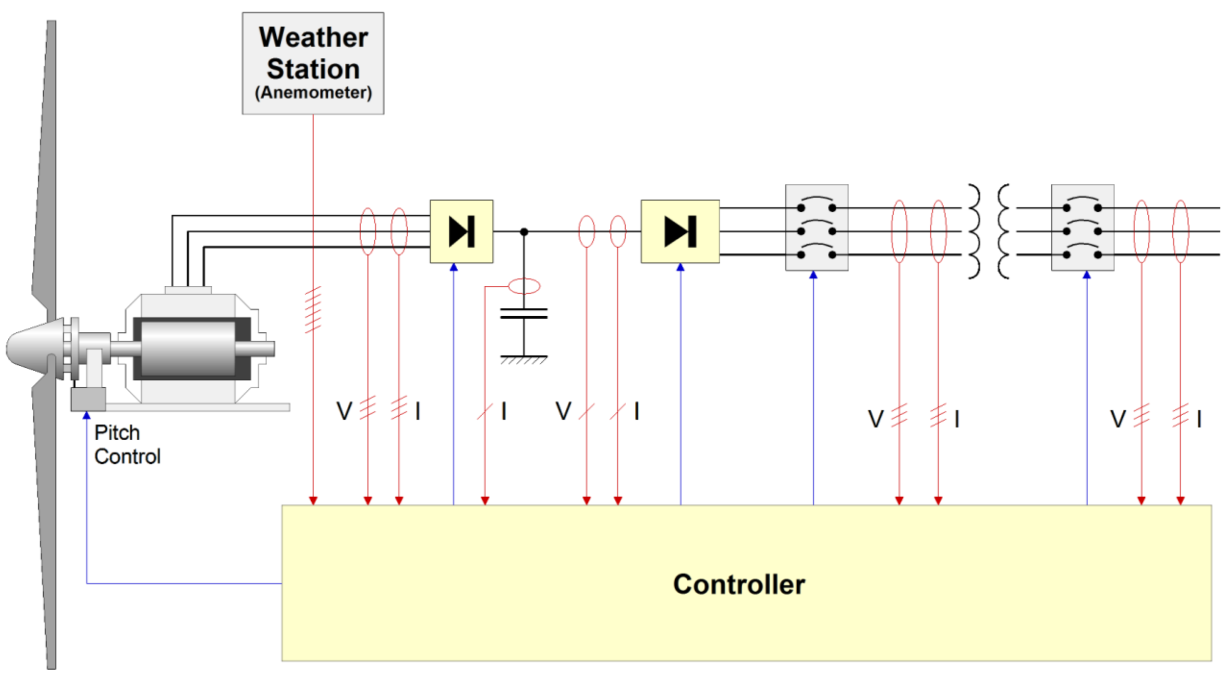


Figure 3.33: A Wind Turbine System with Full Rectification and Inversion

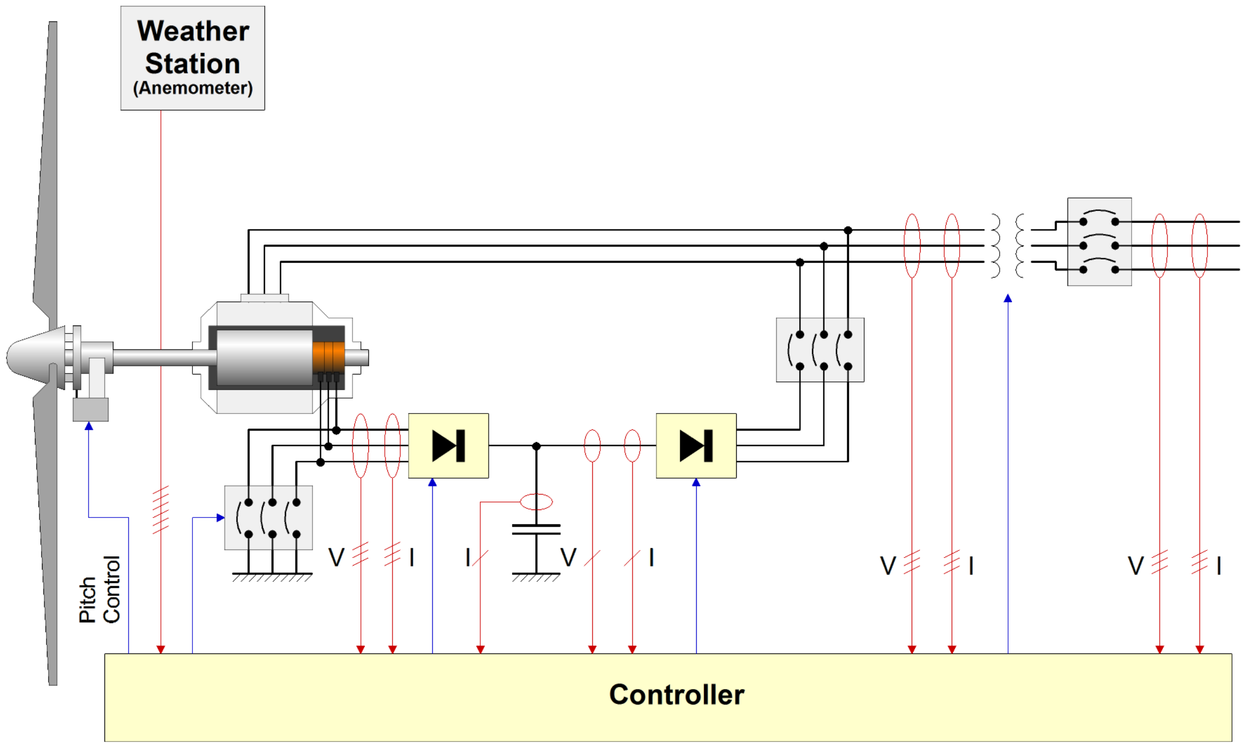


Figure 3.34: A Wind Turbine System with Double Fed Induction Machine and Rotor Winding Control

3.7 Electric Load Modeling

Electric loads have experienced a dramatic change in recent years. The reason for these changes is the proliferation of new technologies (especially power electronics) in the utilization of the electric energy. Electric loads consist of a variety of apparatus such as traditional induction motors, variable speed drives, computers, rectifiers, electronic ballast fluorescent lighting, dimmers, printers, air-conditioners, power conditioners, etc. Each of these load types has specific characteristics. In general, the load model to be selected depends on the intended application. For example, for traditional power flow applications, the load is modeled as a constant power load, or a constant current load or a constant impedance load or combination of these. There is some rational for each one of them. For example, a traditional feeder with many different loads on it will have a voltage regulator that maintains almost constant voltage at the start of the feeder while conditions may change elsewhere in the system. In this case the load of the feeder will be approximately constant for a specific set of customers connected to the feeder (no customer switching in and out). Similar arguments can be made for the constant current (specific loads) and constant impedance loads (typically no voltage regulation). Because a typical feeder may have a variety of loads, then occasionally a combination of these models is used, referred to as ZIP model (Z stands for constant impedance, I stands for constant current, and P stands for constant power). These models are approximations for steady state operation of the system. During transients, these models fail to represent the actual behavior of the system, even in the presence of voltage regulators (voltage regulators are typically electromechanical and have delayed response).

Here we present few examples, such as induction motors, dimmers, power supplies, fluorescent lighting, and converters.

3.7.1 Induction Motors

The majority of electric loads are electric motors, typically two thirds of electric loads are electric motors. Electric motors are dominated by induction type motors. It is estimated that about 50% to 60% of the total electric load consists of induction type motors. Induction motors behave in a manner that affects the steady state operation of the system (typically the required reactive power depends on the voltage magnitude in a manner that if the voltage is reduced, the motor will require more reactive power that will cause further voltage reduction and possibly voltage instability). They also affect the transient response of the system as any transient may result in speed reduction of the motor which alters the characteristics of the motor. In this case the electromechanical transients become very important in determining the response of the system. In any case, electric motors do affect the stability of the system as well as protective relaying practices.

The issue of modeling the dynamics of the load, which by enlarge depend on electric motors, is an issue of intense research. In this section we examine the main characteristics of induction motors with emphasis on the impact on protection functions.

The most well-known induction motor designs are the NEMA designs A, B, C and D. Of course in recent years many new designs have been developed to the point where we have a large number of designs. For simplicity, we should discuss the standard designs A, B, C and D. An equivalent model that captures fairly accurately the characteristics of the standard designs is shown in Figure 3.x in terms of the positive, negative and zero sequence circuits of the induction motor (or generator).

In Figure 3.x the subscript s refers to stator, the subscript r refers to rotor and the subscripts 1, 2 and 0 refer to positive, negative and zero sequence respectively. In terms of energy conversion, the amount of energy consumed by the speed dependent resistors equals the amount of energy transformed from electrical into mechanical (or vice versa if the machine operates as a generator).

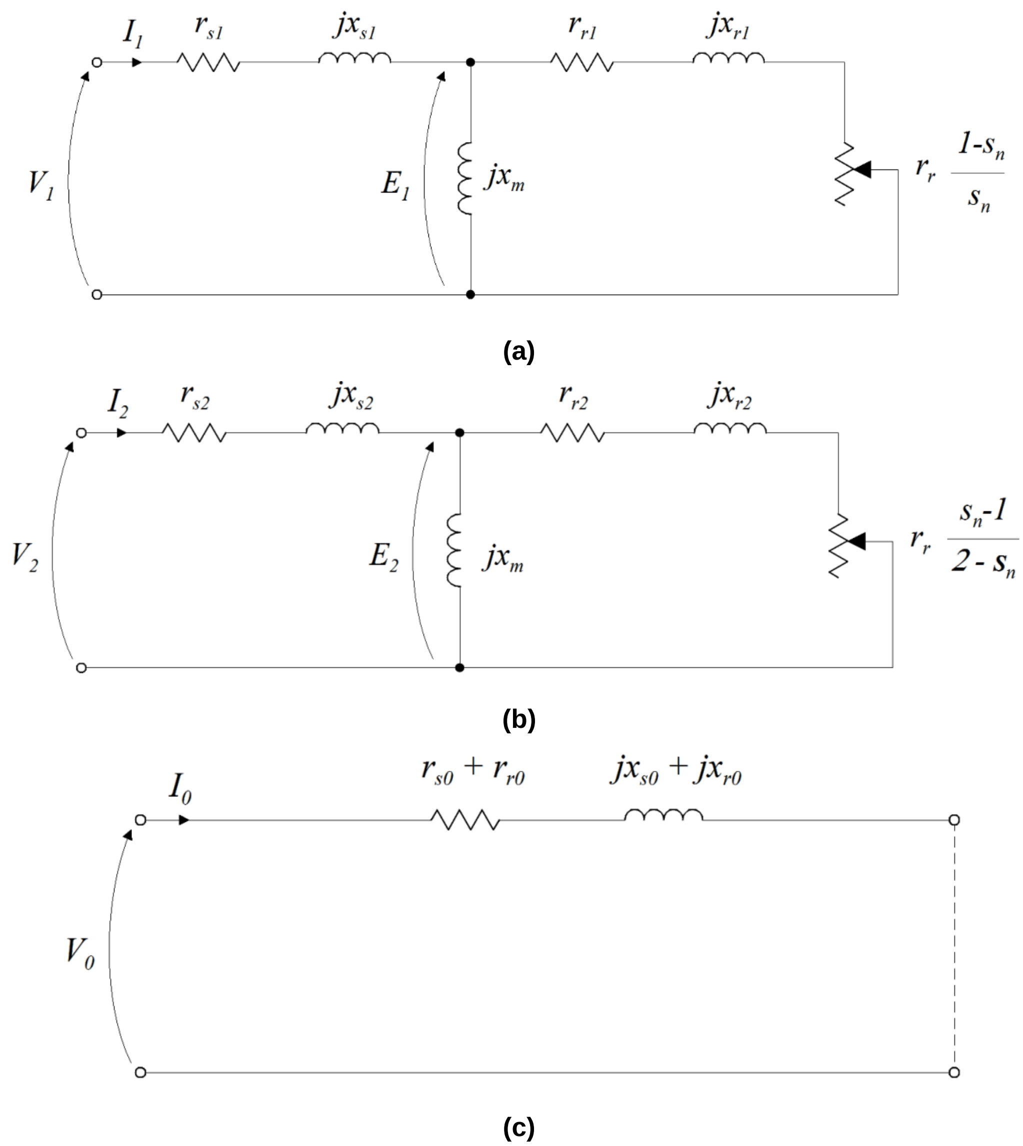


Figure 3.35 Sequence Model of Standard Induction Machine Designs.

(a) positive sequence, (b) negative sequence, (c) zero sequence

3.7.2 Inverter/Rectifier Modeling

One of the most common devices in power systems is the converter. The converter is an electronic switch based device that converts electric power from one form into another by switching operations. Converters can operate as rectifiers (AC to DC) or as inverters (DC to AC) or as general converters (AC to AC). The number of converter designs is enormous. For practical reasons we limit the discussion and presentation to one specific design. The reader is encouraged to consult books that are focused on modeling inverters.

For high power applications, a six valve converter design is often used. Figure 3.z illustrates the topology of the six valve converter. It consists of six switches implemented using SCRs. Each

SCR is protected by an R-C snubber circuit which reduces the transient voltage across each SCR during the transition from conducting to non-conducting state. A six-pulse converter operation is characterized by six switching operations per cycle. 12, 18, and 24 pulse converter topologies are obtained by connecting two, three or four six-valve converters in series with appropriate control signals and phase shifted A/C sources. Table 3.2 shows typical values of current waveform harmonics (in per-unit) generated by 6-pulse, 12-pulse, 18-pulse, and 24-pulse converters. Note that harmonic levels are decreasing with higher pulse topologies.

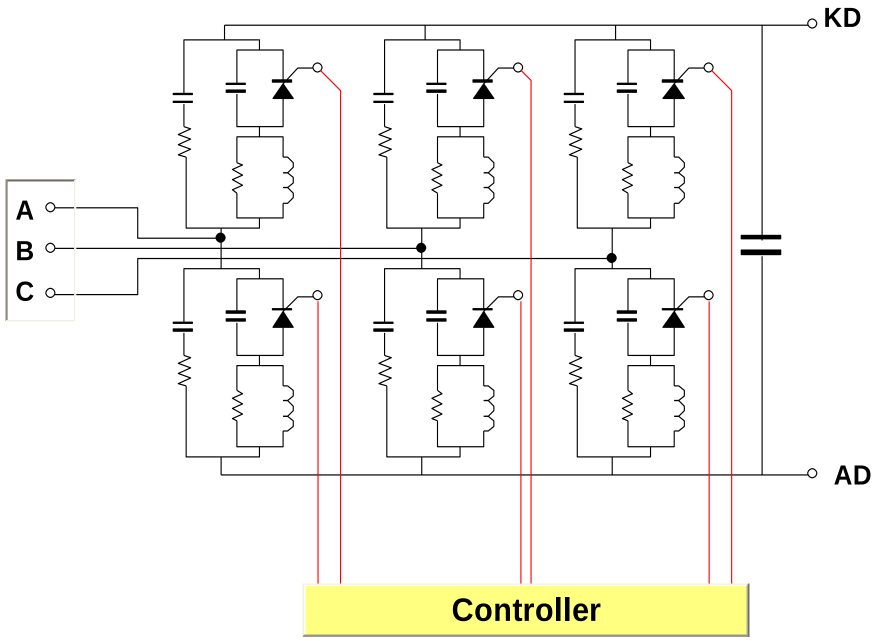


Figure 3.36: Generic Structure of a Converter

Averaging Model: For the averaging model of the two-level converter, we present equivalent equations of the two-level converter in phasor representation. The model equations are:

External-state equations are:

$$\tilde{I}_a = -jB(\tilde{V}_a - \tilde{E}_a) \quad (0.1)$$

$$\tilde{I}_b = -jB(\tilde{V}_b - \tilde{E}_b) \quad (0.2)$$

$$\tilde{I}_c = -jB(\tilde{V}_c - \tilde{E}_c) \quad (0.3)$$

$$I_{KD} = G(V_{KD} - U_{KD}) \quad (0.4)$$

$$I_{AD} = G(V_{AD} - U_{AD}) \quad (0.5)$$

The real power balancing equation is:

$$0 = (V_{KD} I_{KD} + V_{AD}\, \mathrm{I}_{AD}) - \frac{I_{AD}^2}{G} - \frac{I_{KD}^2}{G} + \mathrm{Re}\left\{\tilde{V}_a \tilde{I}_a^* + \tilde{V}_b \tilde{I}_b^* + \tilde{V}_c \tilde{I}_c^*\right\} \quad (0.6)$$

The DC-current balancing equation is:

$$0 = G\left(V_{KD} - U_{KD}\right) + G\left(V_{AD} - U_{AD}\right) \tag{0.7}$$

The relationship between DC voltage and AC voltages is:

$$0 = \sqrt{3}\left|\tilde{E}_a\right| \cdot m_a - 0.707\left(U_{KD} - U_{AD}\right) \tag{0.8}$$

The constraint equation for the modulation index is:

$$0 \le m_a \le 1 \tag{0.9}$$

Alternate Controller (V,P): DC -voltage control & power control.

$$0 = \sqrt{3}\left|\tilde{E}_a\right| - 0.707 U_{dc.\max} \tag{0.10}$$

$$0 = -U_{dc.\max} \cdot m_a + V_{dc.ref} + U_{KD} - V_{KD} - U_{AD} + V_{AD} \tag{0.9}$$

$$0 = P_{ref} - \mathrm{Re}\left\{\tilde{V}_a \tilde{I}_a^* + \tilde{V}_b \tilde{I}_b^* + \tilde{V}_c \tilde{I}_c^*\right\} \tag{0.10}$$

Inverters may generate harmonics depending on their design. For example Pulse Width Modulation inverters generate very little harmonics and most of it at higher frequencies. Pulse inverters (6, 12, 18, etc. pulse inverters) generate much more harmonics and lower frequency harmonics. Table 3.2 provides typical values of generated harmonics by pulse inverters.

Table 3.2: Typical Values of Generated Harmonics (in pu)

Converter Pulses	5^{TH}	7^{TH}	11^{TH}	13^{TH}	17^{TH}	19^{TH}	23^{RD}	25^{TH}	THD
6	0.175	0.110	0.045	0.029	0.015	0.010	0.009	0.008	0.215
12	0.026	0.016	0.045	0.029	0.002	0.001	0.009	0.008	0.063
18	0.026	0.016	0.007	0.004	0.015	0.010	0.001	0.001	0.037
24	0.026	0.016	0.007	0.004	0.002	0.001	0.009	0.008	0.034

3.7.3 Conventional Power Supplies

Electronic devices obtain power from the power system network typically convert AC to low voltage DC. The circuits used for this purpose can be broadly classified as conventional (also referred to as *linear*) and switched-mode. Conventional power supplies use a simple rectifier circuit typically followed by a filter capacitor and optionally a linear voltage a regulator circuit. A typical conventional power supply is illustrated in Figure 3.37. The diode bridge D1 conducts current charging the filter capacitor C1 as long as the voltage across the capacitor is lower than the instantaneous voltage of the AC source feeding the circuit. This results in the current waveform *i(t)* illustrated in Figure 3.38 (red plot trace). Note that the duration and amplitude of the current pulses as well as the capacitor voltage ripple depend on the filter capacitance value and the load current. Increasing the capacitor size results in lower capacitor voltage ripple, but also shortens the input current pulse width and increases their amplitude, thus increasing also the input current harmonic content.

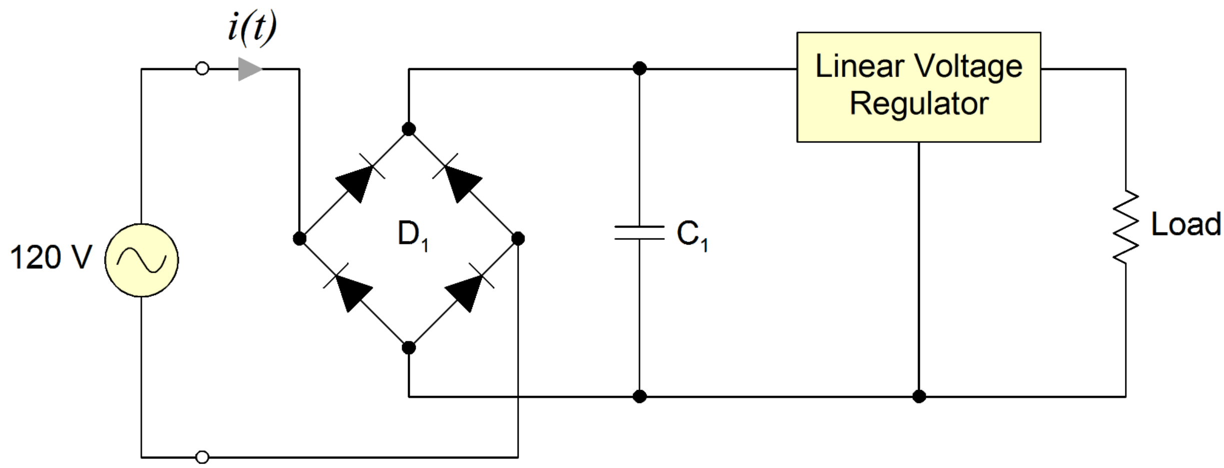


Figure 3.37: Conventional Power Supply Circuit Diagram

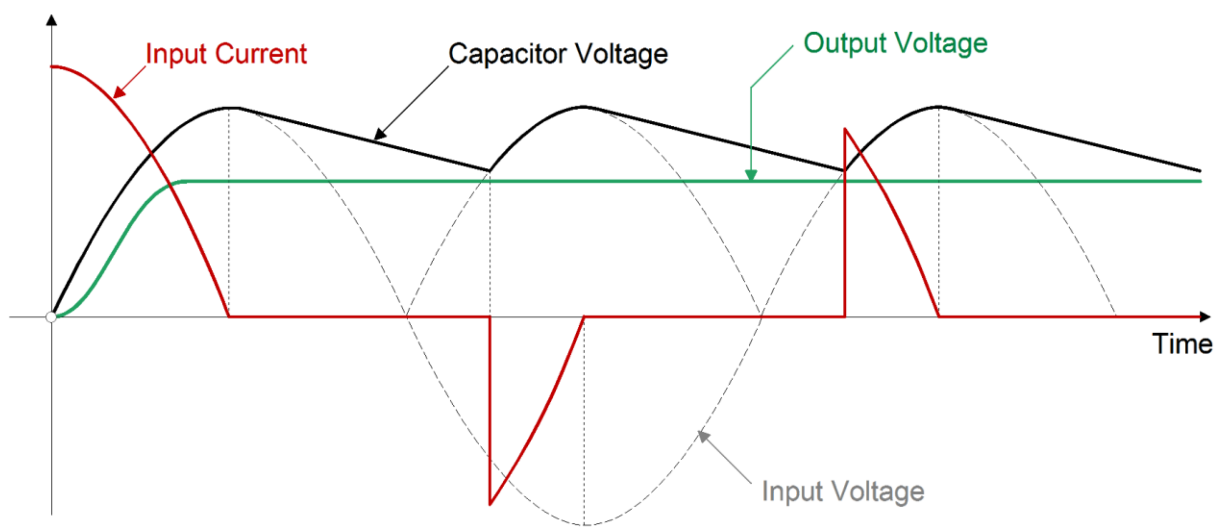


Figure 3.38: Conventional Power Supply Voltage and Current Waveforms

In order to generate a constant output voltage conventional power supplies often include a linear voltage regulator. The linear voltage regulator typically contains a series power transistor that adds significant power loss. Specifically the ripple voltage (i.e. the difference between the capacitor voltage and the desired constant output voltage) appears across the transistor collector-emitter terminals while the current trough the transistor is equal to the load current. Thus the power loss in the transistor is equal to the product of the ripple voltage multiplied by the load current.

Modern electronic devices often include switch-mode power supplies in which the linear regulator is replaced by a switching regulator. Switching regulators control the output voltage by high frequency switching schemes and can be significantly more efficient and also more compact than linear regulators. However, switch-mode power supplies may generate high frequency electromagnetic noise, causing radio interference.

3.7.4 Fluorescent Lighting

Florescent lights provide higher efficiency than incandescent lights and thus they have been widely used in commercial and industrial applications. Conventional florescent consist of a glass tube containing mercury vapor. Two electrodes located at the ends of the tube initiate a discharge arc through the mercury vapor which produces ultraviolet light. A phosphor coating inside the glass tube converts the ultraviolet light to visible light. The electric arc is initiated and extinguished every half cycle resulting in highly distorted current waveform. A typical florescent light current waveform is illustrated in Figure 3.39.

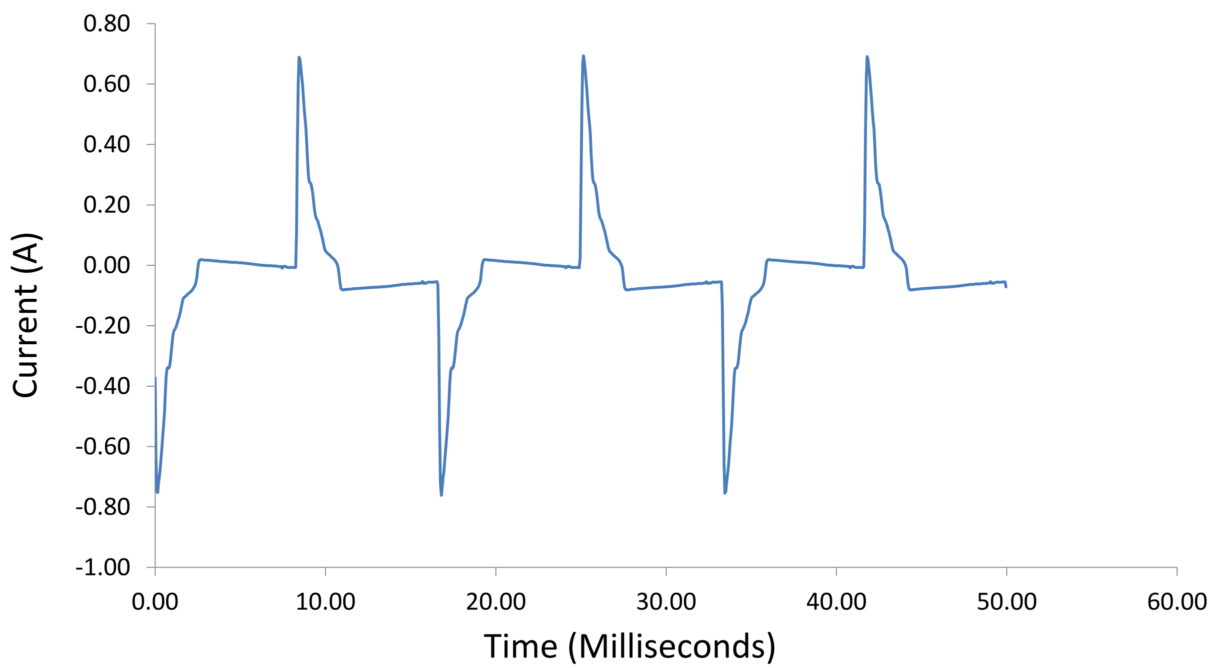


Figure 3.39: Fluorescent Light Current Waveform

The initiation of the electric arc in a fluorescent light requires a relatively high voltage. Furthermore, once the arc is initiated the voltage across the arc must be reduced in order to limit the current. Thus, fluorescent lights require additional hardware that provides these functions known as the *ballast circuit*.

A fluorescent light with a conventional ballast circuit is illustrated in Figure 3.40. The circuit contains a starter switch consisting of a bimetallic switch enclosed in a neon gas tube. When the power is first applied to the circuit the starter switch is closed. This allows electric current to flow through the heating filaments which warm up the mercury into vapor. After a short delay the bimetallic element temperature rises causing the starter switch to open. The current flowing through the ballast inductor is interrupted causing a high voltage transient across the two heater filaments igniting an electric arc through the mercury vapor. Note that the ballast inductor is in series with the mercury arc, providing current limiting impedance.

Conventional ballast circuits have been displaced by "electronic ballasts" which have several advantages over conventional ballasts:

- Provide more efficient operation
- Eliminate fluorescent light flicker
- Prolong the life of fluorescent lights
- Are lighter and more compact than conventional ballasts

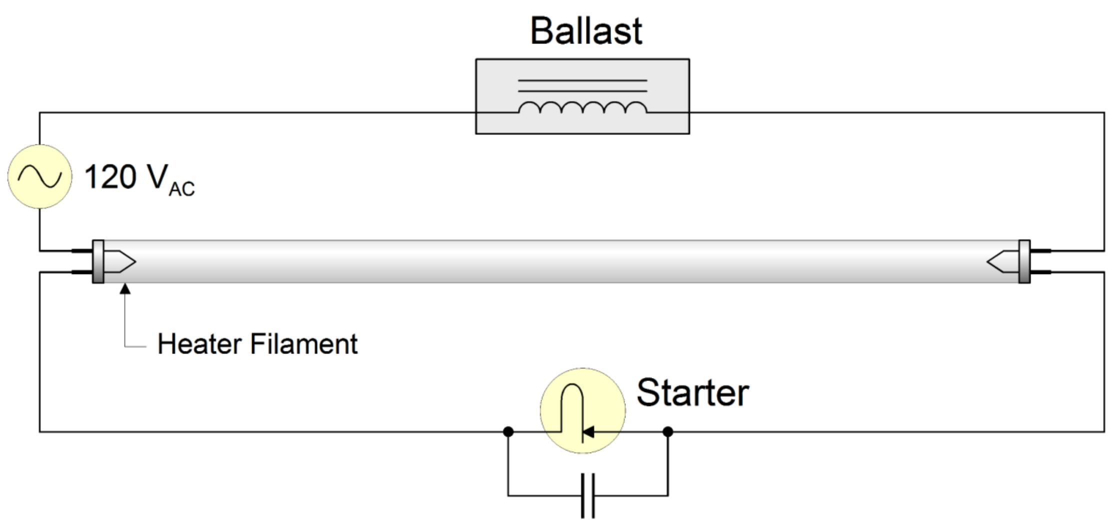


Figure 3.40: Fluorescent Light Conventional Ballast Circuit

A basic electronic ballast circuit is illustrated in Figure 3.41. It consists of back-to-back rectifier and inverter circuits. The inverter typically operates at frequencies in the order of 10 to 20 kHz. The high frequency applied across the fluorescent light electrodes provides a continuous arc thus eliminating any flicker and also increasing light output efficiency.

A variety of electronic ballasts are presently available. Some implementations include a microprocessor based controller which provides alternative starting methods which do not employ the heating filaments. They may also adjust the applied frequency and voltage for optimal operation under various temperatures thus prolonging the lamp life. Furthermore microprocessor based implementations may perform diagnostics that detect lamp failures lamp presence etc. Additionally, advanced implementations may include circuitry that reduce input current harmonics and provide near unity power factor.

Recently compact fluorescent and light emitting diode (L.E.D.) based technologies have been developed yielding devices which directly replace incandescent light bulbs. These devices include electronic interface circuitry within the light bulb base. Note that L.E.D. based lights provide even higher efficiency and longer life than compact fluorescent lights. Both compact fluorescent and LED based lights obtain power from the AC power network through a rectifier. Thus all such devices draw distorted current waveforms and thus generate various levels of current harmonics. Figure 3.42 illustrates typical current waveforms of two types of LED based lights.

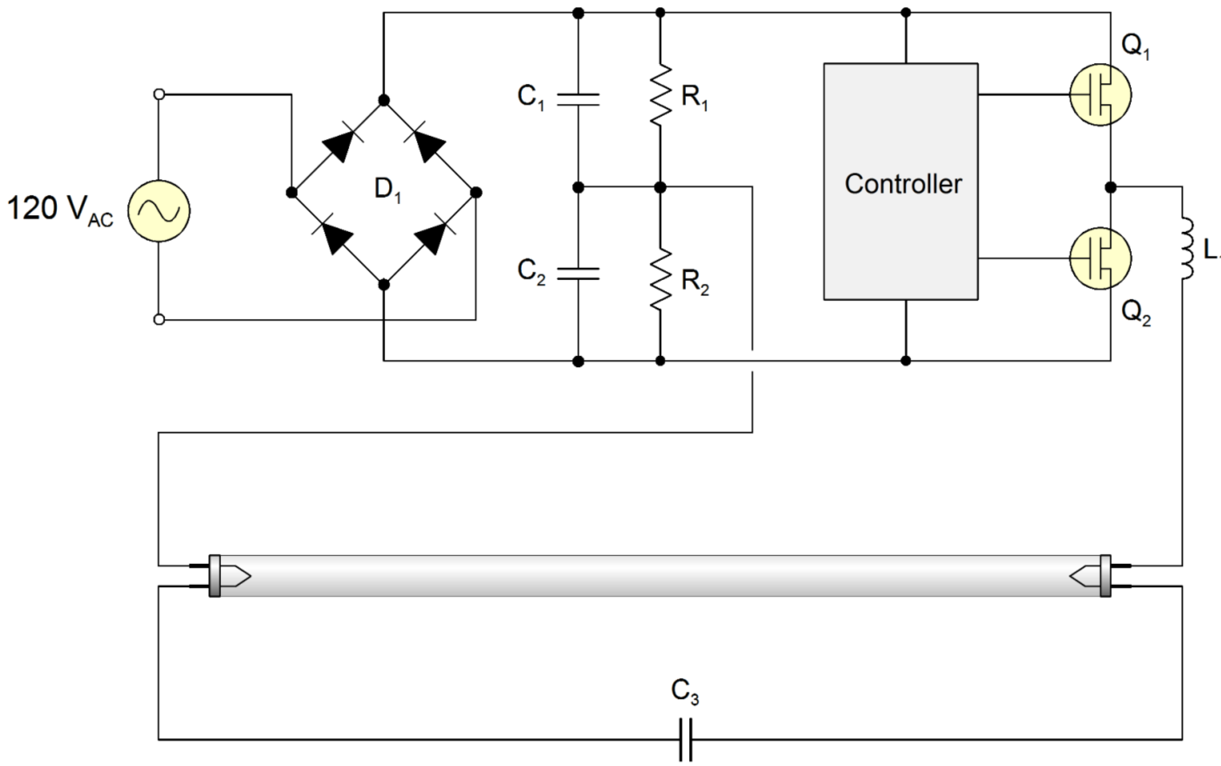
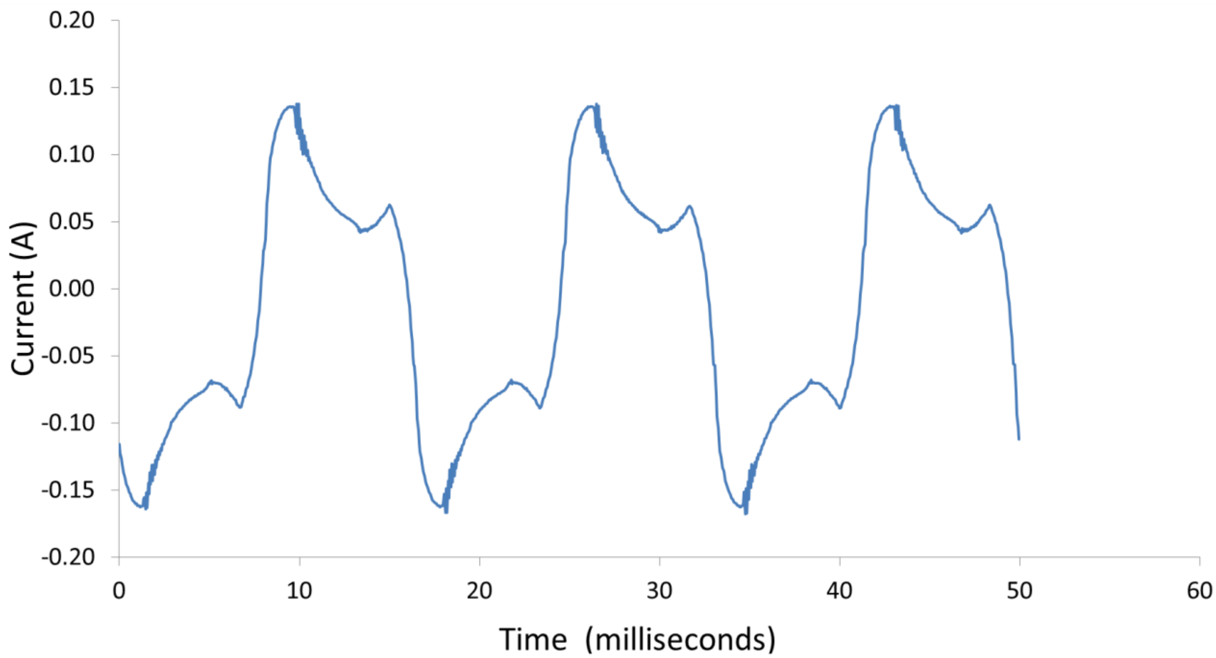
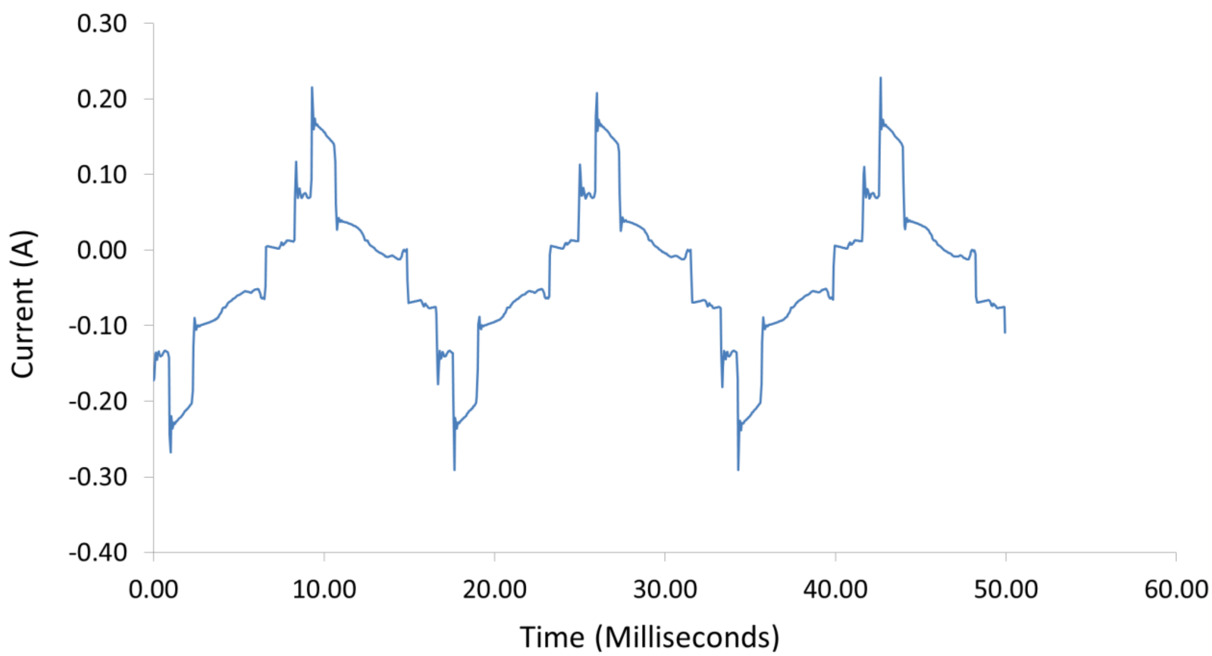


Figure 3.41: Fluorescent Light Electronic Ballast Circuit



(a)



(b)

Figure 3.42: Light Emitting Diode Based Lamp Current Waveforms

(a) Non-Dimmable Type, (b) Dimmable Type

3.7.5 Dimmers

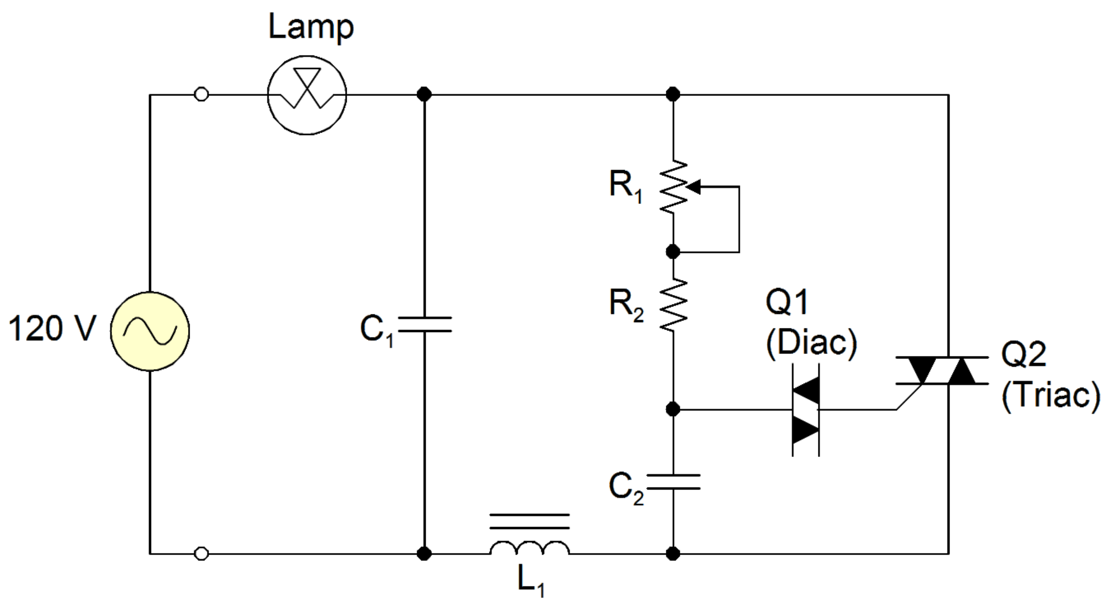
There is a large array of products providing appliance power level control using power electronics. Specifically, these devices use repetitive switching to control the power output of various appliances, such as fans, heaters, lighting fixtures, etc. Typically, the power is switched on and off every half cycle so that the device is connected to the power supply for only a certain portion of each half cycle. We refer to these devices as dimmers since their main application is in controlling the light output of lighting devices.

A typical incandescent light dimmer circuit and the resulting electric current waveform is illustrated in Figure 3.43. The circuit contains a triac (Q2) which switches on after a certain delay from the waveform zero crossing. The delay time is controlled the potentiometer R_1. By increasing the delay angle, the RMS value of the current is decreased. Depending on the selected delay the current waveform will contain certain harmonic levels, with the total harmonic distortion generally increasing with the delay angle. The capacitor C1 and the inductor L1 provide a filtering function which reduces the harmonic levels. The current harmonic levels for several delay angles, neglecting the effect of the harmonic filter are shown in Table 3.3.

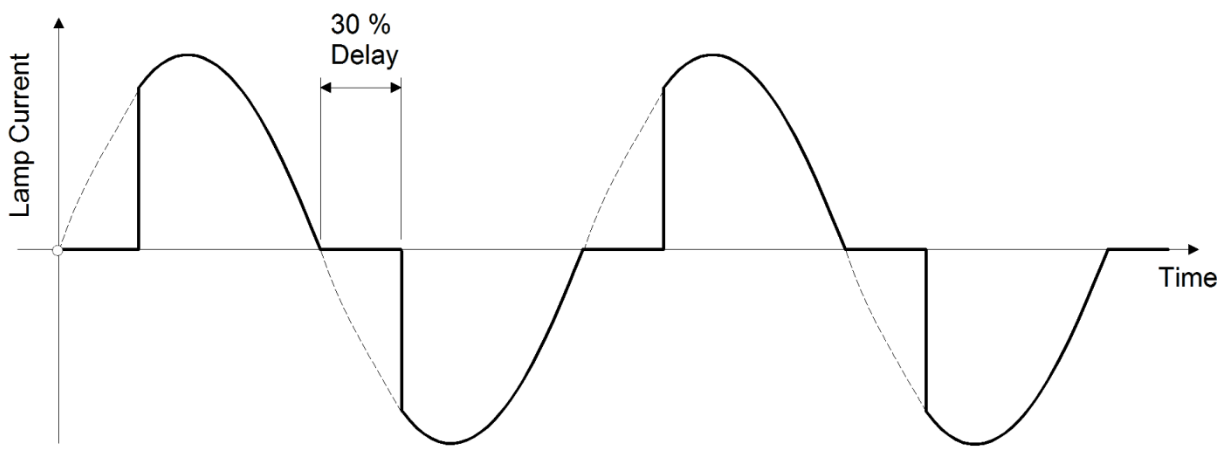
Depending on the quality of the filter design such simple dimmer circuits may generate significant harmonic levels and may result in radio and TV interference. More advanced dimmer circuits are available (at a higher cost) that generate very low harmonic levels, based on high frequency switching and pulse width modulation techniques.

Table 3.3: Typical Harmonics Generated By Light Dimmers

Delay Angle	1st (%)	3rd (%)	5th (%)	7th (%)	9th (%)	11th (%)	13th (%)	15th (%)
10	1.00	0.83	0.82	0.81	0.79	0.77	0.74	0.71
30	1.00	8.00	7.10	5.89	4.57	3.32	2.37	1.91
50	1.00	22.6	15.8	9.10	5.75	5.60	5.06	3.86
70	1.00	46.9	22.1	13.0	12.6	8.58	7.84	7.02



(a)



(b)

Figure 3.43: Light Dimmer Example

(a) Circuit Diagram, (b) Current Waveform

3.8 Application Examples

This section presents few application examples of the models described in this chapter.

3.8.1 Inrush Currents During Transformer Energization

To be added.

3.8.2 Transformer Performance during Direct Current Flow in Neutral (Geomagnetically Induced)

To be added.

3.8.3 Harmonic Currents in Converter Transformers

To be added.

3.8.4 Induced Voltage on Parallel Conductors/Power Lines

Conductors placed parallel to power lines are subject to induced voltages. The level of the voltages can be computed with the models developed so far. Specifically, the voltage on a conductor parallel to a power line (telephone wire, fence, etc.) is given by

$$\tilde{V}_t = \sum_k z_{tk} \tilde{I}_k$$

The mutual impedance z_{tk} can be computed with any of the three models presented earlier. The procedure will be demonstrated with an example.

Example E3.8: Consider a three phase 25 kV overhead transmission line. A telephone line (wire pair) parallels the power line for a distance of one mile. The relative position of the power line and telephone line is illustrated in Figure E3.8. The power line carries the following electric currents.

$$i_a(t) = \sqrt{2}200\cos\omega t + \sqrt{2}50\cos(5\omega t + 10^0) + \sqrt{2}40\cos(7\omega t + 30^0)$$
$$i_b(t) = \sqrt{2}200\cos(\omega t - 120^0)$$
$$i_c(t) = \sqrt{2}200\cos(\omega t - 240^0) + \sqrt{2}50\cos(5\omega t + 250^0) + \sqrt{2}40\cos(7\omega t - 210^0)$$

$$\omega = 2\pi 60\sec^{-1}$$

(a) Compute the electric current in the neutral (for simplicity assume that the electric current in the earth is zero).
(b) Compute the induced voltage on the telephone line per unit length.

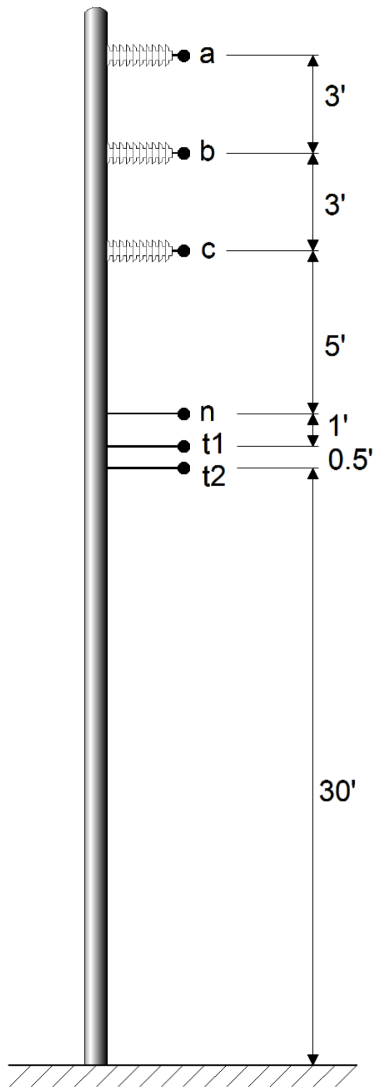


Figure E3.8: A Communication Line Suspended on a Power Pole

Solution: (a) the electric current in the neutral will be the negative sum of all the currents:

$$i_n(t) = -i_a(t) - i_b(t) - i_c(t) = \sqrt{2}50\cos(5\omega t + 130^0) + \sqrt{2}40\cos(7\omega t - 90^0)$$

(b) The induced voltage on the telephone circuit is computed separately for each harmonic.

Fundamental:

$$\tilde{V}_{t1} - \tilde{V}_{t2} = \frac{j\omega\mu}{2\pi}\left[\ln\left(\frac{12.0}{12.5}\right)200 + \ln\left(\frac{9.0}{9.5}\right)200e^{-j120^0} + \ln\left(\frac{6.0}{6.5}\right)200e^{-j240^0}\right] = 0.521e^{-j40.6^0}\ mV/m$$

5th Harmonic:

$$\tilde{V}_{t1} - \tilde{V}_{t2} = \frac{j5\omega\mu}{2\pi}\left[\ln\left(\frac{12.0}{12.5}\right)50e^{j10^0} + \ln\left(\frac{6.0}{6.5}\right)50e^{-j250^0} + \ln\left(\frac{1.0}{1.5}\right)50e^{-j130^0}\right] = 6.535e^{-j66.35^0}\ mV/m$$

7th Harmonic:

$$\tilde{V}_{t1} - \tilde{V}_{t2} = \frac{j7\omega\mu}{2\pi}\left[\ln\left(\frac{12.0}{12.5}\right)40e^{j30^0} + \ln\left(\frac{6.0}{6.5}\right)40e^{-j210^0} + \ln\left(\frac{1.0}{1.5}\right)40e^{-j90^0}\right] = 7.319e^{-j84.4^0}\ mV/m$$

3.9 Problems

Problem P3.1: Consider the three-phase overhead transmission line illustrated in Figure P3.1. The line is constructed in an area that has soil resistivity equal to 185 ohm.meters. The phase conductors are ACSR, BITTERN and the shield wires are ALUMOWELD, 3#7AW. The resistance, geometric mean radius and diameter of these conductors can be obtained from tables and they are provided below at 60 Hz.

ACSR, BITTERN: $r = 0.0729\, ohms/mile, \quad GMR = 0.04447\ feet, \quad d = 1.345\, inches$
ALUMOWELD, 3#7AW: $r = 4.420\, ohms/mile, \quad GMR = 0.002351\, feet, \quad d = 0.311\, inches$

(a) For simplicity, neglect the shield wires and compute the positive, negative and zero sequence parameters of the line per unit length. In other words compute the following parameters: $pos\ seq:(R_1, L_1, C_1), \quad neg\ seq:(R_2, L_2, C_2), \quad and\ zero\ seq:(R_0, L_0, C_0)$.
Your answer should be in ohms per meter, henries per meter and farads per meter.
(b) Using the parameters from (a) compute the nominal pi-equivalent positive, negative and zero sequence circuits at 60 Hz. The total line length is 94.5 miles.
(c) Use the computer program WinIGS to model this line and compute the positive, negative and zero sequence pi-equivalent circuit of the line. Compare the computer results to your results in part (b).

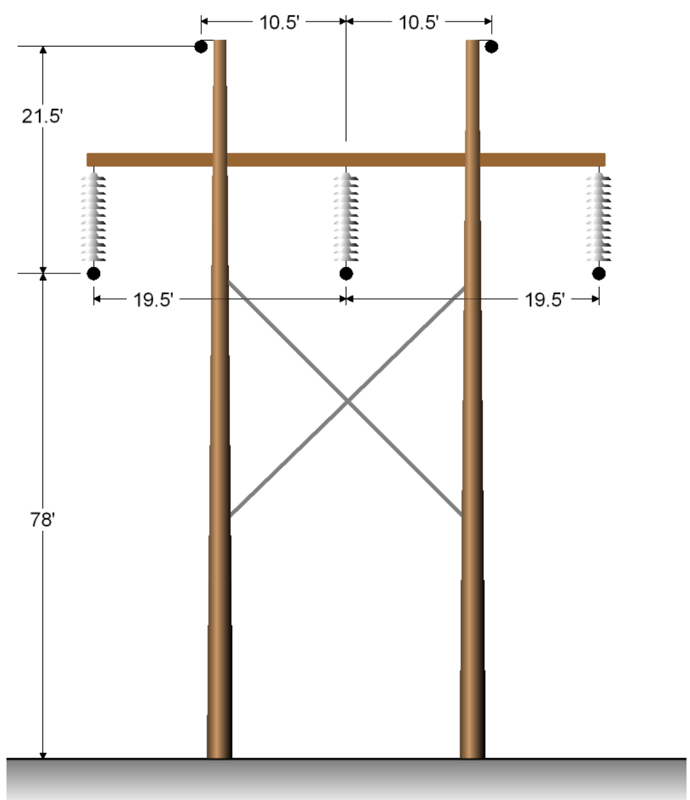


Figure P3.1

Solution: (a) First we compute the R, L and C matrices:

r_c = 0.0729 Ω/mi = 45.3 μΩ/m, r_e = 0.00159f / 1609 = 59.29 μΩ/m

$$R = \begin{bmatrix} 104.59 & 59.29 & 59.29 \\ 59.29 & 104.59 & 59.29 \\ 59.29 & 59.29 & 104.59 \end{bmatrix} \times 10^{-6} \Omega/m$$

$$D_e = 2160 \sqrt{\frac{\rho}{f}} \text{ ft} = 3{,}792.84 \text{ ft}$$

$$L_{ij} = \frac{\mu}{2\pi} ln \frac{D_e}{d_{ij}}$$

where d_{ij} = distance i – j, and d_{ii} = GMR_i

$$L = 2 \begin{bmatrix} 11.354 & 5.27 & 4.577 \\ 5.27 & 11.354 & 5.27 \\ 4.577 & 5.27 & 11.354 \end{bmatrix} \times 10^{-7} \mathrm{H}/m$$

→ L_s = 2.27 μH/m, L_m = 1.007 μH/m

$$C'_{ij} = \frac{1}{2\pi\varepsilon} ln \frac{d_{ij'}}{d_{ij}}$$

$$C' = \frac{1}{2\pi\epsilon} \begin{bmatrix} 7.931 & 2.087 & 1.416 \\ 2.087 & 7.931 & 2.087 \\ 1.416 & 2.087 & 7.931 \end{bmatrix}$$

$$C = C'^{-1} = 2\pi\epsilon \begin{bmatrix} 0.1373 & -0.0319 & -0.0161 \\ -0.0319 & 0.1428 & -0.0319 \\ -0.0161 & -0.0319 & 0.1373 \end{bmatrix}$$

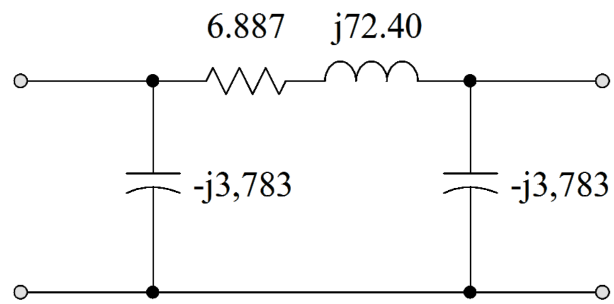
→ C_s = 7.74 pF/m, C_m = -1.481 pF/m

→ $R_1 = R_2 = 45.3 \times 10^{-6}$ Ω/m, $L_1 = L_2 = 1.263 \times 10^{-6}$ H/m, $C_1 = C_2$ = 9.221 pF/m

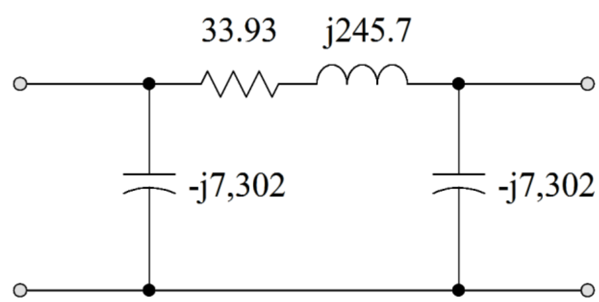
→ $R_0 = 223.17 \times 10^{-6}$ Ω/m, $L_0 = 4.2864 \times 10^{-6}$ H/m, C_0 = 4.778 pF/m

(b)

Positive or Negative Sequence Network



Zero Sequence Network



Problem P3.2: The wires of a fence "run" parallel with a power line as it is illustrated in Figure P3.2. During a certain phase to ground fault, the electric currents in the power line are as follows:

phase A: $\tilde{I}_a$ = negligible
phase B: $\tilde{I}_b$ = negligible
phase C: $\tilde{I}_c$ = 10,000 A

(a) Compute the earth current.
(b) Compute the induced voltage in <u>volts per meter</u> on the fence wire F1. The soil resistivity is 100 Ω·meter.

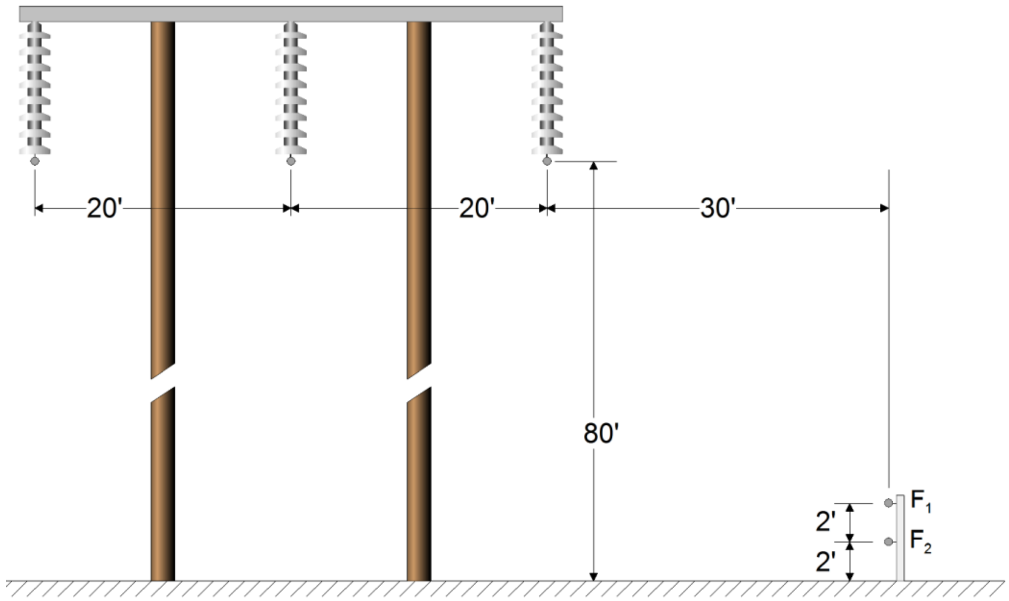


Figure P3.2

Solution:

(a) $\tilde{I}_e = -(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c) = -10kA$

(b) $\tilde{V}_{F1} = j\dfrac{\mu\omega}{2\pi}\tilde{I}_c \ln\dfrac{D_e}{D_{F1c}}$

where

$$D_e = 2160\sqrt{\frac{100}{60}} = 2788.548\ ft$$

$$D_{F1c} = \sqrt{30^2 + 76^2} = 81.7\ ft$$

$$V_{F1c} = 377 \times 2 \times 10^{-7} \times 10^4 \times \ln\frac{2788.548}{81.7} = 2.66\,V/m$$

Problem P3.3: Consider the three-phase, 60 Hz, transmission line of the Figure P3.3. The phase conductors are ACSR with the following parameters:

Radius:	0.85 inches
Geometric Mean Radius:	0.059 feet

The ground wire is neglected. The soil resistivity is 135 ohm.meters. Compute the positive sequence series inductance in <u>Henries per meter.</u>

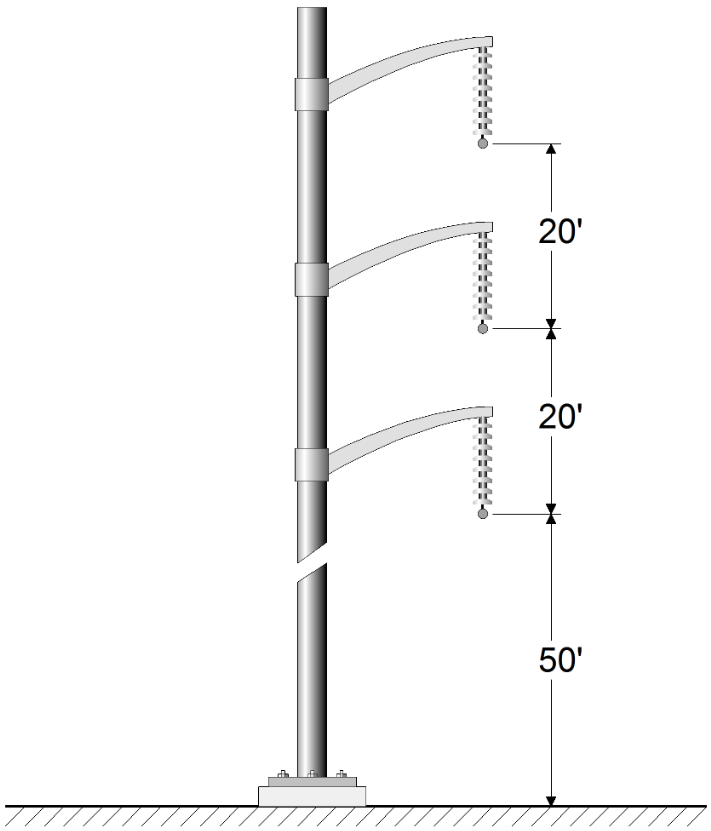


Figure P3.3

Solution: First the inductance matrix is computed:

$$L = \frac{\mu}{2\pi}\begin{bmatrix} ln\frac{D_e}{d} & ln\frac{D_e}{d_{ab}} & ln\frac{D_e}{d_{ac}} \\ ln\frac{D_e}{d_{ab}} & ln\frac{D_e}{d} & ln\frac{D_e}{d_{bc}} \\ ln\frac{D_e}{d_{ac}} & ln\frac{D_e}{d_{bc}} & ln\frac{D_e}{d} \end{bmatrix}$$

$D_e = 2160\sqrt{\frac{\rho}{f}} = 3240$, d = 0.059 ft, d_{ab} = 20 ft, d_{ac} = 40 ft, d_{bc} = 20ft

$$\Rightarrow \quad L = 2 \times 10^{-7}\begin{bmatrix} 10.9135 & 5.0876 & 4.3944 \\ 5.0876 & 10.9135 & 5.0876 \\ 4.3944 & 5.0876 & 10.9135 \end{bmatrix}$$

$\Rightarrow$ $L_1 = L_s - L_m = 1.211\ \mu H / m$

Problem P3.4: Consider the three-phase overhead transmission line illustrated in Figure P3.4. For simplicity, assume that the phase conductors of the line are solid aluminum conductors with diameter 1.0 inch and conductivity 40,000,000 S/m.

(a) Compute the pi-equivalent positive, negative and zero sequence circuits for 60 Hz, 180 Hz and 540 Hz. The line length is 56.5 miles. Neglect the ground wires. The soil resistivity is 225 ohm-meters.

(b) Use the computer program WinIGS to model this line and compute the positive, negative and zero sequence pi-equivalent circuit of the line for 60 Hz, 180 Hz and 540 Hz.

The shield wires are ALUMOWELD, Compare the computer results to your results in part (a). Note that one procedure neglects the shield wires and the other does not.

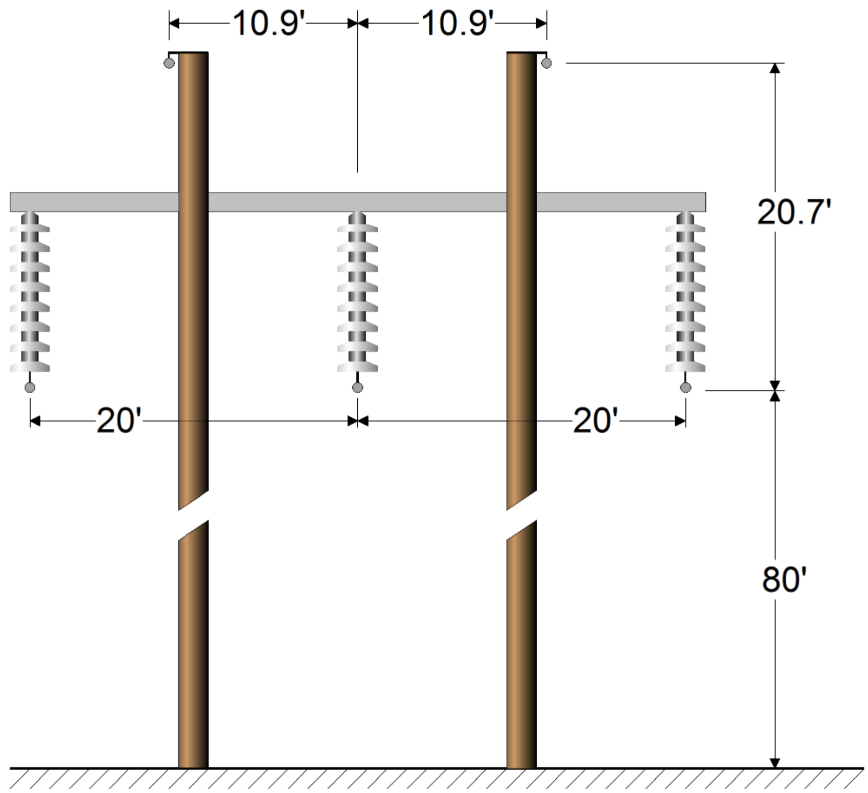


Figure P3.4

Problem P3.5: Consider a three phase, 480 V (line to line) power line feeding a rectifier. Assume that the rectifier generates (a) third harmonic currents which are 42% of the fundamental and zero sequence, (b) fifth harmonic currents which are 38% of the fundamental and positive sequence, and (c) seventh harmonic currents which are 28% of the fundamental and negative sequence. The power line consists of four 1.5 inch diameter solid copper conductors: three phase conductors and one neutral conductor.

(a) Compute the resistance ratio r_{ac}/r_{dc} for the third, fifth and seventh harmonics.
(b) Compute the ohmic losses of this circuit.

The copper conductivity is 57,000,000 S/m

Problem P3.6: Compute the inductance and resistance of a 1,000,000 cm solid copper conductor located 10 meters above 100 ohm.meter soil in the frequency range 1.0 to 420 Hz. Assume that the conductor carries the following currents which return though the soil:

500 A of 60 Hz
50 A of 300 Hz, and
50 A of 420 Hz.

Compute the total ohmic losses in watts per meter. The conductivity of the copper conductor is σ=54,000,000 S/m

Problem P3.7: Consider a 277 V power line (single phase) feeding a rectifier. Assume that the rectifier generates third harmonic currents which are 58% of the fundamental. The power line consists of two 2-inch diameter copper conductors: one phase conductor and one neutral conductor. The line is 300 meters long.

At the present operating condition, the fundamental frequency current of the rectifier is 850 Amperes.

Compute the Ohmic losses in the entire length (300 meters) of the power line.

The resistivity of copper is: 1.8×10^{-8} ohm-meters.

Problem P3.8: Consider the three-phase overhead distribution line illustrated in Figure P3.8. The phase conductors of the line are solid aluminum conductors with diameter 1.0 inch. The line length is 5 miles. The soil resistivity is 225 ohm-meters.

(a) Compute the zero sequence pi-equivalent circuit for the 11th harmonic.
(b) Compute the positive, negative and zero sequence pi-equivalent circuit for the 9th harmonic.

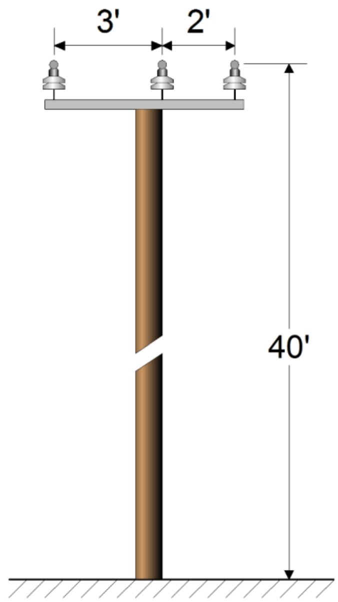


Figure P3.8

The resistivity of aluminum is: 2.8×10^{-8} ohm-meters

Problem P3.9: The two wires of a pilot relaying scheme "run" parallel with the power line as it is illustrated in Figure P3.9. During a certain phase to ground fault, the electric currents in the power line are as follows:

phase A: $\tilde{I}_a$ = negligible

phase B: $\tilde{I}_b$ = negligible

phase C: $\tilde{I}_c$ = 12,500 A

Compute the induced voltage in <u>volts per meter</u> on the pilot wire P1. If the line is 1.8 miles long, what is the total voltage induced on P1?
Compute the induced voltage between the wires P1 and P2. (i.e. compute the induced voltage on wire P1 and P2 an then take the difference).
The soil resistivity is 185 ohm-meters. For simplicity neglect the shield wires.

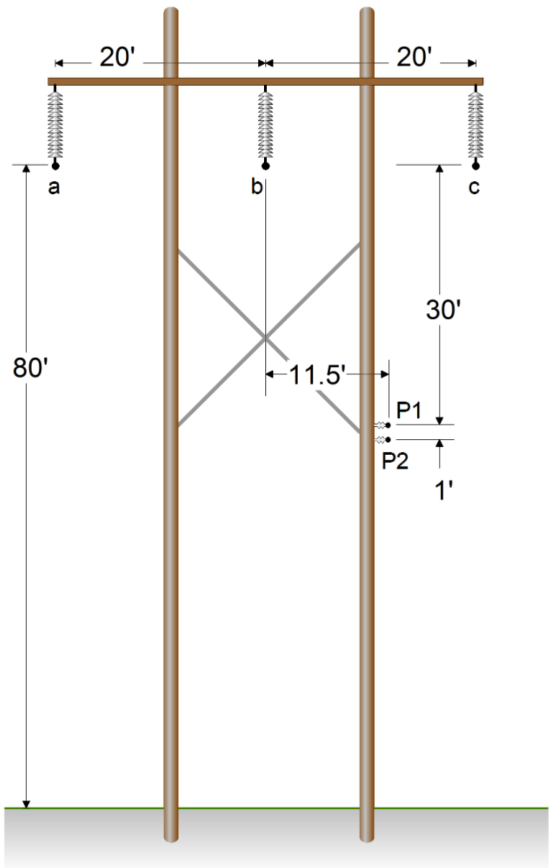


Figure P3.9

Solution:

$$V = \frac{j\omega\mu}{2\pi}\ I\ ln\frac{D_e}{d_{ab}}$$

$$D_e = 2160\sqrt{\frac{\rho}{f}} = 3{,}792.8\ ft$$

$$d_{ab} = \sqrt{30^2 + 8.5^2 = 31.18\ ft}$$

$\rightarrow$ V = 4.525 V/m

1.8 miles = (1.8) (1609 m/mi) = 2,896.2 m

$\rightarrow$ V = (4.525) (2,896.2) V = 13,105 Volts

DRAFT and INCOMPLETE

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from

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Chapter 4
Power System Fault Analysis

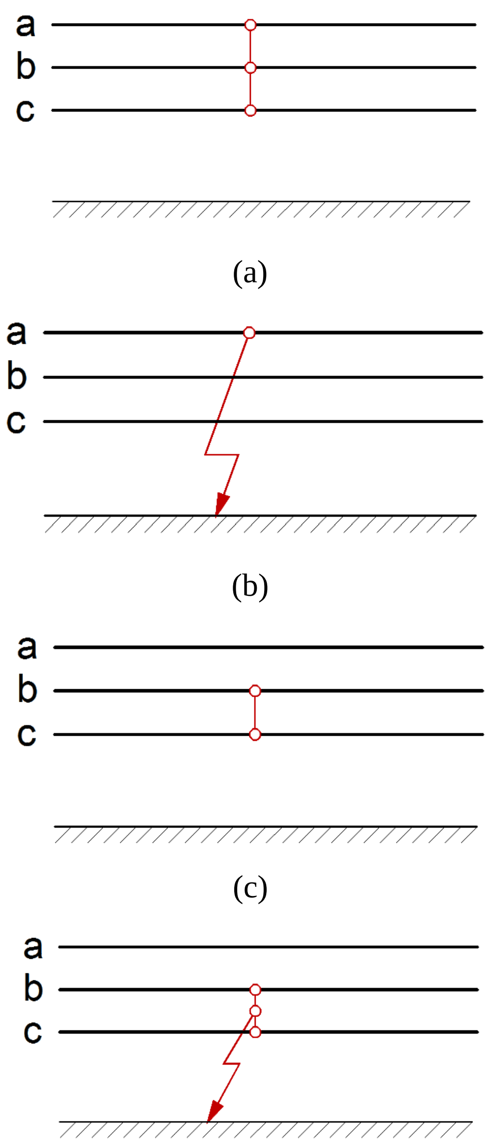
4.1 Introduction

Electric power systems are subject to faults (or short circuits) resulting from insulator failures. Specifically, faults are caused by the breakdown of insulation between a phase conductor and a grounded structure or between any two phase conductors, and the creation of a conductive path. Insulation failure faults are caused by high voltages or insulation defects or combination of the two, or when a conducting medium shorts one or more phases of the system. In general, the root causes of faults are many: lightning, tree limbs falling on lines, wind damage, insulation deterioration, vandalism, etc. In a three-phase system, a fault can involve two or more conductors. Depending on the number of conductors involved in a fault, we characterize the faults as three phase faults, double line faults, single line faults, etc. The fault types mentioned are illustrated in Figure 4.1.

Faults on electric power systems cause a number of undesirable effects: (a) flow of excessive electric current, which, in general, can damage equipment; (b) abnormal voltages (overvoltages or undervoltages); (c) system neutral or ground voltage elevation, which presents a hazard to human beings and animals (Note that system neutral voltage elevation is caused only by asymmetrical ground faults such as single line to ground faults and double line to ground faults); and (d) transferred or induced voltages on neighboring metallic structures and/or communication circuits which may be of significant level.

It is important to isolate the faulted power system element as soon as possible after fault initiation. For this purpose, power systems are equipped with a protective system comprising relays, breakers and fuses. The design of the protective system requires knowledge of the performance of the system during normal and faulty conditions. Specifically, the protection system should be able to distinguish between normal and fault conditions and in the latter case should successfully disconnect the faulty component from the system. Thus, it is necessary to determine the values of system voltages and currents during faulted conditions. This information is utilized in the design of the protection logic and protection settings, the selection of circuit breakers, fuse size, etc.

Proper design of the protection system (protective relays, breakers, fuses, etc.) enables the effective detection and successful isolation of faulted power system elements. Electric power faults are typically cleared within 2 to 90 cycles of the power frequency (0.03 to 1.5 seconds). For certain types of faults, a substantial voltage rise may develop at the system neutral, which affects the safety of human beings and animals near the power system. To mitigate the hazardous effects of power system faults, power systems must be properly grounded. Analysis of power faults and neutral voltage elevation is necessary for the proper design of grounding systems, ensuring safety of personnel.



(d)

Figure 4.1. Four Types of Power System Faults

(a) Three phase fault, (b) single line to ground fault, (c) line to line fault, (d) double line to ground fault

In this chapter we discuss the problem of power system fault analysis. By fault analysis we imply the computational methods that allow us to determine fault current levels, voltage levels, induced voltages, transfer voltages, and other quantities of interest during the fault. Two distinct approaches are presented. First, fault analysis of three-phase systems is approached in the conventional way, through the use of symmetrical components. The basic assumptions and limitations of the method of symmetrical components are explicitly stated. The method of symmetrical components is extended to analyze current distribution among sky wires, neutrals and earth. This extension of the method provides the basis for the computation of ground potential rise. Second, a general approach to the problem of fault analysis is presented. This method is applicable to symmetrical as well as asymmetrical three-phase systems and non-three-phase systems. It provides the current distribution among overhead circuits and earth as well as the ground potential rise of the system neutrals. While the symmetrical components method requires a simplified (approximate) system model, the second method is not based on any simplifying assumptions and therefore it is more accurate and realistic.

The effects of a fault on the power system are numerous. Specifically, upon initiation of a fault the following phenomena occur:

1. Electric current transients: These transients are characterized by the generation of a transient DC component as well as fast (high frequency) transients.
2. Voltage transients: During a fault, the system experiences voltage sags at certain points and voltage swells at other points. In addition, the grounds of the system are typically elevated to a substantial voltage, called *the ground potential rise*.
3. Generating unit transients: These transients involve electromechanical oscillations of generator rotor shafts coupled with the electrical system. Similar phenomena occur in motors if present.

For relaying applications, the fast electrical transients are not considered because they are filtered and thus do not affect the operation of the relays.

Before we embark in fault analysis methodologies, we discuss few issues related to fault analysis in general terms. These issues will be later addressed quantitatively.

Fault Current Distribution. The complexity of a typical power system circuit topology results in complex patterns of fault current flows. When a fault occurs, the fault current flow may follow a number of alternate paths. In general the fault current will split among various paths and the portion of the fault current in each of the alternate paths will be determined by the impedances (and mutual impedances) of these paths. Part of the fault current flows via phase wires, neutral wires, ground wires and part of the fault current flows via the earth. The fault current distribution depends on the relative sizes of transmission line conductors, mutual coupling between conductors, transmission line asymmetries, the resistances of the various system grounds, etc. Thus, it is expedient to perform extensive fault analysis studies in order to correctly determine appropriate protective relay settings. The analysis techniques used for this purpose should be based on full three-phase system models so that system asymmetries are taken into account.

Induced/Transferred Voltages. During a ground fault, fault currents flowing through the phase wires, neutrals and shields, as well as the earth path, induce voltages to any nearby circuits. These voltages may be substantial and can damage sensitive equipment such as control, relays, communication devices, etc. The induced/transferred voltages are caused by three mechanisms (a) ground potential differences caused by current conduction effects, (b) magnetically induced voltages (current induction effects), and (c) capacitive coupled voltages.

Ground Faults and the Effects of Ground Impedance. A power system is a multi-ground system, i.e. it contains many grounding electrodes, which are interconnected via neutrals and shield wires. Thus, when a ground fault occurs (i.e. a fault between a phase and a neutral or a ground wire), the fault current will eventually return to the source via the return path that may involve several paths in parallel. Specifically, the return path may typically include the neutral wire, the ground wires, and the earth path in parallel. The ratio of the neutral/shield current to the earth current is known as the fault current split factor. The split factor mainly depends on the resistance of the power system grounds and the neutral and shield wire sizes. The ground potential rise (GPR) that occurs during a ground fault depends on the split factor, and the ground system impedance. High GPR is undesirable since it generates human safety hazards, and can cause equipment damage.

Overvoltages on Unfaulted Phases. During a ground fault, the voltage of the unfaulted phases with respect to the neutral or ground wire may be elevated above the nominal value. This condition is obviously undesirable, since it may cause equipment damage. Most notably, surge arrester failure may occur if the arresters are not properly sized for this condition. The level of unfaulted phase overvoltage depends on the quality of the grounding system and the impedance of neutral and shield wires. Specifically, the unfaulted phase overvoltage is reduced by increasing the size of the neutral and shield wires of transmission and distribution circuits and by reducing the ground resistance of the transmission and distribution tower and pole grounds. The term *Coefficient of Grounding* has been defined to characterize the performance of the grounding system with respect to unfaulted phase overvoltage. The coefficient of grounding is defined as the ratio of the unfaulted phase overvoltage (with respect to the system neutral or ground voltage) over the nominal phase to ground voltage and it will be discussed later in more detail.

High Impedance faults. Occasionally, the impedance of the fault current return path is very high and in this case the fault current may be small. Low fault currents adversely impact the ability of protective relaying schemes to reliably detect the fault. We refer to these faults as high-impedance faults. For example, a high impedance fault occurs when an overhead line conductor falls on high resistivity soil, or ice. In this case the impedance between the downed conductor and the soil (or ice) is very high and it may limit the flow of current to levels lower than the usual load current. Obviously, such faults are extreme safety hazards, and present a formidable challenge to power system protection engineering. One approach for the detection of high impedance faults is based on the identification of arcing current signatures that are typically associated with fallen conductors. However, these techniques have not been proven to be always effective. Low amplitude arcing fault currents may be masked by high normal load currents, thus failing to detect the occurrence of a high impedance fault. Conversely, nonlinear loads such as arc welders and high frequency variable speed drives may generate current waveforms that can be falsely identified as arcing fault currents, thus causing *nuisance trippings* of the protective system. The possibility of high impedance fault can be also generated by improper design of grounding systems. Specifically, if the grounding system has relatively high impedance, then any phase to ground fault may result in low fault currents because the impedance of the ground will limit the fault current.

Above discussion illustrates the fact that during fault conditions we are concern not only with the levels of faults currents but also with a number of effects that may affect safety of people in and around electrical systems as well as damage of equipment. As we embark to analysis methods, some of these phenomena will be addressed in more detail.

4.2 Symmetrical Fault Analysis – Sequence Component Method

In this section we discuss short-circuit analysis techniques based on the sequence model representation of power system elements. We discuss symmetrical faults in this section followed by asymmetrical faults in the next section. Possible fault types are illustrated in Figure 4.1. The principles of short-circuit analysis using the sequence models are illustrated in Fig. 4.2. Specifically, Figure 4.2a illustrates a power system that is subjected to a fault. The power system is represented as a block with four terminals, the three phase conductors, and the neutral conductor. The fault, in general, is represented by a circuit connected to the four terminals of the power system. Let Y_{abc}^{f} be the admittance matrix of the fault circuit and Y_{abc}^{S} be the admittance matrix of the power system. With reference to Figure 4.2a, the models of the power system and the fault are expressed by the equations

$$\tilde{I}_{abc}^{S} = Y_{abc}^{S}\tilde{V}_{abc} - \tilde{I}_{S,abc} \tag{4.13}$$

$$\tilde{I}_{abc}^{f} = Y_{abc}^{f}\tilde{V}_{abc} \tag{4.14}$$

Where $\tilde{I}_{abc}^{S}, \tilde{I}_{abc}^{f}, \tilde{V}_{abc}$ are vectors of the phase currents and voltages, and $\tilde{I}_{S,abc}$ are Norton equivalent current sources, as has been the usual notation.

Equations (4.13) and (4.14) are transformed through the use of a symmetrical component transformation, yielding

$$\tilde{I}_{120}^{S} = Y_{120}^{S}\tilde{V}_{120} - \tilde{I}_{S,120} \tag{4.15}$$

$$\tilde{I}_{120}^{f} = Y_{120}^{f}\tilde{V}_{120} \tag{4.16}$$

where:

$$Y_{120}^{s} = T^{-1}Y_{abc}^{S}T \tag{4.17a}$$

$$Y_{120}^{f} = T^{-1}Y_{abc}^{f}T \tag{4.18a}$$

We know that Equation (4.15) represents the sequence networks of the power system, while Equation (4.16) represents the sequence model of the fault admittance matrix. The transformed system is illustrated in Figure 4.2b. Depending on the specific fault type, the sequence model of the fault admittance matrix represents a specific set of connections among the three sequence networks. In the subsequent sections we consider various fault types and derive the specific set of connections.

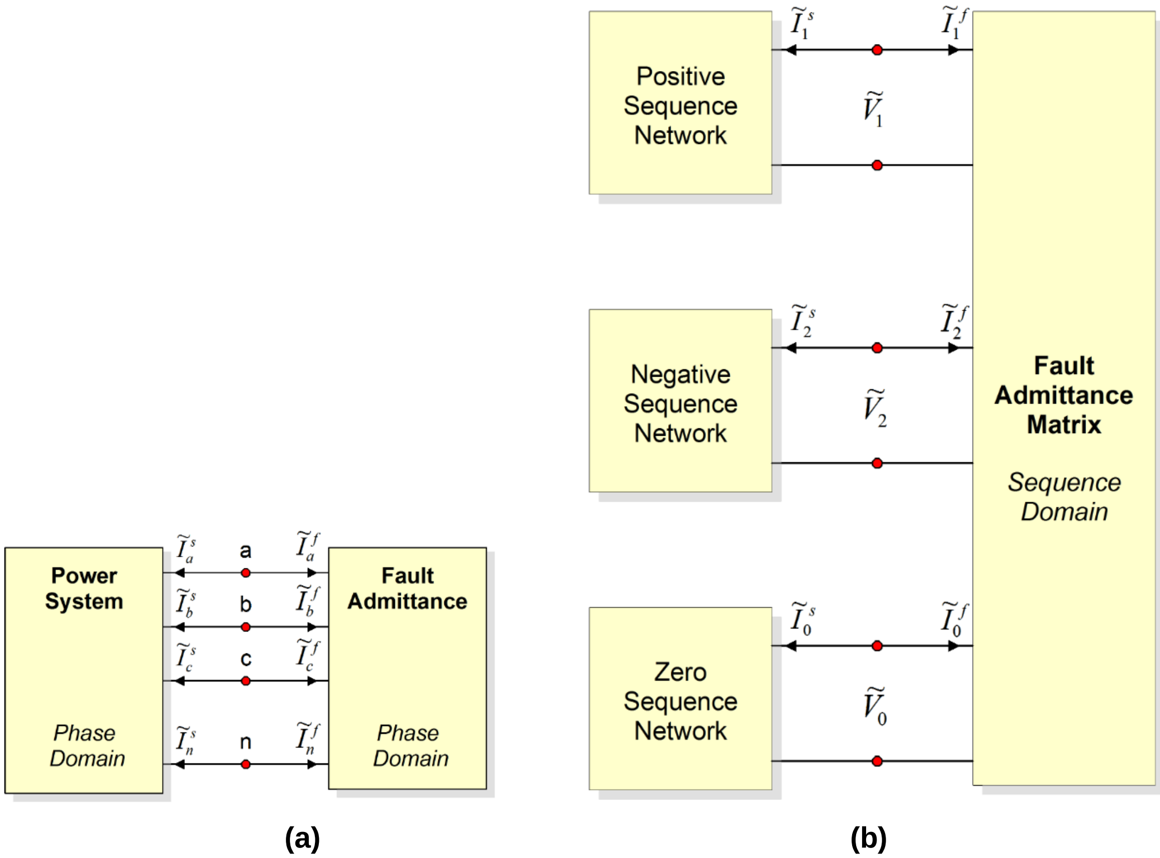


Figure 4.2 Basis of Fault Analysis with Symmetrical Components
Physical System (a, b, c domain or phase domain), (b) Symmetrical Component Domain

Three-Phase Fault

A three-phase fault occurs when all three phases of a power system are connected to the system neutral through a very low impedance, Z_f. In this case

$$\tilde{I}_a^f = \frac{\tilde{V}_a}{Z_f} = Y_f \tilde{V}_a \tag{4.18a}$$

$$\tilde{I}_b^f = \frac{\tilde{V}_b}{Z_f} = Y_f \tilde{V}_b \tag{4.18b}$$

$$\tilde{I}_c^f = \frac{\tilde{V}_c}{Z_f} = Y_f \tilde{V}_c \tag{4.18c}$$

The equations above, written in compact matrix notation, provide the fault admittance matrix, Y_{abc}^f:

$$\begin{bmatrix} \tilde{I}_a^f \\ \tilde{I}_b^f \\ \tilde{I}_c^f \end{bmatrix} = \begin{bmatrix} Y_f & 0 & 0 \\ 0 & Y_f & 0 \\ 0 & 0 & Y_f \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} \tag{4.19}$$

Upon transformation of above equation into the sequence model, we have

$$\begin{bmatrix} \tilde{I}_1^f \\ \tilde{I}_2^f \\ \tilde{I}_0^f \end{bmatrix} = \begin{bmatrix} Y_f & 0 & 0 \\ 0 & Y_f & 0 \\ 0 & 0 & Y_f \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix} \tag{4.20}$$

Equation (4.20) suggests that a three-phase fault in the sequence models is represented with three impedances Z_f connected across each of the sequence networks. These connections are illustrated in Figure 4.2. This analysis procedure is illustrated by example, next.

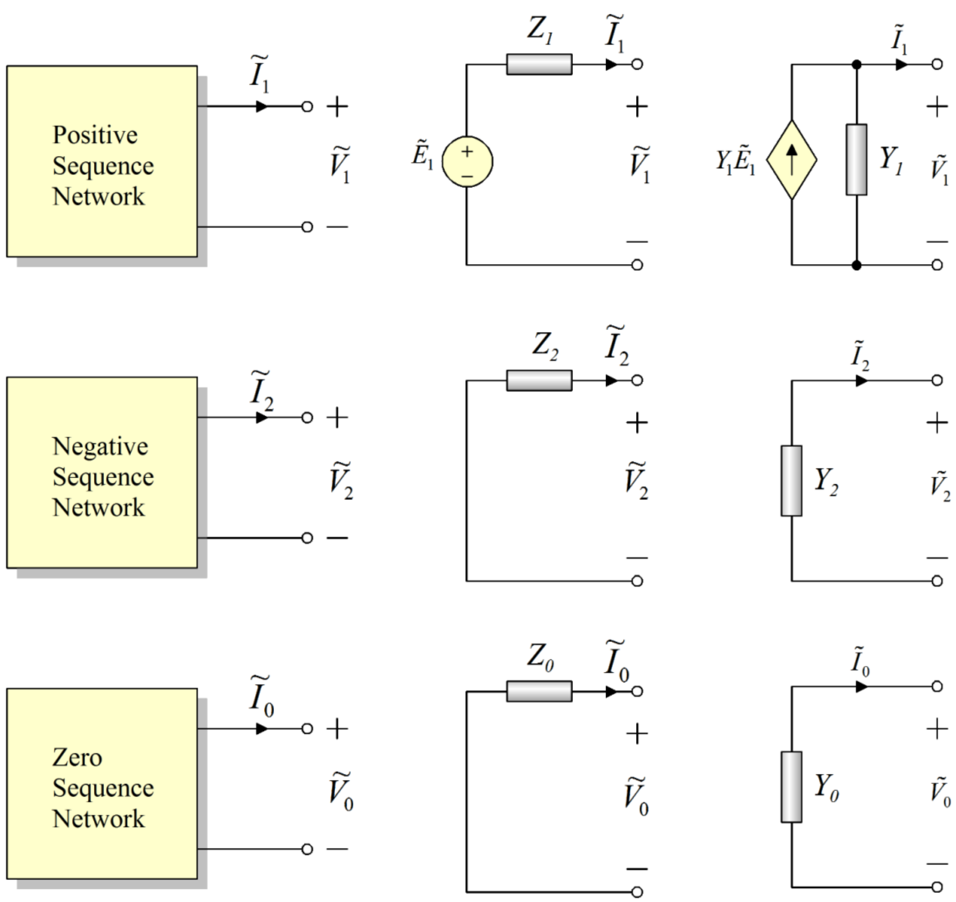
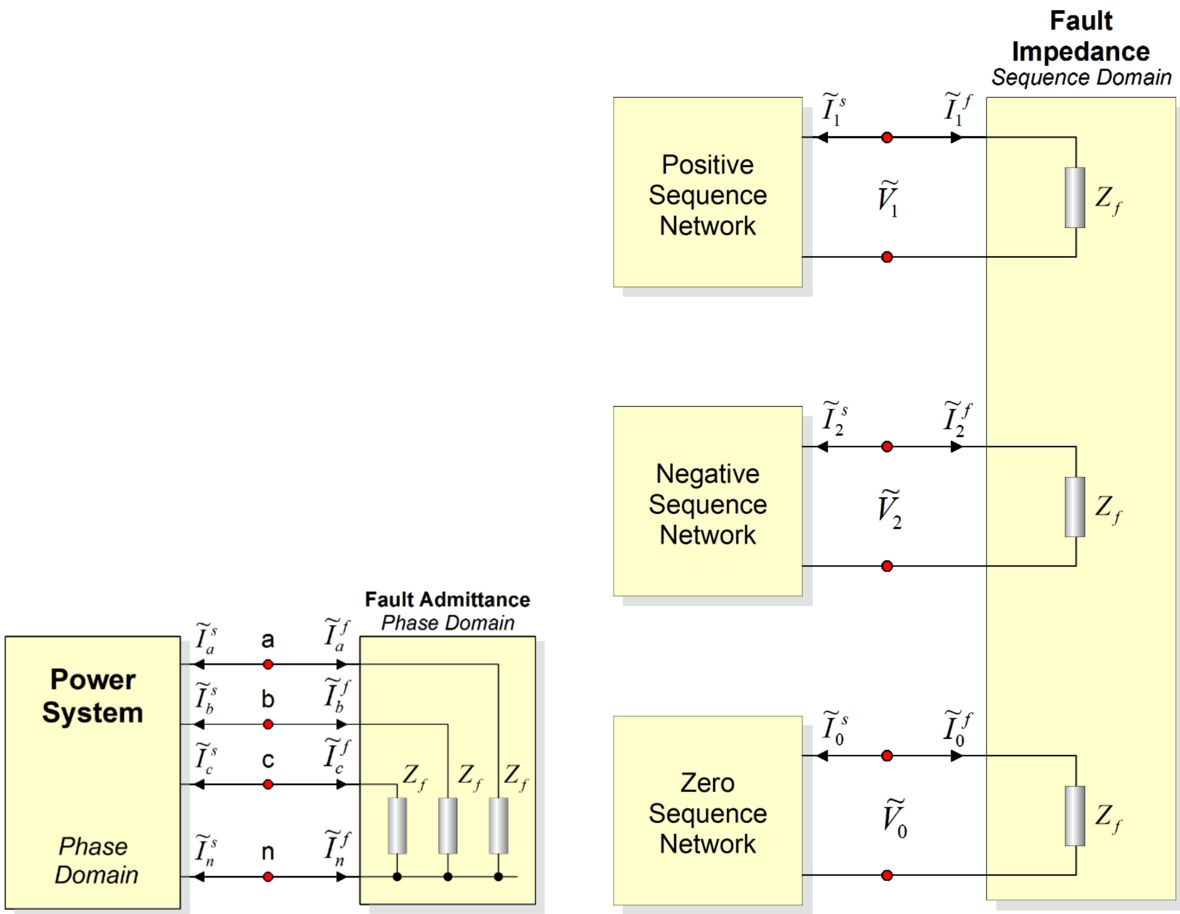


Figure 4.2 Sequence Networks and the Corresponding Thevenin and Norton Equivalents ($Y_1=1/Z_1$, $Y_2=1/Z_2$, $Y_0=1/Z_0$)



$$\tilde{I}_1 = \frac{\tilde{E}}{Z_1 + Z_f}, \quad \tilde{I}_2 = 0, \quad \tilde{I}_0 = 0$$

Figure 4.3: Equivalent Sequence Network and Fault Model for a Three Phase Fault

Example E4.1: Consider the electric power system of the Figure E4.1a. The system consists of an 80 MVA, 60 Hz, 15 kV generator, a 15kV/115kV, 80 MVA step-up transformer, a 28-mile 115 kV line and a delta connected constant impedance electric load. Assume a three-phase fault at point A which is located on the line 18 miles from the transformer. Compute the fault current: (a) neglecting the load current, and (b) without neglecting the load current. Use symmetrical component theory in the computations. System data are as follows:

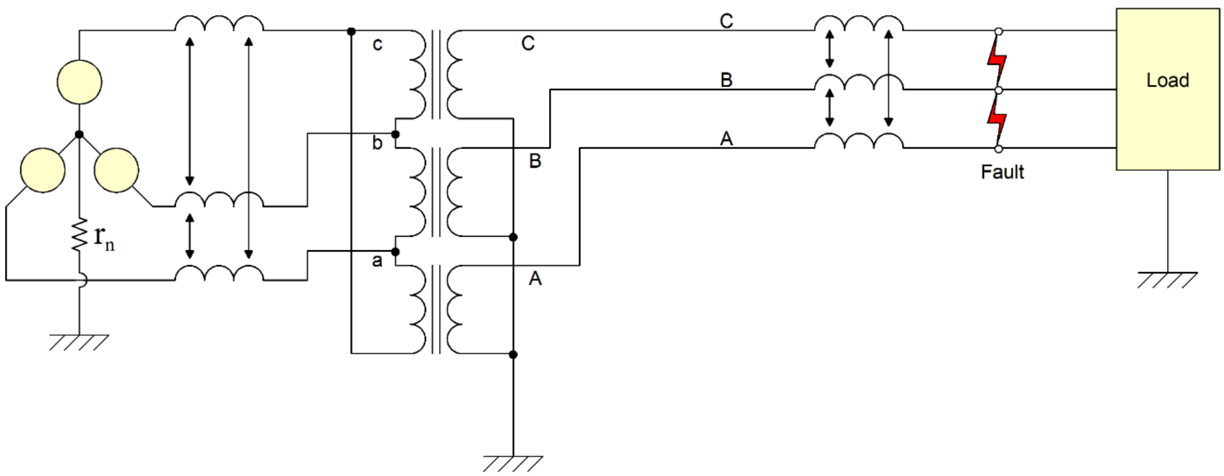
Generator: $x_1^{'} = j0.185\, pu, \quad x_2 = j0.28\, pu, \quad x_0 = j0.06\, pu, \quad r_N = 200\, ohms$

Transformer: $x_1 = x_2 = x_0 = j0.10\, pu$

Transmission line: $z_1 = z_2 = 0.3 + j0.7334\, ohms/mile, \quad z_0 = 0.34 + j1.55\, ohms/mile$

Electric load (three phase, delta connected): $z = 500 + j50\, ohms, \quad each\ leg$

Transformer shunt impedance and transmission line capacitive shunt impedance are to be neglected.



Change figure: phase sequence should be a-b-c and A-B-C.

Figure E4.1a: Faulted Power System Example – Three Phase Fault

Solution: First the sequence network of the system will be developed. We use a common base of 80 MVA and the corresponding nominal voltages. Note that at the 115 kV side the base impedance is:

$$Z_b = \frac{115^2}{80} = 165.3\, ohms$$

The resulting positive sequence network is given in Figure E4.1b.

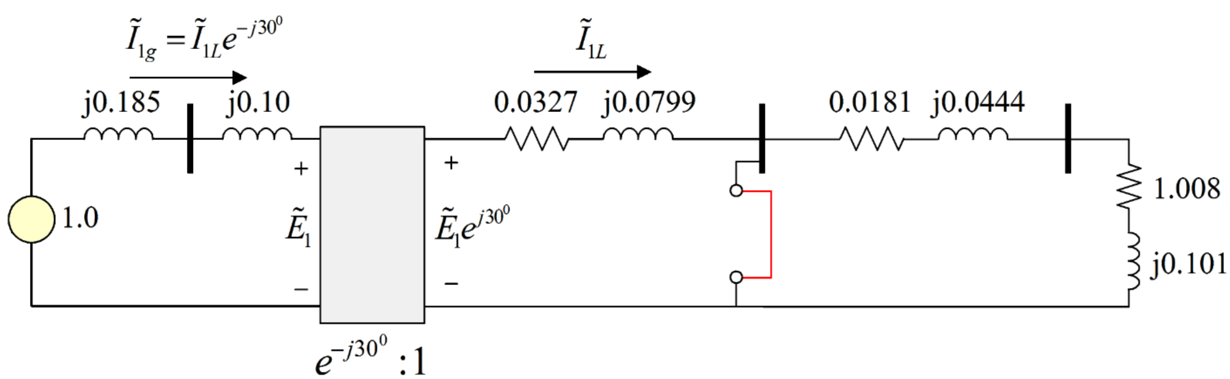


Figure E4.1b: Positive Sequence Network of the Power System of Figure E4.1a

The positive sequence fault current is:

$$\tilde{I}_{1g} = \frac{1.0}{0.0327 + j0.3649} = 2.7295 e^{-j84.88^0} \; pu$$

The phase currents at the generator side are:

$$\begin{bmatrix} \tilde{I}_{ag} \\ \tilde{I}_{bg} \\ \tilde{I}_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_{1g} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.7295e^{-j84.88^0} \\ 2.7295e^{-j204.88^0} \\ 2.7295e^{-j324.88^0} \end{bmatrix} \quad pu$$

The electric currents at the line side can be computed as follows: At the ideal transformer: $\tilde{E}_1 \tilde{I}_{1g}^* = \tilde{E}_1 e^{j30^0} \tilde{I}_{1L}^* \quad \rightarrow \quad \tilde{I}_{1g} = \tilde{I}_{1L} e^{-j30^0}$

Therefore the positive sequence fault current on the line side is: $\tilde{I}_{1L} = 2.7295e^{-j54.88^0}$ pu .

The phase currents on the line are:

$$\begin{bmatrix} \tilde{I}_{aL} \\ \tilde{I}_{bL} \\ \tilde{I}_{cL} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_{1L} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.7295e^{-j54.88^0} \\ 2.7295e^{-j174.88^0} \\ 2.7295e^{-j294.88^0} \end{bmatrix} \quad pu$$

4.3 Asymmetrical Fault Analysis – Sequence Component Method

In this section we examine asymmetrical faults. Several types of asymmetrical faults can occur in the system. We provide a systematic way to analyze these faults

4.3.1 Line-to-Line Fault

A line-to-line fault occurs whenever a very low impedance, Z_f, or very high admittance, $Y_f = 1/Z_f$, is connected across two phases. Assuming that the faulted phases are b and c, the fault currents are

$$\tilde{I}_a^f = 0 \tag{4.21a}$$

$$\tilde{I}_b^f = Y_f \left(\tilde{V}_b - \tilde{V}_c\right) \tag{4.21b}$$

$$\tilde{I}_c^f = -Y_f \left(\tilde{V}_b - \tilde{V}_c\right) \tag{4.21c}$$

Equations (4.21) written in compact matrix notation provide the fault admittance matrix, Y_{abc}^f:

$$\begin{bmatrix} \tilde{I}_a^f \\ \tilde{I}_b^f \\ \tilde{I}_c^f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_f & -Y_f \\ 0 & -Y_f & Y_f \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} \tag{4.22}$$

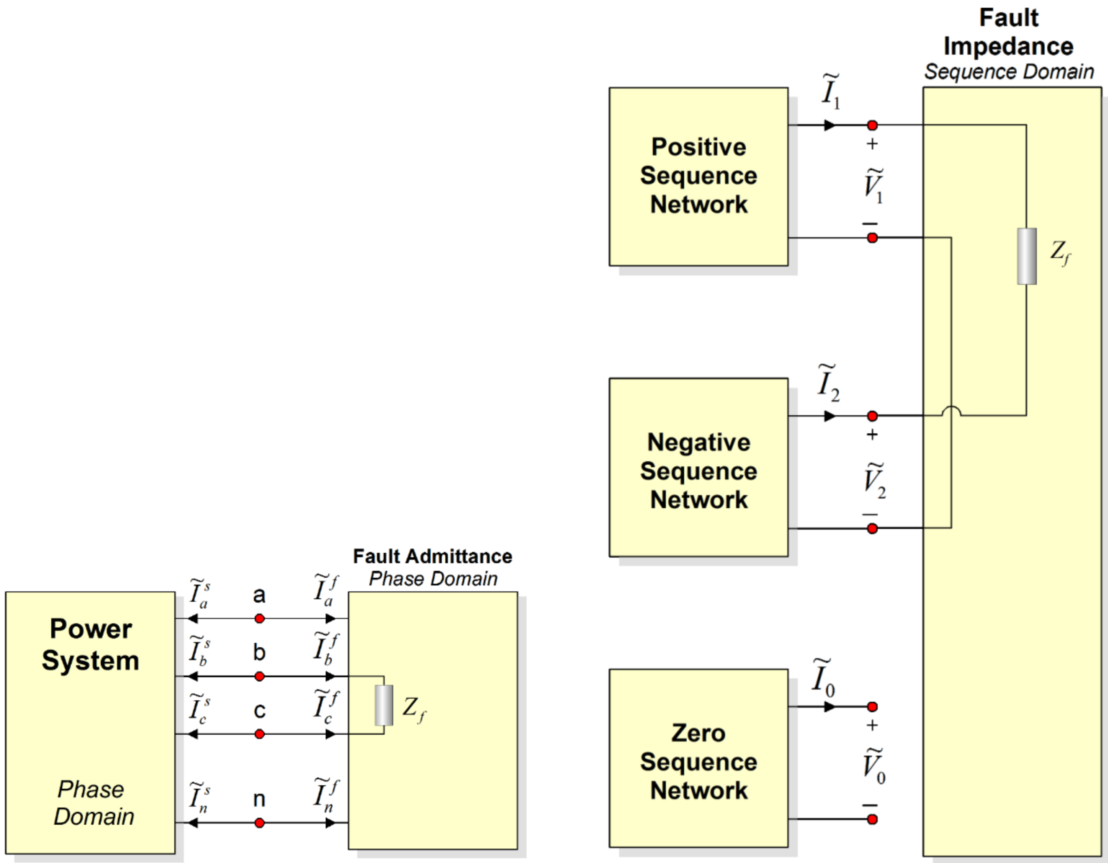


Figure 4.4: Equivalent Sequence Network and Fault Model for a Line-to-Line Fault

Upon transformation of Equation (4.22) into the sequence model, we have

$$\begin{bmatrix} \tilde{I}_1^f \\ \tilde{I}_2^f \\ \tilde{I}_0^f \end{bmatrix} = \begin{bmatrix} Y_f & -Y_f & 0 \\ -Y_f & Y_f & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix} \qquad (4.23)$$

The admittance matrix above corresponds to an admittance Y_f connected between the positive and negative sequence networks as illustrated in Figure 4.4.

Example E4.2: Consider the electric power system of the Figure E4.2a. The system consists of an 80 MVA, 60 Hz, 15 kV generator, a 15kV/115kV, 80 MVA step-up transformer, a 28-mile 115 kV line and a delta connected constant impedance electric load. Assume a line to line fault at point A which is located on the line 18 miles from the transformer. Compute the fault current: (a) neglecting the load current, and (b) without neglecting the load current. Use symmetrical component theory in the computations. System data are as follows:

Generator : $x_1' = 0.185\, pu, \quad x_2 = j0.28\, pu, \quad x_0 = j0.06\, pu, \quad r_N = 200\, ohms$

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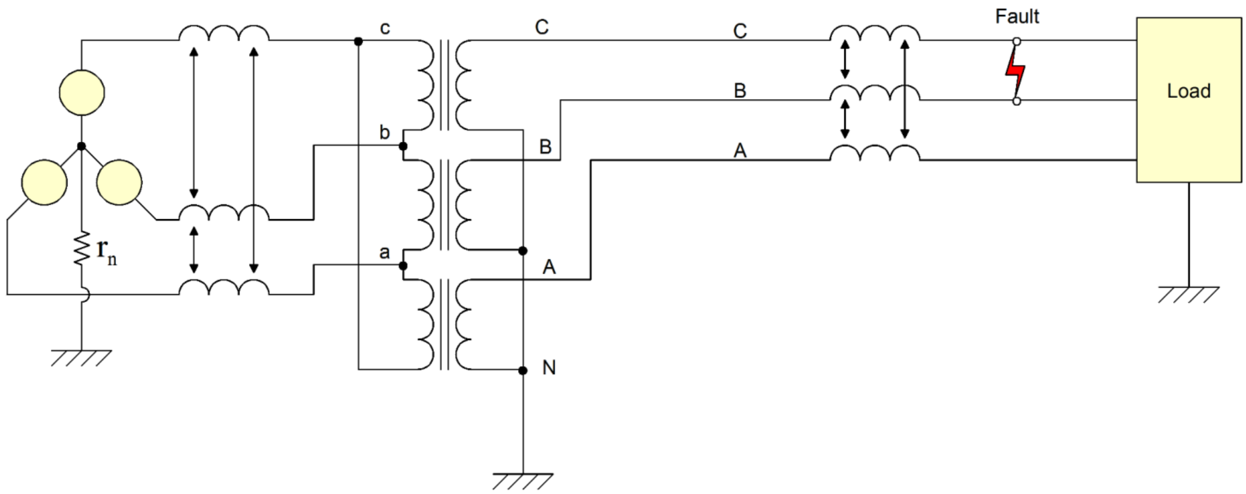


Change figure: phase sequence should be a-b-c and A-B-C.

Figure E4.2a: Faulted Power System Example – Line to Line Fault

Solution: First the sequence network of the system is developed and illustrated in Figures E4.2b and E4.2c for the case without the electric load and with the electric load respectively. Note that for this fault type only the positive and negative sequence models are involved. Subsequently the positive and negative sequence models are connected as shown in the Figures.

(a) Solution of the circuit of Figure E4.2b yields the following symmetrical component currents and voltages (for completeness we include the zero sequence values that will be all zero):

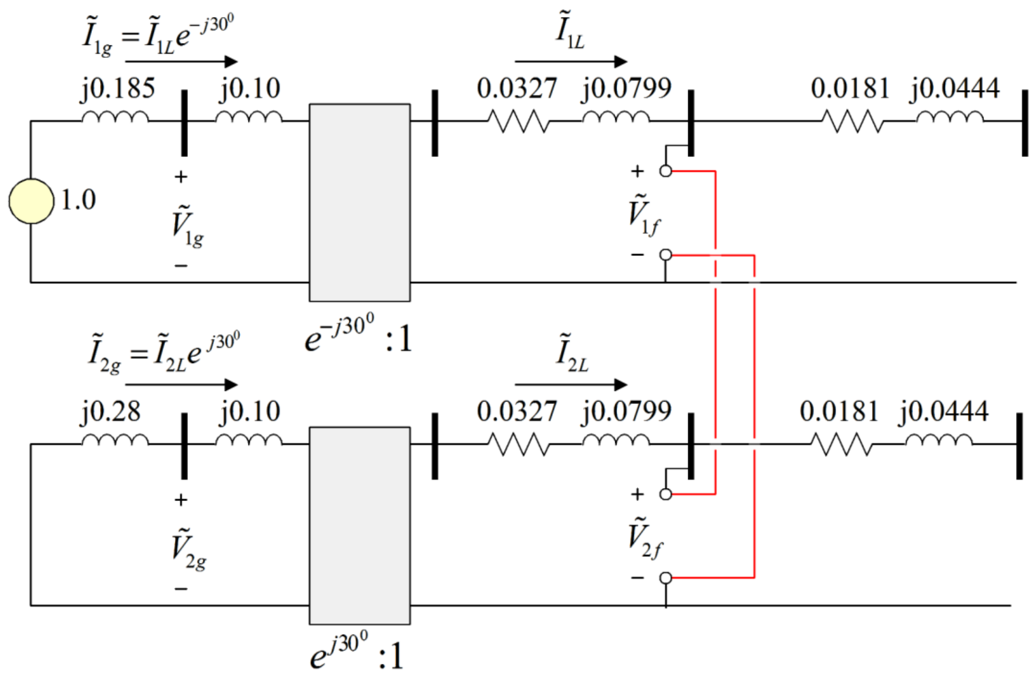


Figure E4.2b: Positive and Negative Sequence Networks of the Power System of Figure E4.2a – Without Load

$$\begin{bmatrix} \tilde{I}_{1L} \\ \tilde{I}_{2L} \\ \tilde{I}_{0L} \end{bmatrix} = \begin{bmatrix} 1.209e^{-j55.47^0} \\ 1.209e^{j124.53^0} \\ 0.0 \end{bmatrix}, \qquad \begin{bmatrix} \tilde{V}_{1L} \\ \tilde{V}_{2L} \\ \tilde{V}_{0L} \end{bmatrix} = \begin{bmatrix} 0.5572e^{j30.47^0} \\ 0.5572e^{j30.47^0} \\ 0.0 \end{bmatrix}$$

$$\begin{bmatrix} \tilde{I}_{1g} \\ \tilde{I}_{2g} \\ \tilde{I}_{0g} \end{bmatrix} = \begin{bmatrix} 1.209e^{-j85.47^0} \\ 1.209e^{j154.53^0} \\ 0.0 \end{bmatrix}, \qquad \begin{bmatrix} \tilde{V}_{1g} \\ \tilde{V}_{2g} \\ \tilde{V}_{0g} \end{bmatrix} = \begin{bmatrix} 0.xxxx\, e^{-jyyy^0} \\ 0.3384e^{j64.53^0} \\ 0.0 \end{bmatrix}$$

$$\tilde{I}_{abcg} = T\tilde{I}_{120g} = \begin{bmatrix} \tilde{I}_{1g} + \tilde{I}_{2g} + \tilde{I}_{0g} \\ a^2\tilde{I}_{1g} + a\tilde{I}_{2g} + \tilde{I}_{0g} \\ a\tilde{I}_{1g} + a^2\tilde{I}_{2g} + \tilde{I}_{0g} \end{bmatrix} = \begin{bmatrix} xxx \\ xxxxe^{-jyyy^0} \\ xxxxe^{jyyy^0} \end{bmatrix}$$

$$\tilde{V}_{abcg} = T\tilde{V}_{120g} = \begin{bmatrix} \tilde{V}_{1g} + \tilde{V}_{2g} + \tilde{V}_{0g} \\ a^2\tilde{V}_{1g} + a\tilde{V}_{2g} + \tilde{V}_{0g} \\ a\tilde{V}_{1g} + a^2\tilde{V}_{2g} + \tilde{V}_{0g} \end{bmatrix} = \begin{bmatrix} xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \end{bmatrix}$$

(b) Solution of the circuit of Figure E4.2c yields the following symmetrical component currents and voltages (for completeness we include the zero sequence values that will be all zero):

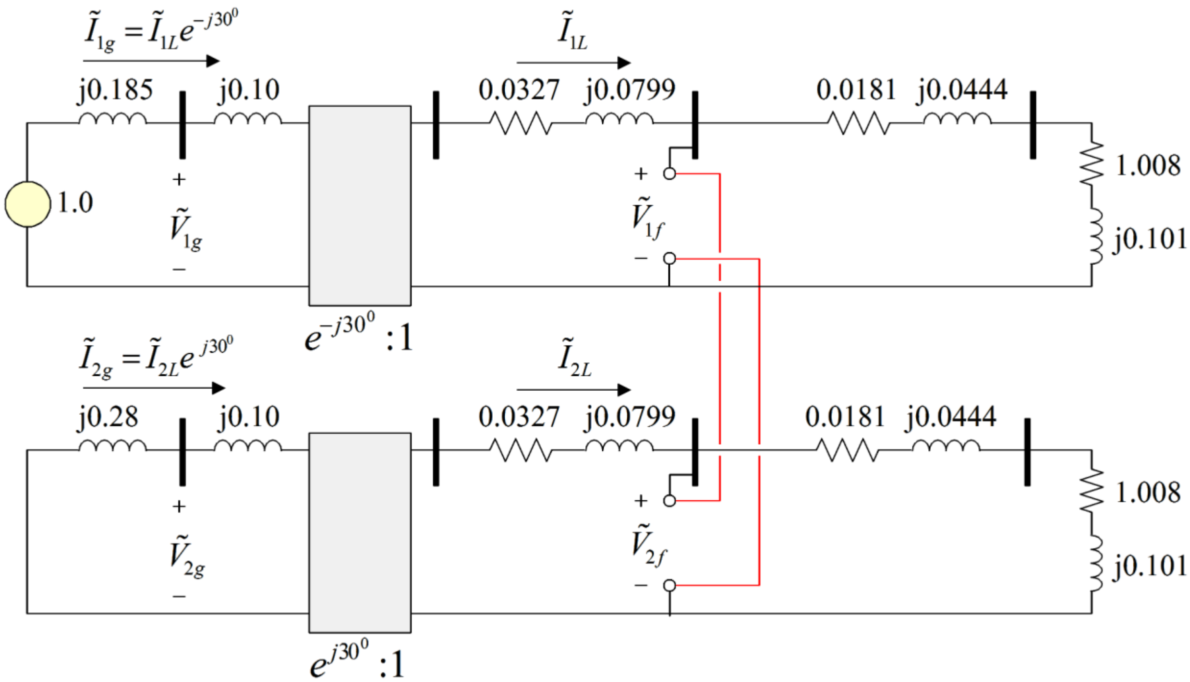


Figure E4.2c: Positive and Negative Sequence Networks of the Power System of Figure E4.2a – With Load

$$\begin{bmatrix} \tilde{I}_{1L} \\ \tilde{I}_{2L} \\ \tilde{I}_{0L} \end{bmatrix} = \begin{bmatrix} 1.543e^{-j38.71^0} \\ 1.049e^{j105.06^0} \\ 0.0 \end{bmatrix}, \quad \begin{bmatrix} \tilde{V}_{1L} \\ \tilde{V}_{2L} \\ \tilde{V}_{0L} \end{bmatrix} = \begin{bmatrix} 0.4834e^{j10.99^0} \\ 0.4834e^{j10.99^0} \\ 0.0 \end{bmatrix}$$

$$\begin{bmatrix} \tilde{I}_{1g} \\ \tilde{I}_{2g} \\ \tilde{I}_{0g} \end{bmatrix} = \begin{bmatrix} 1.543e^{-j68.71^0} \\ 1.049e^{j135.06^0} \\ 0.0 \end{bmatrix}, \quad \begin{bmatrix} \tilde{V}_{1g} \\ \tilde{V}_{2g} \\ \tilde{V}_{0g} \end{bmatrix} = \begin{bmatrix} xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \end{bmatrix}$$

$$\tilde{I}_{abcg} = T\tilde{I}_{120g} = \begin{bmatrix} \tilde{I}_{1g} + \tilde{I}_{2g} + \tilde{I}_{0g} \\ a^2\tilde{I}_{1g} + a\tilde{I}_{2g} + \tilde{I}_{0g} \\ a\tilde{I}_{1g} + a^2\tilde{I}_{2g} + \tilde{I}_{0g} \end{bmatrix} = \begin{bmatrix} xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \end{bmatrix}$$

$$\tilde{V}_{abcg} = T\tilde{V}_{120g} = \begin{bmatrix} \tilde{V}_{1g} + \tilde{V}_{2g} + \tilde{V}_{0g} \\ a^2\tilde{V}_{1g} + a\tilde{V}_{2g} + \tilde{V}_{0g} \\ a\tilde{V}_{1g} + a^2\tilde{V}_{2g} + \tilde{V}_{0g} \end{bmatrix} = \begin{bmatrix} xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \\ xxxe^{-jyyy^0} \end{bmatrix}$$

4.3.2 Line-to-Line-to-Ground Fault

This fault may occur in different ways: for example, when a very low impedance is connected between phases b and c and another low impedance between phase b and neutral; or when a very low impedance is connected between phase b and neutral and another between phase c and neutral. Consider the latter line-to-line-to-ground fault and assume that the fault impedance is Z_f . The fault currents will be

Power System Fault Analysis

$$\tilde{I}_a^f = 0 \tag{4.24a}$$
$$\tilde{I}_b^f = Y_f \tilde{V}_b \tag{4.24b}$$
$$\tilde{I}_c^f = Y_f \tilde{V}_c \tag{4.24c}$$

Equations (4.24) written in compact matrix notation provide the fault admittance matrix, Y_{abc}^f:

$$\begin{bmatrix} \tilde{I}_a^f \\ \tilde{I}_b^f \\ \tilde{I}_c^f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_f & 0 \\ 0 & 0 & Y_f \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = Y_{abc}^f \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} \tag{4.25}$$

Upon transformation of Eq. (4.25) into the sequence model, we obtain

$$\begin{bmatrix} \tilde{I}_1^f \\ \tilde{I}_2^f \\ \tilde{I}_0^f \end{bmatrix} = \begin{bmatrix} 2Y_f/3 & -Y_f/3 & -Y_f/3 \\ -Y_f/3 & 2Y_f/3 & -Y_f/3 \\ -Y_f/3 & -Y_f/3 & 2Y_f/3 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix} \tag{4.26}$$

The admittance matrix equation (4.26) represents a circuit that connects an admittance of $Y_f/3 = 1/3Z_f$ between any pair of sequence networks. These connections are illustrated in Figure 4.5. If the fault admittance is infinity (fault impedance is zero) Figure 4.9 suggests that the three sequence networks are connected in parallel.

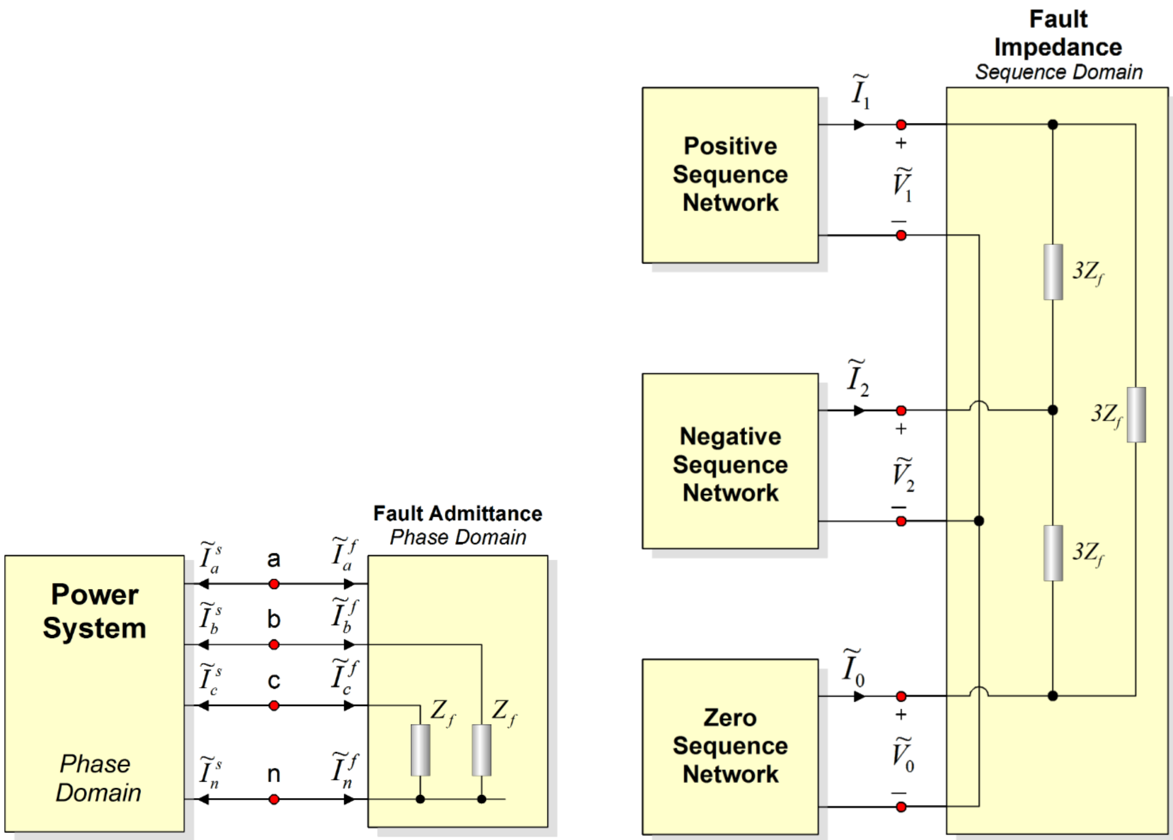


Figure 4.5: Equivalent Sequence Network and Fault Model for a Line-to-Line-to-Ground Fault

Example E4.3: Consider the electric power system of the Figure E4.3a. The system consists of an 80 MVA, 60 Hz, 15 kV generator, a 15kV/115kV, 80 MVA step-up transformer, a 28-mile 115 kV line and a delta connected constant impedance electric load. Assume a line to line to ground fault at point A which is located on the line 18 miles from the transformer. Compute the fault current: (a) neglecting the load current, and (b) without neglecting the load current. Use symmetrical component theory in the computations. System data are as follows:

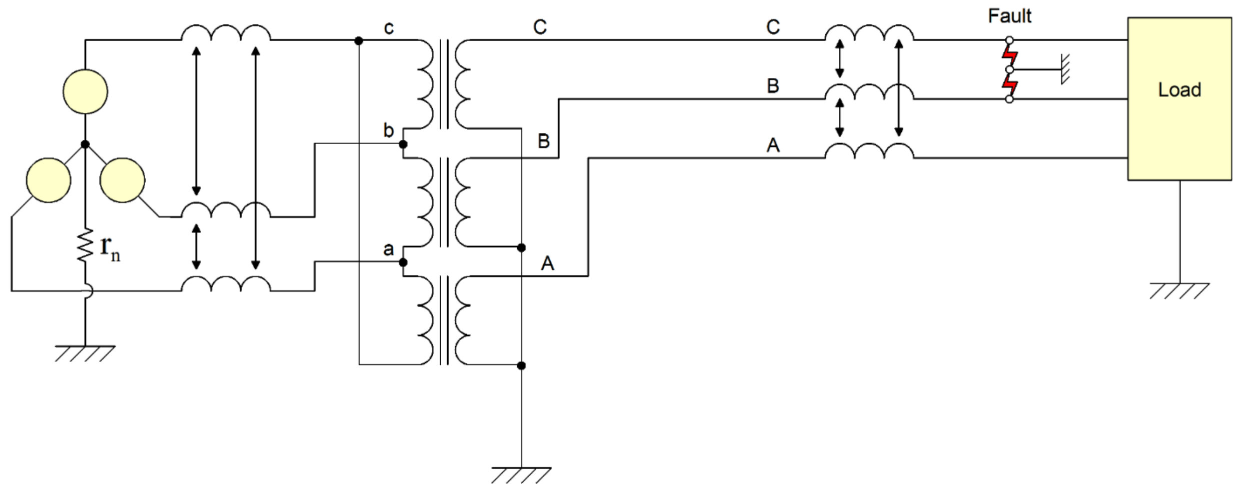
Generator : $x_1^{'} = j0.185\,pu, \quad x_2 = j0.28\,pu, \quad x_0 = j0.06\,pu, \quad r_N = 200\,ohms$

Transformer : $x_1 = x_2 = x_0 = j0.10\,pu$

Transmission line: $z_1 = z_2 = 0.3 + j0.7334\,ohms/mile, \quad z_0 = 0.34 + j1.55\,ohms/mile$

Electric load (three phase, delta connected): $z = 500 + j50\,ohms, \quad each\,leg$

Transformer shunt impedance and transmission line capacitive shunt impedance are to be neglected.



Change figure: phase sequence should be a-b-c and A-B-C.

Figure E4.3a: Faulted Power System Example – Double Line to Ground Fault

Solution: First the positive, negative and zero sequence network of the system are developed. The resulting networks are shown in Figure E4.3b. The connections at the fault location for a double line to ground fault are also indicated in the figure.

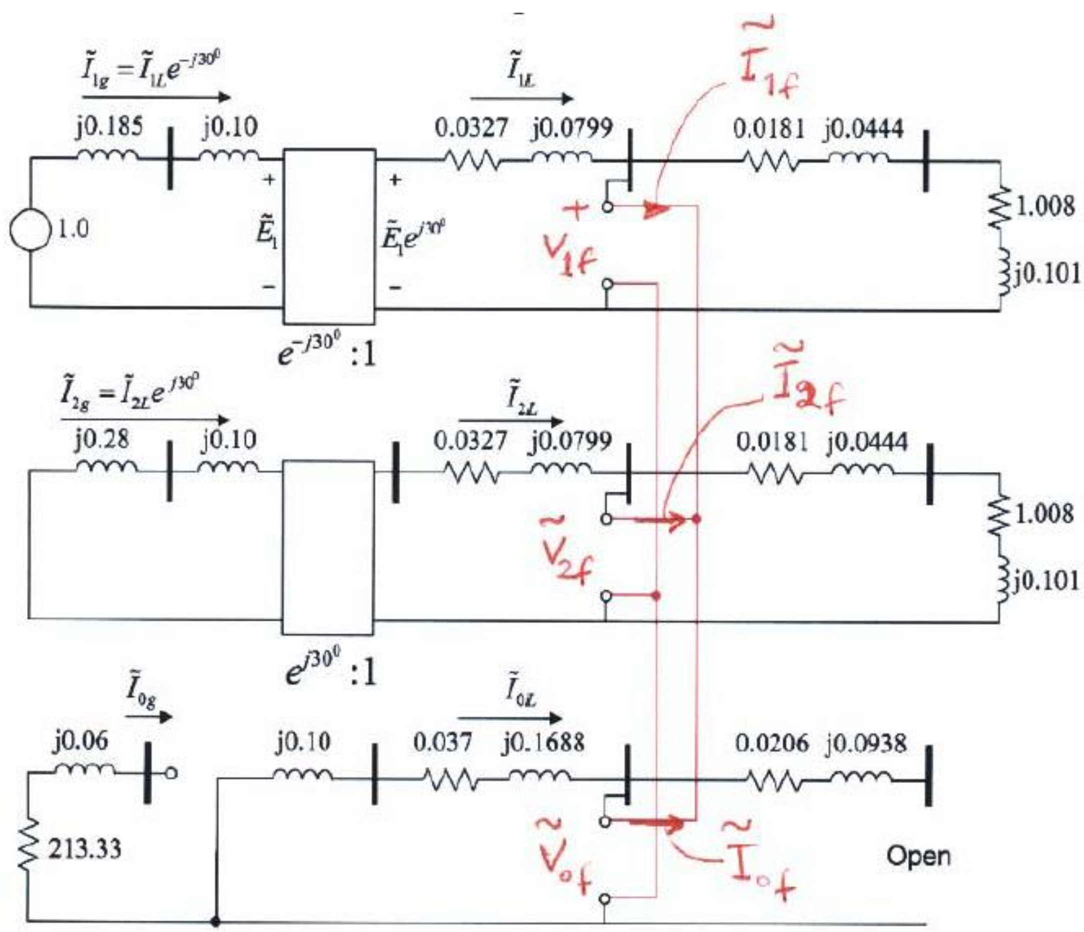


Figure E4.3b: Sequence Networks of the Power System of Figure E4.1a

For case (a) the circuit to the right of the fault will be neglected since no electric current will flow there if the load is removed. For case (b) the circuit of Figure E4.3b applies.

Case a: The applicable circuit is shown in Figure E4.3c. Since the positive, negative and zero sequence voltages at the location of the fault are equal, one can write a KCL equation that will have only this voltage as an unknown. The equation follows.

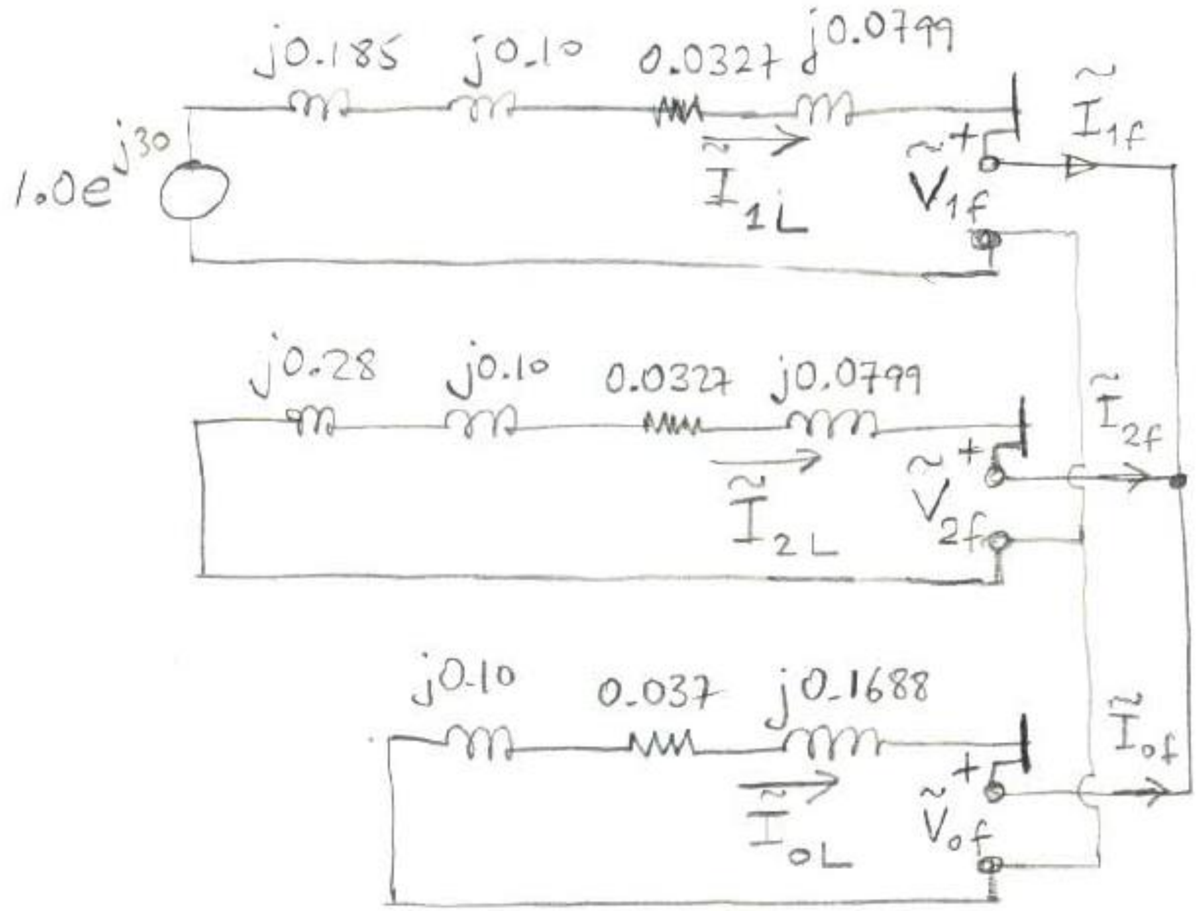


Figure E4.3c: Sequence Circuit Applicable to Case (a)

$$\frac{\tilde{V}_{1f} - 1.0e^{j30^0}}{0.0327 + j0.3649} + \frac{\tilde{V}_{1f}}{0.0327 + j0.4599} + \frac{\tilde{V}_{1f}}{0.037 + \mathrm{j}0.2688} = 0$$

Solution of above equation yields: $\tilde{V}_{1f} = 0.31811e^{j29.1^0}$. Subsequently the following are computed:

$$\tilde{I}_{1f} = \tilde{I}_{1L} = -\frac{\tilde{V}_{1f} - 1.0e^{j30^0}}{0.0327 + j0.3649} = 1.861402 e^{-j54.459^0},$$

$$\tilde{I}_{2f} = \tilde{I}_{2L} = -\frac{\tilde{V}_{1f}}{0.0327 + j0.4599} = 0.689952 e^{j123.167^0},$$

$$\tilde{I}_{0f} = \tilde{I}_{0L} = -\frac{\tilde{V}_{1f}}{0.037 + \mathrm{j}0.2688} = 1.17239 e^{j126.937^0}$$

The fault currents are:

$$\tilde{I}_{abcL} = T\tilde{I}_{120L} = \begin{bmatrix} \tilde{I}_{1L} + \tilde{I}_{2L} + \tilde{I}_{0L} \\ a^2\tilde{I}_{1L} + a\tilde{I}_{2L} + \tilde{I}_{0L} \\ a\tilde{I}_{1L} + a^2\tilde{I}_{2L} + \tilde{I}_{0L} \end{bmatrix} = \begin{bmatrix} 0.0 \\ 2.872178 e^{-j177.172^0} \\ 2.77428 e^{-j74.207^0} \end{bmatrix}$$

Case b: The applicable circuit is shown in Figure E4.3b. Since the positive, negative and zero sequence voltages at the location of the fault are equal, one can write a KCL equation that will have only this voltage as an unknown. The equation follows.

$$\frac{\tilde{V}_{1f} - 1.0e^{j30^0}}{0.0327 + j0.3649} + \frac{\tilde{V}_{1f}}{1.0261 + j0.1454} + \frac{\tilde{V}_{1f}}{0.0327 + j0.4599} + \frac{\tilde{V}_{1f}}{1.0261 + j0.1454} + \frac{\tilde{V}_{1f}}{0.037 + \mathrm{j}0.2688} = 0$$

Solution of above equation yields: $\tilde{V}_{1f} = 0.295349 e^{j17.414^0}$. Subsequently the following are computed:

$$\tilde{I}_{1f} = -\frac{\tilde{V}_{1f} - 1.0e^{j30^0}}{0.0327 + j0.3649} - \frac{\tilde{V}_{1f}}{1.0261 + j0.1454} = 1.820633 e^{-j57.429^0},$$

$$\tilde{I}_{2f} = -\frac{\tilde{V}_{1f}}{0.0327 + j0.4599} - \frac{\tilde{V}_{1f}}{1.0261 + j0.1454} = 0.753862 e^{j133.172^0},$$

$$\tilde{I}_{0f} = -\frac{\tilde{V}_{1f}}{0.037 + \mathrm{j}0.2688} = 1.088507 e^{j115.252^0}$$

The fault currents are:

$$\tilde{I}_{abcf} = T\tilde{I}_{120f} = \begin{bmatrix} \tilde{I}_{1f} + \tilde{I}_{2f} + \tilde{I}_{0f} \\ a^2\tilde{I}_{1f} + a\tilde{I}_{2f} + \tilde{I}_{0f} \\ a\tilde{I}_{1f} + a^2\tilde{I}_{2f} + \tilde{I}_{0f} \end{bmatrix} = \begin{bmatrix} 0.0 \\ 2.50795 e^{j175.856^0} \\ 2.985593 e^{j68.209^0} \end{bmatrix}$$

4.3.3 Single Line-to-Ground Fault

This fault occurs whenever a low impedance, Z_f, is connected between a phase and the neutral. Assuming that the faulted phase is phase a, the fault currents are

$$\tilde{I}_a^f = \frac{\tilde{V}_a}{Z_f} = Y_f \tilde{V}_a \tag{4.27a}$$

$$\tilde{I}_b^f = 0 \tag{4.27b}$$

$$\tilde{I}_c^f = 0 \tag{4.27c}$$

In compact matrix notation the equation above read

$$\begin{bmatrix} \tilde{I}_a^f \\ \tilde{I}_b^f \\ \tilde{I}_c^f \end{bmatrix} = \begin{bmatrix} Y_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} \tag{4.28}$$

Upon transformation of the equation above into the equation model, we have

$$\begin{bmatrix} \tilde{I}_1^f \\ \tilde{I}_2^f \\ \tilde{I}_0^f \end{bmatrix} = \begin{bmatrix} Y_f/3 & Y_f/3 & Y_f/3 \\ Y_f/3 & Y_f/3 & Y_f/3 \\ Y_f/3 & Y_f/3 & Y_f/3 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix} \tag{4.29}$$

The connectivity of the sequence networks from Equation (4.29) is not obvious. Observe that Equation (4.29) states that all the sequence currents are identical, that is,

$$\tilde{I}_1^f = \tilde{I}_2^f = \tilde{I}_0^f = \frac{Y_f}{3}\left(\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0\right) = \frac{1}{3Z_f}\left(\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0\right)$$

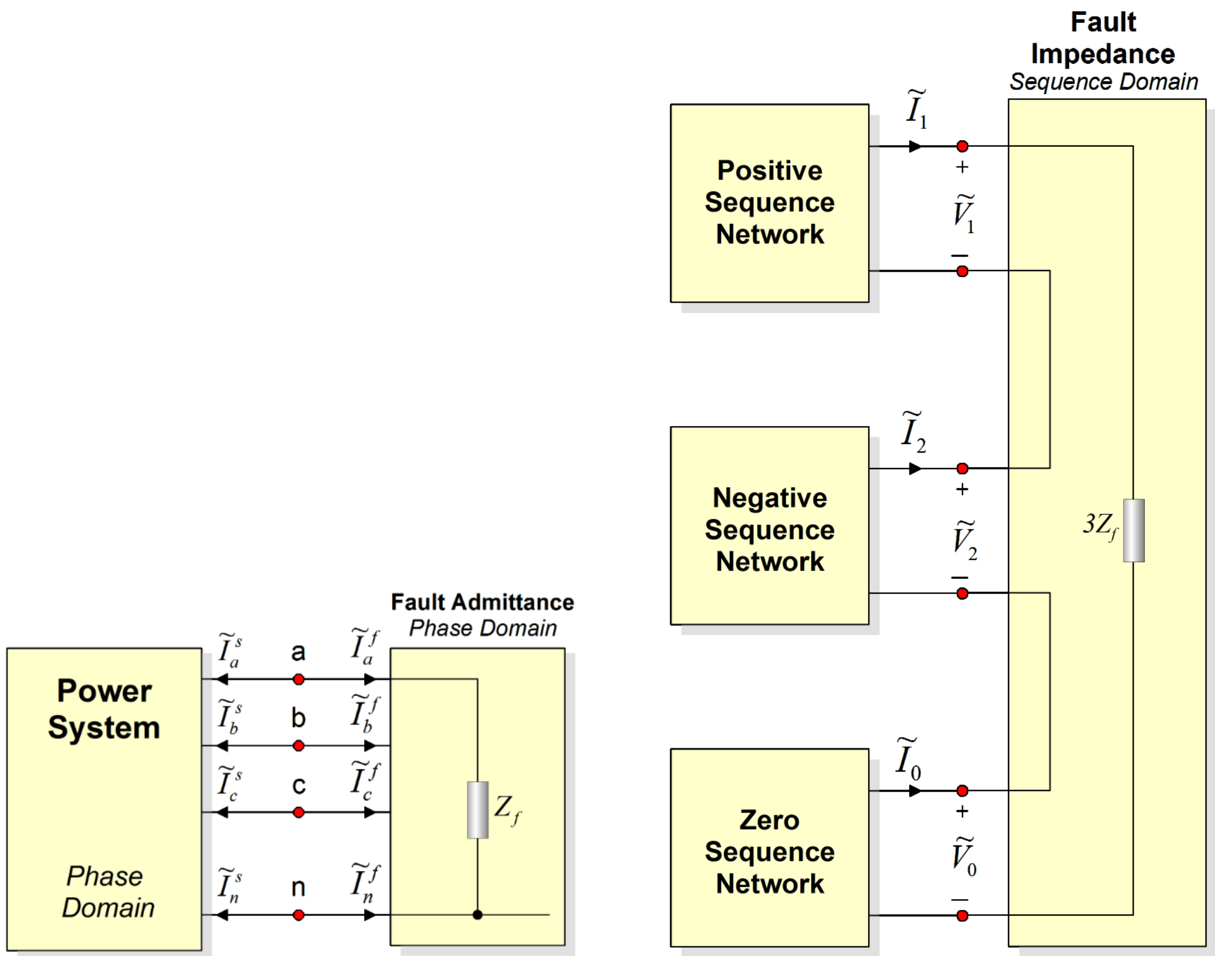


Figure 4.6 Equivalent Sequence Network and Fault Model for a Line-to-Ground Fault

These equations are satisfied only if the sequence networks are connected in series with the impedance $3Z_f$. The connections are illustrated in Figure 4.6.

Example E4.4 Consider the electric power system of the Figure E4.4a. The system consists of a 80 MVA, 60 Hz, 15 kV generator, a 15kV/115kV, 80 MVA step-up transformer, a 28-mile 115 kV line and a delta connected constant impedance electric load. Assume a single phase-to-ground fault at point A which is located on the line 18 miles from the transformer. Compute the fault current: (a) neglecting the load current, and (b) without neglecting the load current. Use symmetrical component theory in the computations. System data are as follows:

Generator : $x_1' = 0.185\,pu, \quad x_2 = j0.28\,pu, \quad x_0 = j0.06\,pu, \quad r_N = 50\,ohms$ on generator ratings

Transformer : $x_1 = x_2 = x_0 = j0.10\,pu$ on transformer ratings

Transmission line: $z_1 = z_2 = 0.3 + j0.7334\,ohms/mile, \quad z_0 = 0.34 + j1.55\,ohms/mile$

Electric load (three phase, delta connected): $z_1 = z_2 = 500 + j50\, ohms, \quad z_0 = \infty$

Transformer shunt impedance and transmission line capacitive shunt impedance are to be neglected.

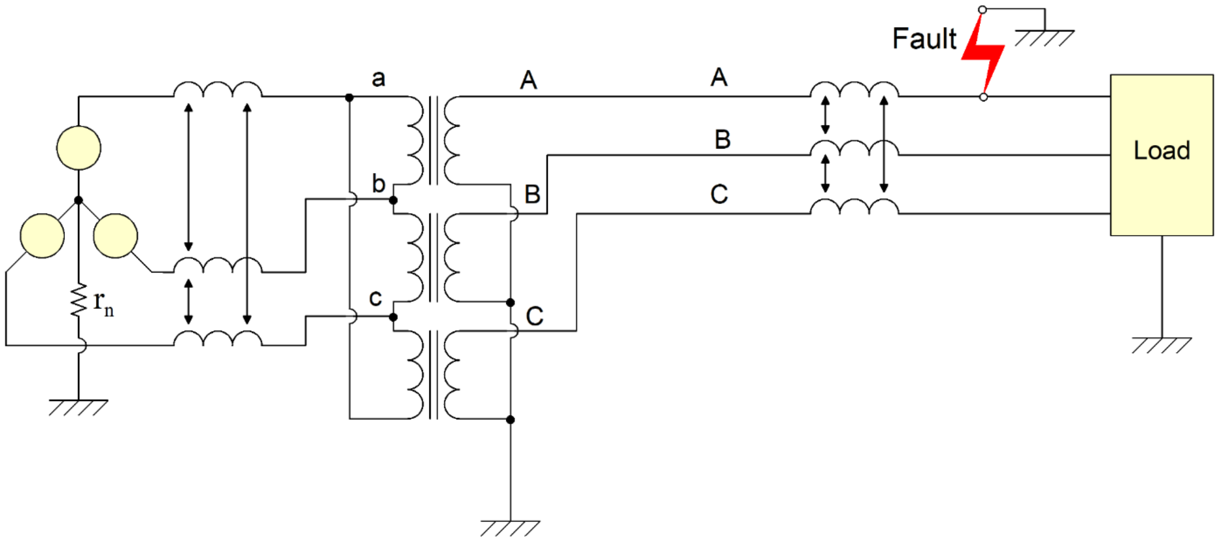


Figure E4.4a: Faulted Power System Example – Single Line to Ground Fault

Solution: First, each element of the power system is replaced with the sequence equivalent circuit. This process creates the sequence network of the system as it is illustrated in Figure E4.4a. Note that all impedances have been converted in pu on an 80-MVA basis. At the 115-kV level, the base impedance is:

$$Z_b = \frac{115^2}{80} = 165.3\, ohms$$

The sequence networks are constructed and illustrated in Figure E4.4b. The figure illustrates the electric load as well.

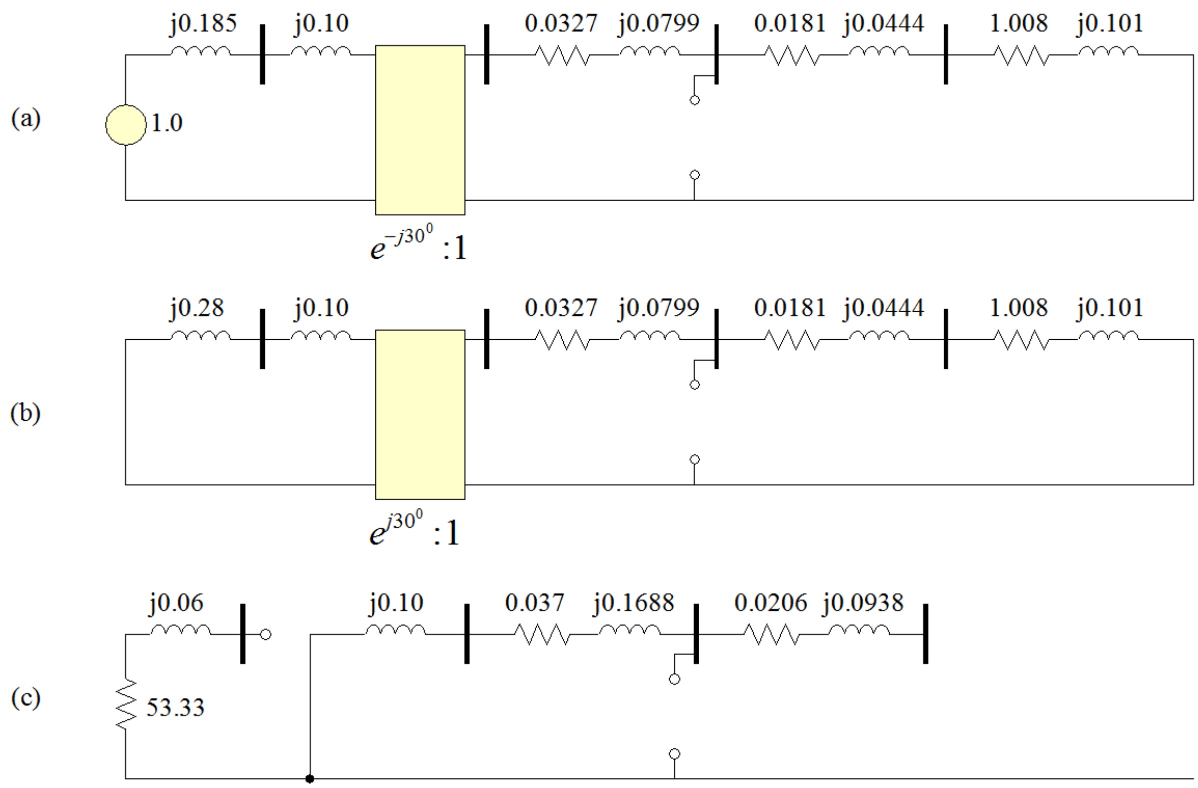


Figure E4.4b. Sequence networks for the power system of Figure E4.2. (a) positive sequence, (b) negative sequence, (c) zero sequence

(a) The electric load is neglected. The Thevenin equivalent sequence networks in this case are constructed from the circuit of Figure E4.4a by removing the electric load. In this case the part of the circuit on the right side of the fault location will not carry electric current and it will not affect the results. The sequence currents are

$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{1.0e^{j30^0}}{0.1024 + j1.0936} = 0.9104e^{-j54.65^\circ} \ pu$$

The electric current distribution is illustrated in Figure E4.4c. The phase electric currents at the fault location are:

$$\tilde{I}_a = 3\tilde{I}_0 = 2.7312e^{-j54.65^\circ} \ pu \quad or \ 1{,}096.9e^{-j54.65^0} \ Amperes$$
$$\tilde{I}_b = 0$$
$$\tilde{I}_c = 0$$

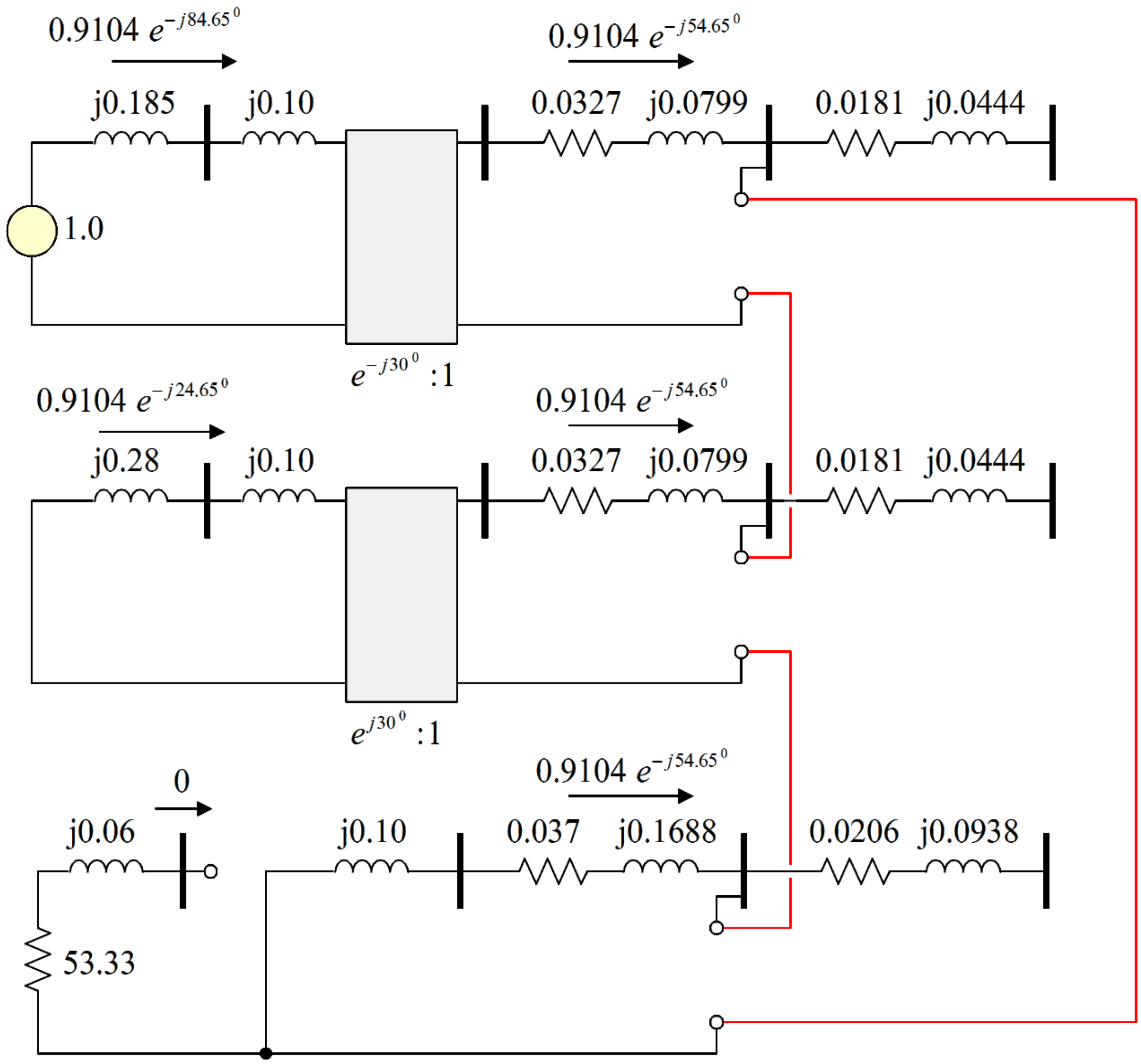


Figure E4.4c Sequence Equivalent Circuits and Electric Current Flow – Electric Load is Neglected

The symmetrical components of the voltages at the location of the fault are:

$$\tilde{V}_{f1} = 1.0e^{j30^0} - (0.0327 + j0.3649)\left(0.9104e^{-j54.65^0}\right) = 0.6665e^{j29.88^\circ}$$
$$\tilde{V}_{f2} = -(0.0327 + j0.4599)\left(0.9104e^{-j54.65^0}\right) = 0.4197e^{-j148.72^0}$$
$$\tilde{V}_{f0} = -(0.0370 + j0.2688)\left(0.9104e^{-j54.65^0}\right) = 0.2470e^{-j152.49^0}$$

The phase voltages at the location of the fault are:

$$\tilde{V}_{fabc} = T\tilde{V}_{f120}$$

$$\tilde{V}_a = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0 = 0.0 + j0.0$$

$$\tilde{\boldsymbol{V}}_{\boldsymbol{b}} = \boldsymbol{a}^2\tilde{\boldsymbol{V}}_1 + \boldsymbol{a}\tilde{\boldsymbol{V}}_2 + \tilde{\boldsymbol{V}}_0 = 0.6665\boldsymbol{e}^{-\boldsymbol{j}90.12^\circ} + 0.4197\boldsymbol{e}^{-\boldsymbol{j}28.72^0} + 0.2470\boldsymbol{e}^{-\boldsymbol{j}152.49^0} = 0.9933\boldsymbol{e}^{-\boldsymbol{j}81.454^0}$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{c}} = \boldsymbol{a}\tilde{\boldsymbol{V}}_1 + \boldsymbol{a}^2\tilde{\boldsymbol{V}}_2 + \tilde{\boldsymbol{V}}_0 = 0.6665\boldsymbol{e}^{\boldsymbol{j}149.88^\circ} + 0.4197\boldsymbol{e}^{-\boldsymbol{j}268.72^0} + 0.2470\boldsymbol{e}^{-\boldsymbol{j}152.49^0} = 1.0283\boldsymbol{e}^{\boldsymbol{j}141.51^0}$$

The generator electric currents are shown in the figure. The symmetrical components of the voltages at the generator terminals are:

$$\tilde{V}_{g1} = 1.0 - (j0.185)\left(0.9104e^{-j84.65^0}\right) = 0.8325e^{-j1.08^0}$$
$$\tilde{V}_{g2} = -(j0.28)\left(0.9104e^{-j24.65^0}\right) = 0.2549e^{-j114.65^0}$$
$$\tilde{V}_{g0} = 0$$

The phase voltages at the generator terminals are:

$$\tilde{V}_{gabc} = T\tilde{V}_{g120}$$

$$\tilde{\boldsymbol{V}}_{\boldsymbol{ga}} = \tilde{\boldsymbol{V}}_{\boldsymbol{g}1} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}0} = 0.8325\boldsymbol{e}^{-\boldsymbol{j}1.08^0} + 0.2549\boldsymbol{e}^{-\boldsymbol{j}114.65^0} = 0.7670\,\boldsymbol{e}^{-\boldsymbol{j}18.81^0}$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{gb}} = \boldsymbol{a}^2\tilde{\boldsymbol{V}}_{\boldsymbol{g}1} + \boldsymbol{a}\tilde{\boldsymbol{V}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}0} = 0.8325\boldsymbol{e}^{-\boldsymbol{j}121.08^0} + 0.2549\boldsymbol{e}^{\boldsymbol{j}5.35^0} = 0.7113\boldsymbol{e}^{-\boldsymbol{j}104.32^0}$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{gc}} = \boldsymbol{a}\tilde{\boldsymbol{V}}_{\boldsymbol{g}1} + \boldsymbol{a}^2\tilde{\boldsymbol{V}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}0} = 0.8325\boldsymbol{e}^{\boldsymbol{j}118..92^0} + 0.2549\boldsymbol{e}^{-\boldsymbol{j}234.65^0} = 1.0862\boldsymbol{e}^{\boldsymbol{j}120.43^0}$$

The phase currents at the generator terminals are:

$$\tilde{I}_{g,abc} = T\tilde{I}_{g,120}$$

$$\tilde{\boldsymbol{I}}_{\boldsymbol{ga}} = \tilde{\boldsymbol{I}}_{\boldsymbol{g}1} + \tilde{\boldsymbol{I}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{I}}_{\boldsymbol{g}0} = 0.9104\boldsymbol{e}^{-\boldsymbol{j}84.65^\circ} + 0.9104\boldsymbol{e}^{-\boldsymbol{j}24.65^\circ} = 1.5768591\boldsymbol{e}^{-\boldsymbol{j}54.65^\circ}$$
$$\tilde{\boldsymbol{I}}_{\boldsymbol{gb}} = \alpha^2\tilde{\boldsymbol{I}}_{\boldsymbol{g}1} + \alpha\tilde{\boldsymbol{I}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{I}}_{\boldsymbol{g}0} = 0.9104\boldsymbol{e}^{\boldsymbol{j}155.35^\circ} + 0.9104\boldsymbol{e}^{\boldsymbol{j}95.35^\circ} = 1.5768591\boldsymbol{e}^{\boldsymbol{j}125.35^\circ}$$
$$\tilde{\boldsymbol{I}}_{\boldsymbol{c}} = \alpha^2\tilde{\boldsymbol{I}}_{\boldsymbol{g}1} + \alpha\tilde{\boldsymbol{I}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{I}}_{\boldsymbol{g}0} = 0.9104\boldsymbol{e}^{\boldsymbol{j}35.35^\circ} + 0.9104\boldsymbol{e}^{-\boldsymbol{j}144.65^\circ} = 0$$

(b) The electric load is not neglected. In this case the equivalent sequence networks and their interconnection for the single line to ground fault is illustrated in Figure E4.4c. This circuit can be solved by any circuit analysis method, i.e. loop analysis, nodal analysis, etc. The solution is shown in Figure E4.4d.

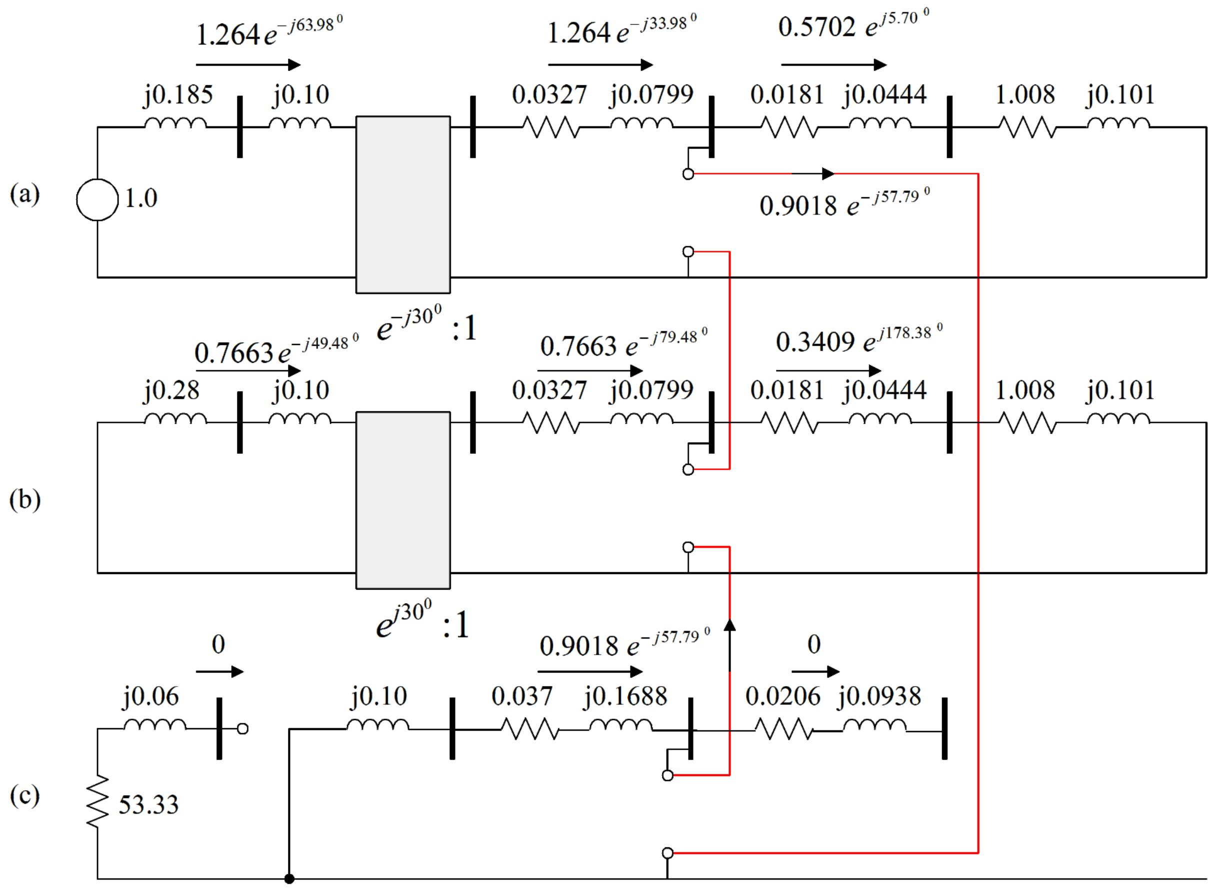


Figure E4.4d. Sequence Equivalent Circuits and Electric Current Flow – Electric Load is Included

The generator electric currents (symmetrical components) are shown in Figure E4.4c. The generator phase electric currents are:

$$\tilde{I}_{g,abc} = T\tilde{I}_{g,120}$$

$$\tilde{\boldsymbol{I}}_{ga} = \tilde{\boldsymbol{I}}_{g1} + \tilde{\boldsymbol{I}}_{g2} + \tilde{\boldsymbol{I}}_{g0} = 1.264\boldsymbol{e}^{-j63.98^0} + 0.7663\boldsymbol{e}^{-j49.48^0} = 2.0150\boldsymbol{e}^{-j58.51^0}$$
$$\tilde{\boldsymbol{I}}_{gb} = \boldsymbol{a}^2\tilde{\boldsymbol{I}}_{g1} + \boldsymbol{a}\tilde{\boldsymbol{I}}_{g2} + \tilde{\boldsymbol{I}}_{g0} = 1.264\boldsymbol{e}^{-j183.98^0} + 0.7663\boldsymbol{e}^{j70.52^0} = 1.2910\boldsymbol{e}^{j141.13^0}$$
$$\tilde{\boldsymbol{I}}_{gc} = \boldsymbol{a}\tilde{\boldsymbol{I}}_{g1} + \boldsymbol{a}^2\tilde{\boldsymbol{I}}_{g2} + \tilde{\boldsymbol{I}}_{g0} = 1.264\boldsymbol{e}^{j56.02^0} + 0.7663\boldsymbol{e}^{-j169.48^0} = 0.9094\boldsymbol{e}^{j92.96^0}$$

The generator voltages (symmetrical components) are:

$$\tilde{V}_{g1} = 1.0 - (j0.185)\left(1.264e^{-j63.98^0}\right) = 0.7965e^{-j7.40^0}$$
$$\tilde{V}_{g2} = -(j0.28)\left(0.7663e^{-j49.48^0}\right) = 0.2146e^{-j139.48^0}$$
$$\tilde{V}_{g0} = 0$$

The generator phase voltages are:

$$\tilde{V}_{gabc} = T\tilde{V}_{g120}$$

$$\tilde{\boldsymbol{V}}_{\boldsymbol{ga}} = \tilde{\boldsymbol{V}}_{\boldsymbol{g}1} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}0} = 0.7965\boldsymbol{e}^{-\boldsymbol{j}7.40^0} + 0.2146\boldsymbol{e}^{-\boldsymbol{j}139.48^0} = 0.6718\boldsymbol{e}^{-\boldsymbol{j}21.11^0}$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{gb}} = \boldsymbol{a}^2\tilde{\boldsymbol{V}}_{\boldsymbol{g}1} + \boldsymbol{a}\tilde{\boldsymbol{V}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}0} = 0.7965\boldsymbol{e}^{-\boldsymbol{j}127.40^0} + 0.2146\boldsymbol{e}^{-\boldsymbol{j}19.48^0} = 0.7584\boldsymbol{e}^{-\boldsymbol{j}111.78^0}$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{gc}} = \boldsymbol{a}\tilde{\boldsymbol{V}}_{\boldsymbol{g}1} + \boldsymbol{a}^2\tilde{\boldsymbol{V}}_{\boldsymbol{g}2} + \tilde{\boldsymbol{V}}_{\boldsymbol{g}0} = 0.7965\boldsymbol{e}^{\boldsymbol{j}112.60^0} + 0.2146\boldsymbol{e}^{-\boldsymbol{j}259.48^0} = 1.0073\boldsymbol{e}^{\boldsymbol{j}110.04^0}$$

For the computation of the fault current, we first compute the positive, negative and zero sequence currents at the fault. Note that:

$$\tilde{I}_{f1} = \tilde{I}_{f2} = \tilde{I}_{f0} = 1.264e^{-j33.98^0} - 0.5702e^{j5.70^0} = 0.9018e^{-j57.79^0}$$

Thus the fault currents are:

$$\tilde{I}_{fabc} = T\tilde{I}_{f120} = \begin{bmatrix} 3\tilde{I}_0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.7054e^{-j57.79^0} \\ 0 \\ 0 \end{bmatrix}$$

Comparing the two cases, it is obvious that the presence of the electric load does not substantially affect the fault current. Of course, since the electric load is represented with a passive R, L equivalent, it does not contribute to the fault current.

In summary, we have discussed procedures by which the sequence networks of a power system must be connected to provide a circuit for the computation of fault currents under specific fault conditions. Specifically, under steady-state but possibly unbalanced conditions, the system is represented with three sequence networks, as in Figure 4.6b. At the point of a fault (symmetric or asymmetric), the sequence networks must be interconnected because it is this area where the sequence quantities interact. The type of interaction, and therefore the way the sequence networks should be interconnected, depend on the fault type. Figures 4.7 through 4.10 illustrate the connections required for the four types of faults mentioned earlier.

Computationally, the analysis of symmetrical or asymmetrical faults requires the following steps:

<u>Step 1:</u> Compute the three equivalent circuits (Norton or Thevenin equivalent) at the fault location (positive, negative, and zero sequence).
<u>Step 2:</u> Interconnect the three equivalent circuits depending on the fault type.
<u>Step 3:</u> Compute the symmetrical components voltages and currents anywhere in the network of step 2. Subsequently compute the phase quantities.

From the tasks above, the most complex and computationally demanding is task 1. It requires the formation of the three sequence networks and subsequent reduction of these to a Norton or Thevenin equivalent. The formation of the three sequence networks is accomplished through interconnection of the sequence models of individual devices. The procedure is straightforward. Four relatively simple examples have illustrated the procedure.

The general approach for fault analysis of large systems is discussed in the next section.

4.4 Fault Analysis of Large Networks

The presented fault analysis method must be applied to the entire network of the power system so that the contributions from all generating sources must be accounted for. For practical reasons this analysis should be fast. In this section we present the application of the sequence component analysis to large scale power systems for the purpose of computing the fault currents. In general the process is quite straightforward and involves the following steps:

Step 1: First each power system component is modeled with their positive, negative and zero sequence models. Subsequently, the three sequence networks are created as follows:
Step 2: The positive sequence model of the network is generated by simply substituting the positive sequence model of each component into the network.
Step 3: The negative sequence model of the network is generated by simply substituting the negative sequence model of each component into the network.
Step 4: The zero sequence model of the network is generated by simply substituting the zero sequence model of each component into the network.
Step 5: Next the fault type to be analyzed is considered. For the specific fault type connect the sequence networks as dictated by the type of the fault (see Figures 4.x, 4.x, 4.x and 4.x). For example for a there phase fault, simply insert a zero impedance element between the positive sequence network at the location of the fault and the reference node; the negative and zero sequence networks will not be involved. Similarly, for a single line to ground fault, simply connect all three networks in series at the location of the fault, etc. The end result is that for a specific fault we will have a lager network to be solved.
Step 6: Solve the network resulting from Step 5. The solution will provide the voltages and currents at any location of the positive, negative, and zero sequence network model.
Step 7: Use the voltages and currents of the solution in step 6 to compute the phase voltages and currents anywhere in the actual network. Note this step simply involves the conversion of the sequence components of the voltages and currents into phase voltages and currents using the inverse symmetrical transformation.

From the computational point of view, the most demanding task is the solution of the network (Task 6). In terms of methods, any circuit analysis method can be used. For example, loop analysis, nodal analysis, hybrid, etc. However, very efficient methods (sparsity coded) have been developed for nodal analysis and it is the preferred method for large systems. For completeness, in the next section we discuss the two major circuit analysis methods, i.e. loop analysis and nodal analysis.

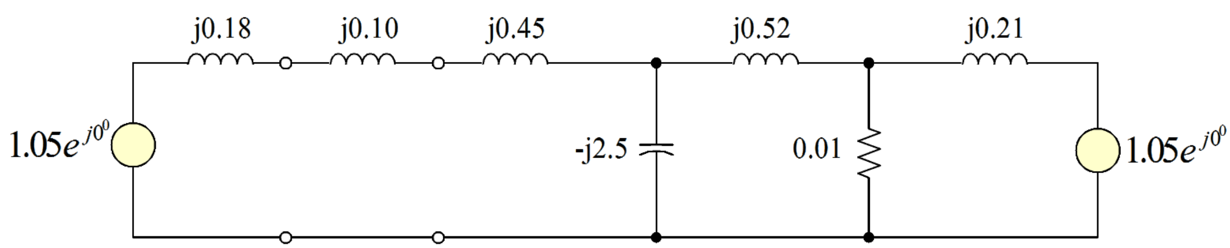
4.4.1 Circuit Analysis Methods

In this section we discuss two circuit analysis methods for fault analysis: (a) the loop analysis and (b) the nodal analysis. The basis of these methods has been also discussed in Chapter 2. Here we will focus on the procedures for creating the network in a form suitable for one of these analysis forms.

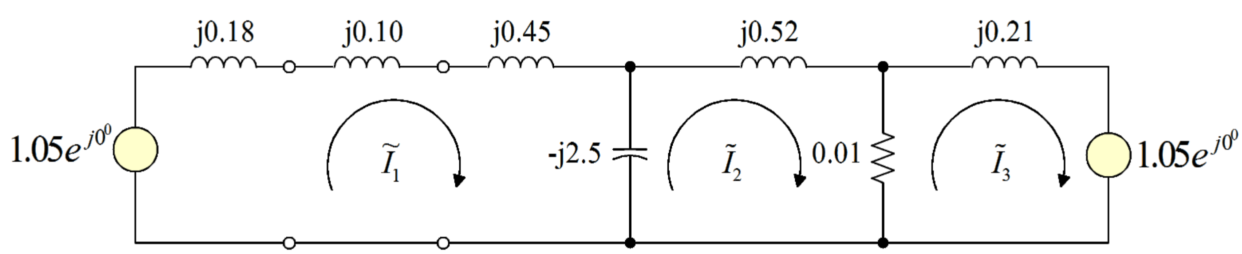
Loop Analysis: The basic idea of the loop analysis is as follows. If we have a large network, we can define a number of electric currents in such a way that if these currents are known, then any other quantity in the network, such as voltages, etc. can be computed. The values of these currents can be computed by a set of equations that are generated as follows: a number of loops in the network are identified in such a way that the number of loops equals the number of the unknown electric currents. Then, Kirchoff's voltage law is written for each one of the loops. This process yields a number of equations which is equal to the number of unknown electric currents. In other words we have a set of equations and an equal number of unknown. Solution of these equations yields the unknown electric currents. Care must be exercised to make sure that these equations are independent.

The process will be illustrated with an example.

Example E4.6: The equivalent circuit of a faulted network is shown in Figure E4.6. The fault is indicated with the 0.01 pu fault resistance. Solve this network using loop analysis.



Solution: Note three loops are readily identified. Defining a loop current in each one of the three loops, results in the system below. Note that the loop currents are defined as the current in the loop branches that are not shared with other loops. In branches that are shared by loops, such as the capacitor branch shown in the figure, the current flow will be a combination of loop currents, in this case I1-I2.



Kirchoff's voltage law around each one of the loops yields the following three equations:

$$1.05 + j0.75\tilde{I}_1 - j2.5\left(\tilde{I}_1 - \tilde{I}_2\right) = 0$$
$$j0.52\tilde{I}_2 + 0.01\left(\tilde{I}_2 - \tilde{I}_3\right) + j2.5\left(\tilde{I}_1 - \tilde{I}_2\right) = 0$$
$$j0.21\tilde{I}_3 - 1.05 - 0.01\left(\tilde{I}_2 - \tilde{I}_3\right) = 0$$

Above equations can be written in matrix form as follows:

$$\begin{bmatrix} -j1.75 & j2.5 & 0 \\ j2.5 & 0.01 - j1.98 & -0.01 \\ 0 & -0.01 & 0.01 + j0.21 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix} = \begin{bmatrix} -1.05 \\ 0 \\ 1.05 \end{bmatrix}$$

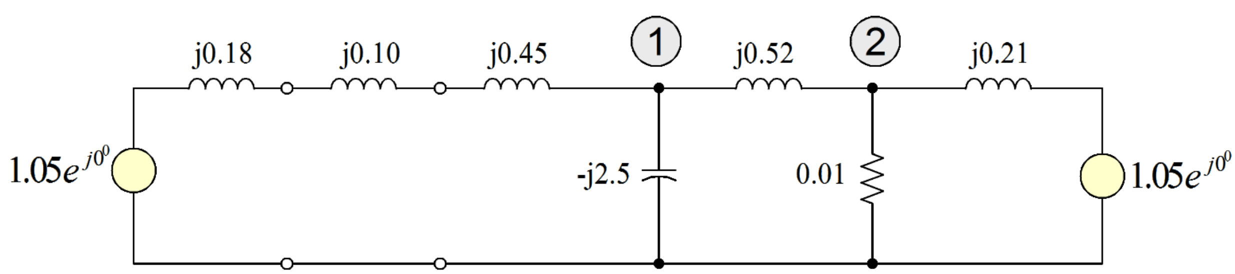
Solution of above equation yields:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix} = \begin{bmatrix} xxe^{jxx} \\ xxe^{jxx} \\ xxe^{jxx} \end{bmatrix}$$

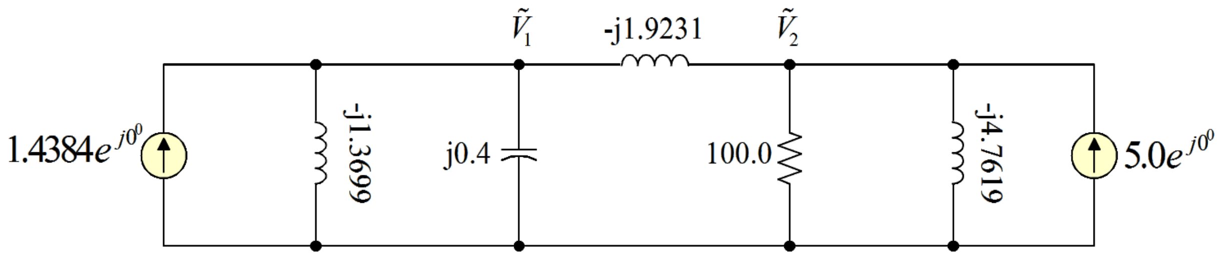
Nodal Analysis: The basic idea of the nodal analysis is the recognition that knowledge of the voltages at each node of a network can provide any other quantity of interest, for example currents, etc. For this reason the method defines the voltages at each node of the network as the unknowns to be computed. Subsequently, Kirchoff's current law is written for each node of the system. Each current is expressed as a function of the unknown node voltages. This process yields as many equations as nodes and they are expressed in terms of the node voltages. In other words we have a set of equations and an equal number of unknown. Solution of these equations yields the unknown node voltages. The process will be illustrated with an example.

Example E4.7: Consider the equivalent circuit of a faulted network, the same as in Example E4.6. Solve this network using nodal analysis.

Solution: First observe that we can identify two nodes (in addition to the reference node) where if the voltages are known at these nodes we can compute anything else in the circuit. These two nodes are identified as 1 and 2 in the figure.



Subsequently we replace the elements of the circuit with Norton equivalents as shown in the figure below, i.e. all impedances are converted to admittances and all voltage sources to current sources.



The nodal equations for above network are:

$$-1.4384 - j1.3699\tilde{V}_1 + j0.4\tilde{V}_1 - j1.9231\left(\tilde{V}_1 - \tilde{V}_2\right) = 0$$

$$-5.0 - j4.7619\tilde{V}_2 + 100.0\tilde{V}_2 - j1.9231\left(\tilde{V}_2 - \tilde{V}_1\right) = 0$$

Above equations can be written in matrix form as follows:

$$\begin{bmatrix} -j2.8930 & j1.9231 \\ j1.9231 & 100.0 - j6.6850 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} 1.4384 \\ 5.0 \end{bmatrix}$$

Solution of above equation yields:

$$\begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} xxe^{jxx} \\ xxe^{jxx} \end{bmatrix}$$

The procedures described above can be applied in a systematic way in any size network. Any electric power network can be thought of consisting of a number of components. For fault analysis based on symmetrical components, the basic and most usual elements that are encountered are: (a) sources, (b) resistors, (c) inductors, (d) capacitors, and (e) magnetically coupled elements. If the selected analysis method is loop analysis, it is expedient to write the model equations in terms of impedances and voltage sources – in this case it is easier to write the loop equations. We refer to this form as the Thevenin equivalent form. Figure 4.7, column three provides the Thevenin equivalent form for the most usual components. If the selected analysis method is nodal analysis, it is expedient to write the model equations in terms of admittances and current sources – in this case it is easier to write the nodal equations. We refer to this form as the Norton equivalent form. Figure 4.7, column four provides the Norton equivalent form for the most usual components.

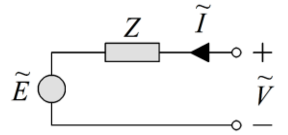
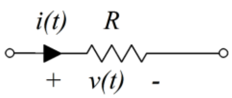
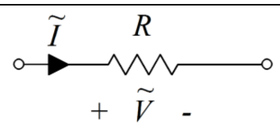
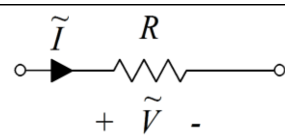
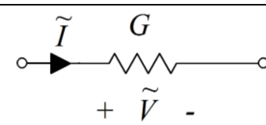
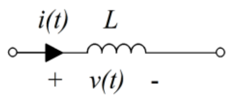
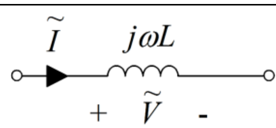
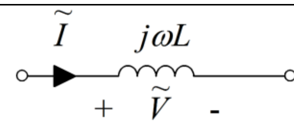
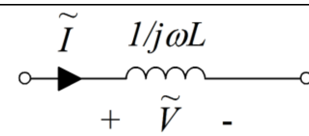
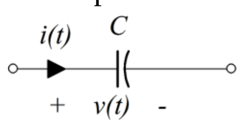
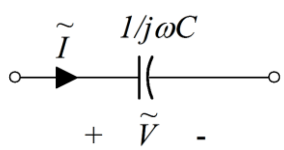
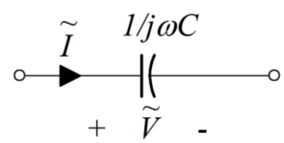
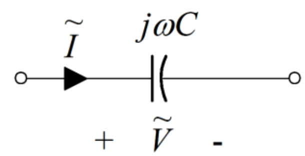
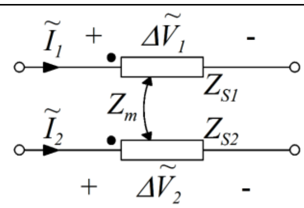
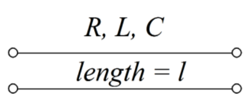
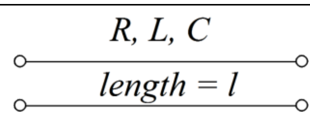
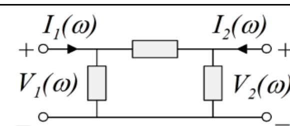
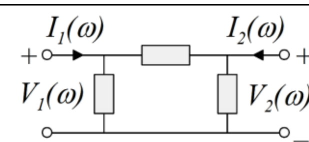
Element	**Model Equations** **Power Frequency**	**Thevenin Equivalent**	**Norton Equivalent**
Source		$\tilde{V} = z\tilde{I} + \tilde{E}$	$\tilde{I} = y\tilde{V} - y\tilde{E}$ $y = 1/z$
Resistor  $v(t) = Ri(t)$	 $\tilde{V} = R\tilde{I}$	 $\tilde{V} = R\tilde{I}$	 $\tilde{I} = G\tilde{V}$
Inductor  $v(t) = L\dfrac{di(t)}{dt}$	 $\tilde{V} = j\omega L\tilde{I}$	 $\tilde{V} = j\omega L\tilde{I}$	 $\tilde{I} = \dfrac{1}{j\omega L}\tilde{V}$
Capacitor  $i(t) = C\dfrac{dv(t)}{dt}$	 $\tilde{I} = j\omega C\tilde{V}$	 $\tilde{V} = \dfrac{1}{j\omega C}\tilde{I}$	 $\tilde{I} = j\omega C\tilde{V}$
Coupled Elements		$\Delta\tilde{V}_1 = z_{s1}\tilde{I}_1 + z_m\tilde{I}_2$ $\Delta\tilde{V}_2 = z_m\tilde{I}_1 + z_{s2}\tilde{I}_2$	$\begin{bmatrix}\tilde{I}_1\\ \tilde{I}_2\end{bmatrix} = A\begin{bmatrix}z_{s2} & -z_m\\ -z_m & z_{s1}\end{bmatrix}\begin{bmatrix}\Delta\tilde{V}_1\\ \Delta\tilde{V}_2\end{bmatrix}$ $A^{-1} = z_{s1}z_{s2} - z_m^2$
Distributed Parameter Circuit  $\dfrac{\partial v(y,t)}{\partial y} = Ri(y,t) + L\dfrac{\partial i(y,t)}{\partial t}$ $\dfrac{\partial i(y,t)}{\partial y} = C\dfrac{\partial v(y,t)}{\partial t}$	 $\dfrac{\partial\tilde{V}(y)}{\partial y} = (R + j\omega L)\tilde{I}(y)$ $\dfrac{\partial\tilde{I}(y)}{\partial y} = j\omega C\tilde{V}(y)$	 $\tilde{V}_1(\omega) = \dfrac{Z_0\cosh(p\ell)}{\sinh(p\ell)}\tilde{I}_1 + \dfrac{Z_0}{\sinh(p\ell)}\tilde{I}_2$ $\tilde{V}_2(\omega) = \dfrac{Z_0}{\sinh(p\ell)}\tilde{I}_1 + \dfrac{Z_0\cosh(p\ell)}{\sinh(p\ell)}\tilde{I}_2$ $Z_\pi = Z_0\sinh(p\ell)$ $Z_\pi' = \dfrac{Z_0\sinh(p\ell)}{\cosh(p\ell)-1}$ $Z_0 = \sqrt{(R+j\omega L)/(j\omega C)}$ $p = \sqrt{(R+j\omega L)(j\omega C)}$	 $\tilde{I}_1(\omega) = \dfrac{\cosh(p\ell)}{Z_o\sinh(p\ell)}\tilde{V}_1 - \dfrac{1}{Z_o\sinh(p\ell)}\tilde{V}_2$ $\tilde{I}_2(\omega) = -\dfrac{1}{Z_o\sinh(p\ell)}\tilde{V}_1 + \dfrac{\cosh(p\ell)}{Z_o\sinh(p\ell)}\tilde{V}_2$ $Y_\pi = \dfrac{1}{Z_0\sinh(p\ell)}$ $Y_\pi' = \dfrac{\cosh(p\ell)-1}{Z_0\sinh(p\ell)}$ $Z_0 = \sqrt{(R+j\omega L)/(j\omega C)}$ $p = \sqrt{(R+j\omega L)(j\omega C)}$

Figure 4.7: Thevenin and Norton Equivalent Circuit Elements

4.4.2 Network Reduction Methods

Many times it is expedient to compute the equivalent positive, negative and zero sequence impedances at the location of the fault. This is most efficiently achieved using the network reduction technique. The process is straightforward. First, the positive, negative and zero sequence networks are constructed by simply replacing each component of the system with the component positive, negative and zero sequence models. The end result of this process will be three networks: the positive, negative and zero sequence network model of the system. Subsequently, the positive sequence network can be reduced to a Thevenin or a Norton equivalent at the location of the fault using network reduction methods. Similarly, the negative sequence network can be reduced to a Thevenin or a Norton equivalent at the location of the fault using network reduction methods. And so for the zero sequence network.

The network reduction method will be illustrated with a simple example.

Example E4.7: Consider the equivalent circuit of a faulted network. The positive sequence network for the system is illustrated in Figure E4.7. Compute the Thevenin equivalent and the Norton equivalent at the location of the fault.

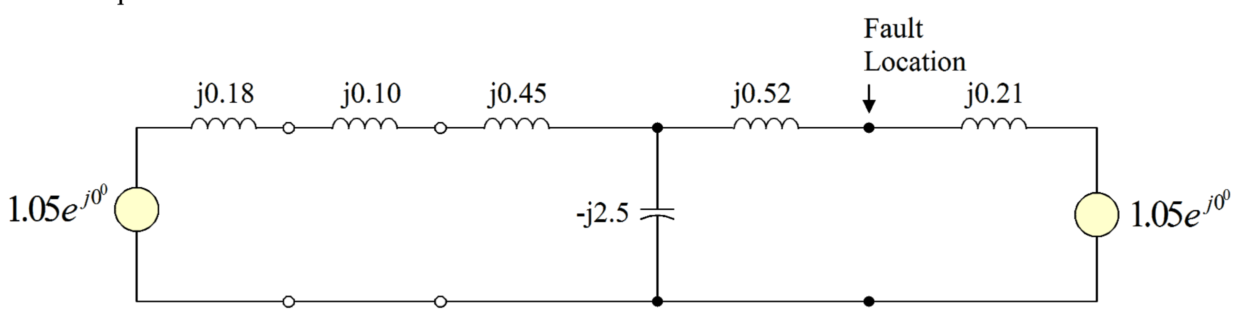


Figure E4.7

Solution: We shall use nodal analysis to compute the Norton equivalent first. For this purpose we convert the positive sequence network into:

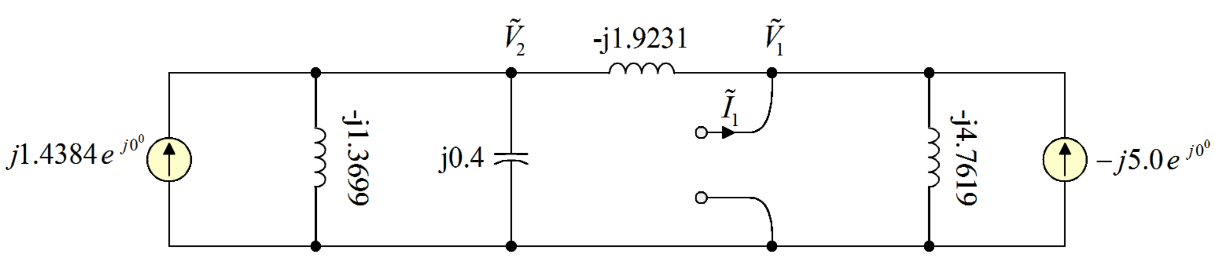


Figure E4.7a

The following nodal equations are written for this network:

$$-\tilde{I}_1 - j1.9231\left(\tilde{V}_1 - \tilde{V}_2\right) - j4.7619\tilde{V}_1 + j5.0 = 0$$

$$-j1.9231\left(\tilde{V}_2-\tilde{V}_1\right)+j0.4\tilde{V}_2-j1.3699\tilde{V}_2+j1.4384=0$$

Above equations can be written in the following compact form:

$$\begin{bmatrix}\tilde{I}_1\\0\end{bmatrix}=\begin{bmatrix}-j6.6850 & j1.9231\\ j1.9231 & -j2.8930\end{bmatrix}\begin{bmatrix}\tilde{V}_1\\\tilde{V}_2\end{bmatrix}+\begin{bmatrix}j5.0\\ j1.4384\end{bmatrix}$$

Upon elimination of $\tilde{V}_2$ from above equations:

$$\tilde{I}_1=-j5.4066\tilde{V}_1-j5.9562 \quad or \quad \tilde{V}_1=j0.185\tilde{I}_1+1.1017$$

The first equation corresponds to the Norton equivalent shown in Figure E4.7b(a) and the second equation corresponds to the Thevenin equivalent shown in Figure E4.7b(b).

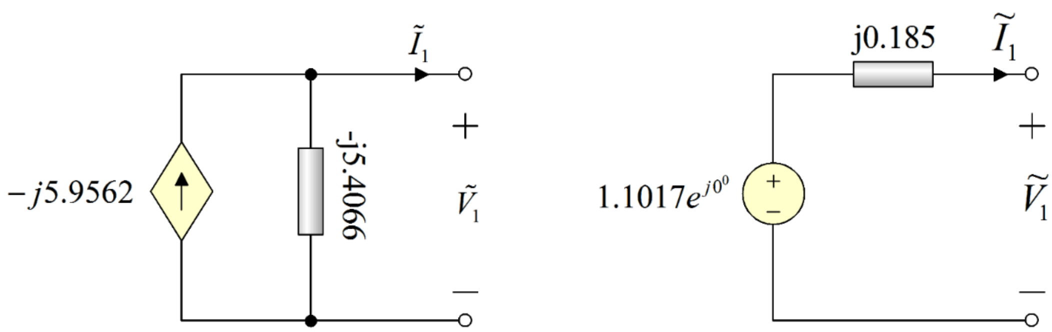


Figure E4.7b Norton and Thevenin Equivalents of the Circuit of Figure E4.7

To summarize, fault analysis of large scale systems can be performed by setting up the positive, negative and zero sequence networks of the system, applying the fault to this network and solving the resulting network problem. The solution can be done directly or by computing the Thevenin or Norton equivalents of the positive, negative and zero sequence network first and then applying the fault model to the equivalents.

The above described procedures will be illustrated with an example.

Example E4.5: Consider the electric power system of Figure E4.5. The parameters of the various system components are given in the Figure. Consider a single line to ground fault at location A. The location A is indicated in the Figure. It is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. Compute the negative sequence current in the generator during this fault. It is also given that prior to the fault the unit operates under nominal voltage at its terminals, rated power and unity power factor. The positive, negative and zero sequence impedances of each line are: $z_1=z_2=j45.5\,ohms$ and $z_0=j106.4\,ohms$. The mutual impedance between the two lines (zero sequence only) is: $z_{0m}=j24.5\,ohms$.

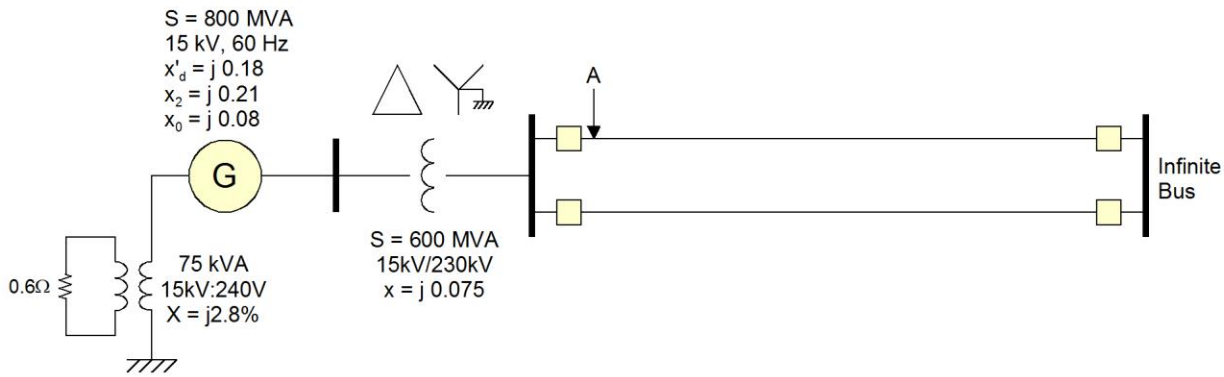


Figure E4.5

Solution: **Pre-fault conditions**: First we need to compute the operating conditions of this system prior to the fault. The system operates under balanced conditions prior to the fault and therefore the positive sequence network applies shown in Figure E4.5a. The figure also shows the given data, specifically the generator operates under nominal voltage, nominal power and unity power factor. Thus, the generator terminal voltage and current is:

$\tilde{V}_1 = 1.0e^{j0}, \quad \tilde{I}_1 = 1.0e^{j0}$

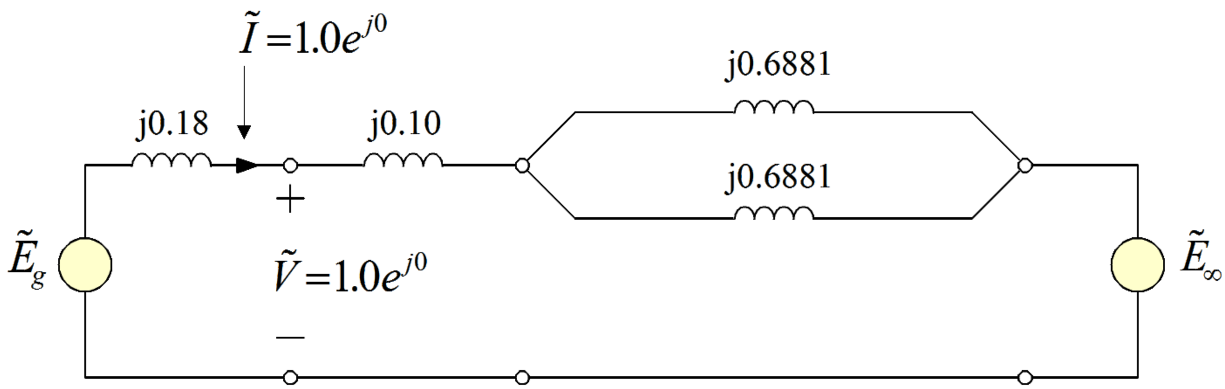


Figure 4.5a: Network Model Prior to Fault

The generator internal voltage and the infinite source voltage are:

$$\tilde{E}_g = 1.0e^{j0} + \mathrm{j}(0.18)\left(1.0e^{j0}\right) = 1.0161\mathrm{e}^{j10.204^0}$$

$$\tilde{E}_\infty = 1.0e^{j0} - j\left(0.10 + \frac{0.6881}{2}\right)\left(1.0e^{j0}\right) = 1.0942e^{-j23.946^0}$$

Fault conditions: Since we have a single line to ground fault we need to construct the positive, negative and zero sequence equivalent networks. Figure E4.5b(a) illustrates the pos, neg and zero sequence equivalent networks and the connections for a single line to ground fault. The sequence networks can be also reduced to the equivalent sequence networks at the location of the fault as shown in Figure E4.5b(b). We present below the analysis of these conditions with two methods: (a) method one simply solves the network of Figure E4.5b(a) and (b) method two first computes the reduced equivalent sequence models and then solves for the fault currents.

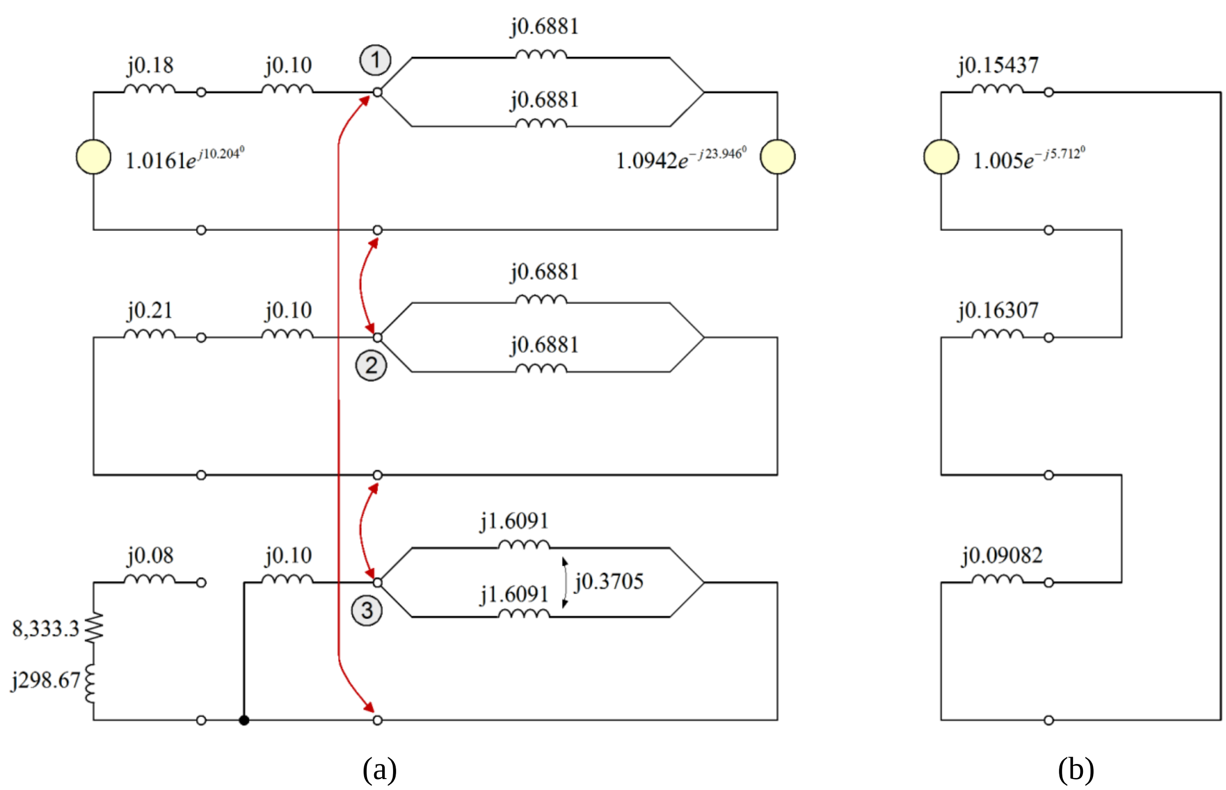


Figure 4.5b: Faulted Network Model
((a) Sequence Networks, (b) Reduced Sequence Networks)

Solution method (a): First the two mutually coupled elements can be replaced with an equivalent impedance of admittance. For this purpose use the equations in Figure 4.7:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = A \begin{bmatrix} z_{s2} & -z_m \\ -z_m & z_{s1} \end{bmatrix} \begin{bmatrix} \Delta\tilde{V}_1 \\ \Delta\tilde{V}_2 \end{bmatrix}, \ A^{-1} = z_{s1}z_{s2} - z_m^2$$

Note that the voltage across each one of the coupled elements is the same and the net current is the sum of the currents, i.e.

$$\Delta\tilde{V}_1 = \Delta\tilde{V}_2 = \Delta\tilde{V}, \quad \rightarrow \quad \tilde{I} = \tilde{I}_1 + \tilde{I}_2 = A\left(z_{s2} + z_{s1} - 2z_m\right)\Delta\tilde{V}$$

This the equivalent admittance is:

$$y_{eq} = A\left(z_{s2} + z_{s1} - 2z_m\right) = -j1.010305, \quad or \quad z_{eq} = j0.9898$$

The nodal equations for the network of Figure E4.5b(a) are:

$$0 = \left(\tilde{V}_1 - \tilde{V}_2 - 1.0161\mathrm{e}^{j10.204^0}\right)\left(\frac{1}{j0.28}\right) + \left(\tilde{V}_1 - \tilde{V}_2 - \tilde{E}_\infty\right)\left(\frac{2}{j0.6881}\right) + \left(\tilde{V}_1 - \tilde{V}_3\right)\left(\frac{1}{j0.1}\right) + \left(\tilde{V}_1 - \tilde{V}_3\right)\left(\frac{1}{j0.9898}\right)$$

$$0 = \left(\tilde{V}_2 - \tilde{V}_3\right)\left(\frac{1}{j0.31}\right) + \left(\tilde{V}_2 - \tilde{V}_3\right)\left(\frac{2}{j0.6881}\right) - \left(\tilde{V}_1 - \tilde{V}_2 - \tilde{E}_g\right)\left(\frac{1}{j0.28}\right) - \left(\tilde{V}_1 - \tilde{V}_2 - \tilde{E}_\infty\right)\left(\frac{2}{j0.6881}\right)$$

$$0 = \left(\tilde{V}_3 - \tilde{V}_1\right)\left(\frac{1}{j0.1}\right) + \left(\tilde{V}_3 - \tilde{V}_1\right)\left(\frac{1}{j0.9898}\right) + \left(\tilde{V}_3 - \tilde{V}_2\right)\left(\frac{1}{j0.31}\right) + \left(\tilde{V}_3 - \tilde{V}_2\right)\left(\frac{2}{j0.6881}\right)$$

Solution of the nodal equations yields:

$$\tilde{V}_1 = xxe^{jxx}, \quad \tilde{V}_2 = xxe^{jxx}, \quad \tilde{V}_3 = xxe^{jxx}$$

The negative sequence current in the generator is:

$$\tilde{I}_2 = -\left(\tilde{V}_2 - \tilde{V}_3\right)\left(\frac{1}{j0.31}\right) = xxe^{jxx}$$

Solution method (b): In this method, the sequence networks are first reduced to a Thevenin equivalent at the location of the fault. Figure E4.5b(b) illustrates the computed Thevenin equivalents. Below we present the computation of the positive sequence Thevenin equivalent.

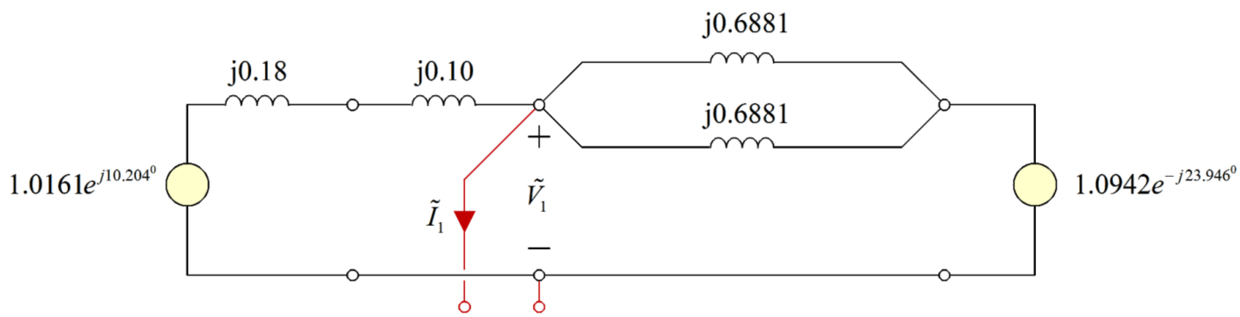


Figure E4.5c: Computation of Positive Sequence Thevenin Equivalent at Fault Location

$$0 = \left(\tilde{V}_1 - 1.0161\mathrm{e}^{j10.204^0}\right)\left(\frac{1}{j0.28}\right) + \left(\tilde{V}_1 - 1.0942e^{-j23.946^0}\right)\left(\frac{2}{j0.6881}\right) + \tilde{I}_1$$

Above equation is cast into the form:

$$\tilde{V}_1 = -j0.15437\tilde{I}_1 + 1.0050e^{-j5.712^0}$$

Above equation corresponds to the positive sequence Thevenin equivalent in Figure E4.5b(b). Similarly the Thevenin equivalents of the negative and zero sequence networks are computed and illustrated in Figure E4.5b(b).

4.5 Electrical Transients

Initiation and clearing of faults triggers transients that may affect the performance of the breakers and their ability to interrupt the circuit. We examine two types of transients that greatly affect breaker performance. The first type of transients affects the electric current magnitude to be interrupted. We refer to these transients as fault current transients. The second affects the voltages to which a breaker is subjected after fault interruption. We refer to these transients as transient voltage recovery. The mechanism of generation of these transients will be described next.

4.5.1 Fault Current Transients

Fault current transients typically include an exponentially decaying DC component as well as a power frequency decaying component. The exponentially decaying DC component can occur anywhere. The exponentially decaying AC component is most profound near a generating unit and is caused by the varying generator impedances during a fault. A typical fault current waveform is shown in Figure 4.9. The fault current transient affects the relay as well as the ability of the breaker to interrupt the fault. The effect on relay operation will be discussed in the next section. The breaker ability to interrupt the fault depends on the rms value of the fault current at the time of the interruption. The rms value must be below the breaker capability. The electric fault current waveform in Figure 4.9 indicates that the rms value of the current varies with time. It is important to analyze the fault current waveform to determine what is the rms value of the fault current at the time of breaker operation. This analysis will be introduced with an example.

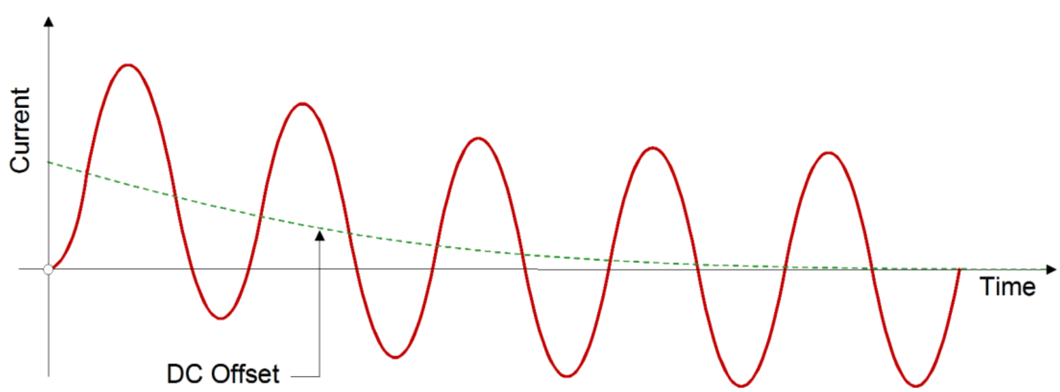


Figure 4.9: Electric Fault Current Transients

Consider an electric power system and assume that the system operates under normal steady state conditions when suddenly a short circuit occurs between a phase and the neutral of the system. We wish to examine the electric fault transients. In order to simplify the analysis, we consider the Thevenin equivalent of the system at the point of the fault, which is illustrated in Figure 4.10. In general the equivalent circuit will consist of an equivalent ideal voltage source behind the Thevenin impedance which for most power systems will consist of a resistance R and inductance L. The equivalent source voltage is:

$$e(t) = \sqrt{2}V\cos\left(\omega t + \phi\right)$$

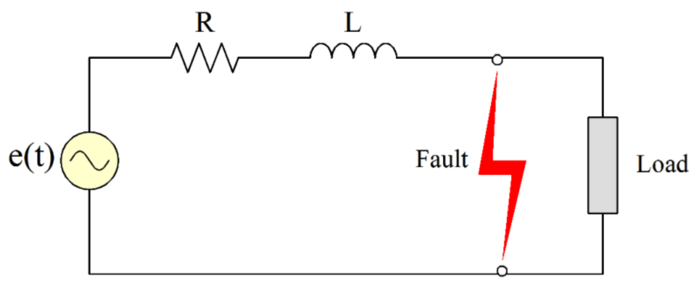


Figure 4.10: Thevenin Equivalent Circuit of a Power System at a Fault Location

The fault current is computed by solving the differential equation describing the equivalent circuit. Specifically, after the fault initiation, the voltage across the fault terminals is zero, thus:

$$e(t) = Ri_F(t) + L\frac{di_F(t)}{dt}$$

The general solution of the above equation, assuming that the fault occurs at t=0 is:

$$i_F(t) = \sqrt{2}I\cos\left(\omega t + \phi + \delta\right) + \left(-\sqrt{2}I\cos\left(\phi + \delta\right) + i(0)\right)e^{-\frac{R}{L}t}$$

where:

$$\tan\delta = -\frac{\omega L}{R}$$

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

Note that the fault current consists of a sinusoidal component plus an exponentially decaying DC component. The amplitude of the DC component mainly depends the source voltage waveform phase at the time of the fault occurrence (angle ϕ). Neglecting the pre-fault current i(0), the maximum DC component amplitude occurs for $\phi = -\delta$. For this condition the DC component becomes:

$$i_{F,DC\max}(t) = -\sqrt{2}Ie^{-\frac{R}{L}t}$$

while the sinusoidal (AC) component is:

$$i_{F,AC}(t) = \sqrt{2}I\cos(\omega t)$$

Thus the RMS Value of the fault current under the above assumption ($\phi = -\delta$) is:

$$I_F = \sqrt{\left(I^2 + 2I^2 e^{-\frac{2R}{L}t}\right)}$$

The above expression is maximum at t=0. Specifically at t=0 the maximum RMS value of the fault current is:

$$I_{F\max} = I\sqrt{3} = \frac{V\sqrt{3}}{\sqrt{R^2 + \omega^2 L^2}}$$

Note that the DC component duration depends on the time constant L/R, i.e. the ratio of the Thevenin equivalent circuit inductance over the resistance at the fault location.

Note that the rms value of the fault current varies with time. This is very important because the ability of circuit breakers to interrupt the fault current depends on the rms value of the fault current at the time of interruption. For example a breaker that is rated 40 kA will be able to interrupt a fault as long as the rms value of the fault is below 40 kAs at the time of interruption. Therefore it is important to compute the fault current at the time of breaker operation. The issue will be illustrated with an example.

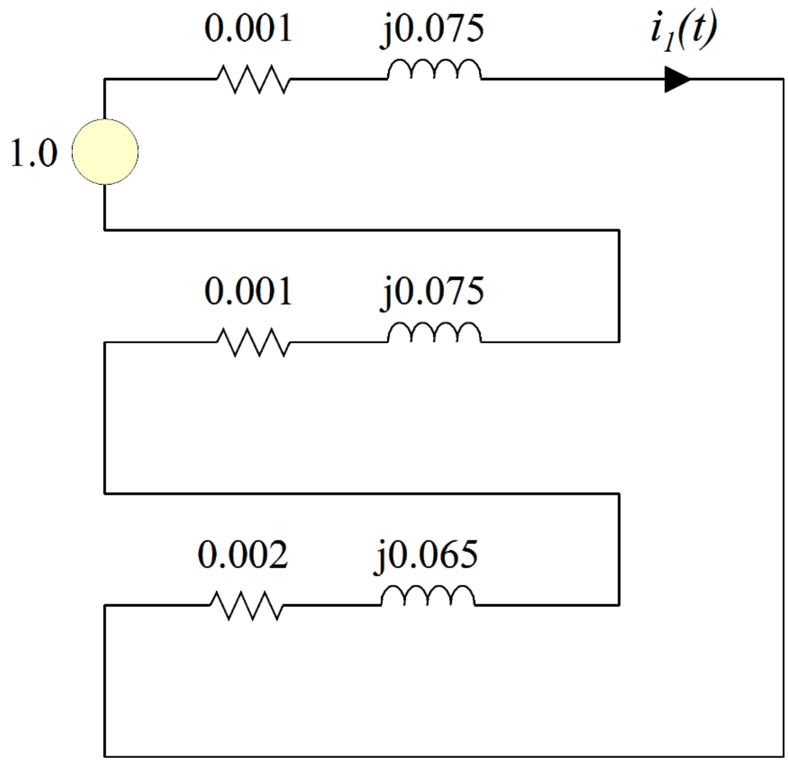
Example E4.9: At a breaker location, the equivalent sequence impedances are:

(a) pos/neg sequence: $R_1 + j\omega L_1 = 0.001 + j0.075\, pu, \quad @\, 13.8kV\ (L-L)\ and\ 36\, MVA$
(b) zero sequence: $R_0 + j\omega L_0 = 0.002 + j0.065\, pu, \quad @\, 13.8kV\ (L-L)\ and\ 36\, MVA$

The breaker is set to clear a fault in four and a half cycles of 60 Hz. Compute the **maximum possible rms value** of a single phase to ground fault current (in Amperes) at the time of operation of the breaker, i.e. four and a half cycles after fault initiation.

Solution: For the purpose of computing the maximum rms value we have to assume maximum DC offset.

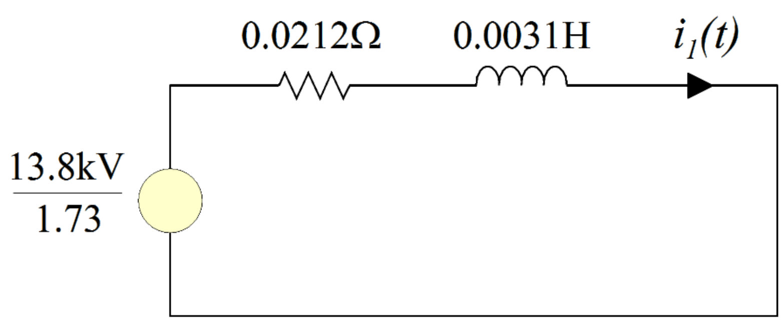
The equivalent circuit is:



The equivalent resistance and inductance converted in actual units are:

$$R = 0.0212\,ohms, \quad L = 0.0031H$$

The equivalent circuit in actual units is:



The fault current is:

$$i_f(t) = 3i_1(t) = 3\sqrt{2}I\cos(\omega t + \theta) + 3\sqrt{2}Ie^{-\frac{0.0212}{0.0031}t}, \quad I = \left|\frac{13.8/\sqrt{3}}{0.0212 + j1.1381}\right| = 7.0004\,kA$$

$$I_{f,rms}(t) = 3\sqrt{I^2 + 2I^2 e^{-\frac{2R}{L}t}}$$

At t=4.5 cycles, t=0.075 seconds. Thus

$$I_{f,rms}(t=0.075)=3\sqrt{I^2+2I^2e^{-\frac{2R}{L}0.075}}=27.51\,kA$$

4.5.2 Fault Current Evolution

The simplified analysis of fault current evolution based on a Thevenin equivalent circuit at the location of the fault, while it is widely used, it does not accurately represent the fault evolution in real systems.

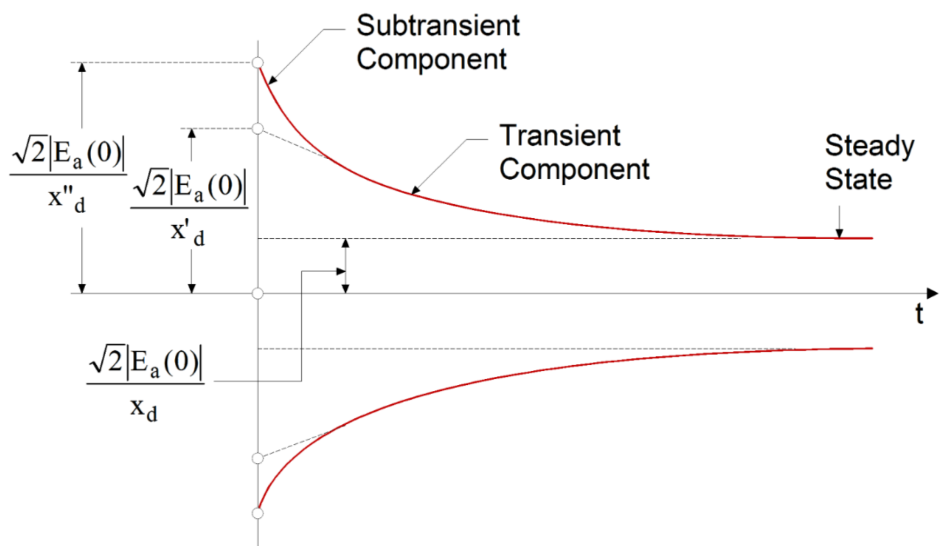


Figure 4.3: Short Circuit Current Components during a Fault near a Generator

4.6 Transient Recovery Voltage

After the occurrence of a fault, the protection system opens the appropriate breaker to interrupt the fault current. A breaker interrupts the fault current by opening a set of contacts. Because the circuit behind the breaker is primarily inductive, the flow of current will not be interrupted until the current crosses a zero value. At this instant, the electric arc ceases to exist and the fault current is interrupted. Once the fault current is interrupted the voltage across the breaker contacts builds up very fast. This phenomenon is called “Transient Recovery Voltage”. If the speed with which this voltage increases is very high (typically in the order of 10 microseconds to reach the nominal circuit voltage), it is possible that the electric arc is reignited. In this case the electric fault current will continue. We refer to this phenomenon as a *breaker restrike*. Breaker restrikes damage the breaker and repeated restrikes may cause breaker failure.

The mathematical model that describes the voltage build up across the plates of a breaker is described with the aid of Figure 4.x. The source *e(t)* and the inductor *L* represent the power system equivalent at the terminals of the breaker. Note this is the Thevenin equivalent at the location of the breaker and the resistance of the equivalent impedance has been neglected for convenience. The capacitor C represents the parasitic capacitance of the power equipment at that location (transformer, breaker, etc.).

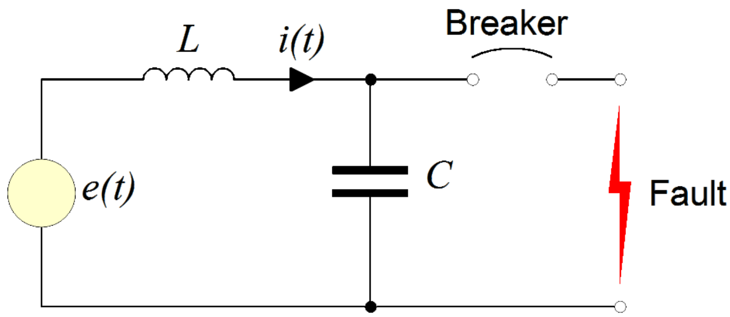


Figure 4.x Equivalent Circuit for Transient Recovery Voltage Analysis

Consider the period during the fault conditions. The fault current is:

$$\tilde{I} = \frac{\tilde{E}}{j\omega L}$$

When the current becomes zero and assuming that in this case the circuit has been interrupted, we will have the following model:

$$e(t) = L\frac{di(t)}{dt} + v_c(t)$$

$$i(t) = C\frac{dv_c(t)}{dt}$$

and initial conditions: $i(t=0)=0, \quad v_c(t=0)=0$.

The solution to this problem is:

$$i(t) = \frac{\sqrt{2}E}{\omega L}\frac{1}{1-\left(\frac{\omega_1}{\omega}\right)^2}\left(\sin(\omega t) - \left(\frac{\omega_1}{\omega}\right)\sin(\omega_1 t)\right), \text{ where } \omega_1 = \frac{1}{\sqrt{LC}}$$

and the voltage across the capacitor is:

$$v_c(t) = e(t) - L\frac{di(t)}{dt} = \sqrt{2}E\left\{\cos(\omega t) - \frac{1}{1-\left(\frac{\omega_1}{\omega}\right)^2}\left(\cos(\omega t) - \left(\frac{\omega_1}{\omega}\right)^2\cos(\omega_1 t)\right)\right\}$$

Note that above voltage consists of three components, two components that vary with the power frequency and another that varies with a much higher frequency.

$$v_{c1}(t) = \sqrt{2}E\{\cos(\omega t)\}$$

$$v_{c2}(t) = \sqrt{2}E\left\{-\frac{1}{1-\left(\frac{\omega_1}{\omega}\right)^2}(\cos(\omega t))\right\}$$

$$v_{c3}(t) = \sqrt{2}E\left\{\frac{1}{1-\left(\frac{\omega_1}{\omega}\right)^2}\left(\left(\frac{\omega_1}{\omega}\right)^2\cos(\omega_1 t)\right)\right\}$$

Typically the angular frequency ω_1 is much greater than the power frequency ω. In this case the component $v_{c2}(t)$ is negligible, while the other two components have almost equal absolute magnitudes but different frequency. A typical graph of the three components and the total voltage is shown in Figure 4.x. It is important to observe in the graph that the peak value of the total voltage occurs at time equal half of the period of the voltage $v_{c3}(t)$. Therefore the time to peak voltage is:

$$\tau = \frac{T_1}{2} = \frac{1}{2f_1} = \frac{2\pi}{2\omega_1} = \pi\sqrt{LC}$$

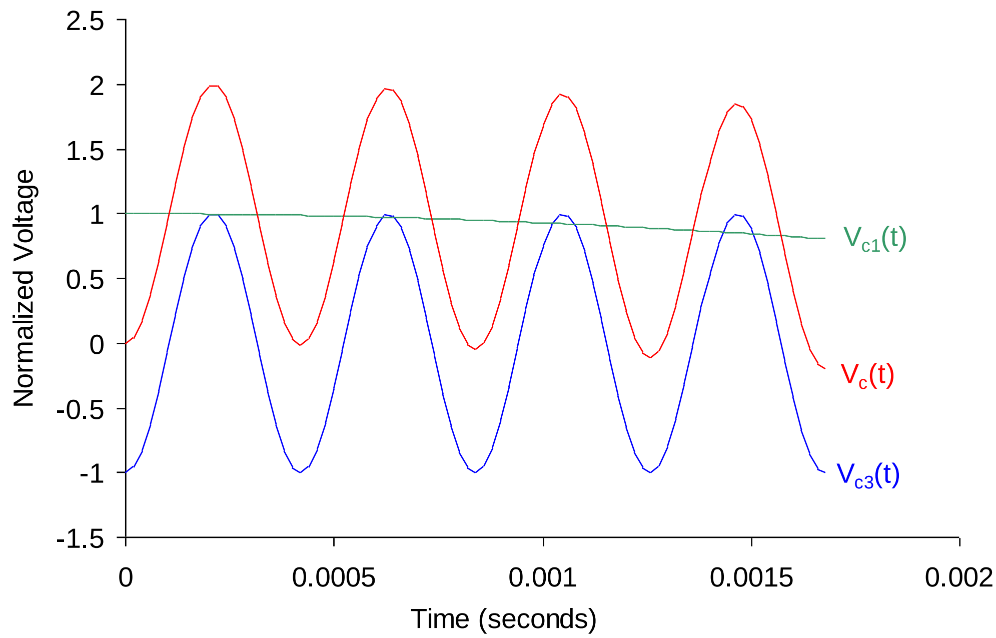


Figure 4.x: Illustration of the Transient Recovery Voltage

Example E4.10: At a breaker location, the Thevenin equivalent inductance is 0.05 pu on a 13.8 kV (L-to-L voltage), 10 MVA (three phase total) basis. The total parasitic capacitance at this point is 0.05 microFarads. Compute the transient recovery voltage of this breaker, i.e. maximum value in volts and rise time to maximum value in microseconds.

Solution: The base impedance at this location is:

$$Z_b = \frac{V_{L-L}^2}{S} = 19.044 \quad Ohms$$

$$L = \frac{(0.05)(19.044)}{377} = 2.5257 \times 10^{-3} \quad Henries$$

The angular frequency is:

$$\omega_1 = \frac{1}{\sqrt{LC}} = 88985.98 \; (s^{-1})$$

The voltage across the breaker is:

$$V_b(t) = e(t) - L\frac{di(t)}{dt} = \sqrt{2}V\left(\cos\omega t - \frac{1}{1-(\omega_1/\omega)^2}\left(\cos\omega t - \left(\frac{\omega_1}{\omega}\right)^2 \cos\omega_1 t\right)\right)$$

$$V_b(t) = \sqrt{2}(13.8\times 10^3/\sqrt{3})(\cos\omega t - \cos\omega_1 t) \; (Volts)$$

The rise time is:

$$t = \frac{\pi}{\omega_1} = 35.304 \times 10^{-6} \text{ (s)}$$

$V_b(t)$ is maximum at $t = 35.304 \times 10^{-6}$. The maximum voltage is:

$$V_b(t) = \sqrt{2}(13.8 \times 10^3 / \sqrt{3})(2) = 22.5353 \times 10^3 \text{ (Volts)}$$

A plot of the voltage waveform is illustrated in Figure E4.10a.

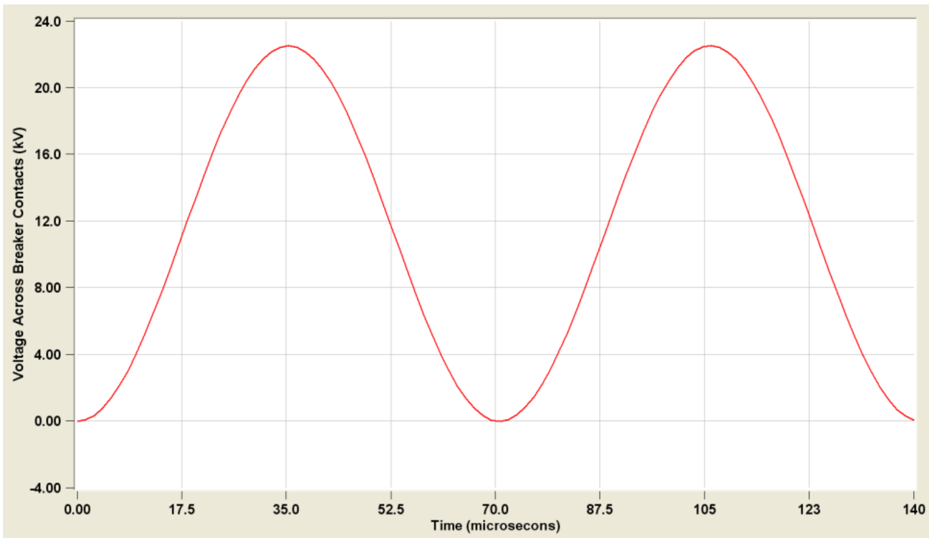


Figure E4.10a: Transient Recovery Voltage, 10 microsecond Plot

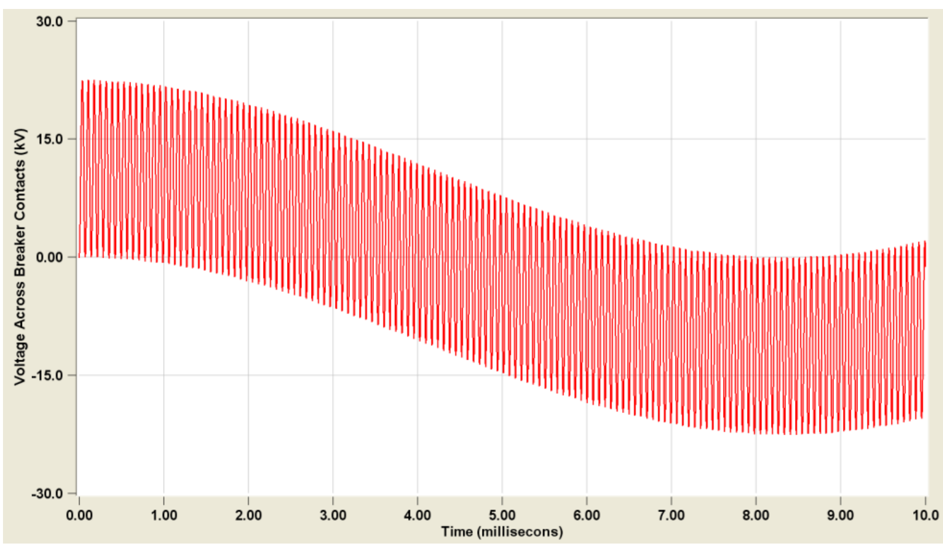


Figure E4.10b: Transient Recovery Voltage, 10 millisecond Plot

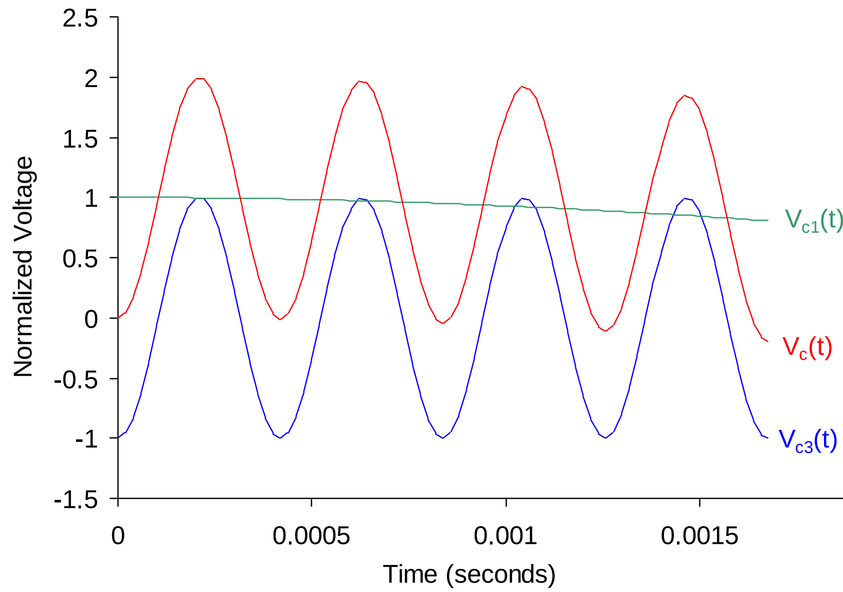


Figure E4.10c. Illustration of the Normalized Transient Recovery Voltage

4.7 Effects of Grounding

Many electrical fault conditions involve the grounds of an electrical installation and the soil as paths of current flow. These paths do have electrical impedance and as such the flow of current generates voltages at the neutrals and the grounded structures of the system. These voltages, most commonly referred to as Ground Potential Rise, may cause electric shocks to persons touching grounded structures and generate safety concerns. The proper analysis of these phenomena is to explicitly model the grounds and the earth path in the circuit used for fault analysis. The analysis of this model will provide the level of the ground potential rise, the level of electric current in ground paths, and other pertinent information. The inclusion of the system grounds, neutrals, ground wires, etc. in the analysis can be achieved in two different ways: (a) using symmetrical components in which case the grounds are included in the model by appropriate application of the symmetrical transformation and (b) using direct phase analysis or physically based systems. In this section we pursue the first approach. Later on, in section 4.7, the second approach is introduced.

As we have seen in previous sections, for fault analysis it is necessary to construct the sequence networks of a given electric power system. In order to be able to determine the fault current distribution among all available paths, it is necessary to explicitly model the grounds, neutrals, shield wires, soil, etc. The question arises what and how the sequence networks are affected by the grounds, neutral wires, shield wires, etc.? Consider the basic assumptions in the definition of sequence circuits. One of the assumptions for sequence models is that the power system elements are symmetric. A consequence of this assumption is that the positive or negative sequence currents flow only in the phase conductors. Electric currents in the grounding system (earth) and shield or neutral conductors are of the zero sequence only. It is concluded that the grounds, neutral wires, etc. will not affect the positive and negative sequence networks but only the zero sequence equivalent network. Therefore, the positive and negative sequence networks are computed in the usual way by ignoring the grounds, neutrals, etc. The zero sequence network, however, must be constructed in such a way as to explicitly represent (a) the shield or neutral wires, (b) the earth path, (c) the tower footing resistances and so on. Subsequently, the sequence networks are connected in a way corresponding to the type of fault (i.e., in series for a single line to ground fault, etc.). This procedure results in a large network problem. Upon solution of this network, the sequence components of the electric current and voltage everywhere in the system are obtained. From this information the actual electric current and voltage everywhere in the system can be computed with the inverse transformation of the symmetrical components.

As has been pointed out, the current distribution among phase conductors, sky wires, neutral conductors, and earth is determined only with the zero sequence network. This observation allows a simplification of the computational procedure. Specifically, we need to analyze only the zero sequence network in order to determine the relative current distribution among all parallel paths. Actual fault current levels are determined through the usual fault analysis. The overall procedure involves the following steps:

Step 1: Compute and lay out the positive sequence network of the system
Step 2: Compute and lay out the negative sequence network of the system.
Step 3: Compute and lay out the zero sequence network for the system under consideration. Earth, shield wires, neutral wires, tower footing resistances, substation grounding impedances, and so on, should be explicitly represented.
Step 4: For a specific fault location and fault type, connect the positive, negative and zero sequence networks as dictated by the fault type.
Step 5: Perform a circuit analysis for the equivalent network of step 4.
Step 6: Use the inverse symmetrical transformation to compute the actual phase quantities from the solution in step 5.

The computational procedure is straightforward. A couple of points, however, need further clarification: (a) the computation of the tower footing resistances and substation ground impedance, and (b) the construction of the zero sequence network in such a way as to retain the identity of shield wires, neutral wires, earth, and so on.

Grounding Structure Impedance: For simple grounding systems, for example a single ground rod, there are approximate formulae for computing the ground resistance. For more complicated grounding systems, such as those of a substation, lattice tower grounds, etc. the analysis is somewhat complex. For a concise review of the computational procedures and further information see reference [???].

Construction of the Zero Sequence Network: The zero sequence network can be constructed with the following procedure: First, each path of zero sequence current flow must be identified and characterized with self-impedance and possible mutual impedance with other paths. Subsequently, the electric currents at the terminals of the device can be expressed as functions of the device terminal voltages or vice versa. Finally, the circuit equations are transformed with the symmetrical component transformation. The resulting equations provide the equivalent zero sequence network. The procedure will be demonstrated with an example. Specifically, we consider the development of the zero sequence network of a transmission line.

The construction of the zero sequence network with explicit representation of the shield wire, earth path, and so on, is illustrated in Figure 4.7. Figure 4.7 illustrates the physical system with a set of unbalanced three-phase currents. The figure is helpful in determining the sequence network representation of the line. Consider one span of the line. In general, if currents $\tilde{I}_a, \tilde{I}_b, \tilde{I}_c, and\ \tilde{I}_g$ flow in the phase conductors and shield wire respectively of a specific span of the line, the induced voltages along the line conductors are:

$$\tilde{V}_{L,a} - \tilde{V}_{R,a} = \left[z_{aa}\tilde{I}_a + z_{ab}\tilde{I}_b + z_{ac}\tilde{I}_c + z_{ag}\tilde{I}_g\right]\ell$$
$$\tilde{V}_{L,b} - \tilde{V}_{R,b} = \left[z_{ba}\tilde{I}_a + z_{bb}\tilde{I}_b + z_{bc}\tilde{I}_c + z_{bg}\tilde{I}_g\right]\ell$$
$$\tilde{V}_{L,c} - \tilde{V}_{R,c} = \left[z_{ca}\tilde{I}_a + z_{cb}\tilde{I}_b + z_{cc}\tilde{I}_c + z_{cg}\tilde{I}_g\right]\ell$$
$$\tilde{V}_{L,g} - \tilde{V}_{R,g} = \left[z_{ga}\tilde{I}_a + z_{gb}\tilde{I}_b + z_{gc}\tilde{I}_c + z_{gg}\tilde{I}_g\right]\ell$$

The parameters appearing in above equations can be computed using the exact Carson's equations or any of the approximate methods discussed in Chapter 3. To simplify the equations we shall use the equivalent depth of return method (see Chapter 3). Using this method above equations are:

$$\tilde{V}_{L,a} - \tilde{V}_{R,a} = \left[(r + r_e + jx_{aa})\tilde{I}_a + (r_e + jx_{ab})\tilde{I}_b + (r_e + jx_{ac})\tilde{I}_c + (r_e + jx_{ag})\tilde{I}_g\right]\ell$$
$$\tilde{V}_{L,b} - \tilde{V}_{R,b} = \left[(r_e + jx_{ba})\tilde{I}_a + (r + r_e + jx_{bb})\tilde{I}_b + (r_e + jx_{bc})\tilde{I}_c + (r_e + jx_{bg})\tilde{I}_g\right]\ell$$
$$\tilde{V}_{L,c} - \tilde{V}_{R,c} = \left[(r_e + jx_{ca})\tilde{I}_a + (r_e + jx_{cb})\tilde{I}_b + (r + r_e + jx_{cc})\tilde{I}_c + (r_e + jx_{cg})\tilde{I}_g\right]\ell$$
$$\tilde{V}_{L,g} - \tilde{V}_{R,g} = \left[(r_e + jx_{ga})\tilde{I}_a + (r_e + jx_{gb})\tilde{I}_b + (r_e + jx_{gc})\tilde{I}_c + (r_g + r_e + jx_{gg})\tilde{I}_g\right]\ell$$

where:

$$x_{aa} = x_{bb} = x_{cc} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{d_p}\right), \quad x_{gg} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{d_g}\right)$$

$$x_{ab} = x_{ba} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{D_{ab}}\right), \quad x_{bc} = x_{cb} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{D_{bc}}\right), \quad x_{ca} = x_{ac} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{D_{ca}}\right)$$

$$x_{ag} = x_{ga} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{D_{ag}}\right), \quad x_{bg} = x_{gb} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{D_{bg}}\right), \quad x_{cg} = x_{gc} = \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{D_{cg}}\right)$$

$$r_e = \frac{\omega\mu}{8}, \qquad D_e = 2160\sqrt{\frac{\rho}{f}}\ (feet) = 658.368\sqrt{\frac{\rho}{f}}\ (meters), \quad \rho\ in\ ohm.m, \quad f\ in\ Hz$$

d_p : is the geometric mean radius of the phase conductors.

d_g : is the geometric mean radius of the shield (ground) conductor.

D_{xy} : is the distance between conductors x and y, x,y can be a, b, c, or g.

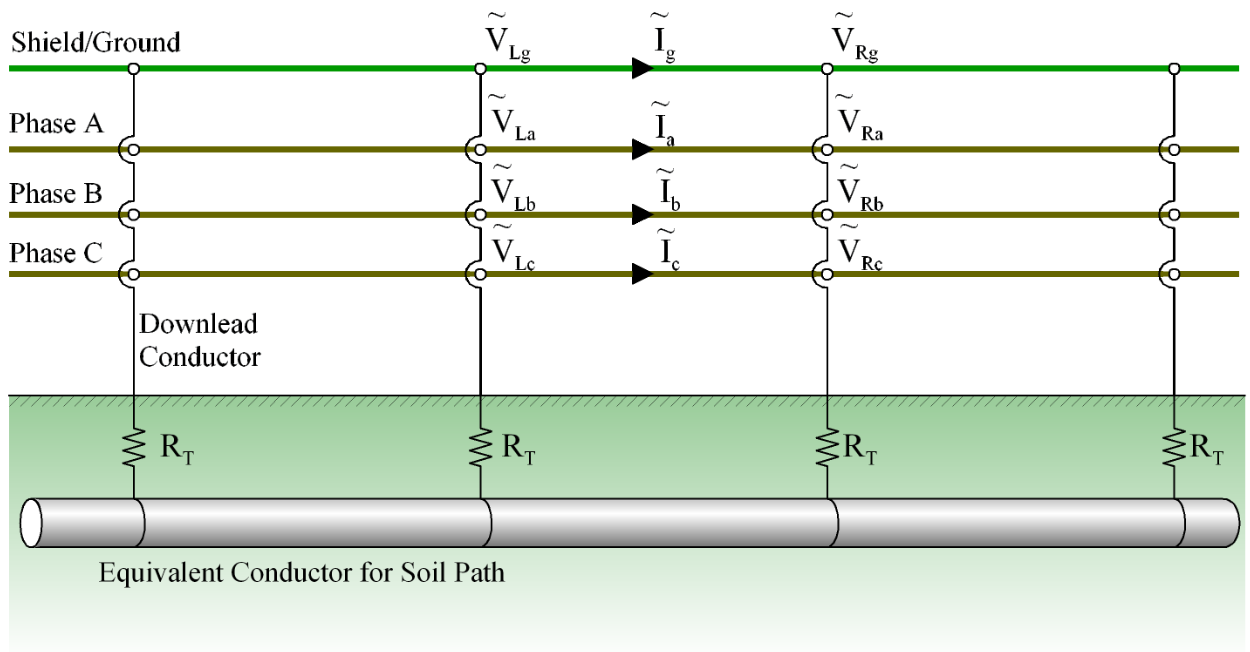


Figure 4.7 Three Phase Transmission Line with One Shield/Ground Conductor

For the purpose of deriving the sequence equivalent circuit of above line, we must assume that the line is symmetric. For this purpose we make the following simplifications:

$$\mathrm{z}_{ab} = \mathrm{z}_{bc} = \mathrm{z}_{ca} = \frac{1}{3}\left(z_{ab} + z_{bc} + z_{ca}\right) = z_m = r_e + jx_m = r_e + \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}\right)$$

$$\mathrm{z}_{ag} = \mathrm{z}_{bg} = \mathrm{z}_{cg} = \frac{1}{3}\left(z_{ag} + z_{bg} + z_{cg}\right) = z_{mg} = r_e + jx_{mg} = r_e + \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_e}{\sqrt[3]{D_{ag}D_{bg}D_{cg}}}\right)$$

Using above approximations, the equations can be cast in compact matrix form as follows:

$$\tilde{\mathrm{V}}_{\mathrm{L,abc}} - \tilde{\mathrm{V}}_{\mathrm{R,abc}} = \left[\left(rI + r_e\prod + jX\right)\tilde{I}_{abc} + \left(r_e + jx_{mg}\right)\Gamma\tilde{I}_g\right]\ell$$

$$\tilde{\mathrm{V}}_{\mathrm{L,g}} - \tilde{\mathrm{V}}_{\mathrm{R,g}} = \left[\left(r_e + jx_{mg}\right)\Gamma^T\tilde{I}_{abc} + \left(r_g + r_e + jx_{gg}\right)\tilde{I}_g\right]\ell$$

where:

$$\tilde{\mathrm{I}}_{\mathrm{abc}} = \begin{bmatrix}\tilde{\mathrm{I}}_{\mathrm{a}}\\ \tilde{\mathrm{I}}_{\mathrm{b}}\\ \tilde{\mathrm{I}}_{\mathrm{c}}\end{bmatrix}, \quad \tilde{V}_{L,abc} = \begin{bmatrix}\tilde{V}_{L,a}\\ \tilde{V}_{L,b}\\ \tilde{V}_{L,c}\end{bmatrix}, \quad \tilde{V}_{R,abc} = \begin{bmatrix}\tilde{V}_{R,a}\\ \tilde{V}_{R,b}\\ \tilde{V}_{R,c}\end{bmatrix}$$

$$\prod = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}, \quad \Gamma = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}, \quad and\ I = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$X = \begin{bmatrix} x_s & x_m & x_m \\ x_m & x_s & x_m \\ x_m & x_m & x_s \end{bmatrix}$$

Equations (7.31) are converted into the symmetrical components framework by substituting the three phase currents and voltages with their symmetrical components. The result is

$$\tilde{\mathrm{V}}_{\mathrm{L,120}} - \tilde{\mathrm{V}}_{\mathrm{R,120}} = \left[T^{-1}\left(rI + r_e \prod + jX\right)T\tilde{I}_{120} + T^{-1}\left(r_e + jx_{mg}\right)\tilde{I}_g\right]\ell$$
$$\tilde{\mathrm{V}}_{\mathrm{L,g}} - \tilde{\mathrm{V}}_{\mathrm{R,g}} = \left[\left(r_e + jx_{mg}\right)\Gamma^T T\tilde{I}_{120} + \left(r_g + r_e + jx_{gg}\right)\tilde{I}_g\right]\ell$$

Note that:

$$T^{-1}\left(rI + r_e \prod + jX\right)T = \begin{bmatrix} r + j(x_s - x_m) & 0 & 0 \\ 0 & r + j(x_s - x_m) & 0 \\ 0 & 0 & r + 3r_e + j(x_s + 2x_m) \end{bmatrix} = \begin{bmatrix} r + jx_1 & 0 & 0 \\ 0 & r + jx_2 & 0 \\ 0 & 0 & r + 3r_e + jx_0 \end{bmatrix}$$

$$T^{-1}\left(r_e + jx_{mg}\right)\Gamma = \begin{bmatrix} 0 \\ 0 \\ r_e + jx_{mg} \end{bmatrix}$$

$$\left(r_e + jx_{mg}\right)\Gamma^T T = \begin{bmatrix} 0 & 0 & 3r_e + j3x_{mg} \end{bmatrix}$$

Upon substitution of these results in above equations and expanding the matrix equation:

$$\tilde{V}_{L,1} - \tilde{V}_{R,1} = \left(r + jx_1\right)\tilde{I}_1\ell \qquad (7.33a)$$
$$\tilde{V}_{L,2} - \tilde{V}_{R,2} = \left(r + jx_2\right)\tilde{I}_2\ell \qquad (7.33b)$$
$$\tilde{V}_{L,0} - \tilde{V}_{R,0} = \left(r + 3r_e + jx_0\right)\tilde{I}_0\ell + \left(r_e + jx_{mg}\right)\tilde{I}_g\ell \qquad (7.33c)$$
$$\tilde{V}_{L,g} - \tilde{V}_{R,g} = 3\left(r_e + jx_{mg}\right)\tilde{I}_0\ell + \left(r_g + r_e + jx_{gg}\right)\tilde{I}_g\ell \qquad (7.33d)$$

The equations derived express the voltages versus the symmetrical component currents in the phases of the line and the actual current in the shield/neutral wire. In order to make the last two equations symmetric, the electric current $\tilde{\mathrm{I}}_{\mathrm{g}}$ is substituted for by $\tilde{\mathrm{I}}_{\mathrm{g}} = 3\tilde{\mathrm{I}}_{\mathrm{g0}}$. Now the equations read:

$$\tilde{V}_{L,1} - \tilde{V}_{R,1} = \left(r + jx_1\right)\tilde{I}_1\ell \qquad (4.33a)$$
$$\tilde{V}_{L,2} - \tilde{V}_{R,2} = \left(r + jx_2\right)\tilde{I}_2\ell \qquad (4.33b)$$

$$\tilde{V}_{L,0} - \tilde{V}_{R,0} = (r + 3r_e + jx_0)\tilde{I}_0 \ell + 3(r_e + jx_{mg})\tilde{I}_{g0}\ell \qquad (4.33c)$$

$$\tilde{V}_{L,g} - \tilde{V}_{R,g} = 3(r_e + jx_{mg})\tilde{I}_0 \ell + 3(r_g + r_e + jx_{gg})\tilde{I}_{g0}\ell \qquad (4.33d)$$

Equations (7.34) represent the equivalent circuit shown in Figure 4.8a, b, and c. Specifically, Figure 4.8a illustrates the positive sequence network, Figure 4.8b illustrates the negative sequence network and Figure 4.8c illustrates the zero sequence network. Comparison of the equations to the equivalent circuit of Figure 4.8 yields

$$z_{1,span} = (r + jx_1)\ell$$
$$z_{0,span} = (r + 3r_e + jx_0)\ell$$
$$z_{0g,span} = 3(r_g + r_e + jx_{gg})\ell$$
$$z_{0pg,span} = 3(r_e + jx_{mg})\ell$$

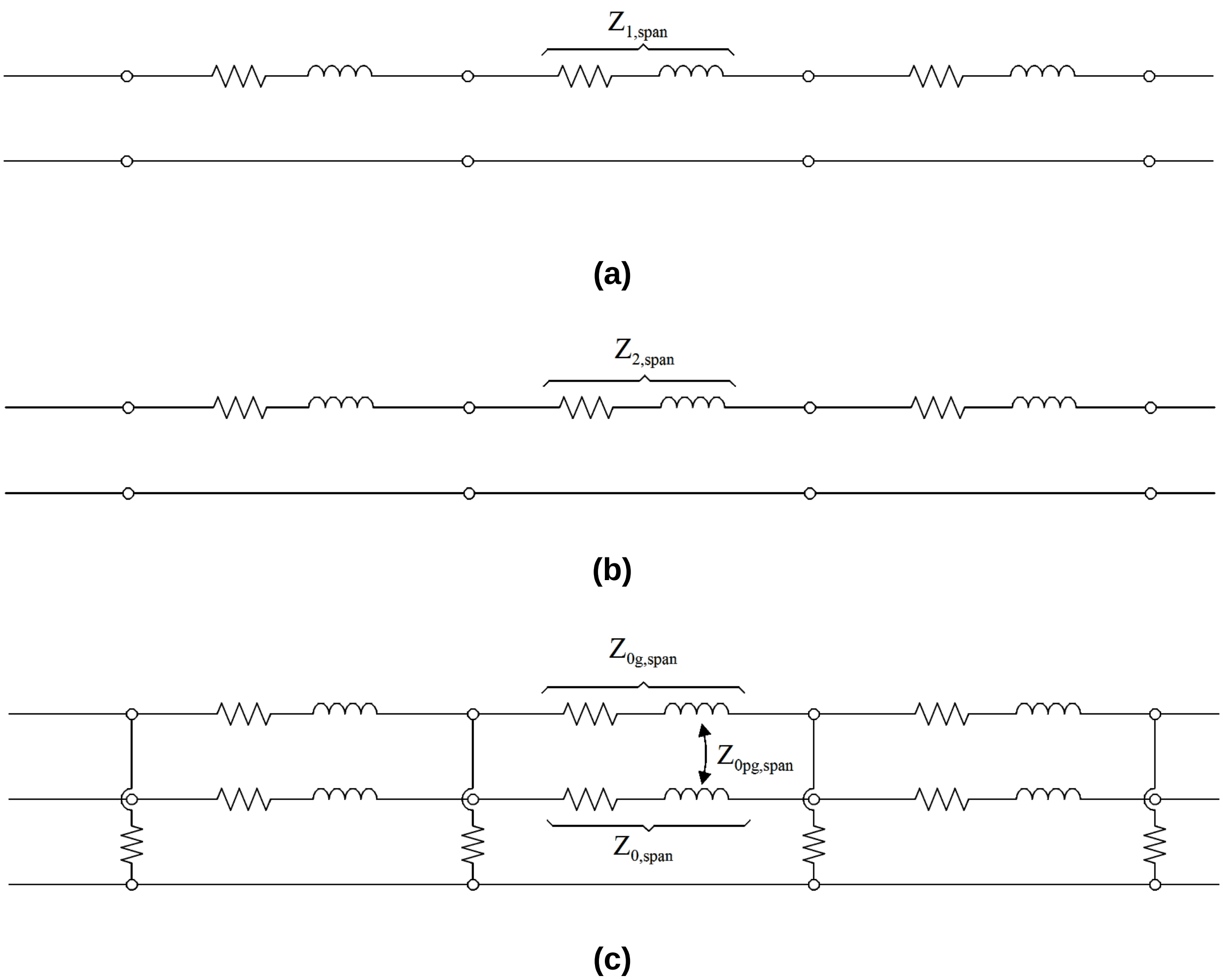


Figure 4.8: Sequence Equivalent Circuits of the Power Line of Figure 4.7
[(a) Positive Sequence Equivalent Circuit, (b) Negative Sequence Equivalent Circuit, (c) Zero Sequence Equivalent Circuit]

The sequence circuits of transmission lines will be demonstrated with an example.

Example E4.7: Consider the distribution line of Figure E4.7. The phase conductors are ACSR, 4/0 with the following parameters: $r = 0.592\, ohms/mile, \quad d = 0.00814\, feet$. The neutral conductor is ACSR, 3/0 with the following parameters: $r = 0.723 ohms/mile, \quad d = 0.006\ feet$. The line is 3.1 miles long and the span length is 0.05 miles, i.e. there are 20 poles per mile. The soil resistivity is 100 ohm.meters. The pole ground resistance is 25 ohms. Compute the sequence networks of this distribution line. The neutral conductor should be explicitly represented in the zero sequence model.

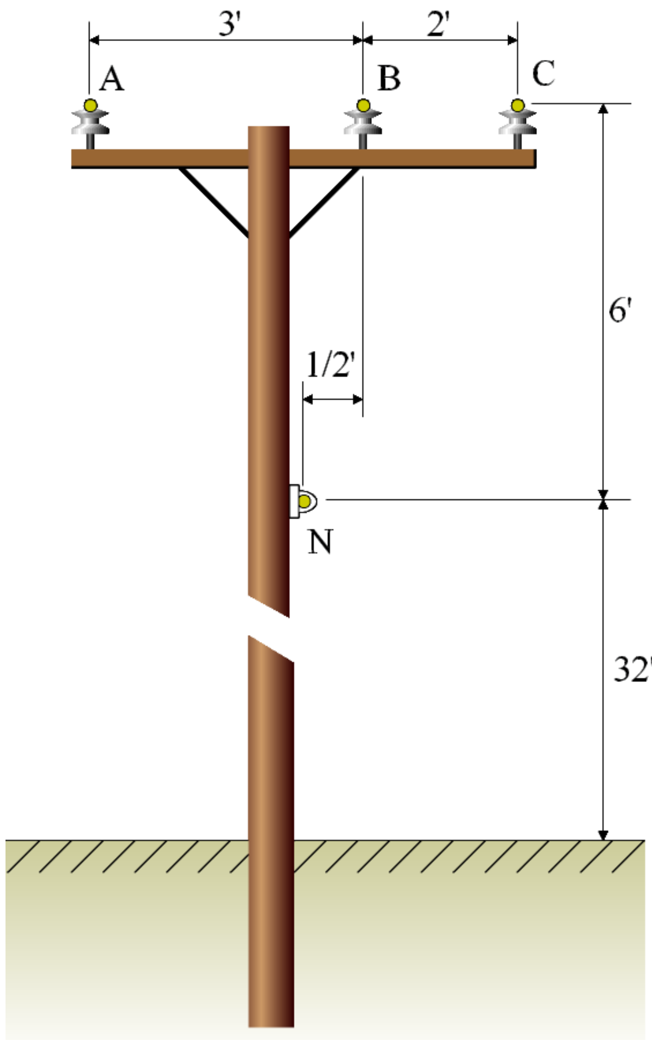


Figure E4.7 Example Distribution Line

Solution: Note that this line will have 62 spans. The sequence equivalent circuits for one span of the line are computed and shown in Figure E4.7a.

In summary, for analysis purposes of fault current distribution and of the ground potential rise of system neutrals, the sequence networks can be employed as an acceptable approximation. For this purpose, the zero sequence network of transmission lines must explicitly represent all the paths available to the flow of zero sequence currents. The zero sequence network of a transmission line that meets this requirement has been developed and illustrated in Fig. 4.8c. Once the sequence networks have been developed, the remaining of the procedure is similar to the usual fault analysis. The procedure will be illustrated with an example.

Example E4.8. Consider the electric power system of Figure E4.8. The system comprises a distribution substation with a delta-wye connected transformer, a transmission line, an equivalent three phase source and a distribution line. The distribution line feeds a house via a distribution transformer. The parameters of the system are:

Equivalent source (115 kV):
$z_1 = j28.5\, ohms, \quad z_2 = j32.5\, ohms, \quad z_0 = j19.5\, ohms, \quad r_N = 50\, ohms$
Transmission Line: (three wire line):
$z_1 = z_2 = 1.5 + j31.5\, ohms, \quad z_0 = 2.8 + j68.5\, ohms$
Transformer (20 MVA, 115kV:12kV): $x_1 = x_2 = x_0 = j0.10\ pu$ on 20 MVA basis
The substation ground impedance is 1.0 ohms. The distribution line is the line described in Example E4.7. The presence of the electrical system of the house can be neglected.

Compute the ground potential rise at the pole ground (near the house) during a single line to ground fault on the high side of the delta-wye connected transformer.

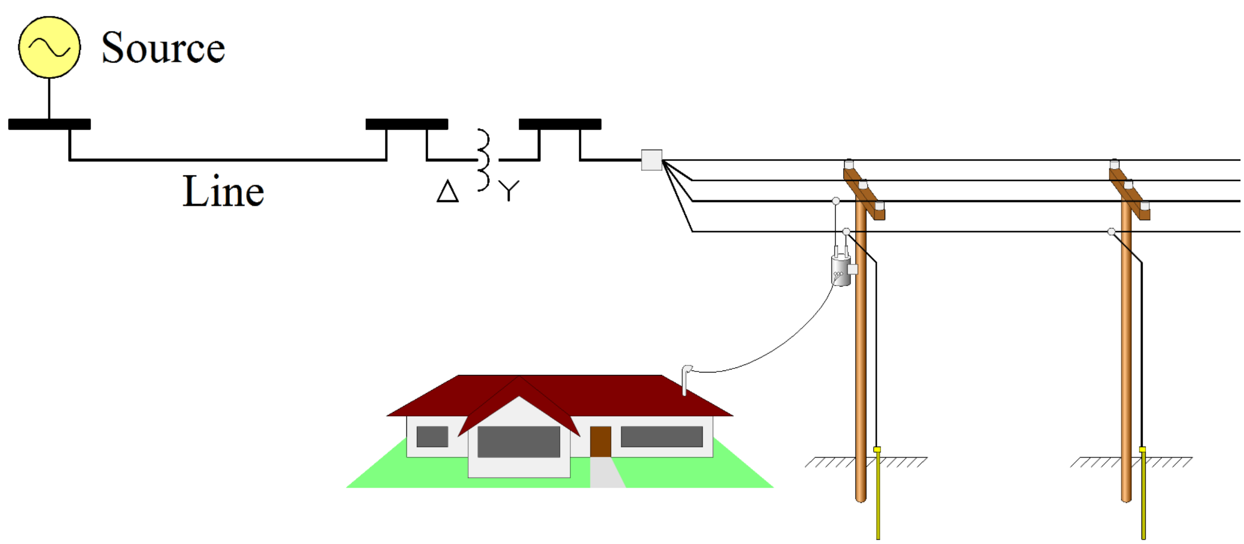
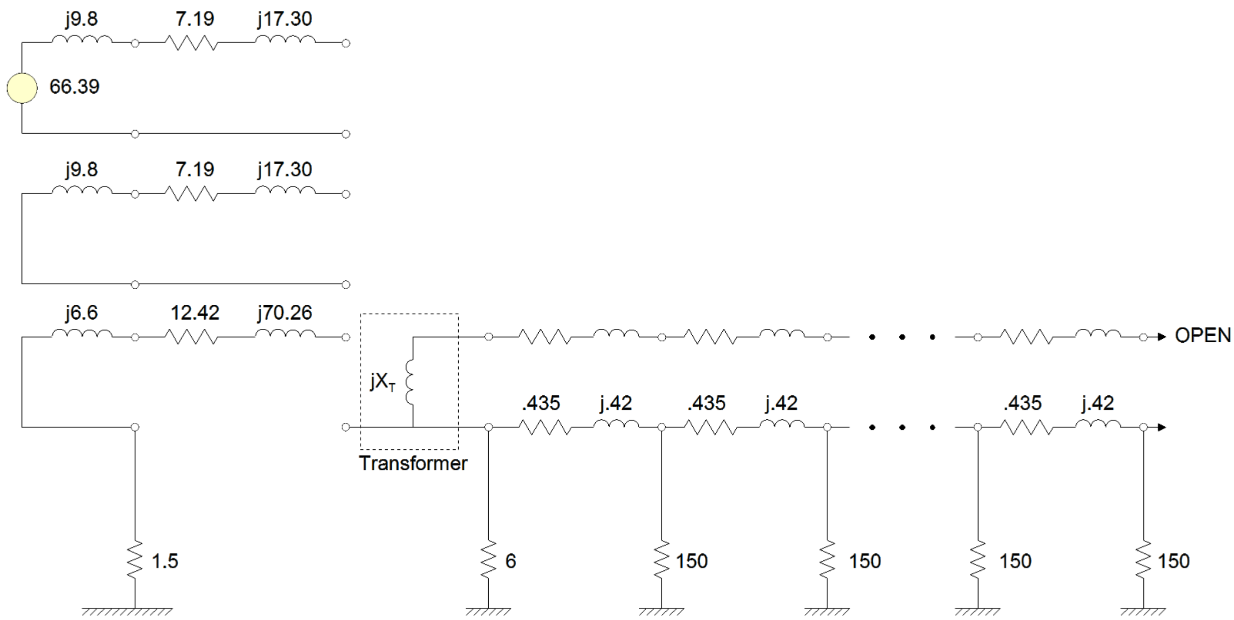
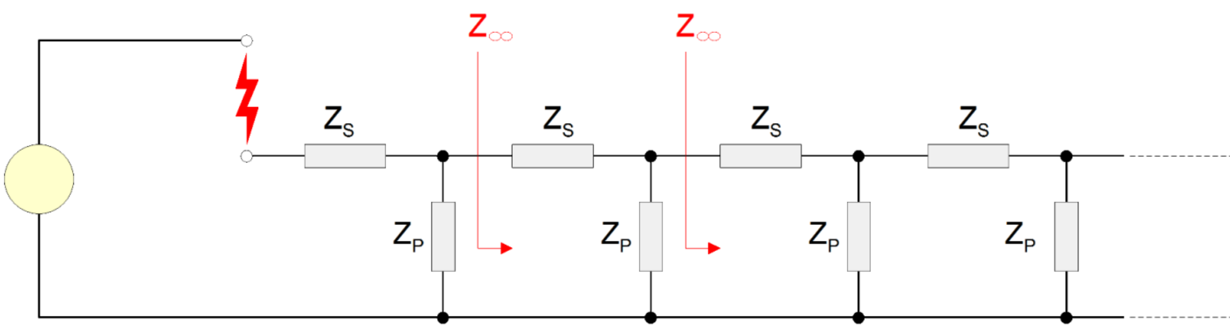
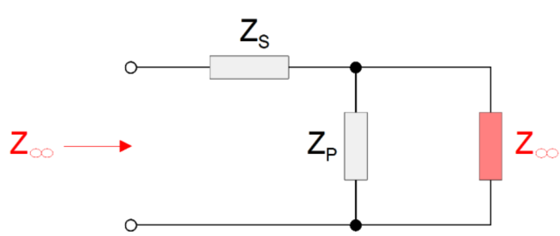


Figure E4.8. Simplified Distribution Substation and Distribution Line Serving a House

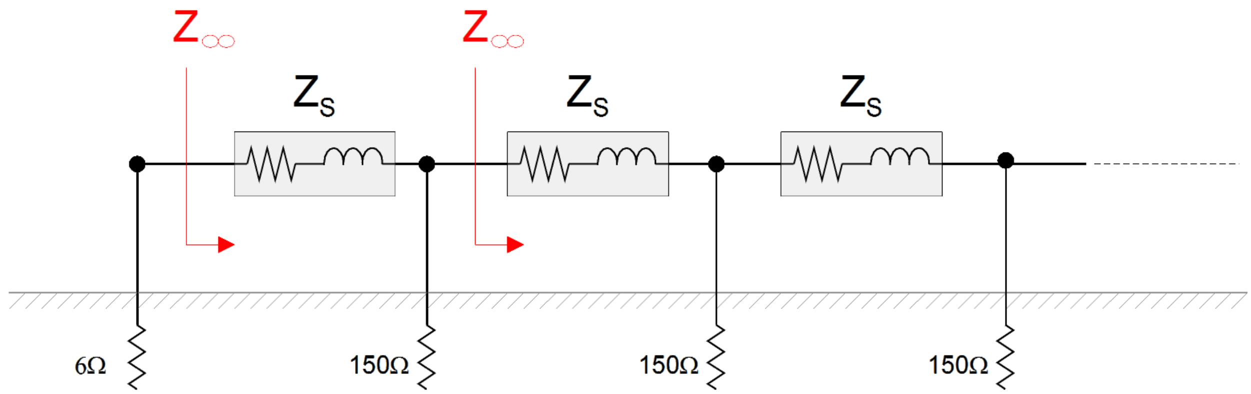
Solution: The sequence equivalent networks for this system are constructed in Figure E4.8a. Note that the distribution line equivalent circuit is taken from Example E4.7.

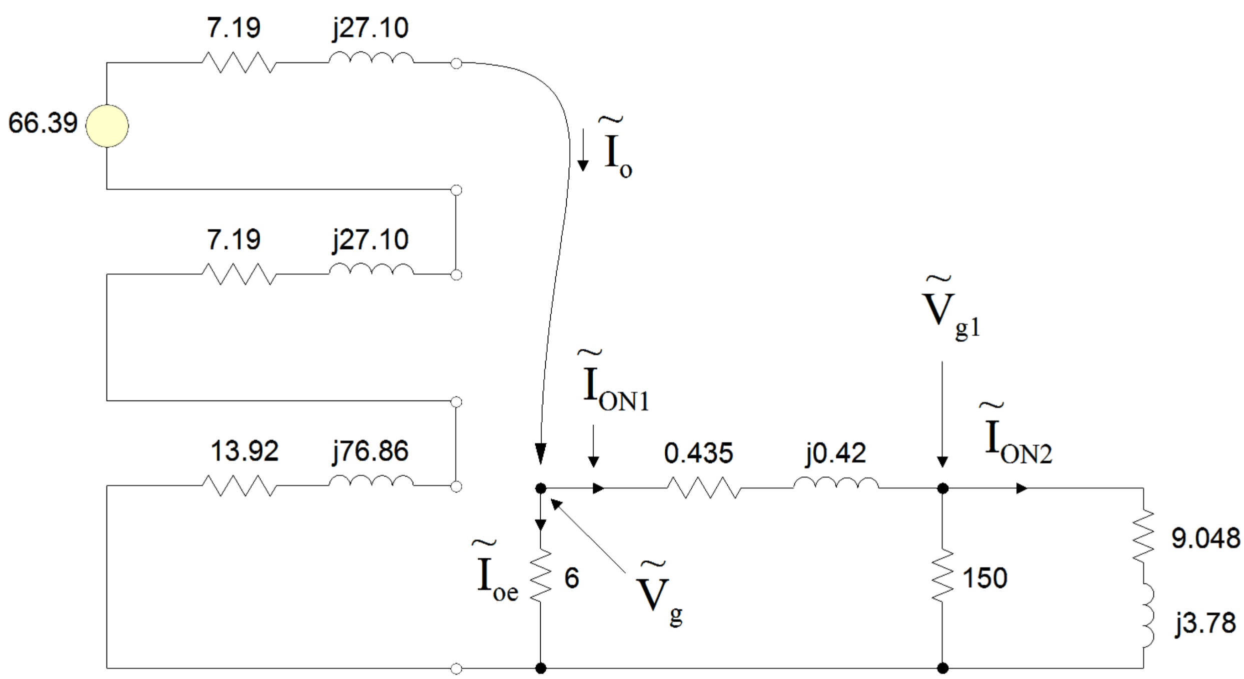






$$Z_\infty = \frac{Z_S}{2} + \sqrt{\left(\frac{Z_S}{2}\right)^2 + Z_S Z_P}$$





$$\tilde{V}_{g1} = 1.752 e^{-j69.11^0} kV \qquad \tilde{I}_{ON1} = 0.1894 e^{-90.42^0} kA \qquad \tilde{V}_g = 1.858 e^{-j67.75^0} kV$$

$$\tilde{I}_{oe} = 0.31 e^{-j67.75^0} kA \qquad \tilde{I}_0 = 0.49 e^{-j76.32^0} kA \qquad \tilde{I}_{ON2} = 0.179 e^{-91.79^0} kA$$

4.8 Fault Analysis - Direct Methods

The symmetrical component method has been widely accepted and used in electric power engineering. However, its applicability is limited to power system analysis problems for which the assumption of symmetric three-phase systems is acceptable. In general the impedances of the three phases may vary by 4 to 6 percent on typical overhead transmission systems. In this case the assumption of symmetrical components incurs an error of about 4 to 6 percent. For many analysis problems such as short-circuit analysis, power flow analysis, transient stability, and so on, this assumption has been widely accepted. For other applications, such as analysis of general multiphase networks or three phase networks with single- and two-phase taps, the symmetrical component method becomes cumbersome. For the purpose of computing fault current distribution among various paths such as shield wires, neutral, earth, etc. and ground potential rise, the symmetrical component method becomes very complex and thus unattractive. For an introduction on how the symmetrical component method can be used for fault current distribution and ground potential rise see reference [???]. The symmetrical component method was developed in an era when computing power was limited. The approximation was quite acceptable considering the benefits and the simplification of the analysis procedure. Today, computing power is readily available and there is no need to make this approximation. We describe here a methodology that is based on direct phase analysis (without any transformations). This approach provides accurate analysis of fault currents, there distribution, ground potential rise, etc.

The direct phase analysis is based on the admittance matrix representation of power system elements. This modeling approach can be considered as a generalization of the Norton's equivalent representation. The method is simple and able to account for (a) asymmetries of power system elements (i.e. untransposed lines, transformer banks with unequal impedances, etc.), (b) single- or two-phase systems, and (c) general multiphase systems. On the other hand, the method is computationally intensive.

4.8.1 Basis of the Method

The methodology is based on an admittance representation of symmetric or asymmetric single- or multiple-phase electric power system elements. For example, consider a single- phase transformer. This element can be viewed as a block with four terminals, as in Fig. 4.12. Neglecting magnetic core saturation, the device is linear. Thus a linear relationship exists among the terminal voltages and the terminal currents. That is,

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix} = Y \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \end{bmatrix}$$

The above equation is a general form of the Norton equivalent of the element under consideration. By definition, the matrix Y is the admittance matrix of the transformer. In case that the device is active, i.e. it contains sources (for example a generator), a similar relationship exists among the terminal currents and voltages with the addition that there will be equivalent sources in the equation. The general form of the equations will be:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix} = Y \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The admittance matrix and equivalent sources if present are determined form consideration of the specific circuits of the element. In Chapter 3, we discussed the computation of the admittance matrix of a transmission line. In general, any linear power system element can assume a similar description. Once all elements are described in this fashion, nodal analysis is applied directly to analyze the electric power system at steady state.

In subsequent sections the modeling procedure is described and the application of nodal analysis is outlined. An example will be employed to demonstrate the fault analysis in direct phase quantities.

4.8.2 Admittance Matrix of Power System Elements

In this section we present the basic modeling procedures by which the admittance matrix of power system elements is computed. The basic idea of these procedures is simple: The circuit equations describing the power system elements are written. Then all variables are eliminated except the terminal voltages. The result provides the admittance matrix. Depending on the complexity of the power system element under consideration, the procedure may be simple or complex. The procedures are illustrated here with typical applications.

Single-Phase Transformer

A single-phase transformer is illustrated in Figure 4.12.

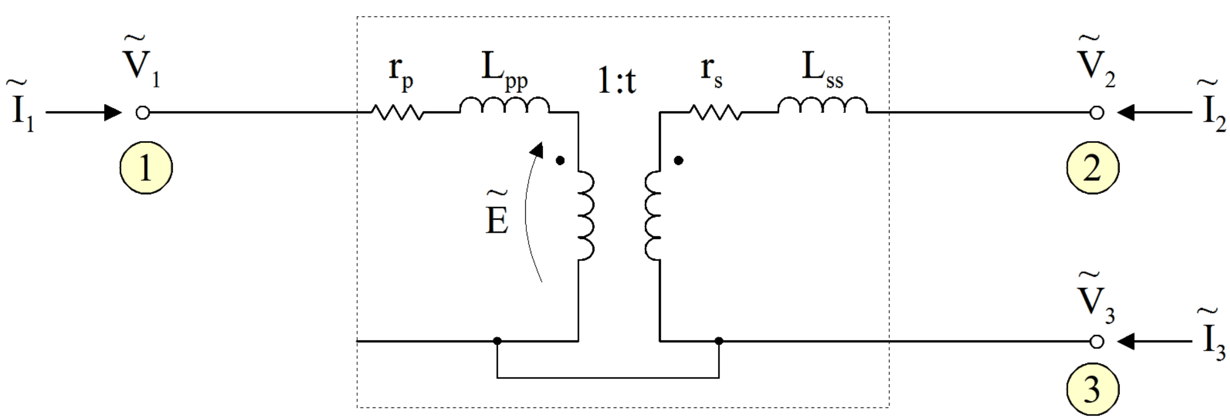


Figure 4.12. A Single Phase Transformer

The equations describing the transformer are

$$\tilde{V}_1 - \tilde{V}_3 = (r_p + j\omega L_{pp})\tilde{I}_1 + \tilde{E}$$
$$\tilde{V}_2 - \tilde{V}_3 = (r_s + j\omega L_{ss})\tilde{I}_2 + t\tilde{E}$$
$$0 = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3$$
$$0 = \tilde{I}_1 + t\tilde{I}_2$$

Define:

$$z_p = r_p + j\omega L_{pp}, \qquad y_p = \frac{1}{z_p}$$
$$z_s = r_s + j\omega L_{ss}, \qquad y_s = \frac{1}{z_s}$$

The last equation can be utilized to eliminate the variable $\tilde{E}$. Specifically, solve first equation for $\tilde{I}_1$ and the second equation of $\tilde{I}_2$, substitute into last equation and solve for $\tilde{E}$:

$$\tilde{I}_1 = y_p(\tilde{V}_1 - \tilde{V}_3 - \tilde{E})$$
$$\tilde{I}_2 = y_s(\tilde{V}_2 - \tilde{V}_3 - t\tilde{E})$$

Substitution into last equation yields:

$$y_p(\tilde{V}_1 - \tilde{V}_3 - \tilde{E}) + t y_s(\tilde{V}_2 - \tilde{V}_3 - t\tilde{E}) = 0$$

Upon solution for $\tilde{E}$:

$$\tilde{E} = (y_p + t^2 y_s)^{-1}(y_p\tilde{V}_1 + t y_s\tilde{V}_2 - (y_p + t y_s)\tilde{V}_3)$$

Upon substitution of the variable $\tilde{E}$ in equations (4.39):

$$\tilde{V}_1 - \tilde{V}_3 - (y_p + t^2 y_s)^{-1}(y_p\tilde{V}_1 + t y_s\tilde{V}_2 - (y_p + t y_s)\tilde{V}_3) = z_p\tilde{I}_1$$
$$\tilde{V}_2 - \tilde{V}_3 - t((y_p + t^2 y_s)^{-1}(y_p\tilde{V}_1 + t y_s\tilde{V}_2 - (y_p + t y_s)\tilde{V}_3)) = z_s\tilde{I}_2$$
$$0 = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3$$

Equations (4.36) are written in compact matrix notation as follows:

$$A\tilde{V} = Z\tilde{I}$$

where

$$A = \begin{bmatrix} 1 - \dfrac{y_p}{y_p + t^2 y_s} & -\dfrac{ty_s}{y_p + t^2 y_s} & \dfrac{y_p + ty_s}{y_p + t^2 y_s} - 1 \\ \dfrac{ty_p}{y_p + t^2 y_s} & 1 - \dfrac{t^2 y_s}{y_p + t^2 y_s} & \dfrac{ty_p + t^2 y_s}{y_p + t^2 y_s} - 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} z_p & 0 & 0 \\ 0 & z_s & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\tilde{I} = \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix}, \qquad \tilde{V} = \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \end{bmatrix}$$

The matrix Z is not singular. Thus Z^{-1} exists. Pre-multiplication of the above Equation by Z^{-1} yields

$$\tilde{I} = Z^{-1} A \tilde{V} = Y \tilde{V}$$

Thus the admittance matrix of the single-phase transformer of Fig. 4.11 is

$$Y = Z^{-1} A$$

Example E4.11: A single-phase transformer is rated 15-kVA, 7200/240 V. The leakage impedance is 2.2% on its rating and the resistance is 0.8%. The magnetizing current is 0.7%. Compute the admittance matrix of the transformer.

Solution: Assuming that the transformer impedance is equally divided between the two windings (on a per unit basis), the parameters r_p, L_{pp}, and so on, are computed as follows:

$$n = \frac{7200}{240} = 30$$

$$r_p = \frac{(0.4)(0.01)(7.2)^2}{0.015} = 13.824\, ohms$$

$$r_s = \frac{(0.4)(0.01)(0.24)^2}{0.015} = 0.01536\, ohms$$

$$\omega L_{ps} = \frac{(100.0/0.7)(7.2)(0.24)}{0.015} = 9600.0\, ohms$$

$$\omega L_{pp} = n\omega L_{ps} + \frac{(1.1)(0.01)(7.2)^2}{0.015} = 288038.016\,ohms$$
$$\omega L_{ss} = \frac{\omega L_{ps}}{n} + \frac{(1.1)(0.01)(0.24)^2}{0.015} = 320.04224\,ohms$$

$$z_p = 13.824 + j288038.016\,ohms$$
$$z_s = 0.01536 + j320.04224\,ohms$$
$$z_m = j9600.0\,ohms$$

Upon substitution into Equation (4.41) and evaluation we have

$$Y = \begin{bmatrix} 0.004224 - j0.011616 & -0.12673 + j0.34851 & 0 \\ -0.12673 + j0.34851 & 3.8017 + j10.4565 & 0 + j0.001823 \\ 0 & 0 + j0.001823 & 0 - j0.001823 \end{bmatrix}$$

Single-Phase Source

The Thevenin equivalent circuit of a single phase source is illustrated in Figure 4.13.

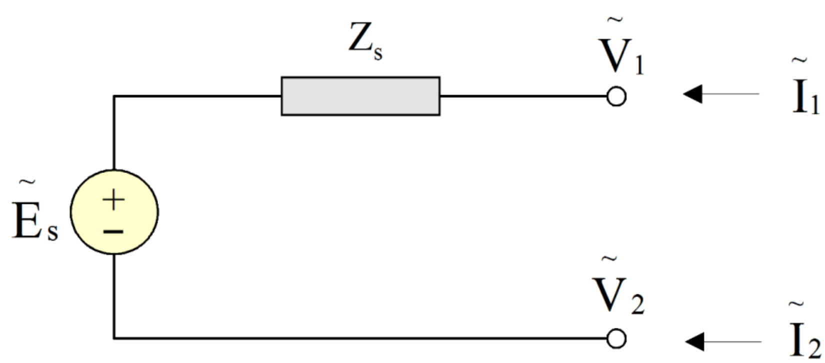


Figure 4.13: A Single Phase Source

The admittance matrix representation of a single-phase source is computed as follows. The equations describing the model of Figure 4.13 are:

$$\tilde{V}_1 - \tilde{V}_2 = z_s \tilde{I}_1 + \tilde{E}_s$$
$$0 = \tilde{I}_1 + \tilde{I}_2$$

Upon solution of Equation (4.42) for $\tilde{I}_1, \tilde{I}_2$:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} y_s & -y_s \\ -y_s & y_s \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} - \begin{bmatrix} y_s \tilde{E}_s \\ -y_s \tilde{E}_s \end{bmatrix}, \quad y_s = \frac{1}{z_s}$$

Equations (4.43) are in the desired form.

Example E4.12: Consider a 7200 V single-phase source of internal impedance j2 Ω. Compute the admittance matrix equation of the source.

Solution: The admittance matrix is

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} -j0.5 & j0.5 \\ j0.5 & -j0.5 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} - \begin{bmatrix} -j3600.0 \\ j3600.0 \end{bmatrix}$$

4.8.3 Nodal Analysis

Consider an electric power system consisting of n interconnected elements. The admittance matrix representation for element k is

$$\tilde{I}^k = Y^k \tilde{V}^k - b^k, \quad \textit{for devices } k = 1, 2, \ldots K$$

where Y^k is the admittance matrix of element k and b^k is the equivalent source of element k. Note the superscript denotes the device.

For fault analysis, the fault is also considered as an element and the admittance matrix representation of the fault is computed. In this case one of Equations (4.44) represents the fault model. Equations (4.44) state that the electric current at any terminal of an electric power system element is a linear combination of the terminal voltages of the element plus a constant. On the other hand, the element may be connected to other elements. Consider an interconnection node j as illustrated in Figure 4.14.

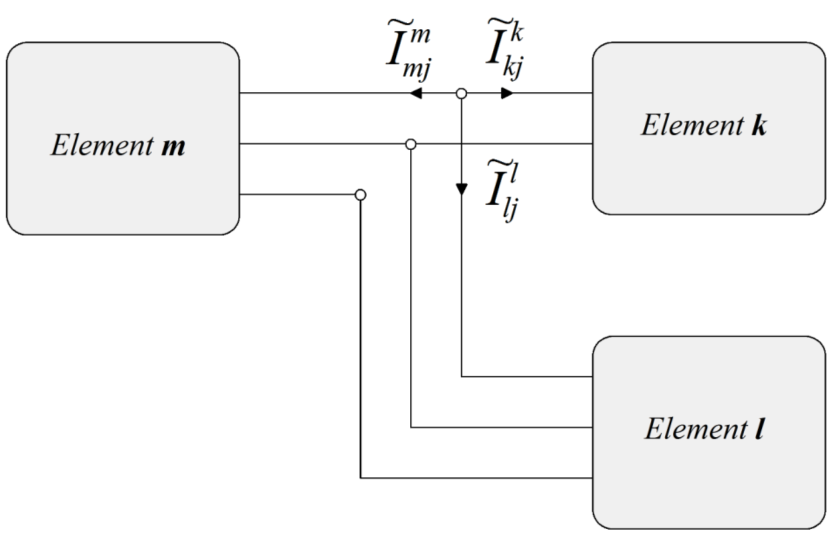


Figure 4.14: A General Interconnection Node

Kirchhoff's current law applied at node j yields

$$\tilde{I}^{k}_{j(k)} + \tilde{I}^{m}_{j(m)} + ... = 0$$

Where: $\tilde{I}^{k}_{j(k)}$ is the electric current flowing into terminal j of device k,

$\tilde{I}^{m}_{j(m)}$ is the electric current flowing into terminal j of device m, etc

Consider a network with many devices. The generalized Norton equation for each device is expressed with equation (). The terminals of device k are in general numbered $1, 2, ..., n(k)$, where $n(k)$ is the number of terminals of device k. Now consider the entire network. Assume that to each node of the network a number is assigned and that the total number of nodes is n. The assigned numbers to the nodes of the network will be $1, 2, ..., n$. We need to establish a correspondence between the network node numbers and the device terminal node numbers. This is achieved with the incidence matrix of device k, A^k. Specifically, the incidence matrix of device k is defined as follows:

$$A^k : \quad \{A^k\}_{ij} = \left| \begin{array}{l} 1, \quad if \ node \ j \ of \ device \, k \ is \ connected \ to \ network \, node \, i \\ 0, \quad otherwise \end{array} \right.$$

Now consider the product of the device terminal current vector and the incidence matrix:

$$A^k \tilde{I}^k$$

Above expression is a vector of dimension equal to the number of network nodes and it contains in entry j the terminal current of device k if device k is connected to network node j, otherwise it is zero. Using this vector, Kirchoff's current law at each network node is expressed as follows:

$$\sum_{k=1}^{K} A^k \tilde{I}^k = 0$$

Upon substitution of the device currents:

$$\sum_{k=1}^{K} A^k \left(Y^k \tilde{V}^k - b^k \right) = 0$$

Now consider the vector of device terminal voltages, $\tilde{V}^k$ and the vector of network node voltages, $\tilde{V}$. The two vectors are interrelated with the incidence matrix as follows:

$$\tilde{V}^k = \left(A^k \right)^T \tilde{V}$$

Upon substitution and rearranging:

$$\sum_{k=1}^{K} A^k Y^k \left(A^k \right)^T \tilde{V} = \sum_{k=1}^{K} A^k b^k$$

This equation can be written as:

$$Y\tilde{V} = b$$

Where:

$Y = \sum_{k=1}^{K} A^k Y^k \left(A^k\right)^T$ is the admittance matrix of the entire network

$b = \sum_{k=1}^{K} A^k b^k$ is a vector of independent current sources

$\widetilde{V}$ is the vector of voltages at the nodes of the system

Solution of the above equation yields the voltage at every node in the system. Back substitution into Equations (xxx) yields the electric current at the terminals of each device. From this information, any other quantity of interest can be computed. For example, the ground potential rise of a substation grounding system is the voltage at the node representing the neutral, and so on. The procedure will be illustrated with an example.

Example E4.13: Consider a simple (made up) power system consisting of a single phase 277 Volt, 60 Hz generator, a single phase 277V/7200V transformer and a single phase 7.2 kV distribution line. The system is illustrated in Figure E4.13. The ground resistance of the generator/transformer ground is 2 ohms and the pole ground resistance is 35 ohms. Assume a phase to neutral fault at the end of the single phase distribution line. For this condition compute (1) the fault current, (2) the current at the generator/transformer ground and (3) the ground potential rise at the transformer and (4) the ground potential rise at the location of the fault.
Repeat the above computations assuming that all grounds are ideal, i.e. their impedance is zero.

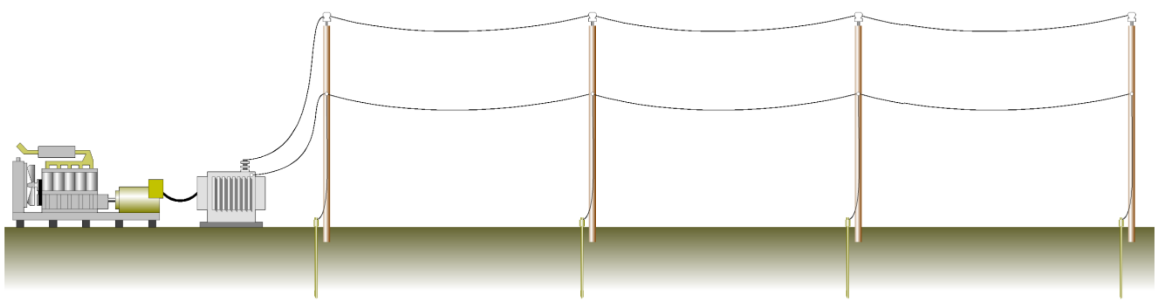
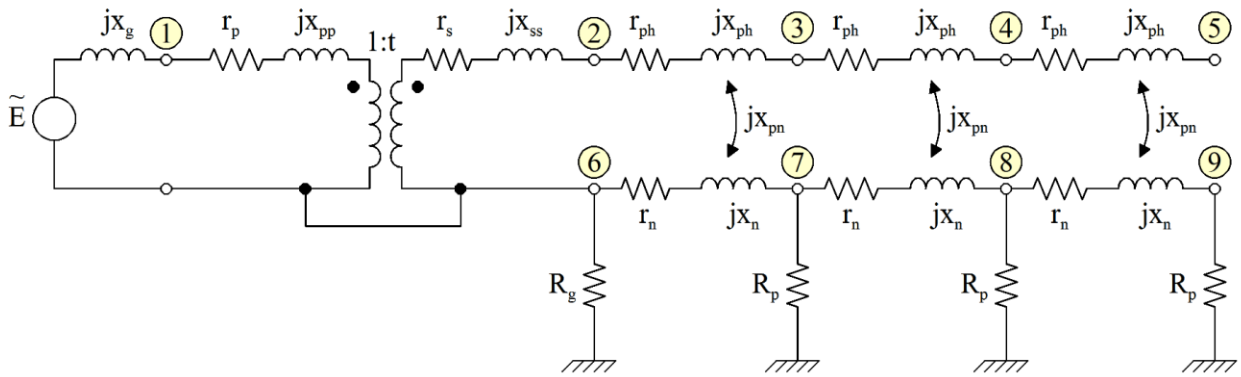


Figure E4.13: A Simple Single Phase Power System

Solution: The equivalent circuit of this system is shown in Figure E4.13a.

to be completed



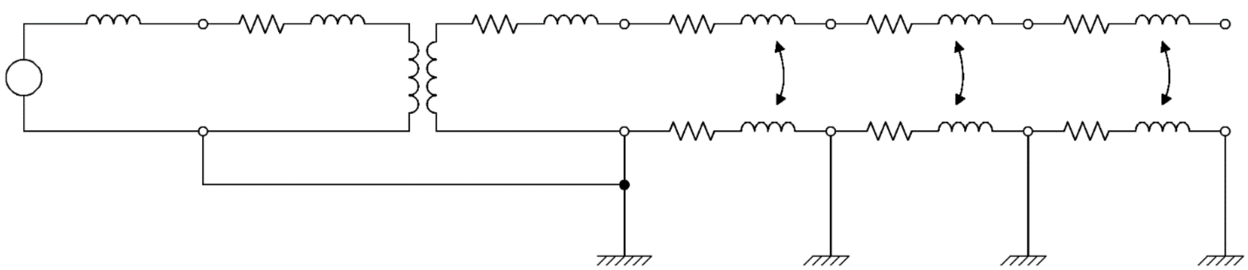


Figure E4.13a: Equivalent Circuit

4.8.4 Discussion of Direct Phase Analysis

In this section we have presented procedures for fault analysis in power systems based on a generalized admittance matrix representation of power system elements (generalized Norton equivalent form). This representation of power system elements is a generalization of Norton equivalents. The only assumption involved in this representation is that the power system elements are linear devices (i.e. the nonlinearities are ignored or a linearized model of power system elements is utilized). The solution method is a straightforward application of nodal analysis. The procedure is computationally intensive and suitable for computer implementation. The advantage of the method stems from the capabilities of the admittance matrix representation. Specifically, the admittance matrix can model (a) transmission line asymmetries, (b) grounding systems, and so on. Thus this method is suitable to study the effects of asymmetries, ground potential rise, fault current distribution, and other factors.

4.9 Temporary Power Frequency Overvoltages

An electric fault initiates transients and abnormal conditions. Typical transients are: immediately after the electric fault has initiated, electrical transients occur that normally decay fast. These transients may cause temporary high overvoltages. Typical durations of these abnormal voltages are fractions of a cycle or several cycles, i.e. typically less than the duration of the electric fault.

Another phenomenon that lasts while the fault is on, is the occurrence of abnormal voltages at the faulted and unfaulted phases. For example, the faulted phases typically will experience voltage sags while the unfaulted phases will experience overvoltage. Another phenomenon is that the presence of an electric fault in a power system disturbs the equilibrium between the mechanical power input of a generator and the ability of the generator to deliver electric power to the network. As a result, generators may accelerate or decelerate during a fault depending on the relationship between the mechanical input power and the electric power output of the generator.

In this section, we examine the temporary overvoltages during the fault conditions. The electromechanical oscillations triggered by a fault are examined in Chapter x. For the electrical transients triggered by a fault see reference [???].

Coefficient of Grounding: The power frequency overvoltages experienced by the unfaulted phases during a fault depend very much on the way the system is grounded. Consider for example a system that is practically ungrounded, i.e. the ground impedances are very high. When a single-phase to ground fault occurs in this system, the point of the fault becomes the system ground and the voltage of the faulted phase at this point will be near zero. The neutral of the system will float and it will be elevated to a voltage approximately equal to the nominal phase voltage. The voltage of the unfaulted phases will be approximately equal to the nominal line to line voltage of the system, i.e. an overvoltage of about 73%. On the other hand consider a system that is perfectly grounded, i.e. the ground impedances are very small. A single phase to ground fault at this system will not affect the voltages of the neutral since the neutral is almost perfectly grounded. The voltage of the neutral will remain near zero and therefore the voltages of the unfaulted phases will remain near the nominal phase to neutral voltages. In other words the overvoltage of the unfaulted phases will be zero for this system. Practical systems are neither perfectly grounded nor ungrounded. For practical systems and in case of a ground fault the unfaulted phases will experience an overvoltage that is somewhere between 0 and 73 %, the exact value depends on the parameters of the grounding system. In order to quantify this overvoltage the coefficient of grounding has been introduced. Specifically the coefficient of grounding is defined as the overvoltage on an unfaulted phase conductor during a ground fault on another phase.

The coefficient of grounding depends on the ground impedance and the other impedances of the system. For a specific system, the coefficient of grounding can be computed by simply analyzing the fault condition, computing the voltages of the unfaulted phases and taking the ratio of the overvoltage over the nominal voltage. We will consider this computations first using symmetrical components and then using direct phase analysis.

Computation of the Coefficient of Grounding by Symmetrical Components: Consider a three phase system. At a certain location, the sequence parameters of the system are $z_1, z_2 \text{ } and \text{ } z_0$. Note that the value of the zero sequence impedance depends of the grounding impedances of the system while the positive and negative sequence impedances are almost independent of the grounding impedances. Consider a single-phase to ground fault at this power system. The fault currents will be:

$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{1}{z_1 + z_2 + z_0}\ pu$$

The sequence voltages will be:

$$\tilde{V}_1 = 1.0 - z_1 \tilde{I}_1 = 1.0 - \frac{z_1}{z_1 + z_2 + z_0}$$

$$\tilde{V}_2 = -z_2 \tilde{I}_2 = -\frac{z_2}{z_1 + z_2 + z_0}$$

$$\tilde{V}_0 = -z_0 \tilde{I}_0 = -\frac{z_0}{z_1 + z_2 + z_0}$$

The actual voltages at the three phases are:

$$\tilde{V}_a = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0 = 0$$

$$\tilde{V}_b = a^2 \tilde{V}_1 + a\tilde{V}_2 + \tilde{V}_0 = a^2 - \frac{a^2 z_1 + a z_2 + z_0}{z_1 + z_2 + z_0}$$

$$\tilde{V}_c = a\tilde{V}_1 + a^2 \tilde{V}_2 + \tilde{V}_0 = a - \frac{a z_1 + a^2 z_2 + z_0}{z_1 + z_2 + z_0}$$

It should be observed hat for many systems the positive and negative sequence impedances are practically the same. For these systems, above expressions are further simplified into:

$$\tilde{V}_b = a^2 \tilde{V}_1 + a\tilde{V}_2 + \tilde{V}_0 = a^2 - \frac{z_0 - z_1}{2z_1 + z_0}$$

$$\tilde{V}_c = a\tilde{V}_1 + a^2 \tilde{V}_2 + \tilde{V}_0 = a - \frac{z_0 - z_1}{2z_1 + z_0}$$

Observe that when we have an ungrounded system, the zero sequence impedance will be extremely high. At the limit when the zero sequence impedance becomes infinity:

$$\tilde{V}_b = \lim_{z_0 \to \infty} \left(a^2 - \frac{z_0 - z_1}{2z_1 + z_0} \right) = \left(\sqrt{3}\right) e^{j210^0}$$

$$\tilde{V}_c = \lim_{z_0 \to \infty} \left(a - \frac{z_0 - z_1}{2z_1 + z_0} \right) = \left(\sqrt{3}\right) e^{j150^0}$$

Note that in this case both unfaulted phases will experience a voltage of 1.73 (square root of 3) pu, or an overvoltage of 73%.

Another extreme case will be a system with equal sequence impedances (positive, negative and zero sequence are all equal). This case corresponds to a perfectly grounded system and with negligible mutual coupling between phases. In this case:

$$\tilde{V}_b = \lim_{z_0 \to z_1} \left(a^2 - \frac{z_0 - z_1}{2z_1 + z_0} \right) = e^{j240^0}$$

$$\tilde{V}_c = \lim_{z_0 \to z_1} \left(a - \frac{z_0 - z_1}{2z_1 + z_0} \right) = e^{j120^0}$$

Note that in this case both phases B and C will have nominal voltage, i.e. they will not experience any overvoltage during the ground fault of phase A.

For any other value of the impedances (and therefore quality of grounds), the voltages of the unfaulted phases can be computed from the equations above.

An approximate graphical representation of these equations is given in Figure 4.x. Note that this figure has been developed by an IEEE committee. The simplifying assumptions in this figure are: (a) the system devices are represented with sequence equivalent circuits (b) the positive and negative sequence impedances are equal at the location of the fault, and (c) the positive/negative sequence resistance is neglected.

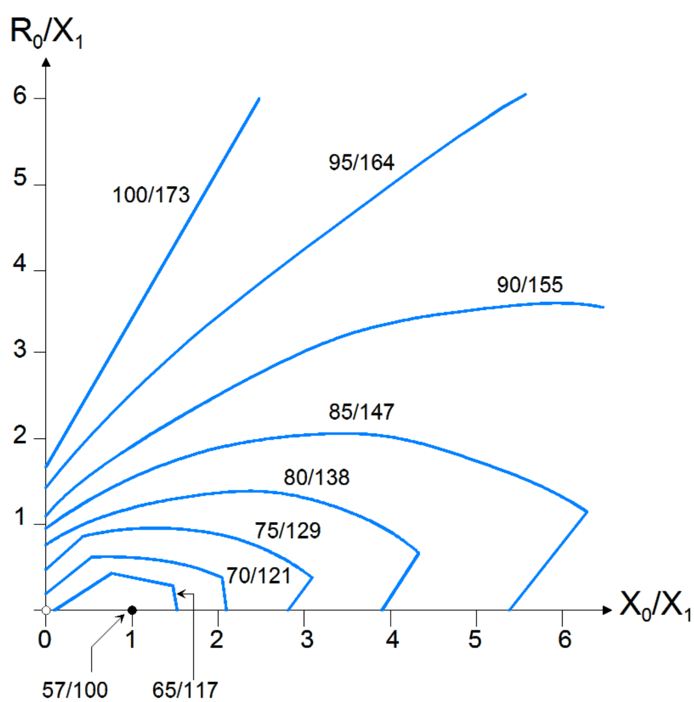


Figure 4.x. Parametric Representation of the Overvoltage of the Unfaulted Phases during a Ground Fault – Simplified Analysis ($z_2 = z_1, \quad r_1 = 0$ **)**

A better representation of overvoltages of unfaulted phases is provided in Figure 4.y. The assumptions used in developing the data of Figure 4.y are: (a) the system devices are represented with sequence equivalent circuits and (b) the positive and negative sequence impedances are equal at the location of the fault.

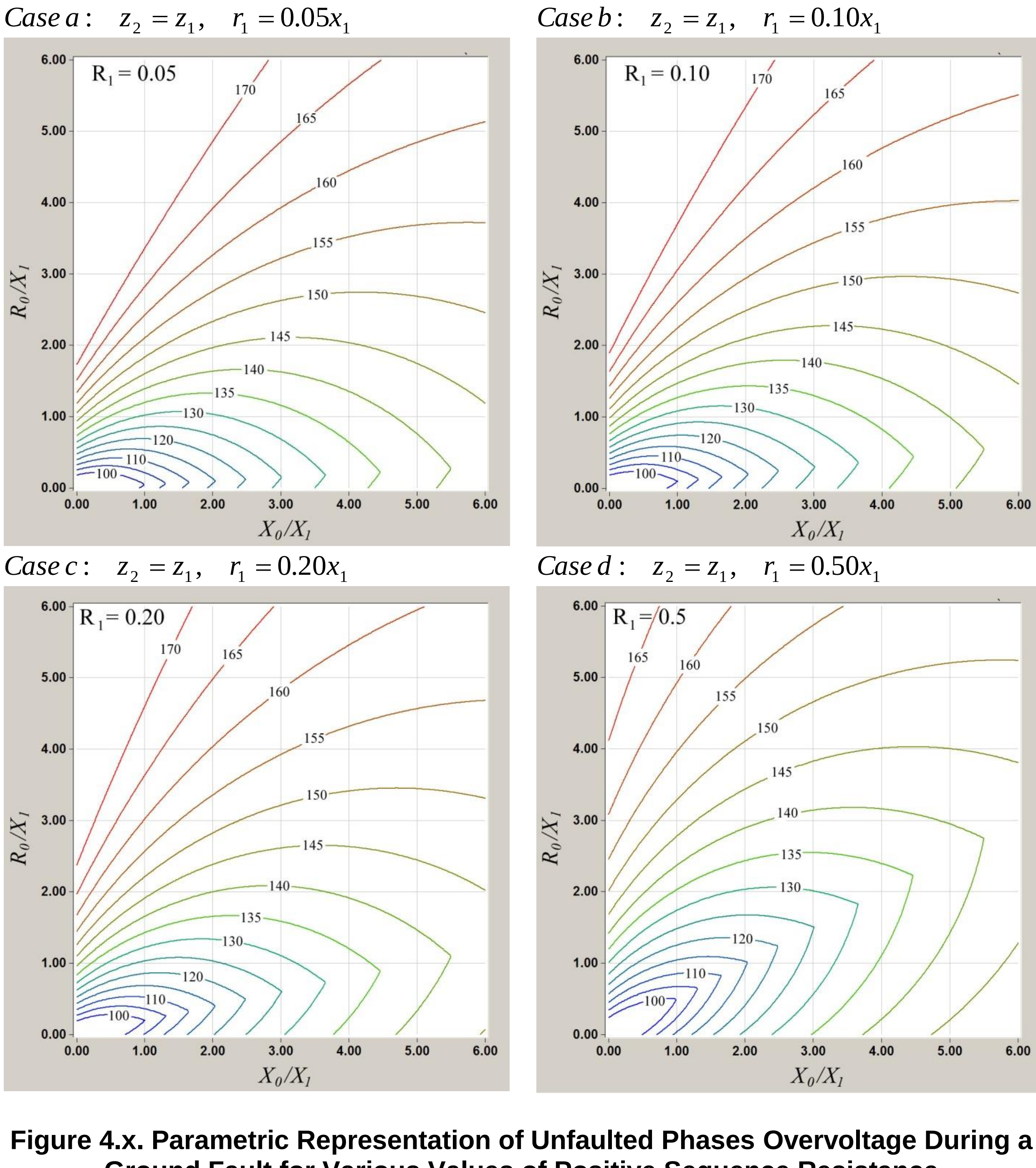


Figure 4.x. Parametric Representation of Unfaulted Phases Overvoltage During a Ground Fault for Various Values of Positive Sequence Resistance

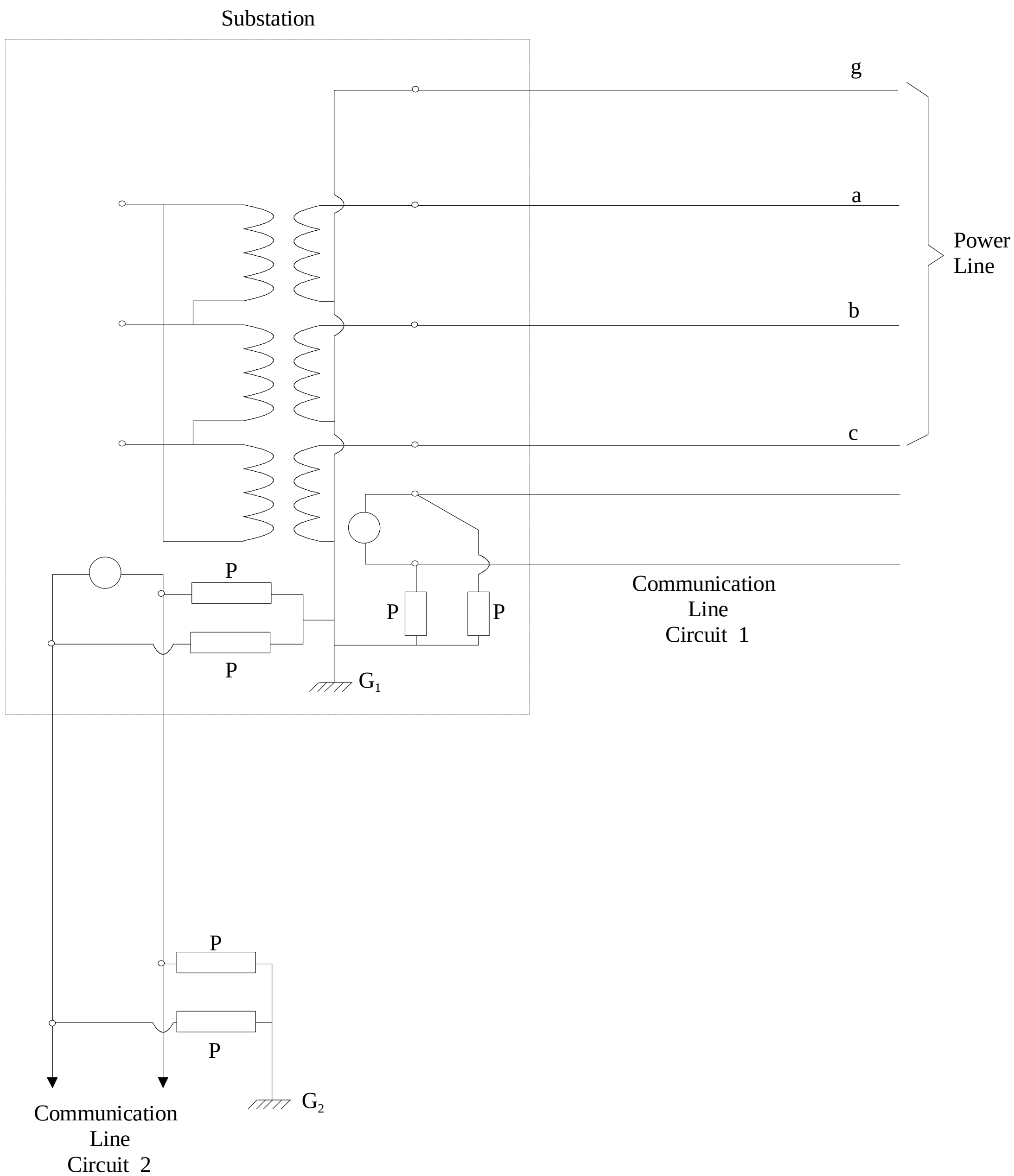
4.10 Transferred and Induced Voltages during Faults

Because the soil is not perfectly conducting, potential differences may be generated between two grounded points. As a matter of fact, electric currents do circulate in earth during even normal operating conditions. During fault conditions (especially asymmetric faults) the earth currents may be substantial, resulting in high-voltage elevation of grounding systems and transfer voltages to nearby metallic structures. These voltages may cause;

1. Safety problems
2. Misoperation or damage of communication circuits

Safety problems are generated whenever the voltages transferred to grounded structures are high enough to cause electric shocks to human beings or animals. Communication circuits are affected in two ways: (a) voltages are induced on these circuits from the electric currents of the power line which corrupt the useful signals with noise, increasing the possibility of misoperation, or (b) voltages may be transferred by conduction to communication circuits, which can cause interference problems or damage of communication equipment. The two interference mechanisms are illustrated in Figure 4.15. The figure illustrates two communication circuits. Circuit 2 may be subjected to a voltage equal to the difference in the ground potential rise at the two grounds G1 and G2. Circuit 1 is parallel to the power line and is therefore subjected to induced voltage from the electric currents of the power line. Induced voltages are mitigated by transposing the communication circuit. In general, communication circuits must be protected against transferred or induced voltages. Protection schemes involve protection blocks, isolation transformers, neutralizing transformers, and so on. Judicious application of protection schemes requires that the maximum voltage on the communication circuit under any adverse conditions be computed.

The basic problem is illustrated by the simplified system of Figure 4.16. A power transmission line is illustrated terminating in two substations. The substations each comprise a delta-wye-connected transformer. To the left of the figure is a source feeding the substation. Another power transmission line starts from the substation to the right of the figure. Both substations are grounded with a ground mat.



Note to myself: Do a pilot wire relaying example

Figure 4.15: Two Mechanisms by Which Power Lines Can Interfere With Communication Circuits

[(a) Line 1 is subject to interference by induction, (b) line 2 is subject to interference by conduction]

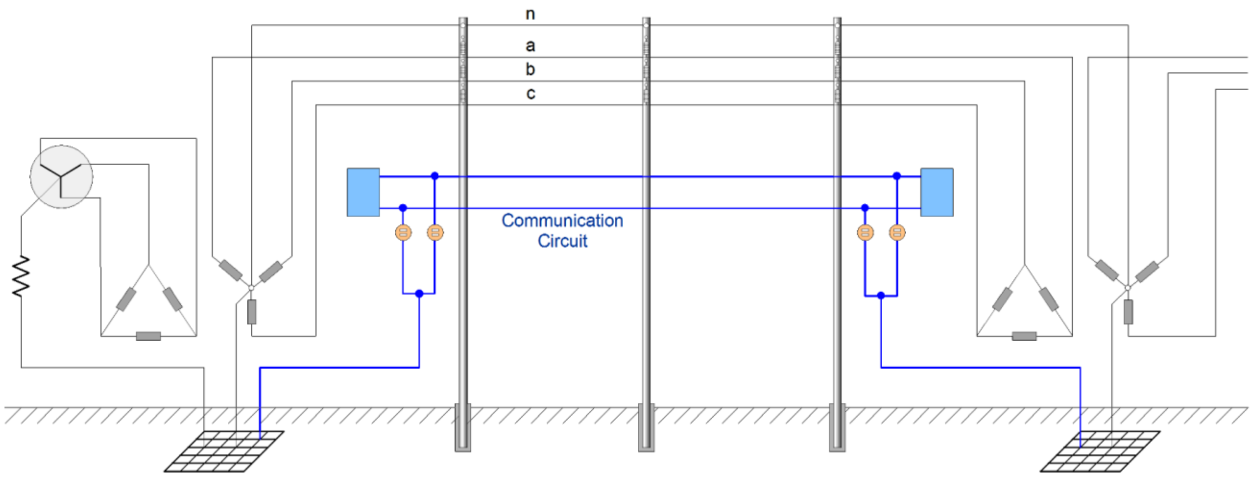


Figure 4.16 Communication Circuit in Power System Environment

A communication circuit is suspended on the same towers as the power line. Typically, the communication circuit is connected to the substation grounding system through protector blocks. Our discussion will focus on the voltage induced or transferred to the communication circuit.

During normal operating conditions, almost balanced three-phase currents flow through the power line. The voltages induced on the communication line form each of the phase currents are approximately equal in magnitude. The phase difference between any two is 120°. Thus the total induced voltage is approximately zero. On the other hand, the ground potential rise at the two grounds, G1 and G2, is small (smaller than the protection level of the protector blocks), and therefore the voltage transferred to the communication circuit is practically zero. However, during unbalanced conditions, the total induced voltage may be substantial. At the same time, the ground potential rise of the grounds, G1 and G2, may be substantial. As a result, the communication circuit may be subjected to a substantial voltage and must be protected. Selection of the protection depends on the expected level of overvoltages during all foreseeable adverse conditions. Specifically, the protection scheme should be able to withstand the maximum voltage that may develop between points G1 and G2. Typical protection schemes of communication circuits are illustrated in Figure 4.17. Scheme (a) involves protector blocks only. It provides protection for voltages up approximately to 300 V. Scheme (b) involves protector blocks and an isolation transformer. This scheme is capable of providing protection against much higher voltages, depending on the insulation level of the isolation transformer. The third winding will carry an electric current proportional to the voltage on the communication circuit approximately equal but of opposite polarity (neutralizing voltage) of the voltage developed across the communication circuit (see Figure 4.17c). Protection scheme (b) is simple, effective, and relatively inexpensive. The selection of the insulation level of the isolation transformer is based on the maximum voltage that can develop along the communication circuit under any foreseeable adverse condition. For a specific condition, the voltage along the communication circuit can be computed using the direct phase analysis method. For this purpose, the wires of the communication circuit are modeled as conductors of the power line.

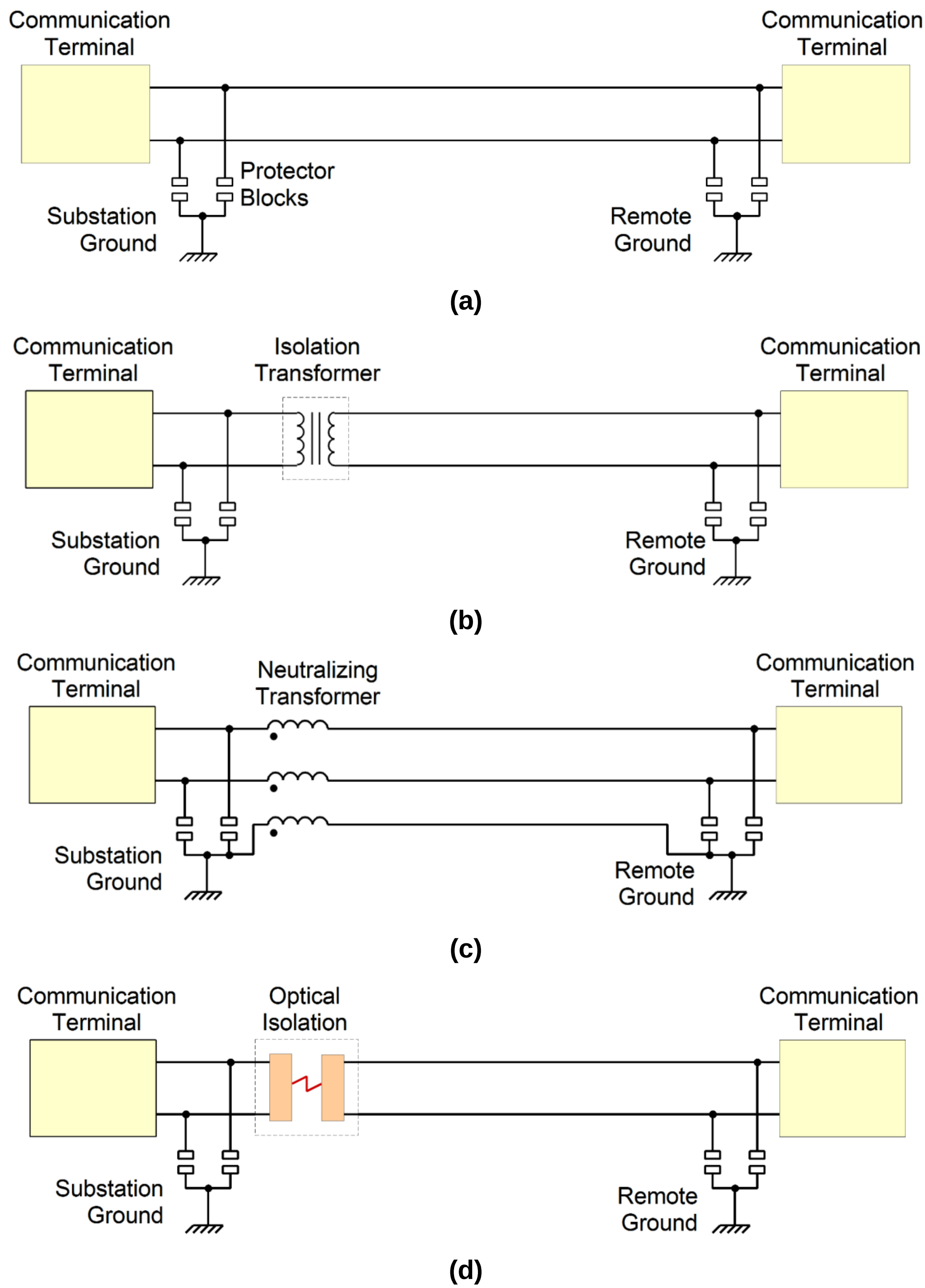


Figure 4.17: Communication Line Protection Schemes

[(a) Protector blocks, (b) protector blocks and isolation transformer, (c) protector blocks and neutralizing transformer, (d) optical Isolation]

4.11 Breaker Selection

The breaker should be capable of interrupting the fault whenever the breaker is tripped. The maximum fault current that a breaker may be called to interrupt depends on the location of the breaker within the network. The breaker rating should be so selected as to be higher than the maximum fault current at the location of the breaker for the projected life of the breaker. Since power systems are continuously expanding and therefore the fault currents are increasing as the system expands, it is important to select the breaker rating to meet the needs of the system for the long expected life of a breaker.

4.12 Wire Sizing for Electric Faults

An electrical circuit may consists of (a) phase conductors, (b) neutral conductors (c) shield wires and (d) ground conductors. This classification is based on the intended function of the conductors. The intended function of the phase conductors is to provide the path for the flow of electric current through the circuit at the operating voltage of the system. The intended function of the neutral conductor is to provide the return path for the flow of electric current – the neutral is intended to operate at near zero voltage. The intended function of shield wires is to provide the termination point of lightning and to guide the lightning current to the ground of the facility thus minimizing the overvoltages or transients in other parts of the system and especially of the phase conductors. Finally, the intended function of the ground conductors is (a) to provide a low impedance return path for the flow of fault currents and lightning currents (or any other abnormal currents in the system) and (b) provide for bonding of any electric equipment case to a low impedance ground so that equipment cases will be always at very low voltage and safe to be touched by humans.

This analysis indicates that phase conductors and neutral conductors carry the load electric current and occasionally (during faults) they will also carry the fault current. The size of these conductors should be so selected that it will be adequate to carry the normal load current continuously and the fault current during the short duration of the fault. In general, sizing phase and neutral conductors to carry the normal load current will be adequate to ensure the adequate sizing of the conductor during faults. However, there are exceptions. For this reasons it is important to determine the size of these conductors for the normal continuous operation as well as the size of the conductors for fault conditions and select the largest of the two. For the first determination there are extensive ampacity tables for all power conductors and therefore the process is as simple as selecting the appropriate entry from the table. The selection of the conductor size for fault conditions is the same as the one discussed below for ground conductors.

The sizing of shield conductors is based on strength considerations. These conductors normally will carry lightning currents. The amount of energy in the flow of lightning currents is relatively low and a relatively small size is adequate. When the proper size wire is selected from mechanical considerations, this size is typically adequate for the lightning currents.

The selection of the ground conductor size is based on the level of fault current. A ground fault always results in fault current flow in grounding conductors. In order that the ground conductors be capable of withstanding the flow of fault current, their size should be appropriately selected. The criteria for selecting ground conductors are simple: the ground conductors should not melt (or better yet should not exceed a permissible temperature that is normally below their melting point) for the duration of the fault current flow. In addition, the National Electrical Code [???] imposes a minimum required size of ground conductors. The selection procedure then should consists of two steps: (a) step one: determine the minimum ground conductor size required by the NEC. (b) step two: compute the conductor size required so that the ground conductor temperature will not exceed the maximum permissible temperature under any adverse conditions, and (c) step three: select the largest ground conductor size from steps one and two.

Selection of Grounding Conductor Size: The NEC states a requirement of minimum size ground conductor as it is shown in Table 4.x. The table provides the minimum size ground conductor permitted by the NEC.

Computation of ground conductor required size from thermal considerations is based on the following procedure. A permissible temperature is assumed for the specific material of the conductor. For example for copper conductors one may select a permissible temperature of 450 degrees Celcius. Recall that the melting temperature of copper is 1,083 degrees Celcius. Thus the permissible temperature is normally a temperature that is below the melting temperature with a considerable safety margin. Then we assume that during the fault all the thermal energy released in the conductor remains in the conductor material and raises the temperature of the conductor (in other words, since the duration of the fault is short, there is not enough time for any of the released thermal energy to dissipate in the ambient). We also assume that the initial temperature of the ground conductor is 40 degrees Celcius – this represents worst conditions such as summer conditions when the temperatures may be high. The analysis of this problem provides us with a relationship between the temperature rise, duration of the fault and conductor cross section. Solution of this equation for the conductor cross section area as a function of fault current and fault duration provides the needed ground conductor size. This procedure has generated the nomogram of Figure 4.y which provides the minimum size conductor required as a function of fault current and duration.

Table 4.x. Minimum Size Equipment Grounding Conductors for Grounding Raceway and Equipment, per NEC, Table 250.222

Rating or Setting of Automatic Overcurrent Device in circuit ahead of equipment, conduit, etc. Not Exceeding (Amperes)	Copper Wire No.	Aluminum Or Copper-Clad Aluminum Wire No.*
15	14	12
20	12	10
30	10	8
40	10	8
60	10	8
100	8	6
200	6	4
300	4	2
400	3	1
500	2	1/0
600	1	2/0
800	1/0	3/0
1000	2/0	4/0
1200	3/0	250 kcmil
1600	4/0	350 kcmil
2000	250 kcmil	400 kcmil
2500	350 kcmil	600 kcmil
3000	400 kcmil	600 kcmil
4000	500 kcmil	800 kcmil
5000	700 kcmil	1200 kcmil
6000	800 kcmil	1200 kcmil

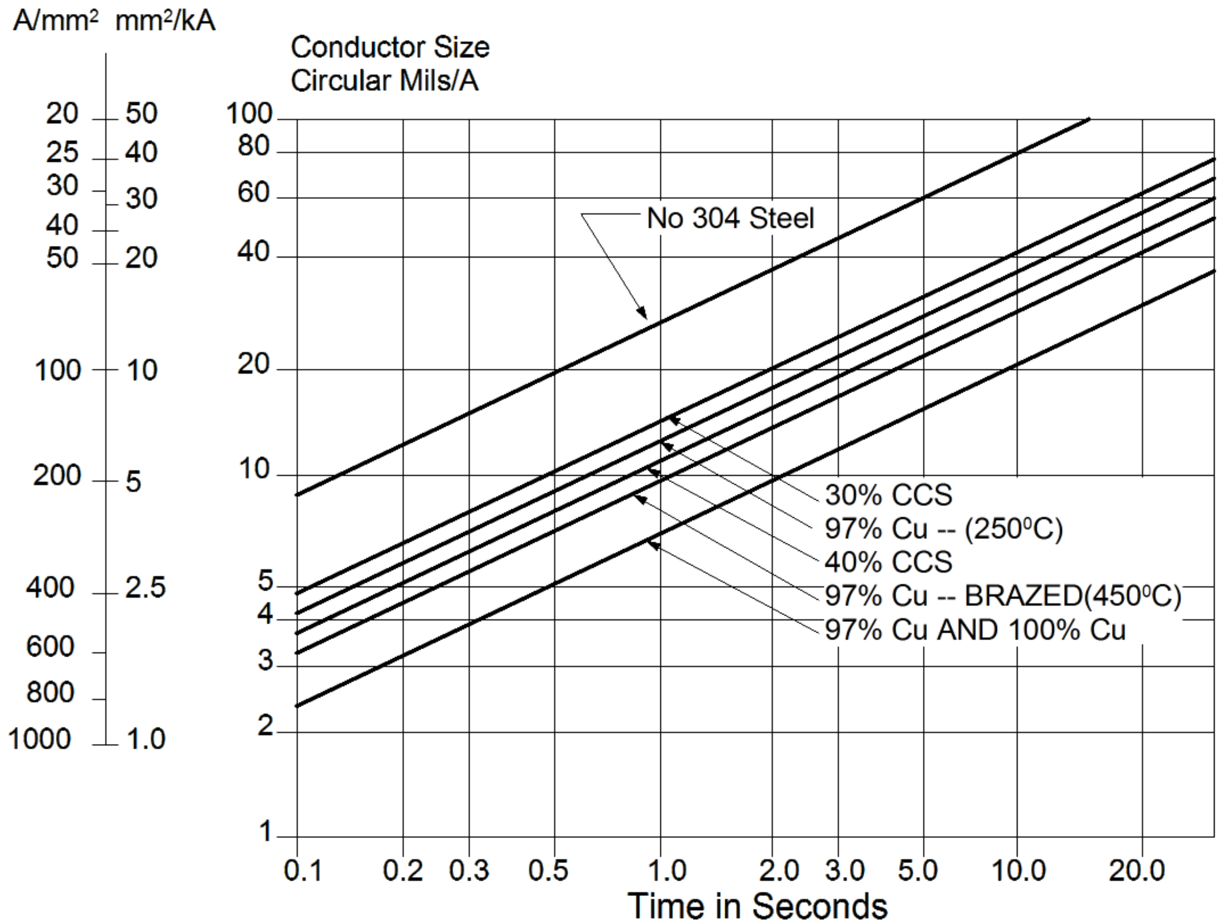


Figure 4.y. Nomogram for Conductor Sizing (IEEE Std 80, 1980 Edition)

Example E4.14: Consider the simplified power system of Figure E4.x. It is desirable to determine the size of the phase conductors and the size of the grounding conductors. It is given that the faults can be interrupted within 15 cycles. Determine the minimum size of the grounding conductors.

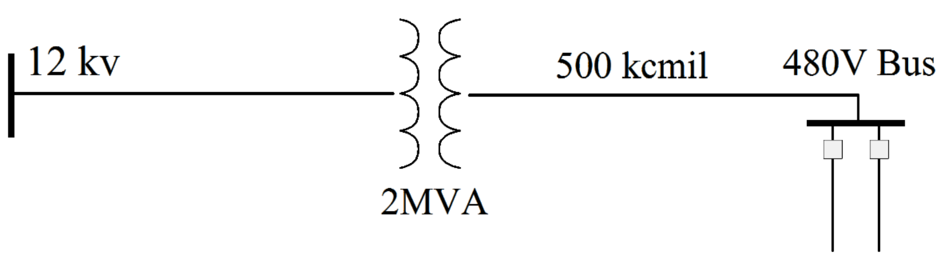


Figure E4.x Simplified Example Power System
(Transformer to 480V bus distance is 30 feet)

Solution: The phase conductor size is determined from (a) ampacity tables and (b) maximum permissible temperature during faults.

to be continued.

4.13 Summary and Discussion

In this chapter we have discussed fault analysis methodologies. The conventional fault analysis method based on symmetrical components has been reviewed. An extension of the method has been presented which enables the computation of the fault current distribution and ground potential rise of grounding systems. The symmetrical component method neglects asymmetries existing in power system elements such as transmission lines. The direct phase analysis method has been presented, which takes asymmetries into consideration. Direct phase analysis is based on the admittance matrix representation of power system elements (or Norton equivalent) and nodal analysis. The method is computationally intensive and thus, by necessity, computer based.

4.14 Problems

Problem P4.1: At a certain location of a three phase system, an engineer measures the following phase currents and phase to neutral voltages:

$$\tilde{I}_a = 200Ae^{j10^\circ} \qquad \tilde{V}_a = 15e^{j4^\circ}\,kV$$
$$\tilde{I}_b = 150Ae^{-j110^\circ} \qquad \tilde{V}_b = 14.5e^{-j120^\circ}\,kV$$
$$\tilde{I}_c = 160Ae^{-j240^\circ} \qquad \tilde{V}_c = 14.8e^{-j235^\circ}\,kV$$

Compute the symmetrical components: $\tilde{I}_1, \tilde{I}_2, \tilde{I}_0, and\, \tilde{V}_1, \tilde{V}_2, \tilde{V}_0$.

Solution: By application of the reverse symmetrical transformation:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_1 \\ \tilde{I}_1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 200e^{j10^0} \\ 150e^{-j110^0} \\ 160e^{j240^0} \end{bmatrix} = \begin{bmatrix} xx \\ xx \\ xx \end{bmatrix}$$

$$\begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 15.0e^{j4^0} \\ 14.5e^{-j120^0} \\ 14.8e^{-j235^0} \end{bmatrix} = \begin{bmatrix} xx \\ xx \\ xx \end{bmatrix}$$

Problem P4.2: The positive, negative and zero sequence Thevenin equivalent impedances at the point of a 115 kV three phase system are (on a 115 kV, 100 MVA base):

$z_1 = j0.085\ pu, \quad z_2 = j0.089\ pu, \quad$ and $z_0 = j0.12\ pu$

Compute the three phase fault current in pu and in actual Amperes for a three phase fault at this location of the system.

Compute the line to line fault current in pu and in actual Amperes for a line to line fault at this location of the system.

Compute the single phase fault current of the system for a single phase to ground fault at this location of the system.

Solution: The fault current for the 3-phase fault is:

3φ Fault $\tilde{I}_1 = \frac{1}{0.085} = 11.7647 \text{ pu} \Rightarrow \quad \tilde{I}_1 = 5.9064 \text{ kA}$

1φ Fault $\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{1}{0.294} = 3.401 \text{ pu} \Rightarrow I_A = 10.204 \text{ pu}$

$\Rightarrow I_A = 5.1229 \text{ kA}$

Problem P4.3: A synchronous 60 Hz, 18 kV, 360 MVA generator is connected to an infinite bus (ideal voltage source- zero internal impedance) through a step up transformer and a series capacitor compensated transmission line as in Figure P4.3. The internal voltage (generated) of the generator is 1.0 pu and the voltage of the infinite source is 1.0 pu. The generator's transient reactance (positive sequence) is 0.20 per unit on the generator ratings. The transformer is 360 MVA, 18 kV/230 kV, delta/wye connected (wye on the 230 kV side) with a leakage impedance of j0.08 pu on the transformer ratings. The transmission line positive sequence impedance is j0.10 per unit on a 100 MVA (3 phases), 230 kV (Line to Line) basis. The absolute value of the capacitor impedance is 60% of the absolute value of the transmission line impedance.

(a) Compute the transmission line impedance in per-unit on the same base as for the 230 kV side of the transformer.
(b) Compute the series capacitor impedance in per-unit on the same base as for the 230 kV side of the transformer.
(c) Compute the three phase fault current for a three phase fault on the high side bus of the transformer in per-unit and in Amperes.
(d) Compute the three phase fault current for a three phase fault in the middle of the transmission line in per-unit and in Amperes.

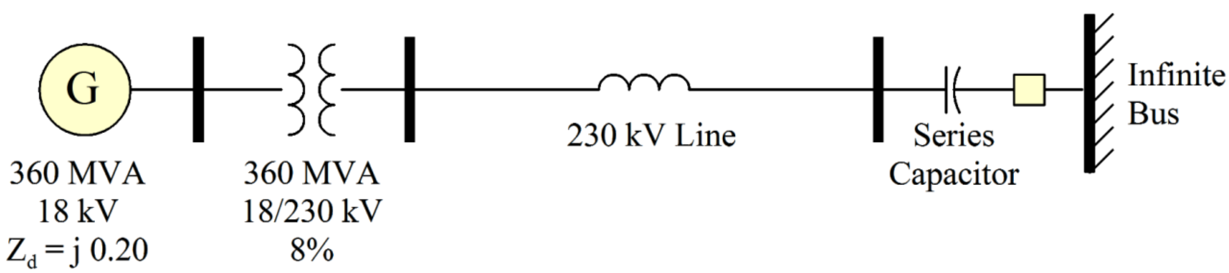


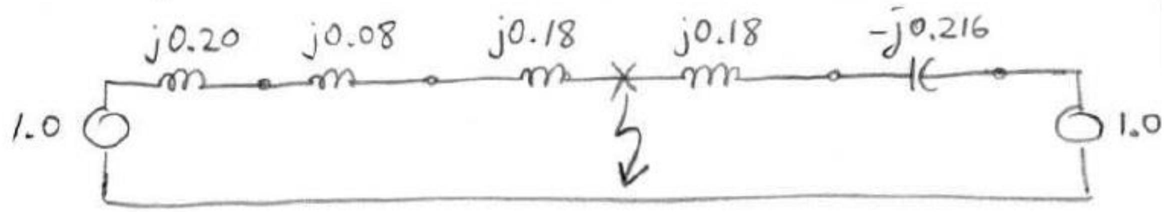
Figure P4.3

Solution:

a) $Z_{Line} = j0.10 \frac{360}{100} = j0.36 \text{ pu}$

b) $Z_{cap} = -(0.60)\, Z_{Line} = -j0.216 \text{ pu}$

c)



$$I_f = \frac{1.0}{j0.46} + \frac{1.0}{-j0.036} = j25.6038 \text{ pu}$$

$$\Rightarrow \quad I_f = (j25.6038)\left(\frac{360/3}{230/\sqrt{3}}\right) = 23.137 \text{ kA}$$

Problem P4.4: Consider the simplified electric power system of Figure P4.4 consisting of a balanced generator, a symmetric transmission line and a symmetric electric load. Each phase of the symmetric line has a self-impedance of $j9.0\,ohms$. The mutual impedance between any two phases is j4 ohms with the indicated polarity. Other system parameters are indicated in the figure. Assume a single phase to ground fault on phase A of the electric load. Compute the fault current in phase A of the generator.

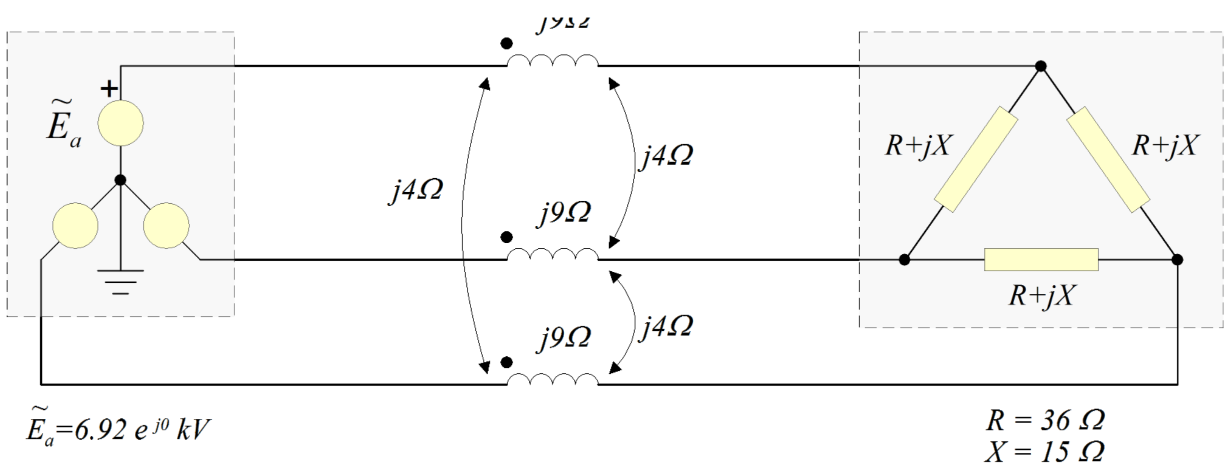
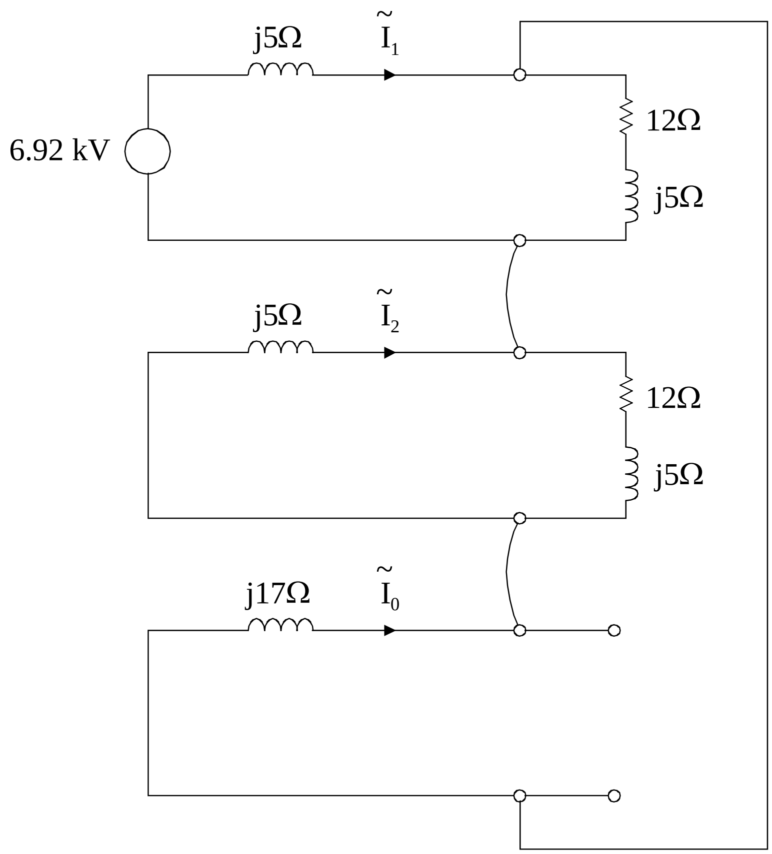


Figure P4.4

Solution: The sequence networks and their connections for a single line to ground fault are shown in the figure below.



Solution of this network yields (the circuit analysis is omitted):

$$\tilde{I}_1 = 646.7e^{-j42.48^0} \ A$$
$$\tilde{I}_2 = 363.6e^{-j83.62^0} \ A$$
$$\tilde{I}_0 = 436.9e^{-j66.43^0} \ A$$

Using the symmetrical transformation, i.e. $\tilde{I}_{abc} = T\tilde{I}_{120}$, the phase A current of the generator is:

$$\tilde{I}_a = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_0 = 1{,}383.99e^{-j59.99^0} \ A$$

Problem P4.5: Consider the simplified system of Figure P4.5. System parameters are given in the figure. The system operates unloaded. The generated voltages of the two generators are:

$Unit\ G1: \quad \tilde{E}_1 = 1.0e^{j0^\circ}$ pu, $\quad Unit\ G2: \quad \tilde{E}_2 = 1.0e^{-j30^0}\ pu$

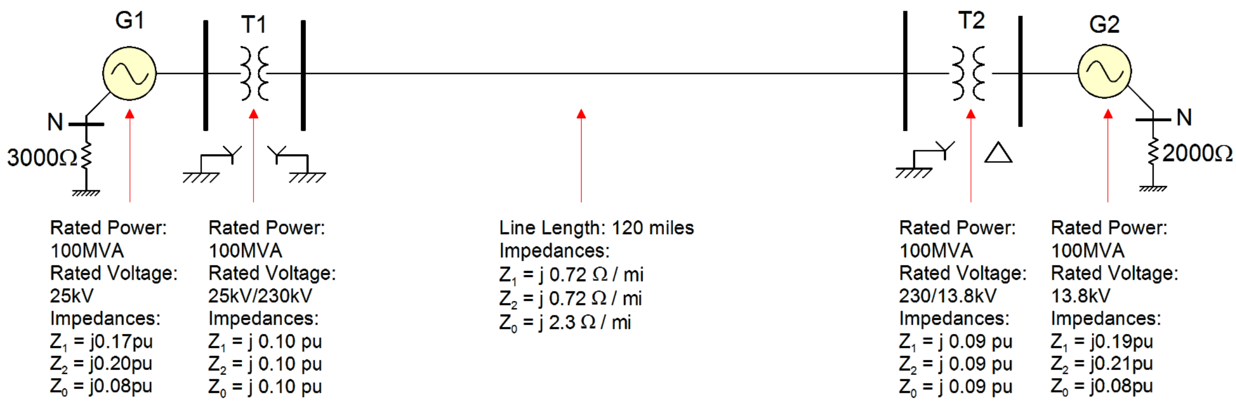
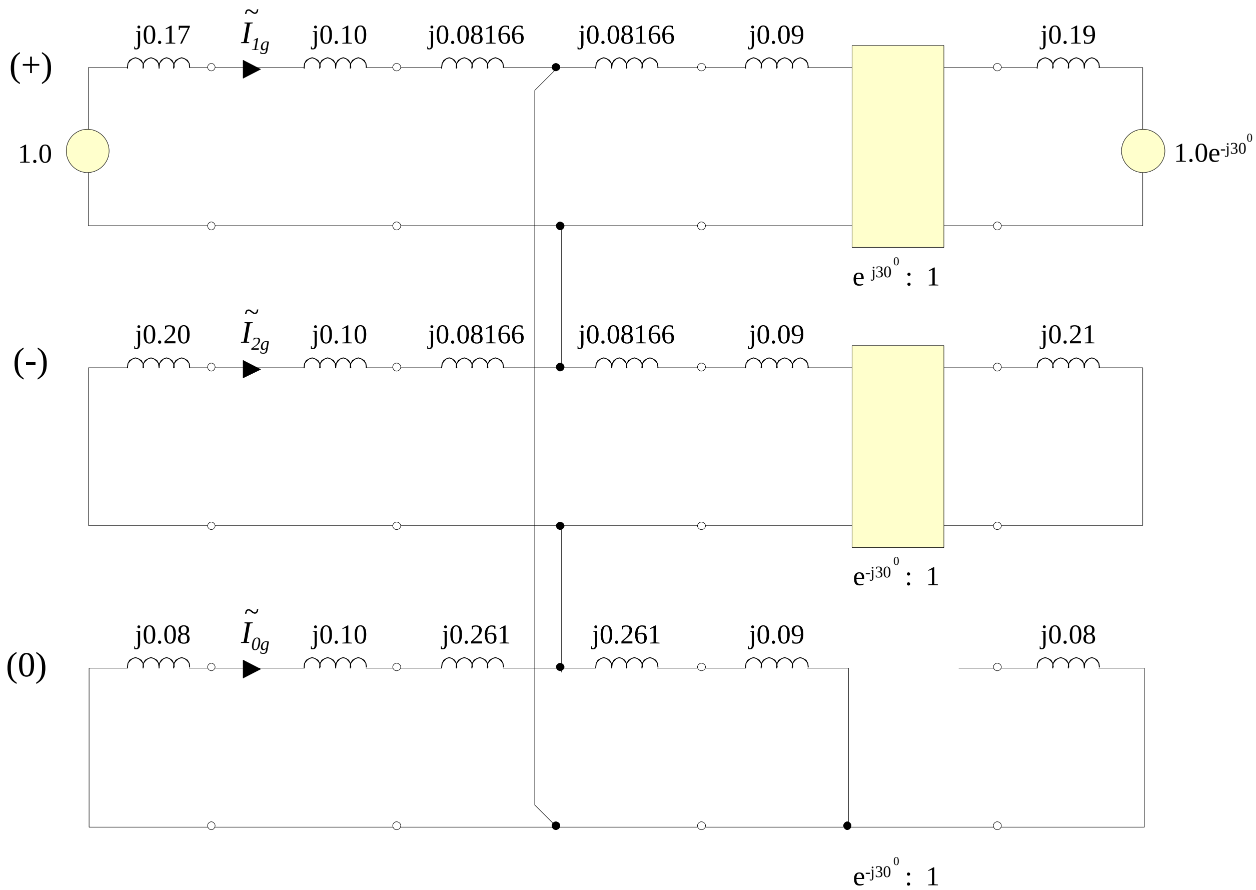


Figure P4.5

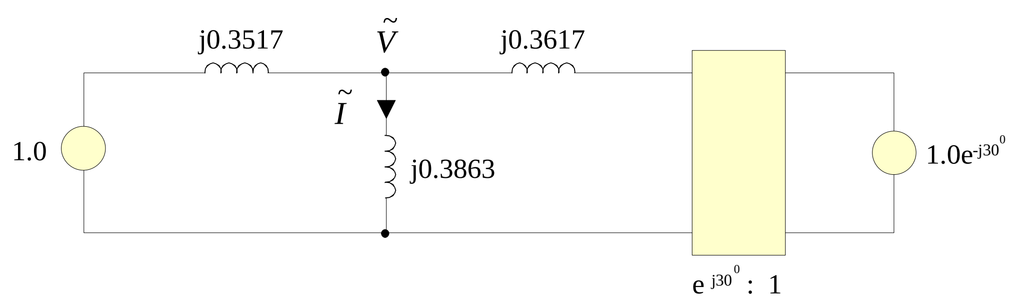
Under these conditions a single phase to ground fault occurs on the middle of the line.
(a) Compute the fault current assuming zero fault impedance.
(b) Compute the electric currents at the terminals of the generator G1.
(c) Compute the voltages at the terminals of the generator G1.
(d) Compute the electric currents at the terminals of the generator G2.
(e) Compute the voltages at the terminals of the generator G2.

Solution: (a) The sequence networks converted on a common base pu system are:

MODIFY SOLUTION TO REFLECT GROUND IMPEDANCES



Network reduction yields:



Solution of the circuit:

$$\frac{\tilde{V}-1.0}{j0.3517}+\frac{\tilde{V}}{j0.3863}+\frac{\tilde{V}-1.0}{j0.3617}=0 \quad \Rightarrow \quad \tilde{V}=0.6842$$

$$\tilde{I}_0=\frac{\tilde{V}}{j0.3863}=-j1.7711$$

Thus:

$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = -j1.7711\,pu \;\;\Rightarrow\;\; \tilde{I}_F = -j5.3133\,pu \;\;\Rightarrow\;\; \tilde{I}_f = -j1.3338\,kA$$

(b) Unit G1 sees the following sequence currents:

Positive sequence: $\tilde{I}_{1g} = \dfrac{1.0 - \tilde{V}}{j0.3517} = -j0.8979$

Negative sequence: $\tilde{I}_{2g} = \dfrac{1}{2}(-j1.7711) = -j0.8856$

Zero sequence: $\tilde{I}_{0g} = \dfrac{j0.351}{j0.351 + j0.441}(-j1.7711) = -j0.7849$

Applying the symmetrical transformation:

$$\tilde{I}_{ag} = -j2.5684\,pu \;\;\Rightarrow\;\; \tilde{I}_{ag} = -j5.9315\,kA$$

$$\tilde{I}_{bg} = 0.10738e^{j95.7^0} \;\;\Rightarrow\;\; \tilde{I}_{bg} = 0.248e^{j95.7^0}\;kA$$

$$\tilde{I}_{cg} = 0.10738e^{j84.3^0} \;\;\Rightarrow\;\; \tilde{I}_{bg} = 0.248e^{j84.3^0}\;kA$$

The relay will see above currents.

Problem P4.6: A three-phase transmission line connects two electrical power systems as in Figure P4.6a. The line configuration is shown in Figure P4.6b. Each phase conductor has the following parameters: r = 0.12 ohms/mile and GMR = 0.035 feet. The operating voltage of the line is 115kV line to line. The line length is 80 miles and the soil resistivity is 500 ohm-meters. Each of the power systems is represented as an equivalent source that is solidly grounded and with the following sequence impedances:

$Z_1 = Z_2 = j0.1\,pu$, $Z_0 = j0.06\,pu$

The voltage sources behind the equivalent impedances are in phase.

(a) Compute the fault currents for a three-phase fault at the middle of the line using symmetrical components.
(b) Compute the fault currents for a three-phase fault at the middle of the line using direct phase analysis.
(c) Compare the results from (a) and (b). State your own conclusions.
(d) Repeat (a), (b) and (c) assuming a 0.5 ohm fault impedance (resistive).

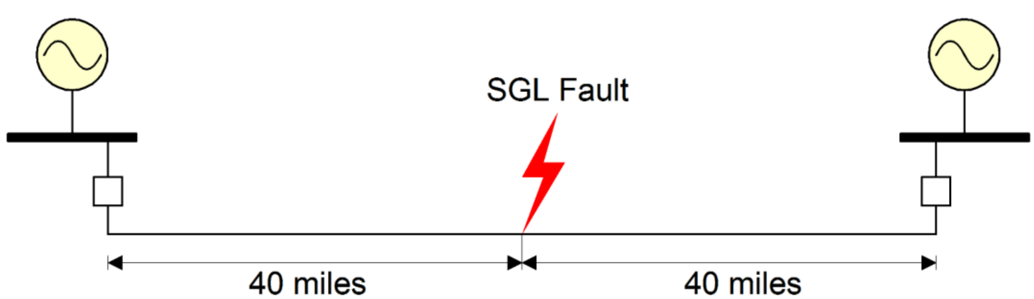


Figure P4.6a

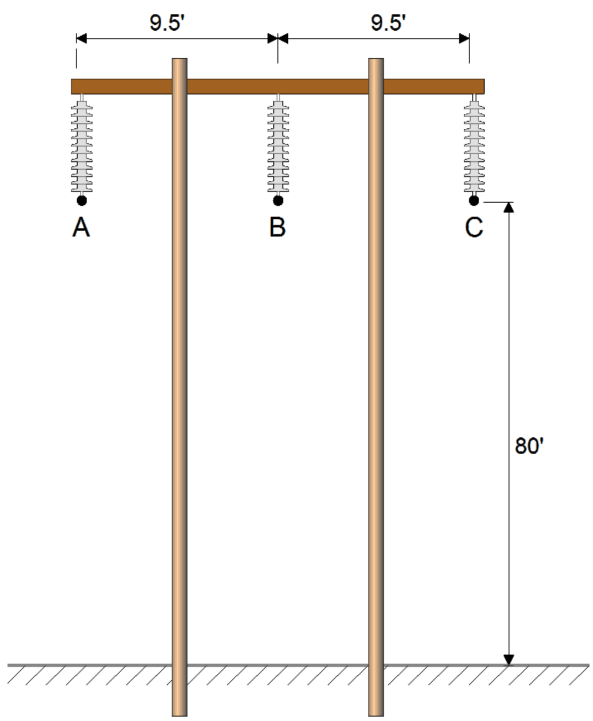


Figure P4.6b

Solution: First the line parameters are computed.

Give more information to make it possible to use the program WinIGS.

(a) Line parameters

$$Z_s = r_{ac,c} + r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{d} = 0.0001338 + j0.000911 \text{ ohms/m}$$

$$Z_{bc} = Z_{ab} = r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{d_{ab}} = 0.00005929 + j0.000489 \text{ ohms/m}$$

$$Z_{ac} = r_e + j\frac{\omega\mu}{2\pi}\ln\frac{D_e}{d_{ac}} = 0.00005929 + j0.000436 \text{ ohms/m}$$

Computing TZT^{-1}, eliminating off diagonals, yields

$$Z_2 = Z_1 = 0.0000745 + j0.000439 \text{ ohms/m}$$

$$Z_0 = 0.000252 + j0.001853 \text{ ohms/m}$$

40 mile section of line $\Rightarrow$ $Z_1 = Z_2 = 4.8 + j28.254$ ohms

$Z_0 = 12.432 + j119.259$ ohms

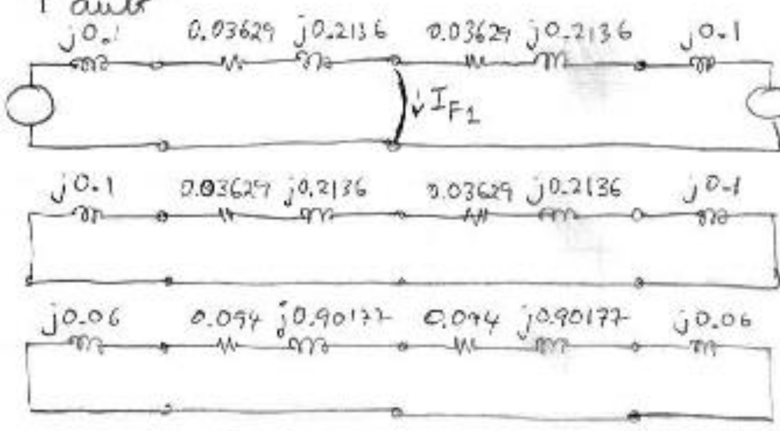
Convert to pu

Impedance Base $= Z_B = 115^2/100 = 132.25$ ohm

$$\Rightarrow \quad Z_{1u} = Z_{2u} = 0.03629 + j0.2136$$

$$Z_{0u} = 0.094 + j0.90177$$

3Φ Fault



$$\tilde{I}_{F1} = 2\,\frac{1}{0.03629 + j0.3136} = 6.3352\,e^{-j83.399^\circ}$$

(Note $\tilde{I}_2 = \tilde{I}_0 = 0$)

$$I_{Base} = (100/3)/(115/\sqrt{3}) = 0.502 \text{ kA}$$

Fault Currents

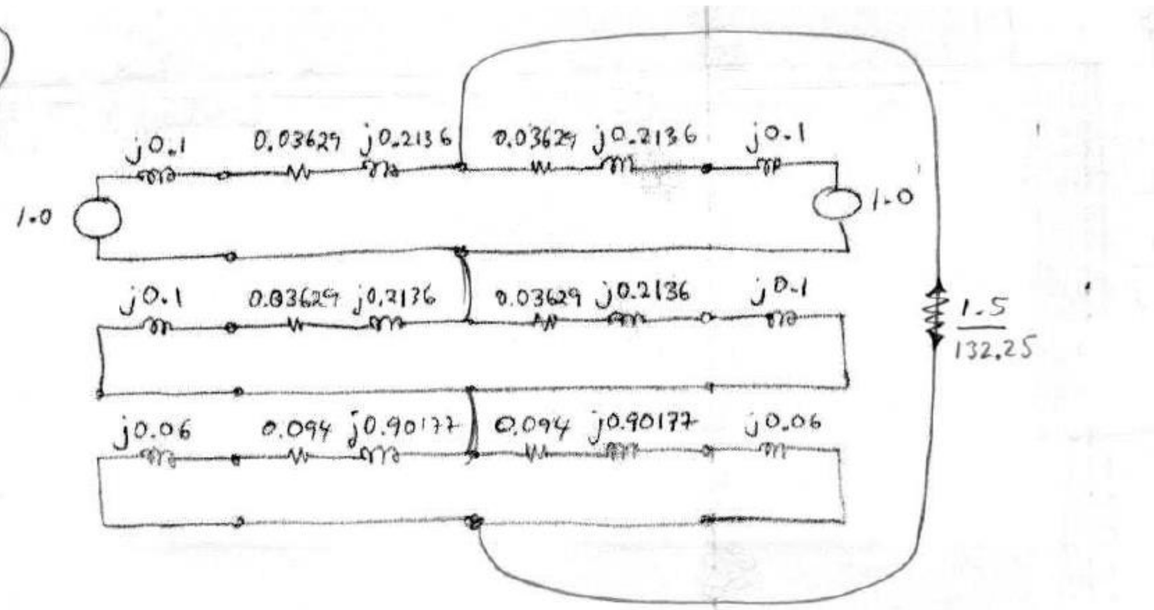
$$\tilde{I}_{aF} = 3.1805 \text{ kV } e^{-j83.399^\circ}$$

$$\tilde{I}_{bF} = 3.1805 \text{ kV } e^{-j203.399^\circ}$$

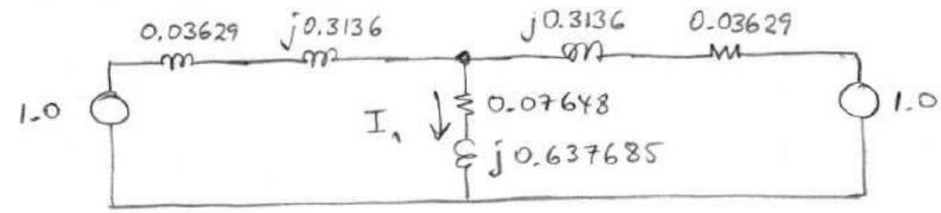
$$\tilde{I}_{cF} = 3.1805 \text{ kV } e^{-j323.399^\circ}$$

Below is solution for (d):

(a)



Reduced to



$$\tilde{I}_1 = \tilde{I}_2 = I_0 = \frac{1.0}{0.094625 + j0.794485} = 1.24984\, e^{-j83.208^\circ}$$

$$\tilde{I}_a = 3.74952\, e^{-j83.208^\circ} \quad \text{or} \quad 1.8824\, e^{-j83.208^\circ}\ \text{kA}$$

(b) Computer generated result

1.883 kA $e^{-j82.377^\circ}$

(c) Magnitude Error

0.03 %

Phase Error

0.831°

Problem P4.7: Consider the electric power system of Figure P4.7a. The geometry of the two mutually coupled transmission lines is illustrated in Figure P4.7b. The parameters of the equivalent sources ES1, ES2, ES3 and ES4 are:

Sources ES1 and ES2:

$\text{Nominal Voltage}: \ 230\,kV, \quad Z_1 = Z_2 = j0.1\ pu, \quad Z_0 = j0.12\ pu$

Sources ES3 and ES4:

$\text{Nominal Voltage}: \ 345\,kV, \quad Z_1 = Z_2 = j0.1\ pu, \quad Z_0 = j0.12\ pu$

Assume that the autotransformers are 200 MVA, 345kV:115kV, 4.5% leakage impedance. Consider a line to ground fault at location A of the 230 kV line. The location A is indicated in the Figure. Compute the zero sequence current of the 115 kV line during the above defined fault.

Hint: Use program WinIGS.

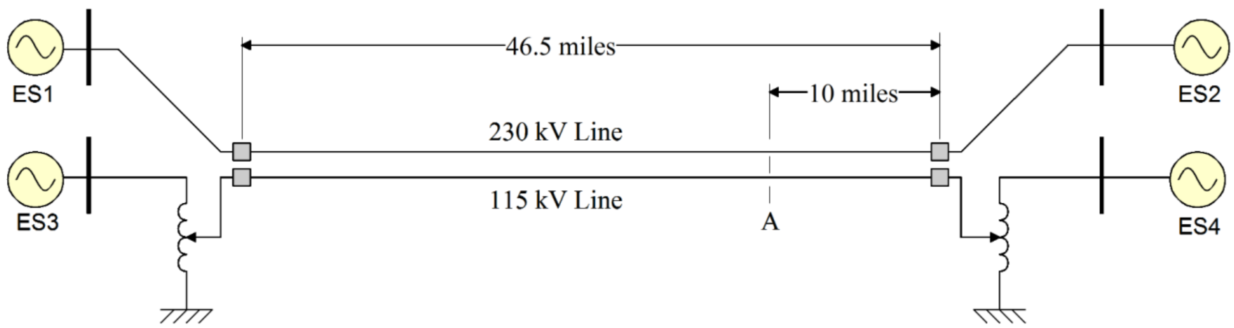


Figure P4.7a

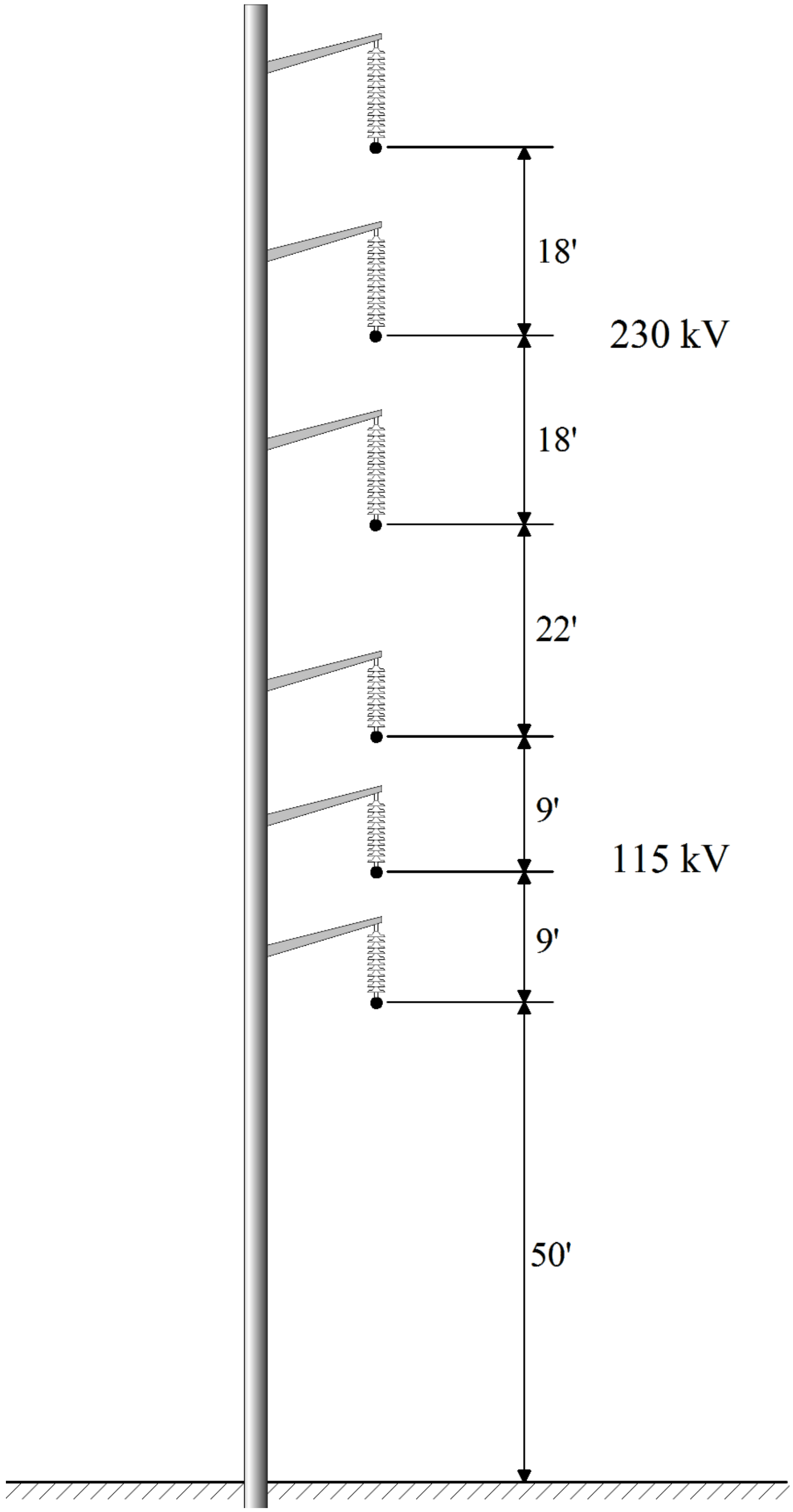
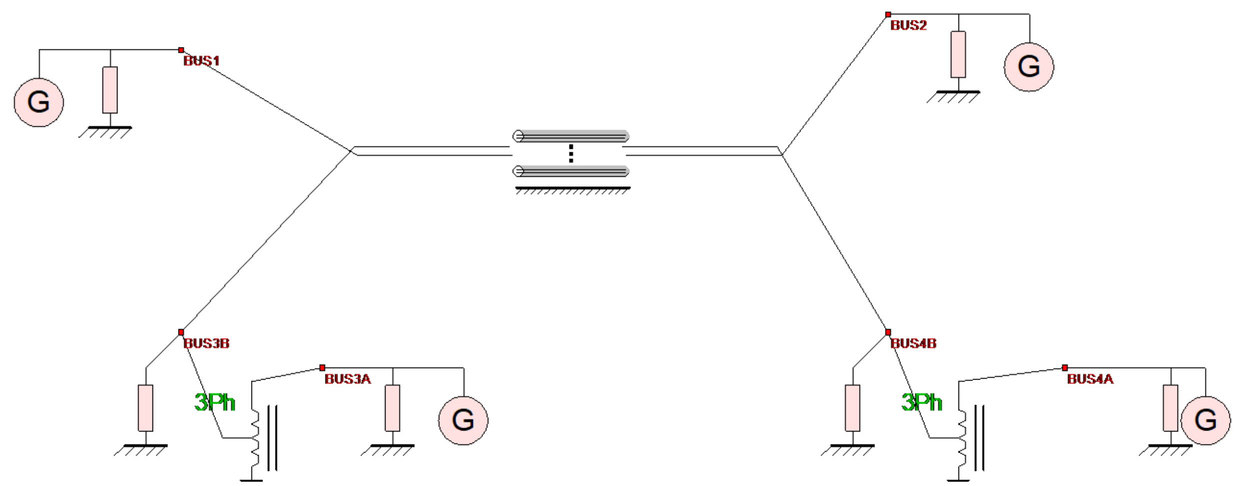
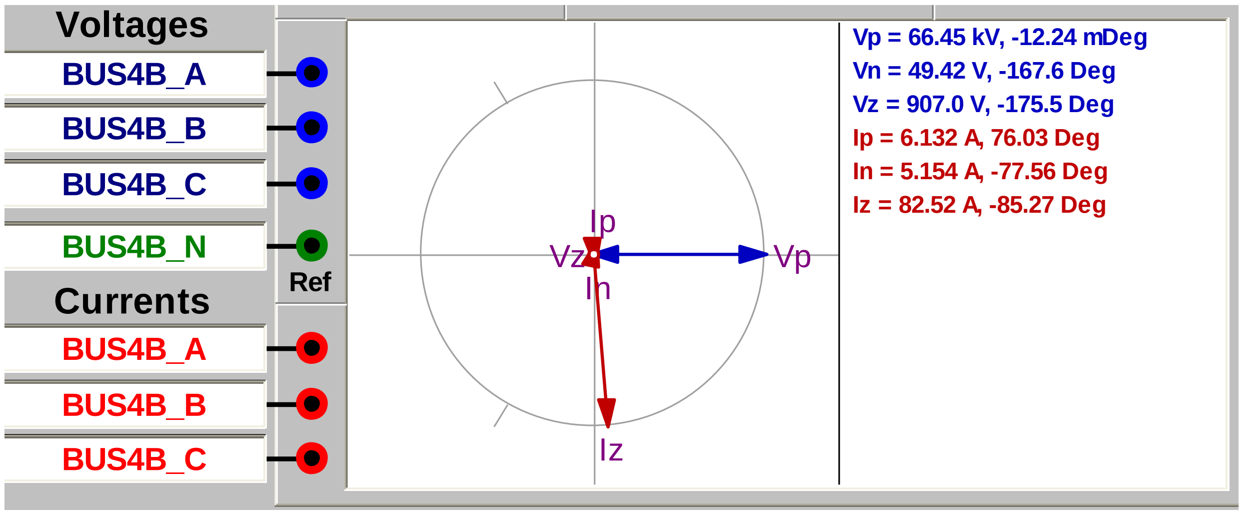


Figure P4.7b

Solution: The system is modeled in WinIGS. The WinIGS model is illustrated in the figure below. MODEL MUST BE UPDATED.



The single line to ground fault on the 230 kV line at the indicated locations has been simulated. The voltages and currents on the 115 kV line during this fault is shown in the figure below. Note that the zero sequence current is 82.52 Amperes.



Problem P4.8: Consider the electric power system of Figure P4.8. The system consists of a generator, a delta-wye connected transformer and a three phase line. The point A of the line is located 25.6 miles from the transformer.

(a) Compute the fault current at point A for a three phase fault,
(b) Compute the fault current at point A for a single phase fault,
(c) Compute the voltage magnitude of phase B and C at point A for the fault condition in (b) above, and
(d) For the fault condition in (b) above, compute the electric currents supplied by the generator and the voltages at the three phases of the generator.

Use symmetrical component theory in the computations. System data are as follows:
Generator (350 MVA, 15kV): $z_1 = j0.175\ pu, \quad z_2 = j0.21\ pu, \quad z_0 = j0.08\ pu$

Transformer (270 MVA, 15/230 kV): $z_1 = j0.08\ pu$, $z_2 = j0.08\ pu$, $z_0 = j0.08\ pu$, (on transformer rating)
Transmission line: $z_1 = z_2 = 0.3 + j0.72\ ohms/mi$, $z_0 = 0.45 + j1.75\ ohms/mi$

Transformer shunt impedance and transmission line capacitive shunt impedance are to be neglected.

ADD BREAKERS TO THE FIGURE

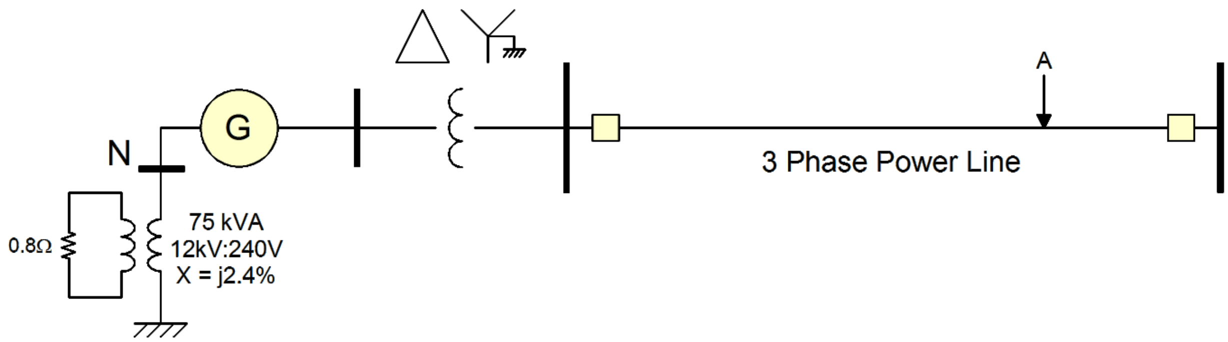


Figure P4.8: A Simplified Power System

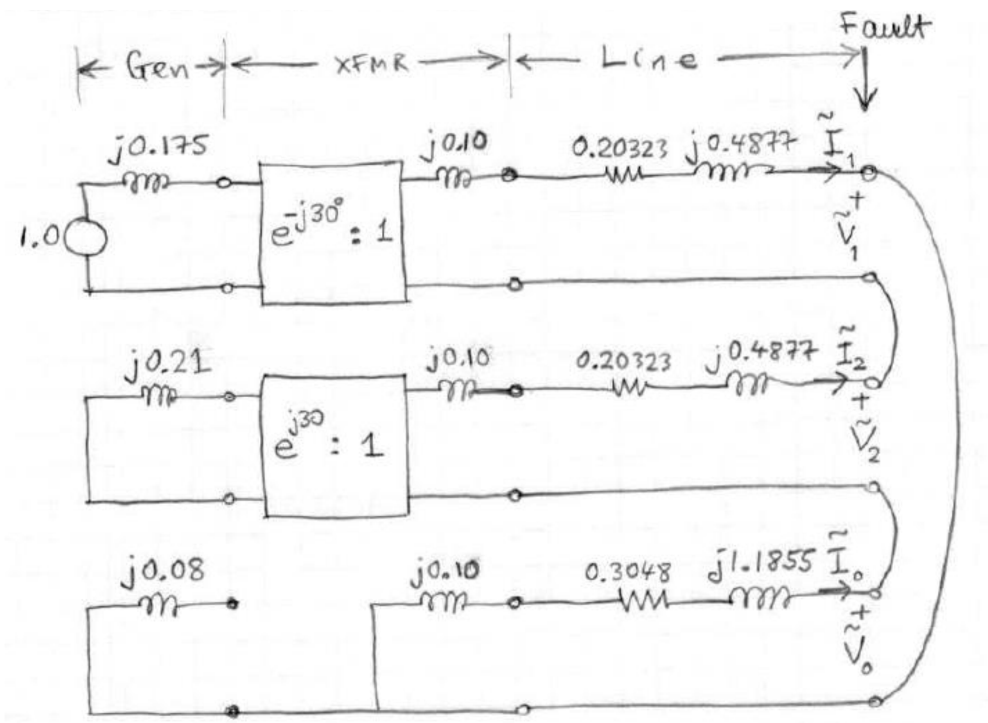
Solution: a) The transmission line parameters in the per unit system are:

$$Z_b = \frac{115^2}{350} = 37.79\,\text{Ohms}$$

$$z_1 = z_2 = (0.3 + j0.72) \times 25.6 \frac{1}{37.79} = 0.20323 + j0.4877\,\text{pu}$$

$$z_0 = (0.45 + j1.75) \times 25.6 \frac{1}{37.79} = 0.3048 + j1.1855\,\text{pu}$$

The model is:



$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{e^{j30^\circ}}{0.71126 + j2.8459} = 0.3409 e^{-j45.97^\circ} = 0.2369 - j0.2451$$

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_0 \end{bmatrix} = \begin{bmatrix} 1.0227 e^{-j45.97^\circ} \\ 0.0 \\ 0.0 \end{bmatrix} \quad pu$$

, or

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_0 \end{bmatrix} = \begin{bmatrix} 1.855 e^{-j45.97^\circ} \\ 0.0 \\ 0.0 \end{bmatrix} \quad \text{kAmperes}$$

b) The voltages of phases B and C at the location of the fault are:

to be completed

c) The electric currents of the generator phases A, B and C are:

to be completed

d) The voltages of phases A, B and C at the generator are:

to be completed

Problem 1

Transmission line $\qquad L_s = \frac{\mu}{2\pi} \ln \frac{D_e}{d} \qquad L_m = \frac{\mu}{2\pi} \ln \frac{D_e}{D}$

$$D_e = 2160 \sqrt{\frac{\rho}{f}} = 6235.4 \text{ feet}$$

$$D = \sqrt[3]{(9.5)(9.5)(19)} = 11.969 \text{ feet}$$

$L_s = 2.418 \times 10^{-6}$ H/m

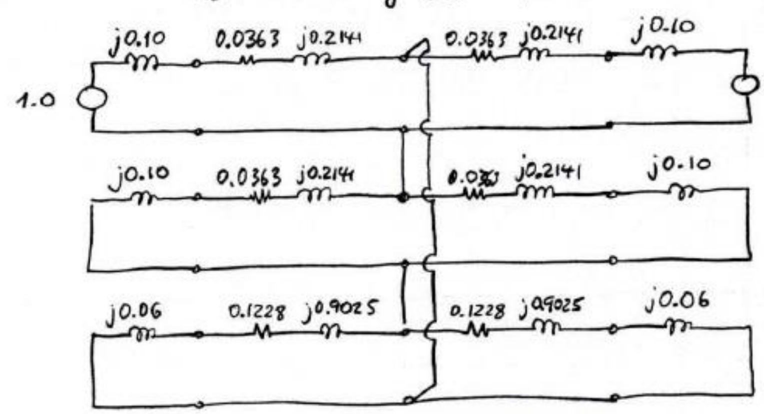
$L_m = 1.251 \times 10^{-6}$ H/m

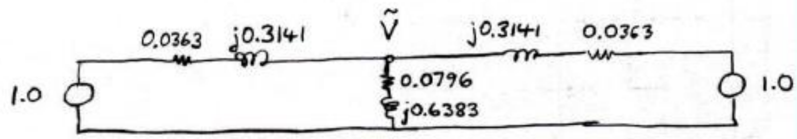
$r_e = 0.00159 f$ Ω/mile $= 0.0954$ Ω/mile

$r = 0.12$ Ω/mile

$$Z_1 = 0.12 + j\omega(L_s - L_m)(1609) \ \Omega/\text{mile} = 0.12 + j0.708 \ \Omega/\text{mile}$$

$$Z_0 = r + 3r_e + j\omega(L_s + 2L_m)(1609) = 0.406 + j2.984 \ \Omega/\text{mile}$$





$$\left(\frac{2}{0.0363 + j0.3141} + \frac{1}{0.0796 + j0.6383}\right)\tilde{V} = \frac{2}{0.0363 + j0.3141}$$

$$\Rightarrow \quad \tilde{V} = 0.8027\, e^{-j0.5°}$$

$$I_1 = I_2 = I_0 = \frac{\tilde{V}}{0.0796 + j0.6383} = 1.248\, e^{-j83.3°}$$

$I_a = 3.7439$ pu.

Problem P4.9: Consider the electric power system of Figure P4.9. The parameters of the various system components are given in the figure. Consider a line to ground fault at location A. The location A is indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. It is given that prior to the fault the unit operates under nominal voltage at its terminals and zero power.

(a) Construct the positive, negative and zero sequence networks of this system in per unit – use as power base 800 MVA (total three-phase) and voltage base the corresponding nominal voltages.
(b) Compute the negative sequence current in the generator during the fault (single line to ground at location A).
(c) Compute the zero sequence current in the unfaulted transmission line.

The positive, negative and zero sequence impedances of each line are: $Z_1 = Z_2 = j37.5\, ohms$ and $Z_0 = j93.7\, ohms$. The transmission lines are not mutual coupled. The "infinite bus" is an ideal-balanced-three-phase voltage source. Assume zero power flow prior to the fault.

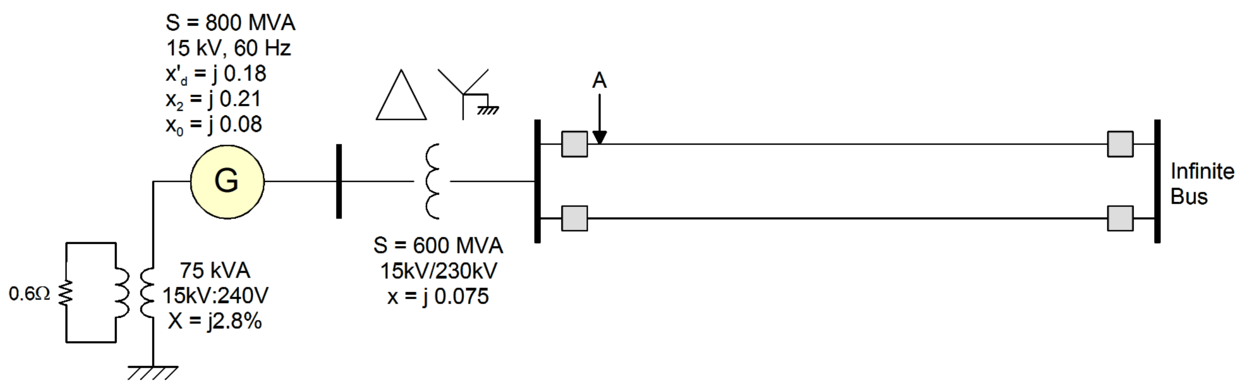


Figure P4.9

Problem P4.10: Consider the two-unit, two transformer, and one line power system of the Figure P4.10. All pertinent parameters are given in the Figure.
(a) Compute the fault current for a single phase to ground fault on Phase A of the high voltage side of the wye-wye connected transformer. Prior to the fault, the generators operate at nominal voltage and the electric current anywhere in the system is zero. Compute the voltage at the three phases of the generator G1 for the above defined condition. Compute the voltages at the three phases of the generator G2 for the above defined condition.
(b) Compute the fault current for a single phase to ground fault at the terminals of the generator G1. Prior to the fault, the generators operate at nominal voltage and the electric current anywhere in the system is zero. Compute the voltage at the three phases of the generator G1 for the above defined condition. Compute the voltages at the three phases of the generator G2 for the above defined condition.

Hint: Convert all parameters on a common 100 MVA base.

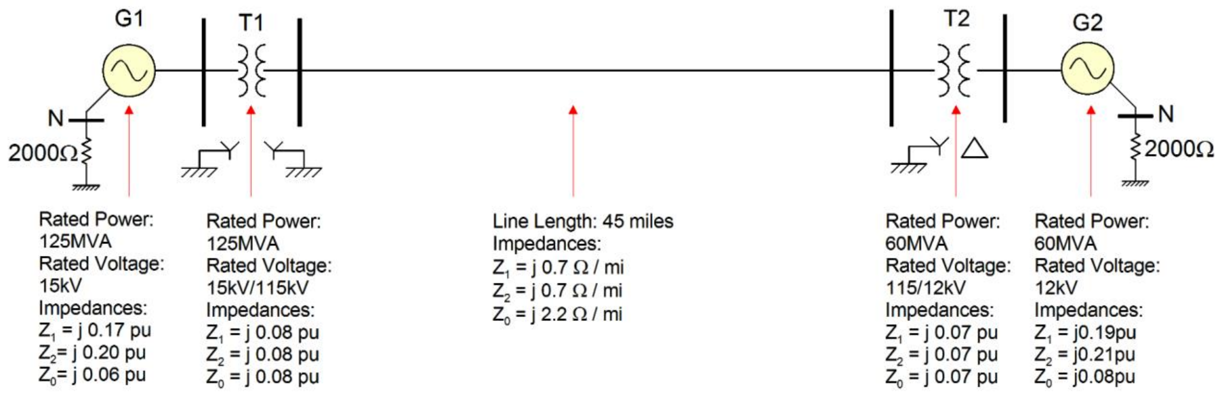


Figure P4.10

Solution: Convert all parameters on a 100 MVA base:

$S_B^{3\phi}$ = 100 MVA

G1:

$$z_1 = j0.17\frac{100}{125} = j0.136\,(\text{p.u.})$$

$$z_2 = j0.20\frac{100}{125} = j0.16\,(\text{p.u.})$$

$$z_0 = j0.06\frac{100}{125} = j0.048\,(\text{p.u.})$$

T1:

$$z_1 = z_2 = z_0 = j0.08\frac{100}{125} = j0.064\,(\text{p.u.})$$

T.L:

$$z_B = \frac{115^2}{100} = 132.25\ (\Omega)$$

$$z_1 = z_2 = \frac{(j0.7)(45)}{132.25} = j0.2382\,(p.u.)$$

$$z_0 = \frac{(j2.2)(45)}{132.25} = j0.7486\,(p.u.)$$

T2:

$$z_1 = z_2 = z_0 = j0.07\frac{100}{60} = j0.1167\,(\text{p.u.})$$

G2:

$$z_1 = j0.19\frac{100}{60} = j0.31676\,(\text{p.u.})$$
$$z_2 = j0.21\frac{100}{60} = j0.35\,(\text{p.u.})$$
$$z_0 = j0.08\frac{100}{60} = j0.1333\,(\text{p.u.})$$

For prefault currents to be zero, the phase angles of the generated voltages must be selected as it is illustrated in Figure P4.11a.

The Sequence Networks of the system are:

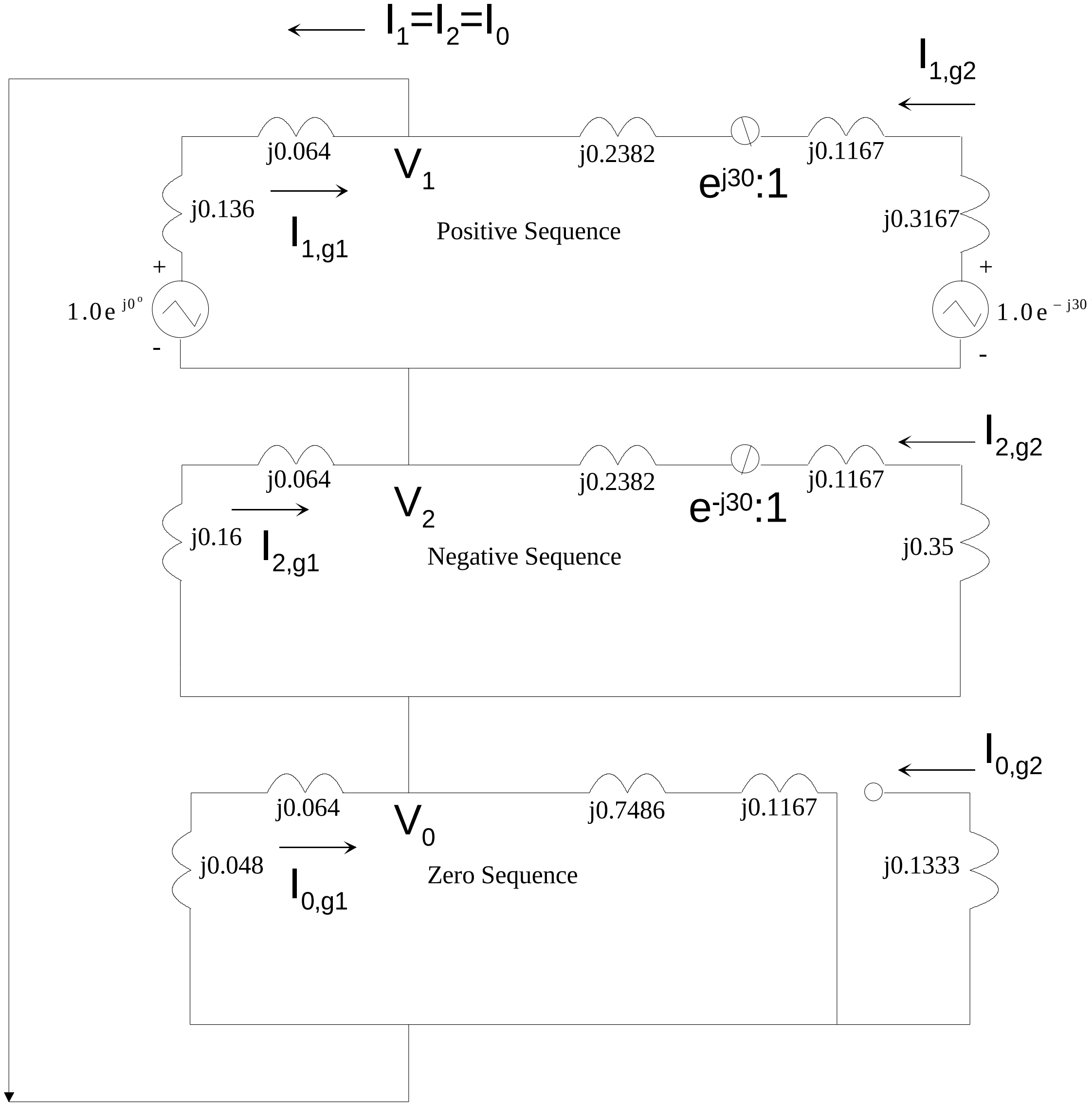


Figure P4.11a. System Sequence Networks

From Figure P4.11a, upon reducing the negative and zero sequence equivalent networks, the following equivalent circuit can be obtained:

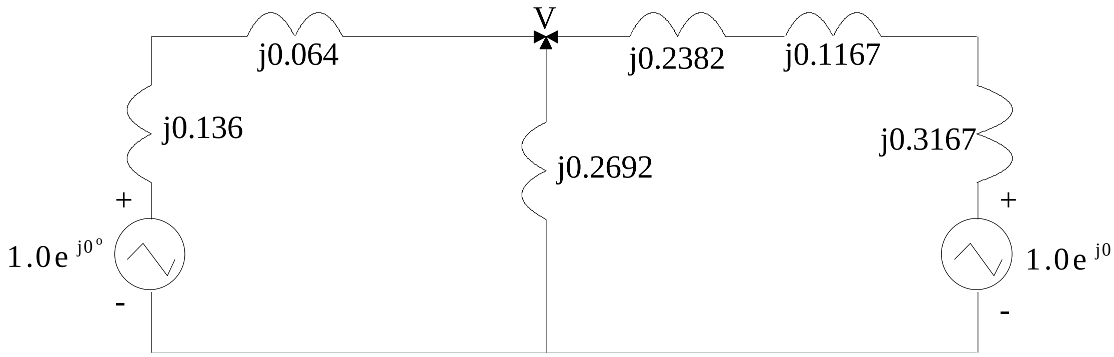


Figure P4.11b. Sequence Networks Equivalent Circuit

Solving for V in Figure P4.11b, as follows:

$$\frac{1.0e^{j0^o}-\mathrm{V}}{\mathrm{j}0.2}-\frac{\mathrm{V}}{\mathrm{j}0.2692}+\frac{1.0e^{j0^o}-\mathrm{V}}{\mathrm{j}0.6716}=0.0$$

Above equation yields the following solution:

$$\tilde{V}=0.6359, \quad and \quad \tilde{I}_1=\tilde{I}_2=\tilde{I}_0=2.3623e^{-j90^0}$$

Form Figure 1, the sequence components of the voltage at the fault location are:

$\mathrm{V}_1=0.6359$ (p.u.), Positive sequence
$\mathrm{V}_2=-0.4016$ (p.u.), Negative sequence
$\mathrm{V}_0=-0.2343$ (p.u.), Zero sequence

From Figure 1, the following sequence currents can be calculated:

Generator # 1 side (no phase shift), currents in the direction "out" of generator:

$$\tilde{I}_{1,g1}=\frac{1.0-0.635}{j0.2}=1.8255e^{-j90^o} \text{ (p.u.)}$$

$$\tilde{I}_{2,g1}=-\frac{-0.401}{j0.224}=1.7897e^{-j90^o} \text{ (p.u.)}$$

$$\tilde{I}_{0,g1}=-\frac{-0.234}{j0.112}=2.0884e^{-j90^o} \text{ (p.u.)}$$

Generator #1 sequence components of terminal voltages are:

$$v_1 = 1.0e^{j0^o} - (1.8255e^{-j90^o})(j0.136) = 0.7517e^{j0^o} \text{ (p.u.)}$$
$$v_2 = -(1.7897e^{-j90^o})(j0.16) = -0.2864e^{j0^o} \text{ (p.u.)}$$
$$v_0 = -(2.0884e^{j90^o})(j0.048) = -0.1002e^{j0^o} \text{ (p.u.)}$$

Generator #1 terminal voltages are:

$$v_{abc} = T^{-1}v_{120} = [0.3651e^{j0^o} \quad 0.9587e^{-j110.32^o} \quad 0.9587e^{j110.32^o}]^T$$

Generator # 2 side (phase shift), currents in the direction "out" of generator:

$$\tilde{I}_{1,g2} = \frac{1-0.635}{j0.6716}1.0e^{-j30^o} = 0.5436e^{-j120^o} \text{ (p.u.)}$$
$$\tilde{I}_{2,g2} = -\frac{-0.401}{j0.7049}1.0e^{+j30^o} = 0.5687e^{-j60^o} \text{ (p.u.)}$$
$$\tilde{I}_{0,g2} = 0.0 \text{ (p.u.)}$$

Generator #2 sequence components of terminal voltages are:

$$v_1 = 1.0e^{-j30^o} - 0.5436e^{-j120^o}\,j0.3167 = 0.8278e^{-j30^o} \text{ (p.u.)}$$
$$v_2 = -0.5687e^{-j60^o}\,j0.35 = -0.199e^{j30^o} \text{ (p.u.)}$$
$$\mathrm{v}_0 = 0.0 \text{ (p.u.)}$$

Generator #2 terminal voltages are:

$$v_{abc} = T^{-1}v_{120} = [0.7484e^{-j43..32^o} \quad 0.7484e^{-j136.68^o} \quad 1.0268e^{j90^o}]^T$$

Problem P4.12: At a certain location of a 115 kV electric power system, the driving point impedances are:

(a) pos/neg sequence: $R_1 + j\omega L_1 = 0.001 + j0.055\ pu, \quad @\,115kV\ (L-L)\ and\ 100\ MVA$

(b) zero sequence: $R_0 + j\omega L_0 = 0.008 + j0.195\ pu, \quad @\,115kV\ (L-L)\ and\ 100\ MVA$

Assume a single phase to ground fault at this point and on phase A. Compute the voltage of the unfaulted phases.

Solution: The sequence fault currents are:

$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{1}{0.01 + j0.305} = 3.2769e^{-j88.12^0}\ pu$$

The sequence voltages are:

$$\tilde{V}_1 = 1.0 - (0.001 + j0.055)\tilde{I}_1 = 0.81976\, e^{-j0.184^0}$$
$$\tilde{V}_2 = -(0.001 + j0.055)\tilde{I}_2 = 0.18025\, e^{-j179.16^0}$$
$$\tilde{V}_0 = -(0.008 + j0.195)\tilde{I}_0 = 0.6395\, e^{-j179.53^0}$$

The voltages of the unfaulted phases are:

$$\tilde{V}_b = a^2\tilde{V}_1 + a\tilde{V}_2 + \tilde{V}_0 = 1.294 e^{-j17.8^0}$$
$$\tilde{V}_c = a\tilde{V}_1 + a^2\tilde{V}_2 + \tilde{V}_0 = 1.291 e^{j18.0^0}$$

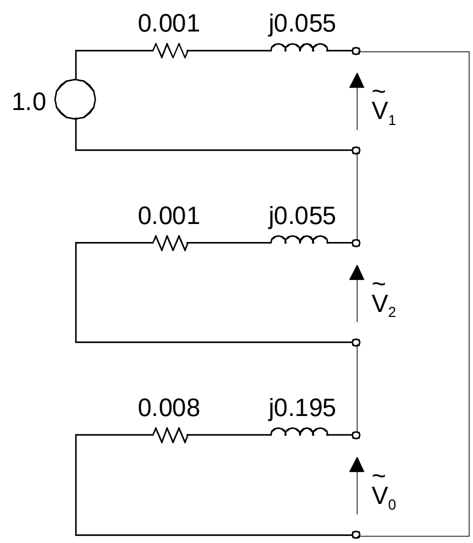


Figure P4.12

Problem P4.13: Consider a 600 A circuit, 25 kV (line to line voltage), two miles long, as it is illustrated in Figure P4.13. The equivalent source impedance (on a 33.33 MVA, 14.45 kV basis) is:

$$z_1 = z_2 = j0.15\, pu, \quad z_0 = j0.11\, pu$$

The impedance of the circuit is

$$z_1 = z_2 = j0.70\, ohms/mile, \quad z_0 = j2.10\, ohms/mile$$

Consider a single phase to ground fault at the end of the circuit on phase A.
(a) Compute the fault current.
(b) Compute the voltage of the unfaulted phases, B and C.

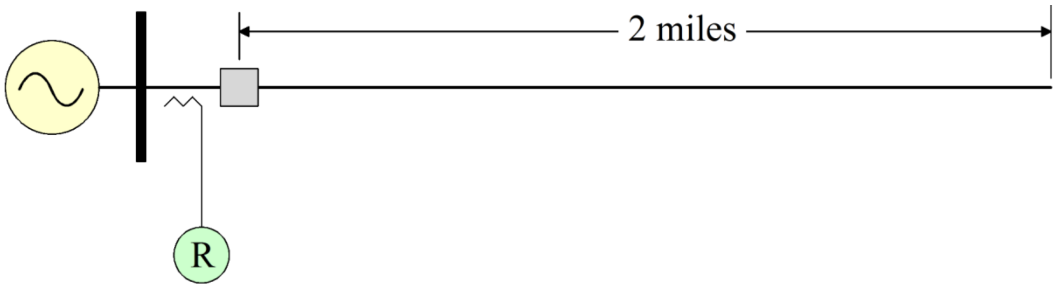
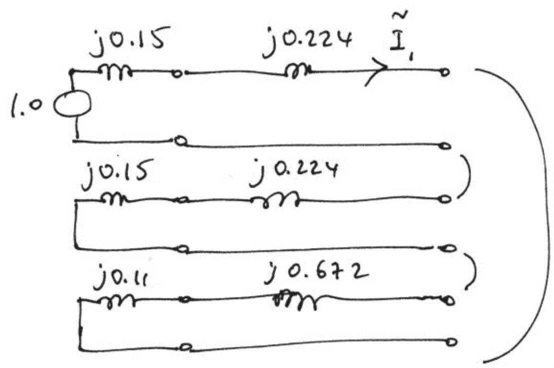


Figure P4.13

Solution: (a)

a)



$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{1}{j1.53} = -j0.6$$

$$I_F = 1.9607 \text{ pu}$$
$$= 4.528 \text{ kA}$$

b) $\tilde{V}_1 = 1.0 - (j0.374)(-j0.6535) = 1.0 - 0.2444 = 0.7556$

$\tilde{V}_2 = -(j0.374)(-j0.6535) = -0.2444$

$\tilde{V}_0 = -(j0.782)(-j0.6535) = -0.511$

$\tilde{V}_b = 0.7556\,a^2 - 0.2444\,a - 0.511 = 1.1565\,e^{-j131.51^\circ}$

$\tilde{V}_c = 0.7556\,a - 0.2444\,a^2 - 0.511 = 1.1565\,e^{j131.51^\circ}$

Problem P4.14: Consider the electric power system of the Figure P4.14. The system consists of a generator, a delta-wye connected transformer and a three phase line. Assume a single phase-to-ground fault occurring at point A of the line, located 25.6 miles from the transformer. The fault analysis for this condition is provided below using symmetrical components.

(a) Compute the voltage magnitude of phases B and C at point A during the fault condition, and
(b) Compute the electric current supplied by the generator (all three phases).
(c) Compute the voltages at the three phases of the generator.

Use symmetrical component theory in the computations. System data are as follows:

Generator (350 MVA, 15kV):

$z_1 = j0.175\ pu,\ \ z_2 = j0.21\ pu, and\ z_0 = j0.08\ pu$ (@ 350 MVA)
Transformer (280 MVA, 15kV/115kV):
$z_1 = z_2 = z_0 = j0.08\ pu$ (@ 280 MVA)
Transmission line:
$z_1 = z_2 = 0.3 + j0.72\ Ohms/mile,\ \ z_0 = 0.45 + j1.75\ Ohms/mile$

Neglect the transformer shunt impedance and the transmission line capacitive shunt impedance.

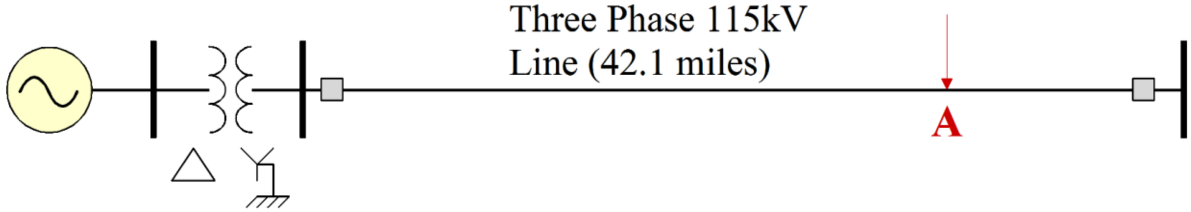


Figure P4.14. Simplified Power System

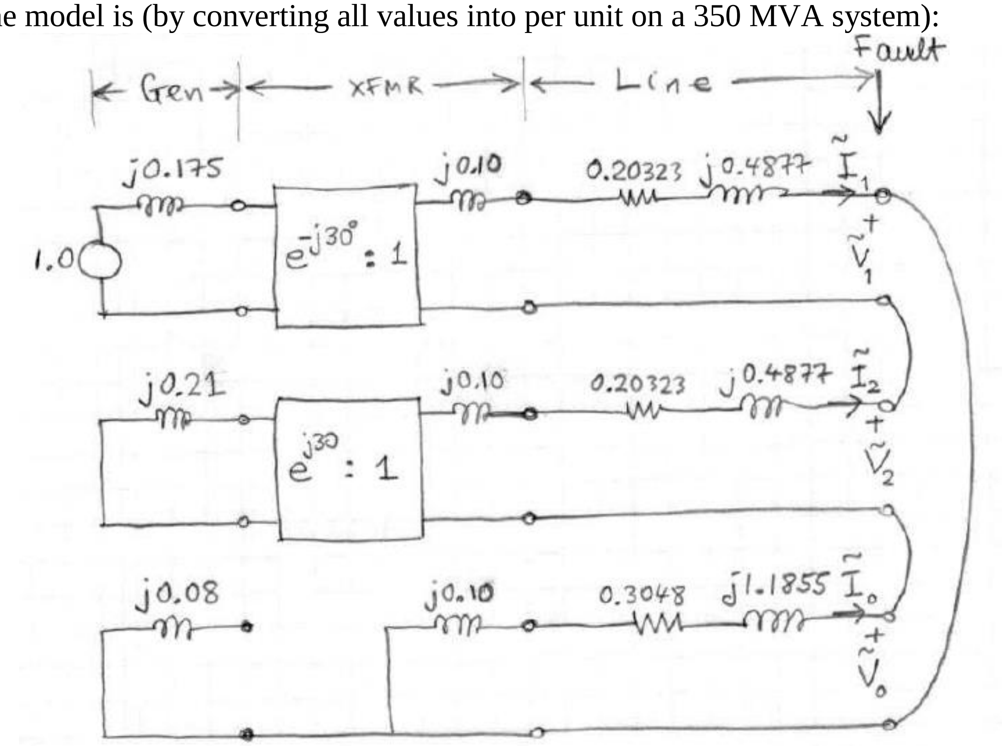
Solution: The transmission line parameters in the per unit system are:

$$Z_b = \frac{115^2}{350} = 37.79\,\text{Ohms}$$

$$z_1 = z_2 = (0.3 + j0.72) \times 25.6 \frac{1}{37.79} = 0.20323 + j0.4877\,\text{pu}$$

$$z_0 = (0.45 + j1.75) \times 25.6 \frac{1}{37.79} = 0.3048 + j1.1855\,\text{pu}$$

The model is (by converting all values into per unit on a 350 MVA system):



$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{e^{j30^\circ}}{0.71126 + j2.8459} = 0.3409 e^{-j45.97^\circ} = 0.2369 - j0.2451$$

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_0 \end{bmatrix} = \begin{bmatrix} 1.0227 e^{-j45.97^\circ} \\ 0.0 \\ 0.0 \end{bmatrix} \quad pu$$

, or

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_0 \end{bmatrix} = \begin{bmatrix} 1.855 e^{-j45.97^\circ} \\ 0.0 \\ 0.0 \end{bmatrix} \quad \text{kAmperes}$$

ECE 4321 | HW # 3 | Solution

Problem 1

(a)

$$\tilde{V}_1 = 1.0e^{j30^\circ} - (0.20323 + j0.7627)\tilde{I}_1 = 0.73096\, e^{j30.327^\circ} \quad (0.630936 + j0.3691)$$

$$\tilde{V}_2 = -(0.20323 + j0.7627)\tilde{I}_2 = 0.26907\, e^{j209.109^\circ}$$

$$\tilde{V}_0 = -(0.3048 + j1.2855)\tilde{I}_0 = 0.45037\, e^{j210.691^\circ}$$

$$\begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_0 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

(b)

$$\tilde{I}_{g1} = 0.3409\, e^{-j15.97^\circ}$$

$$\tilde{I}_{g2} = 0.3409\, e^{-j75.97^\circ}$$

$$\tilde{I}_{g0} = 0$$

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}_{g1} \\ \tilde{I}_{g2} \\ \tilde{I}_{g0} \end{bmatrix} = \begin{bmatrix} 0.59045\, e^{-j45.97^\circ} \\ 0 \\ 0.59045\, e^{j134.03^\circ} \end{bmatrix}$$

(c)

$$\tilde{V}_{g1} = 1.0 - (j0.175)(0.3409\, e^{-j15.97^\circ}) = 0.98358 - j0.05735$$

$$\tilde{V}_{g2} = -(j0.21)(0.3409\, e^{-j75.97^\circ}) = 0.071589\, e^{j194.03^\circ}$$

$$\tilde{V}_{g0} = 0$$

$$\begin{bmatrix} \tilde{V}_{ga} \\ \tilde{V}_{gb} \\ \tilde{V}_{gc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \tilde{V}_{g1} \\ \tilde{V}_{g2} \\ \tilde{V}_{g0} \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Part (a):

$$\begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = \begin{bmatrix} 0.0 \\ 1.1112 e^{-j97.2760^0} \\ 1.1013 e^{j157.9971^0} \end{bmatrix} \quad \text{pu}$$

Problem P4.15: At a breaker location, the Thevenin equivalent resistance is 0.2 pu and the equivalent inductance is 0.006 pu on a 13.8 kV (L-to-L voltage), 100 MVA (three phase total) basis. A fault at this location is interrupted in 4.5 cycles. Compute the maximum possible rms value of the fault current at the time of the interruption.

Solution: The impedance in pu is:

$$z = 0.2 + j\omega(0.006) = 0.2 + j2.2619$$

The base impedance, resistance and inductance are:

$$Z_b = \frac{(13.8)^2}{100} = 1.9044\, ohms,$$

$$R = (0.2)(1.9044) = 0.3809\, ohms,$$

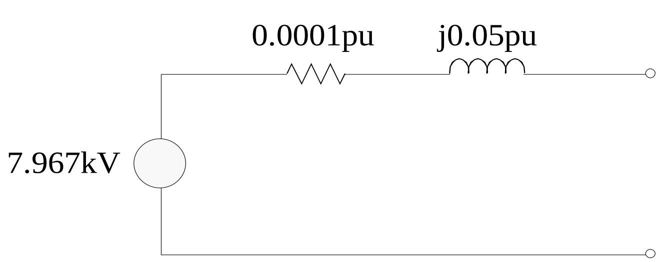
$$L = \omega(0.006)\frac{Z_b}{\omega} = 0.114\, Henries$$

The fault current at the time of interruption (t=4.5/60 seconds) is:

$$I_{f,\max} = \left| \frac{13.8/\sqrt{3}}{0.3809 + j4.3076} \right| \sqrt{1 + 2e^{-\frac{2R}{L}t}} = 1.9872\, kA$$ note to myself: check result/equation

Problem P4.16: At a breaker location, the Thevenin equivalent impedance consists of a series R and L and the Thevenin equivalent source is 7.967 kV. The inductive reactance (jωL) is 0.05 pu and the resistance (R) is 0.0001 pu on a 7.967 kV, 3.333 MVA basis. The breaker is set to clear a fault in six cycles of the 60 Hz. Compute the **maximum possible** rms value of the fault current (in Amperes) at the time of operation of the breaker, i.e. six cycles after fault initiation.

Solution: The equivalent circuit is shown below.



The symmetrical fault current is:

$$I_{ac,s} = \left|\frac{1}{0.0001 + j0.5}\right| \frac{10}{13.8\sqrt{3}}\, kA = 8.3674\, kA$$

The maximum rms value is

$$I_{rms,\max} = I_{ac,s}\sqrt{1 + 2e^{-\frac{2Rt}{L}}}$$

where:

$$R = 0.0001\frac{13.8^2}{10} = 0.0019\, ohms$$
$$L = \left(\frac{0.05}{377}\right)\frac{13.8^2}{10} = 0.0025\, Henries$$
$$\sqrt{1 + 2e^{-\frac{2Rt}{L}}} = 1.6493 \quad at \quad t = 0.1\sec$$

$$I_{rms,\max} = 13{,}799\ Amperes$$

Problem P4.17: At a breaker location, the Thevenin equivalent impedance consists of a series R and L and the Thevenin equivalent source is 14.45 kV. The inductive reactance (jωL) is 0.08 pu and the resistance (R) is 0.002 pu on a 14.45 kV, 3.333 MVA basis. The breaker is set to clear a fault in six cycles of the 60 Hz. Compute the **maximum possible** rms value of the fault current (in Amperes) at the time of operation of the breaker, i.e. six cycles after fault initiation.

Solution: The symmetrical fault current is:

$$I_{ac,s} = \left|\frac{1}{0.002 + j0.08}\right| \frac{3.333}{14.45}\, kA = 2.882\, kA$$

The maximum rms value is

$$I_{rms,\max} = I_{ac,s}\sqrt{1 + 2e^{-\frac{2Rt}{L}}}$$

where:

$$R = 0.002\frac{14.45^2}{3.333} = 0.1253\, ohms$$
$$L = \left(\frac{0.08}{377}\right)\frac{14.45^2}{3.333} = 0.01329\, Henries$$

$$\sqrt{1+2e^{-\frac{2Rt}{L}}} = 1.1417 \quad at \quad t = 0.1\sec$$

$$I_{rms,\max} = 3{,}290 \ Amperes$$

Problem P4.18: At a breaker location, the equivalent sequence impedances are:
(a) pos/neg sequence: $R_1 + j\omega L_1 = 0.001 + j0.075 \, pu, \quad @ \, 13.8kV \ (L-L) \ and \ 36 \, MVA$
(b) zero sequence: $R_0 + j\omega L_0 = 0.003 + j0.055 \, pu, \quad @ \, 13.8kV \ (L-L) \ and \ 36 \, MVA$
The breaker is set to clear a fault in five and a half cycles of 60 Hz. Determine the **maximum possible rms value** that the breaker will be called upon to interrupt during a three-phase fault or a single phase to ground fault.
Hint: Compute the maximum rms value of the current (in Amperes) for (a) three phase fault and (b) single phase to ground fault at the time of operation of the breaker, i.e. five and a half cycles after fault initiation. By inspection of the results select the worst case.

Solution: The base current is

$$I_b = 1.5061 kA$$

The three phase fault current is: $I_{sym,rms} = 13.3321, \quad X/R = 75$

The single phase fault current is: $I_{sym,rms} = 14.6298, \quad X/R = 41$

Assuming maximum DC offset, the electric current rms value at the time of the interruption is:

Three phase fault: $I_{F,rms} = 13.3321\sqrt{1+2e^{-\frac{2R}{L}t}} = 17.866 \, pu, \quad or \ 26.908 \, kA$

Single phase to ground fault: $I_{F,rms} = 14.6298\sqrt{1+2e^{-\frac{2R}{L}t}} = 17.1275 \, pu, \quad or \ 25.7995 \, kA$

Problem P4.19: Consider the electric power system of Figure P4.19. The parameters of the various system components are given in the figure. Consider a line to ground fault on phase A, high voltage side of the transformer. Thus the fault is practically on the 230 kV bus. This fault is supposed to be cleared by opening the generator breaker at about 6 cycles (total time) after fault initiation. For simplicity, assume that the line breakers are open and therefore there will be no contribution to the fault current from the transmission lines. Compute the **maximum possible rms value** of the fault current in the generator breaker at the time of interruption.

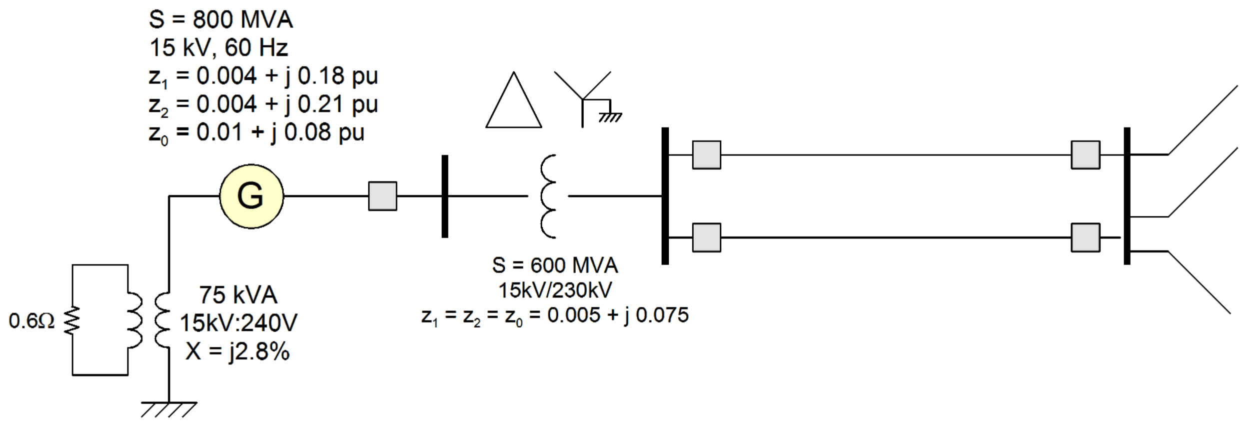
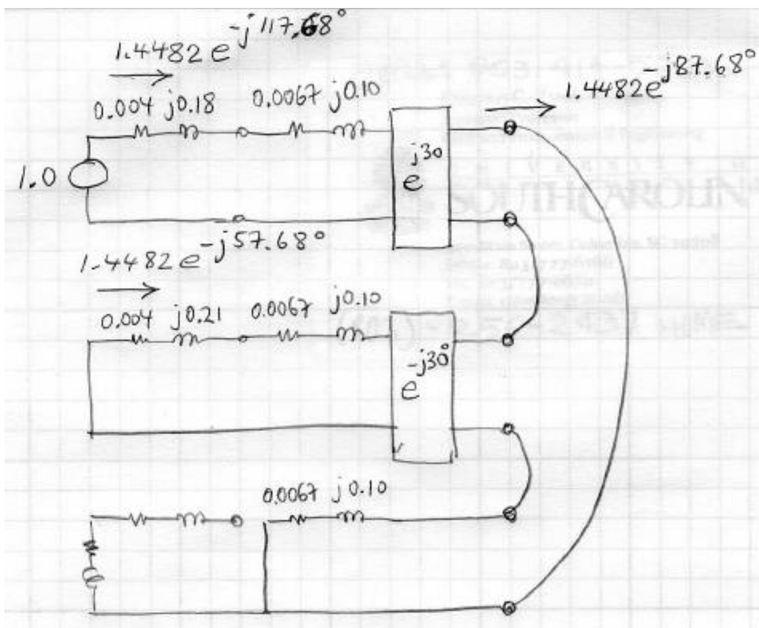


Figure P4.19

Solution: Since we have a line to ground fault we need the positive, negative and zero sequence equivalent networks. The figure below illustrates the equivalent circuits and the connections for a line to ground fault. Note the circuit is expressed in per unit on the generator ratings. The solution is superimposed on the Figure.



Thus at the generator breaker the following currents flow (symmetrical fault current):

$\tilde{I}_p = 1.4482e^{-j117.68^0}\ pu, \quad \tilde{I}_n = 1.4482e^{-j57.68^0}\ pu, \quad \tilde{I}_0 = 0.0$

Converting these values in actual units and in phase quantities via the symmetrical transformation, we obtain:

$\tilde{I}_a = 77{,}235e^{-j87.68^0}\ A, \quad \tilde{I}_b = 77{,}235e^{j92.33^0}\ A, \quad \tilde{I}_c = 0.0$

Now the X to R ratio is obtained from the Figure, i.e. R=0.028, and X=0.69. Thus:

$$\frac{X}{R} = 24.6429$$

Assuming maximum DC offset, at the time of interruption (6 cycles) the rms value will be:

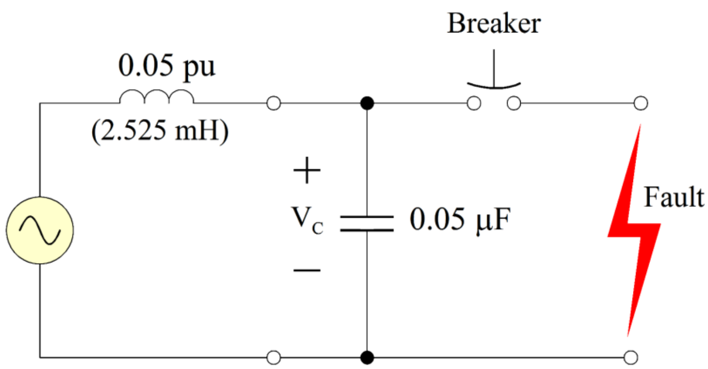
$$I_{F,rms} = 77{,}235\sqrt{1 + 2e^{-\frac{2R}{L}t}} = 80{,}776\ A$$

Problem 4.20: At a breaker location, the Thevenin equivalent inductance is 0.05 pu on a 7.96 kV, 3.3333 MVA basis. The total parasitic capacitance at this point is 0.05 microFarads. Compute the transient recovery voltage of this breaker, i.e. maximum value in volts and rise time to maximum value in microseconds.

Solution: The numerical values of the capacitance and inductance are:

$$C = 0.05\ \mu F$$

$$L = \frac{0.05}{\omega}\frac{13.8^2}{10} = 2.5\ mH$$



$$\omega_1 = \frac{1}{\sqrt{LC}} = 88{,}985\ rad/\sec$$

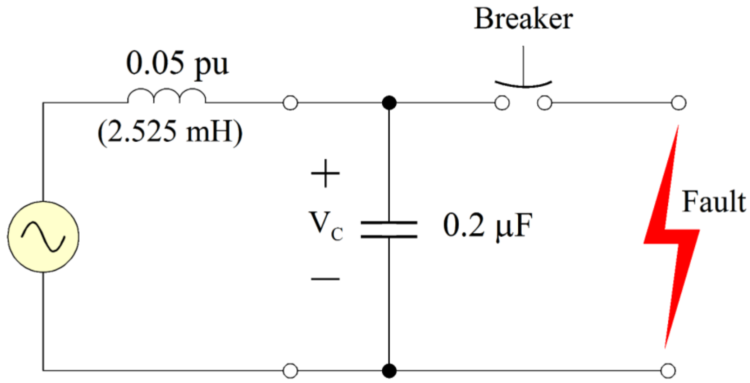
$$\frac{T}{2} = \frac{2\pi}{2\omega_1} = 35.3\ \mu\sec$$

$$TRV = 2\sqrt{2}\frac{13.8}{\sqrt{3}}kV = 22{,}536V \quad or \quad 2.0\ pu$$

Problem P4.21: At a breaker location, the Thevenin equivalent inductance is 0.05 pu on a 7.96 kV, 3.3333 MVA basis. The total parasitic capacitance at this point is 0.05 microFarads. An engineer decides that the parasitic capacitance is not enough and adds a capacitor of value 0.15 microFarads in parallel with the parasitic capacitance. Compute the transient recovery voltage of this breaker, i.e. maximum value in volts and rise time to maximum value in microseconds. The voltage of the source is 7.96 kV. Neglect the resistance of the circuit.

Solution: $C = 0.2\ \mu F$

$$L = \frac{0.05}{\omega}\frac{13.8^2}{10} = 2.5\,mH$$



$$\omega_1 = \frac{1}{\sqrt{LC}} = 44{,}492\,rad/\sec$$

$$\frac{T}{2} = \frac{2\pi}{2\omega_1} = 70.6\,\mu\sec$$

$$TRV = 2\sqrt{2}\frac{13.8}{\sqrt{3}}\,kV = 22{,}536V \quad or \quad 2.0\ pu$$

Problem P4.22: A TRV problem with clearing an out of step generator.

Problem P4.23: At a breaker location and for a specific fault close to the breaker of a 60 Hz system, the Thevenin equivalent voltage is 1.0 pu and the Thevenin equivalent impedance is

Z= 0.002+j0.08 pu on a 7.96 kV, 33.33 MVA basis.

A fault at this location is interrupted by the breaker in 5.0 cycles. Compute the maximum possible rms value of the fault current at the time of the interruption.

Solution:

$$I_{F_{rms}} = I \sqrt{1 + 2e^{-\frac{2R}{L}t_1}}$$

$$I = I_{pu} I_b = \frac{1}{\sqrt{0.002^2 + 0.08^2}} \cdot \frac{33.33}{7.96} \ kA$$

$$= 52.323 \ kA$$

$$\frac{2R}{L}t_1 = \frac{2R\omega}{\omega L}t_1 = (18.85)(0.08333)$$

$$= 1.57$$

$$\sqrt{1 + 2e^{-\frac{2R}{L}t}} = 1.18985$$

$$I_{F_{rms}} = 62.256 \ kA$$

Problem P4.24: Consider the electric power system of Figure P4.24. The parameters of the various system components are given in the figure. Consider a single line to ground fault at the load location (near the middle of one of the lines).

The two transmission lines parallel each other for the entire length of 62.7 miles. Each line is suspended on single poles (name: AGC-P-230), uses phase conductors ACSR, BITTERN, shield wires ALUMOWE, 3#7AW, soil resistivity 175 ohm.meters, the poles are spaced 0.1 miles apart, the ground impedance at each pole is 25 ohms and the distance between the two lines is 40 feet (pole to pole).

(a) Compute the positive/negative and zero sequence models of each of the lines and the mutual impedance model between the two lines. For this computation neglect the electric load near the middle of one of the lines.
(b) Compute the single line to ground fault at the load location by modeling the two transmission lines as two mutually coupled lines. During this fault compute the negative sequence current in the generator.
(c) Compute the single line to ground fault at the load location by modeling the two transmission lines as two independent (not mutually coupled) lines. During this fault compute the negative sequence current in the generator.
(d) Compare the results in cases (b) and (c) above.

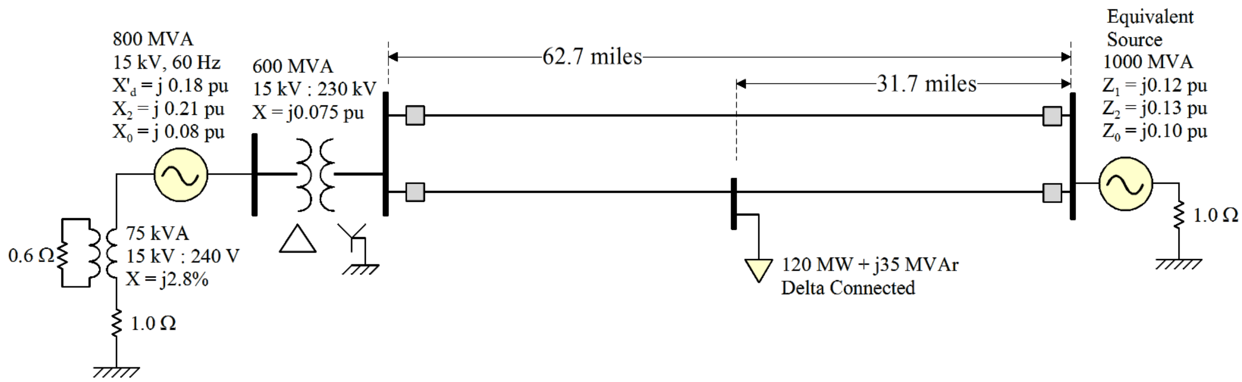


Figure P4.24

Problem 2 Data File available on course web site.

(a) $Z_1 = Z_2 = 4.597 + j43.770$ ohms

$Z_0 = 39.435 + j150.962$ ohms

Mutual Impedances

$Z_{1m} = Z_{2m} = 0$

$Z_{0m} = 34.5190 + j84.7059$ ohms.

(b) $\tilde{I}_{1P\text{-}G\text{-}Fault} = 5.1956\, e^{-j77.1639}$ kA

$\tilde{I}_{g2} = 9.728\, e^{j128.7^\circ}$ kA

(c) $\tilde{I}_{1P\text{-}G\text{-}Fault} = 5.0703\, e^{-j78.4445^\circ}$ kA

$\tilde{I}_{g2} = 9.460\, e^{j127.5^\circ}$ kA

(d) Fault Current Error

Magnitude : 2.4% Phase : 1.2806°

Generator Negative Sequence Error

Magnitude : 2.75% Phase : 1.2°

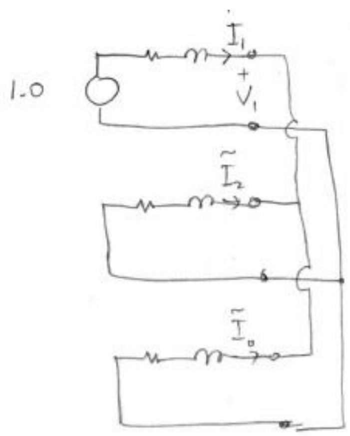
Problem P4.25: At a certain location of a 115 kV electric power system, the driving point impedances are:

(a) pos/neg sequence: $R_1 + j\omega L_1 = 0.001 + j0.055\ pu, \quad @\,115kV\ (L-L)\ and\ 100\ MVA$
(b) zero sequence: $R_0 + j\omega L_0 = 0.008 + j0.195\ pu, \quad @\,115kV\ (L-L)\ and\ 100\ MVA$

Assume a line-to-line-to-ground fault at this point and on phases B and C.

1. Compute the fault currents of phases B and C.
2. Compute the voltage of phase A during the fault.

Solution:



$$Z_2 \| Z_0 = \frac{(0.001+j0.055)(0.008+j0.195)}{0.009+j0.25}$$
$$= 0.042915384\, e^{j88.67085}$$
$$= 0.00099546 + j0.0429038$$

$$\tilde{I}_1 = \frac{1}{Z_1 + Z_2\|Z_0} = \frac{1.0}{0.00199546 + j0.0979038}$$
$$= 10.21198\, e^{-j88.8323^\circ}$$

$$\tilde{V}_1 = \tilde{I}_1 (Z_2 \| Z_0) = 0.43825\, e^{-j0.16145^\circ}$$

$$\tilde{I}_2 = -\frac{\tilde{V}_2}{Z_2} = 0.1219 - j7.9656$$

$$\tilde{I}_0 = -\frac{\tilde{V}_0}{Z_0} = 0.0856 - j2.2478$$

$$\tilde{I}_a = 0$$
$$\tilde{I}_b = 15.8696 + j3.0838$$
$$\tilde{I}_c = 15.6116 + j3.6524$$
$$\tilde{V}_a = 1.31475\, e^{-j0.1645^\circ}$$

Problem 1

Transmission line $\quad L_s = \frac{\mu}{2\pi} \ln \frac{D_e}{d} \quad L_m = \frac{\mu}{2\pi} \ln \frac{D_e}{D}$

$$D_e = 2160 \sqrt{\frac{\rho}{f}} = 6235.4 \text{ feet}$$

$$D = \sqrt[3]{(9.5)(9.5)(19)} = 11.969 \text{ feet}$$

$L_s = 2.418 \times 10^{-6}$ H/m

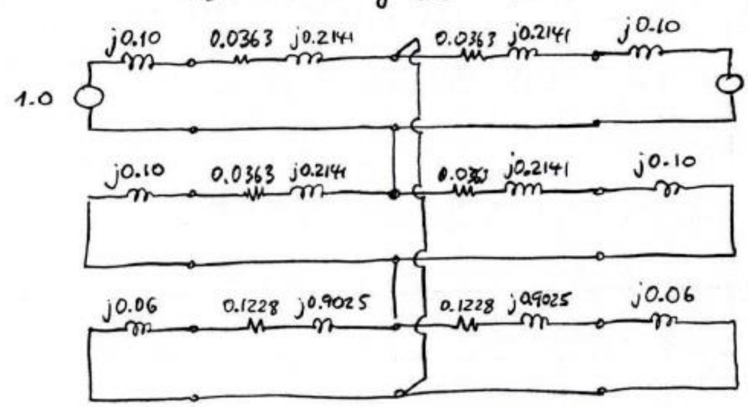
$L_m = 1.251 \times 10^{-6}$ H/m

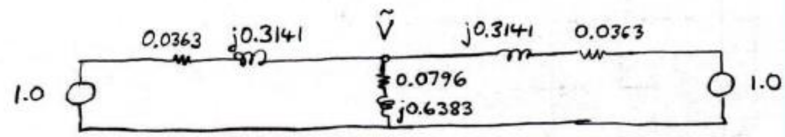
$r_e = 0.00159 f$ Ω/mile $= 0.0954$ Ω/mile

$r = 0.12$ Ω/mile

$$Z_1 = 0.12 + j\omega(L_s - L_m)(1609) \ \Omega/\text{mile} = 0.12 + j0.708 \ \Omega/\text{mile}$$

$$Z_0 = r + 3r_e + j\omega(L_s + 2L_m)(1609) = 0.406 + j2.984 \ \Omega/\text{mile}$$





$$\left(\frac{2}{0.0363 + j0.3141} + \frac{}{0.0796 + j0.6383} \right) \tilde{V} = \frac{2}{0.0363 + j0.3141}$$

$$\Rightarrow \quad \tilde{V} = 0.8027 \, e^{-j\,0.5^\circ}$$

$$I_1 = I_2 = I_0 = \frac{\tilde{V}}{0.0796 + j0.6383} = 1.248 \, e^{-j\,83.3^\circ}$$

$I_a = 3.7439$ pu.

Problem P4.26: Consider the electric power system of Figure P4.26. The parameters of the various system components are given in the figure. Consider a line to ground fault at location A. The location A is indicated in the figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. It is given that prior to the fault the unit operates under nominal voltage at its terminals and zero power.

(a) Construct the positive, negative and zero sequence networks of this system in per unit – use as power base 800 MVA (total three-phase) and voltage base the corresponding nominal voltages.
(b) Compute the negative sequence current in the generator during the fault (single line to ground at location A).
(c) Compute the zero sequence current in the unfaulted transmission line.

The positive, negative and zero sequence impedances of each line are: $Z_1 = Z_2 = j37.5\, ohms$ and $Z_0 = j93.7\, ohms$. The transmission lines are not mutual coupled. The "infinite bus" is an ideal-balanced-three-phase voltage source. Assume zero power flow prior to the fault.

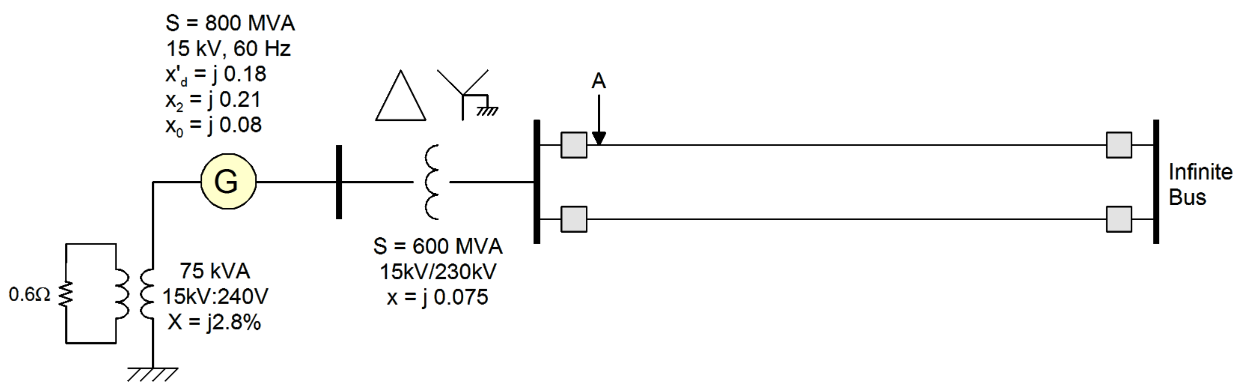
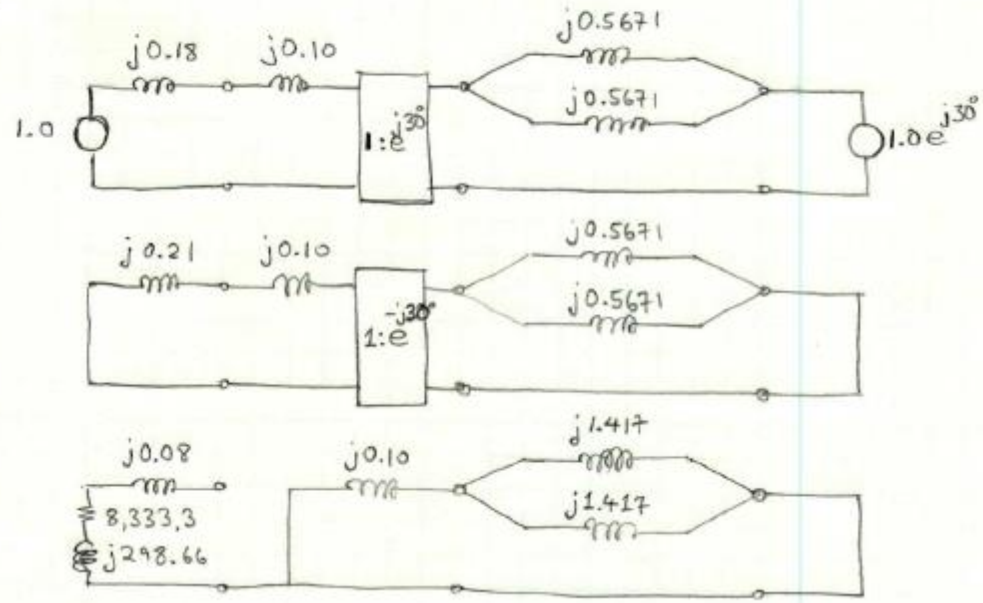


Figure P4.26

Solution:

(a) $Z_{B,15} = 0.28125$ Ohms $\qquad Z_{B,230} = 66.125$ Ohms



(b) $\tilde{I}_{g,neg} = 1.268\, e^{-j90^\circ}$

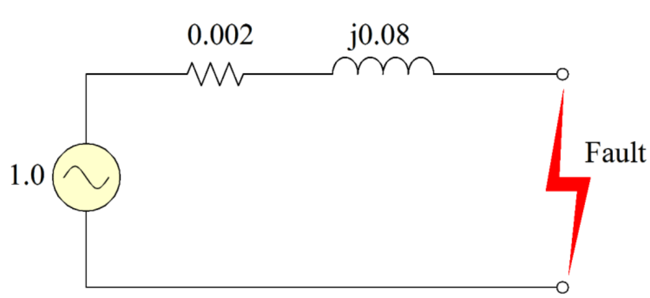
(c) $\tilde{I}_{line2,zero} = 0.1642\, e^{j120.0^\circ}$

Problem P4.27: At a breaker location and for a specific fault close to the breaker of a 60 Hz system, the Thevenin equivalent voltage is 1.0 pu and the Thevenin equivalent impedance is

Z= 0.002+j0.08 pu on a 7.96 kV, 33.33 MVA basis.

A fault at this location is interrupted by the breaker in 5.0 cycles. Compute the maximum possible rms value of the fault current at the time of the interruption.

Solution: The equivalent circuit model is:



The rms value of the fault current is:

$$I_{F_{rms}} = I\sqrt{1+2e^{-\frac{2R}{L}t_1}}$$

$$I = I_{pu} I_b = \frac{1}{\sqrt{0.002^2+0.08^2}} \cdot \frac{33.33}{7.96}\ \text{kA}$$

$$= 52.323\ \text{kA}$$

$$\frac{2R}{L}t_1 = \frac{2R\omega}{\omega L}t_1 = (18.85)(0.08333)$$

$$= 1.57$$

$$\sqrt{1+2e^{-\frac{2R}{L}t}} = 1.18985$$

$$I_{F_{rms}} = 62.256\ \text{kA}$$

Problem P4.28: A synchronous 60 Hz, 18 kV, 360 MVA generator is connected to an infinite bus (ideal voltage source- zero internal impedance) through a step up transformer and a series capacitor compensated transmission line as in Figure P4.28. The voltage at the terminals of the generator is 1.0 pu, the generated real power is 0.95pu and the power factor is 0.95 current lagging. The generator's transient reactance is 0.20 per unit on the generator ratings. The transformer is 360 MVA, 18 kV/230 kV, with a leakage impedance of j0.08 pu on the transformer ratings. The transmission line impedance is j0.10 per unit on a 100 MVA (3 phases), 230 kV (Line to Line) basis. The absolute value of the capacitor impedance is 60% of the absolute value of the transmission line impedance.

(a) Compute the transmission line impedance in per-unit on the same base as for the 230 kV side of the transformer.
(b) Compute the series capacitor impedance in per-unit on the same base as for the 230 kV side of the transformer.
(c) Compute the infinite bus voltage phasor and the generated voltage phasor of the generator.
(d) Assume a single phase to ground fault in the middle of the 230 kV transmission line. Further assume that the generated voltage and the infinite bus voltage remain constant. Compute the fault current in per-unit and in actual amperes.

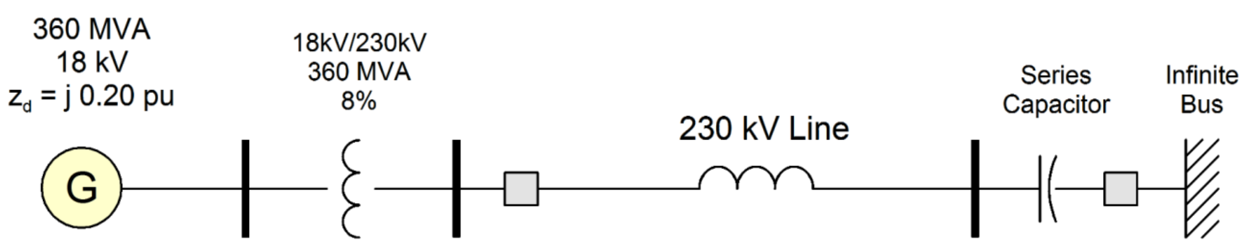


Figure P4.28

The transformer is delta-wye connected (delta on generator side).
Assume positive, negative and zero sequence impedances to be same for each device.

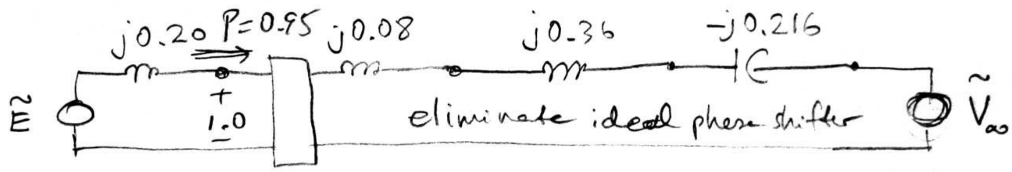
Solution:

ECE6323 FINAL | Problem 1 | Solution

(a) $$Z_{Line} = j0.1 \frac{230^2/100}{230^2/360} = j0.36 \text{ pu}$$

(b) $$Z_{cap} = -j0.36 \times 0.60 = -j0.216 \text{ pu}$$

(c)



$$\tilde{I} = 1.0\, e^{-j18.195^\circ}$$

$$\tilde{E} = 1.0 + j(0.2)(1.0\, e^{-j18.195^\circ}) = 1.08\, e^{j10.139^\circ}$$
$$\tilde{V}_\infty = 1.0 - j(0.224)(1.0\, e^{-j18.195^\circ}) = 0.954\, e^{-j12.887^\circ}$$

(d) $$\tilde{I}_{fault} = 7.9341\, e^{j44.954^\circ} \text{ pu}$$

$$I_{fault} = 7.17\, e^{j44.954^\circ} \text{ kA}$$

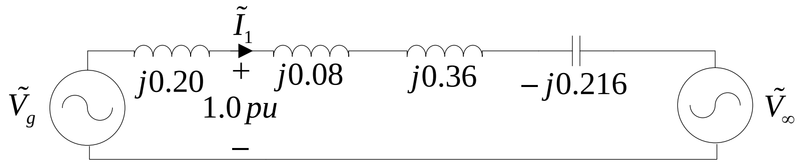
a)

b)

c) $Z_{line} = j0.1 \times \frac{360MVA}{100MVA} = j0.36 pu$

d) $Z_{cap} = -j0.6 \times Z_{line} = -j0.216 pu$

e) The positive phase equivalent circuit is as follows,



$$\tilde{I}_1 = \left|\frac{P}{0.95\tilde{V}_1}\right| e^{-j\arccos 0.95} = \frac{0.95}{0.95\times 1} e^{-j\arccos 0.95} = 1.0e^{-j18.1949^\circ}\,pu$$

$$\tilde{V}_g = \tilde{V}_1 + j0.2\times\tilde{I}_1 = 1.0 + j0.2\times 1.0e^{-j18.1949^\circ} = 1.0 + j0.2\times 1.0e^{-j0.3176}$$

$$= 1.0624 + j0.19 = 1.0793e^{j0.177}\,pu = 1.0793e^{j10.1391^\circ}\,pu$$

$$\tilde{V}_\infty = \tilde{V}_1 - j(0.08+0.36-0.216)\times\tilde{I}_1 = 1.0 - j0.224\times 1.0e^{-j18.1949^\circ} = 1.0 - j0.224\times 1.0e^{-j0.3176}$$

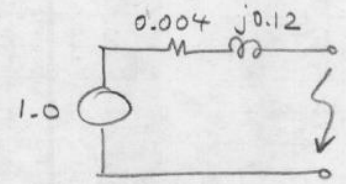
$$= 0.9301 - j0.2128 = 0.9541e^{-j0.2249}\,pu = 0.9541e^{-j12.8876^\circ}\,pu$$

Problem P4.29: At a breaker location and for a specific fault close to the breaker of a 60 Hz system, the Thevenin equivalent voltage is 1.0 pu and the Thevenin equivalent impedance is

Z= 0.004+j0.12 pu on a 14.43 kV, 33.33 MVA basis.

A fault at this location is interrupted by the breaker in 4.0 cycles. Compute the maximum possible rms value of the fault current at the time of the interruption.

Solution:



$$\tilde{I}_{sym} = \frac{1.0}{0.004+j0.12} = 8.3287\, e^{-j88.09^\circ} \text{ pu} = I e^{j\alpha}$$

$$I_{dc,\,max} = \sqrt{2}\, I\, e^{-\frac{R}{L}t} = 11.7786\, e^{-12.5667t}$$

$$I_{rms}(t) = \sqrt{I^2 + 2I^2 e^{-\frac{2R}{L}t}} = \sqrt{69.3672 + 138.7354\, e^{-25.133t}}$$

$$I_{rms}\left(t = \tfrac{4}{60}\text{ sec}\right) = 9.7642 \text{ pu}$$

$$I_{Base} = \frac{S_b}{V_b} = 2.3098 \text{ kA}$$

$$I_{rms}\left(t = \tfrac{4}{60}\text{ sec}\right) = 22.5531 \text{ kA}$$

Problem P4.30: Consider the electric power system of Figure P4.30. The parameters of the various system components are given in the figure. Consider a line to ground fault on phase A, high voltage side of the transformer (location F). Thus the fault is practically on the 230 kV bus. This fault is supposed to be cleared by opening the generator breaker at about 6 cycles (total time) after fault initiation. For simplicity, assume that the line breakers are open and therefore there will be no contribution to the fault current from the transmission lines.

(a) Compute the **maximum possible rms value** of the fault current at location F at the time of interruption. Assume the transformer is solidly grounded, i.e. R=0. All pu values are on the respective component ratings.

(b) What is the **maximum possible rms value** of the fault current at the generator breaker at the time of interruption (any phase)?

Hint: Construct the pos, neg and zero sequence network on a common pu system or in actual quantities.

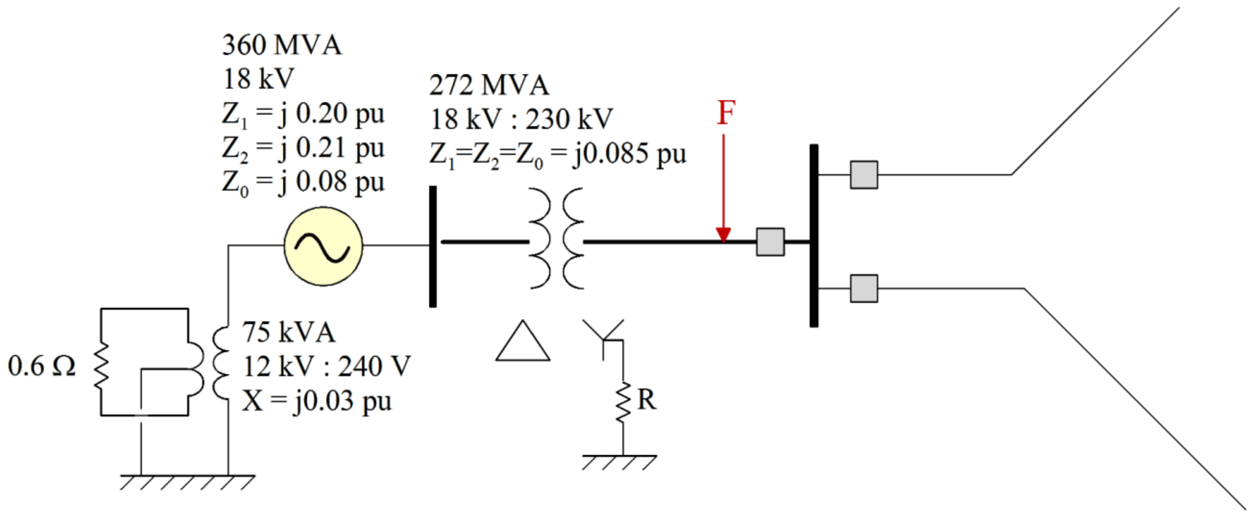
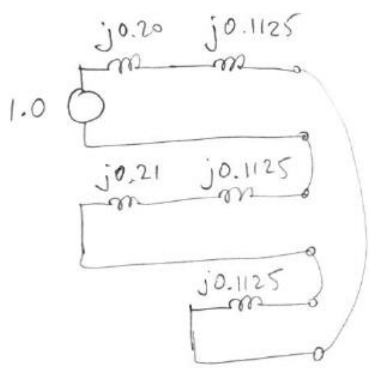


Figure P4.30

Solution:



$$I_1 = I_2 = I_0 = \frac{1.0}{j0.7475} = -j1.3378 \text{ pu} \rightarrow -j1.2089 \text{ kA}$$

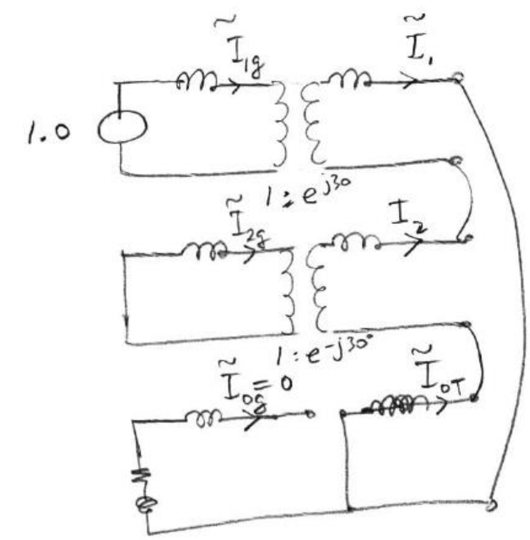
$$I_F = -j3.6268 \text{ kA}$$

$$I_{F_{max}} = 6.2818 \text{ kA}$$

3

Part (b) (Problem 1)

The following diagram is helpful in determining the fault current at the generator



The generator will experience pos and neg seq currents only. The phase currents are

$$\tilde{I}_a = \tilde{I}_{1g} + \tilde{I}_{2g}$$

$$\tilde{I}_b = a^2\tilde{I}_{1g} + a\tilde{I}_{2g}$$

$$\tilde{I}_c = a\tilde{I}_{1g} + a^2\tilde{I}_{2g}$$

Note that $\tilde{I}_1 = \tilde{I}_2 = -j1.3378$ pu $\Rightarrow \tilde{I}_{1g} = \tilde{I}_1 e^{-j30°}$, $\tilde{I}_{2g} = \tilde{I}_2 e^{+j30°}$

$$\Rightarrow \quad \tilde{I}_a = -j1.3378\sqrt{3} \text{ pu}$$

$$\tilde{I}_b = 0$$

$$\tilde{I}_c = j1.3378\sqrt{3} \text{ pu}$$

Offset will be same

$$I_{F_{max}} = 4.0134 \text{ pu} \Rightarrow I_{F_{max}} = 46.3428 \text{ kA}$$

DRAFT and INCOMPLETE

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from

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Chapter 5
Protection Fundamentals

“Find out why! To accept something the way it has always been done is simply not acceptable. Furthermore, it is boring.”

Walter A. Elmore, 1925-2010

5.1 Introduction

Power systems are exposed to a variety of disturbances that generate abnormal voltages and currents or in general abnormal conditions. Some of the abnormal conditions may cause severe damage to equipment and expose bystanders to danger. These conditions lead to outages which are localized when the protection system performs as designed. Sometimes the protection system may mis-operate or protection equipment may fail (i.e. breaker failure) leading to cascading and/or wide spread outages. The cause of these disturbances must be eliminated as soon as possible to (a) protect humans in the vicinity of the electrical power system, (b) protect equipment and avoid damage to equipment and (c) minimize customer disruptions and avoid spreading the disturbance to a larger area of the power system. On the other hand, some of the abnormal conditions may be tolerable in the sense that they do not create dangerous conditions or damage to equipment and they may fade away as time progresses. For tolerable conditions it is not necessary to act. The objective of power system protection is to identify abnormal conditions, classify them into tolerable and non-tolerable and for the non-tolerable to isolate the cause of the disturbance as fast as possible so as to minimize damage to equipment, minimize the risk to bystanders and avoid spreading the disturbance to a wider area. For tolerable conditions, the protection system should allow the system to return to normal conditions without any additional action. At the same time it is also desirable to minimize the number of customers that will be affected as a result of protective relaying action. This desirable feature always translates to the requirement that the protective system should isolate the smallest possible portion of the system that contains the disturbance.

The practice in electric power system protection has evolved over the years as power systems started from small radial systems and grew into large interconnected and complex systems. The protection function becomes increasingly challenging as the complexity of the power system increases. For example, in networked systems containing many generating sources, the task of correctly identifying which part of the system is causing the disturbance and whether this disturbance is tolerable cannot be accomplished by monitoring a single quantity of the system (such as electric fault current) or monitoring information at one location only. More sophisticated techniques have been developed based on monitoring both voltage and current, utilizing both magnitude and phase information, and using measurements from both local and remote locations (telemetered). System oscillations may also cause abnormal conditions which must be evaluated to determine whether they are tolerable or detrimental to the system. On the other hand, in radial

power systems (for example distribution systems), it is generally easier to achieve the protection objectives. Yet, in recent years, we have seen new challenges in distribution systems as the potential of distributed generation is becoming a reality. New challenges arise from the fact that legacy distribution system protection has been designed assuming radial system operation. New distributed generating resources along distribution systems tend to invalidate the protection system designs of the past. Additional complexities arise from the fact that distributed generation is normally interfaced via converters that they are presenting totally different characteristics from the traditional power apparatus, such as lack of inertia and limited fault currents. The end result is that power system protection, both at the transmission and distribution levels, is a challenging and exciting field of study.
Recall that in order to simplify the practice and the design of protection systems, we classify the protection problems into two broad categories. In the first category we place all protection problems that deal with the protection of a single device or devices forming a unit, i.e. a generator, a transmission line, a transformer, a generator/stepup transformer unit, etc. We refer to the device or unit under protection as a protection zone. The protection schemes in this category are designed to determine faulted conditions in the protection zone only, respond to these faults only and do not respond to faults outside the protection zone. In the second category we deal with protection problems dealing with abnormal system conditions and they may involve more than one protection zone. The technologies we use for both protection problems have commonalities, in other words we use similar devices to protect individual protection zones and to protect against system problems. The logic used to decide on protective action, i.e. the settings of the algorithms, will be different depending on the application. The technologies involved include interrupting devices (fuses, breakers, automatic switches) and control devices (relays, logic circuits, etc.). The logic used to take decisions is referred to as protection function. Many different protection functions have been developed over the years, such as overcurrent, differential, distance, traveling wave, etc. In this chapter we will examine the basic technologies used for protection as well as the different protection functions.

The main protection technologies used in protective systems are:

- Fuses
- Breakers
- Reclosers
- Sectionalizers

The main protective function/logic used in order of increasing complexity are:

- Overcurrent Relays
- Time Delay Overcurrent Relays
- Directional Relays
- Distance Relays
- Pilot Relaying

This chapter provides a concise overview of the interrupting device technologies and discusses the principle of operation of protection functions with commentary about their advantages and disadvantages. In subsequent chapters, the relay functions are utilized for the design of protection schemes of specific protection zones (generators, transmission lines, transformers, etc.) as well as system protection problems.

5.2 Fuses

Fuses provide overcurrent protection in one single device without the need of any additional subsystems such as monitoring, decision logic and settings. The principle of operation is very simple: the fuse provides a path for the electric current through a wire that is appropriately sized so that when the electric current exceeds a certain value the wire will melt in a time that depends on the level of the electric current. Once the wire melts, the electric circuit is interrupted.

Fuses may be used for overcurrent protection of any electrical circuit, i.e. power transformers, capacitors, distribution circuits, residential circuits, etc. Typical fuses are illustrated in Figure 5.1. Figure 5.1a illustrates a transmission level fuse, Figure 5.1b shows a distribution level fuse and Figure 5.1c shows a medium voltage capacitor fuse. The distribution class fuse shiwn in Figure 5.1b is typically pole-mounted and consists of a fuse holder and a replaceable fuse element.

(a) (b) (c)

Figure 5.1: Fuses: (a) Typical Distribution Fuse (show a transmission/transformer fuse, and a capacitor fuse in same figure)

Fuse operating characteristics are specified by their load carrying capability and via time-current characteristic (melting) curves. Fuse selection is based on two criteria: (a) load carrying capability and (b) coordination with other protection devices. These criteria are discussed next.

Load carrying capability of distribution class fuses: The load carrying capability should be sufficient for the expected load current. Manufacturers typically specify the load carrying capability using the following three parameters:

- Continuous Load (for 3 hours or more)
- Cold Load Pickup
- Hot Load Pickup

Continuous Load is the maximum RMS current that the fuse can withstand on a continuous basis, for at least 3 hours, without being damaged. *Cold load pickup* is the maximum RMS current that the fuse can withstand for a short time typically 30 seconds starting from a "cooled state", i.e. the fuse does not carry current prior to this current (for at least 30 minutes). Cold load pickup refers to distribution feeder outage events: an outage of a feeder for a relatively long time, such as 30 minutes, will result in all equipment and loads in the feeder to "cool down", i.e. the temperature of equipment will drop, motors will go to standstill, appliances will stop, etc. When the feeder is energized after an event like this, the amount of current that will be drawn by the various loads of the feeder will be substantially higher than the nominal load current of these devices. Note for example an electric motor starting after this event may draw a startup current several times its rated current. We refer to this current as the cold load pickup current. The fuses connected to the feeder will experience this current. During the cold load pickup the fuses should not operate if there is no fault in the system. The cold load pickup current is generally larger than the continuous load. *Hot Load Pickup* is the maximum RMS current that the fuse can withstand for a short time after a momentary interruption (less than 3 minutes) assuming that the fuse was supplying maximum continuous load before the interruption. This means that the hot load pickup is defined for a fuse that is at the rated operating temperature of the fuse. As can be inferred, hot load pickup refers to distribution events that follow a temporary feeder outage. Upon energization of a feeder after a brief interruption, the current in the feeder will be higher than the current before the interruption because many motors have lost speed during the interruption and when the voltage is restored they will start accelerating drawing startup current. The hot load pickup current is generally higher than the continuous load current and lower than the cold load pickup current. For selecting a fuse, the expected continuous load, hot pickup load current and cold load pickup current must be known for the feeder.

Fuses used for the protection of transformers should be also selected so that they can withstand inrush currents during the energization of the transformer.

Typical time-current characteristic curves for a distribution fuse is shown in Figure 5.2. Note that the fuse is described in terms of the minimum melting time versus the electric current through the fuse and the total clearing time of the current. Note that the difference between the minimum melting time and the total clearing time can be substantial. The characteristics suggest that there is always an uncertainty about the exact melting time of the fuse element depending on many factors. It is easy to understand the statistical distribution of these times if one considers the following facts: (a) the initial temperature of the fuse and its housing will vary and it will affect the time at which the wire will reach the melting temperature, (b) the size (cross section) of the wire is not constant because of manufacturing imperfections and therefore the thinner parts will reach melting temperature faster than the other parts, and (c) once the wire of the fuse melts, an electric arc is generated in the location of the wire. The housing of the fuse element is designed to quench this electric arc. Various designs are available for this purpose. Once the electric arc has been extinguished, then the fault has been cleared. The time between the initiation of the fault and the extinguishing of the electric arc is the total clearing time of the fuse. The variability of the total clearing time must be considered when fuses are coordinated with other protection devices.

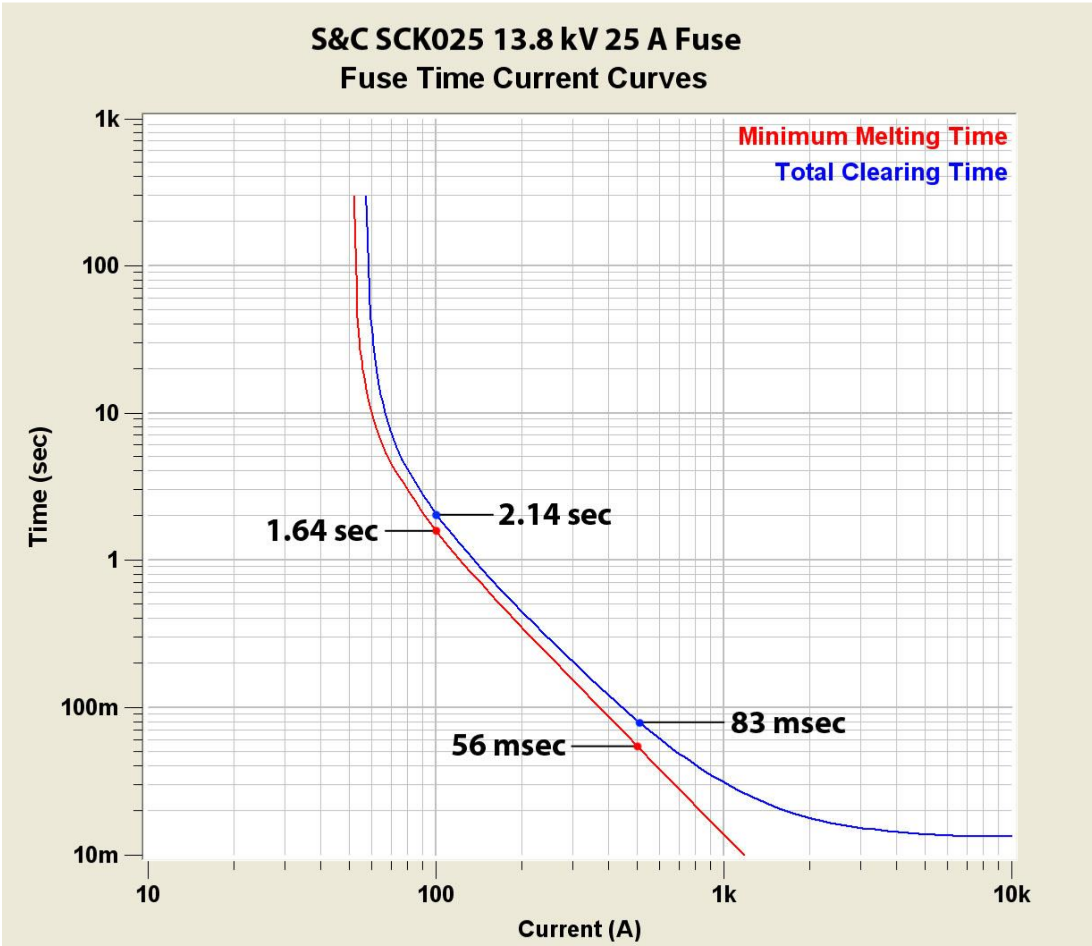


Figure 5.2: Minimum Melting Time and Total Clearing Time for a Specific Fuse (SCK025 13.8 kV, 25 A Fuse – Courtesy S&C Electric Company)

Figure 5.3 show the minimum melting time of a certain class of fuses versus the fault current through the fuse. Similar curves provide the maximum melting time of a fuse versus fault current through the fuse. The exact time characteristics for all available fuses can be found at the manufacturers' web sites.

In many applications it is desirable to limit the level of fault current. The main reason will be to protect electrical equipment. For example, consider a circuit that has been designed to withstand for a short time an electric current of 10,000 Amperes. This is very common for distribution circuits, i.e. transfomers, conductors, connectors, etc. that we may use to construct the circuit are rated for a maximum current of 10,000 Amperes. In case that the source for this circuit increases because of expansion, it is possible that the available fault current may increase, for example to 20,000 Amperes. In this case we may have two options: (a) limit the fault current by means of additional impedance inserted in the circuit, or (b) using specially designed fuses that limit the fault current, i.e. a current limiting fuse. A current limiting fuse is designed to operate within one half cycle if the current exceeds the limit value. The design also provides a scheme to immediately extinguish the electric arc. For example the fuse wire may be embedded in sand which will absorb the energy of the electric arc and it will extinguish the arc.

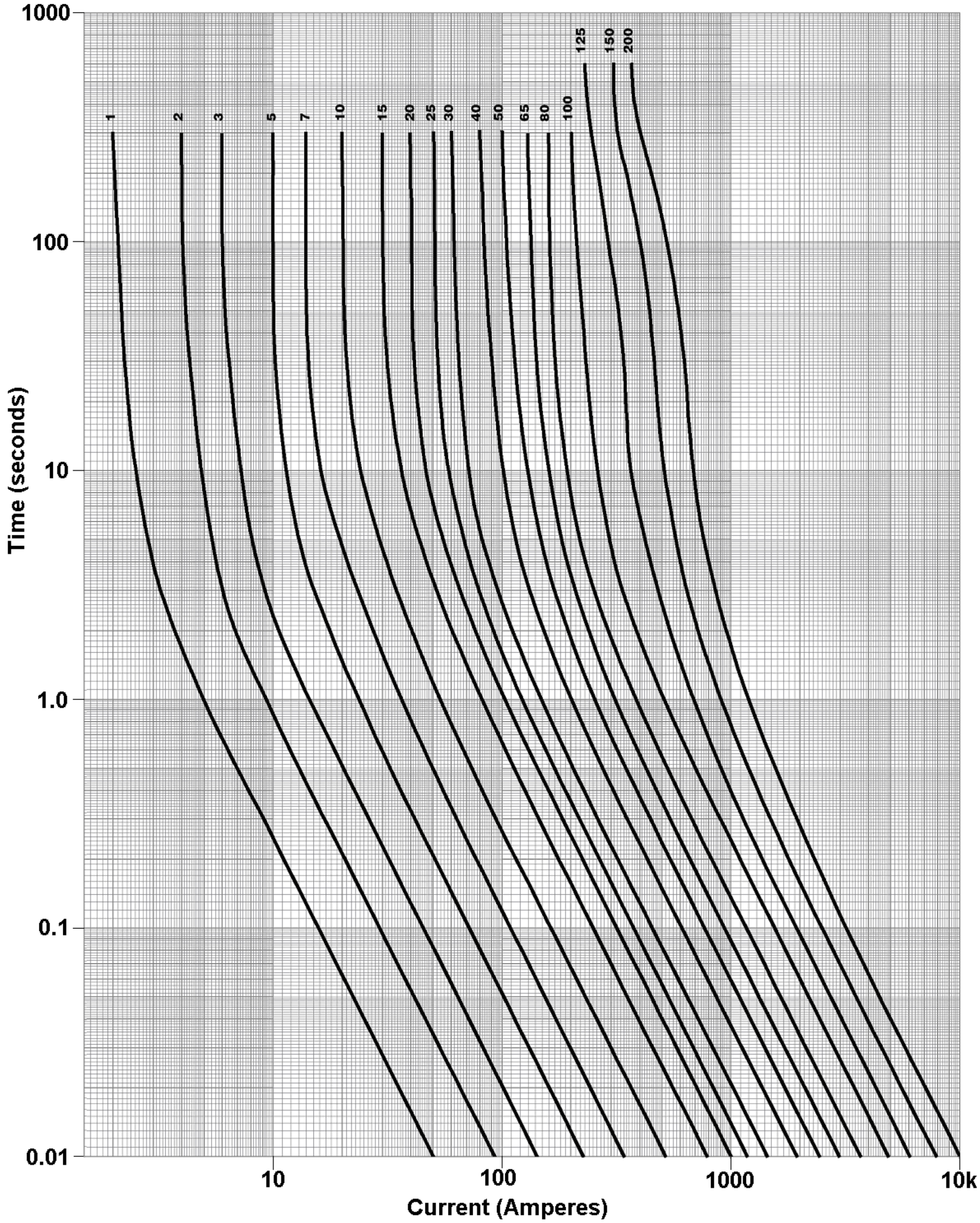


Figure 5.3: Distribution Fuse Time-Current Curves

Coordination with other protective devices: Fuses should be coordinated with the other protective devices that may be present in a circuit. The objective of the coordination is to minimize the amount of load disconnected for any given fault condition. In the process of coordination, one needs to remember that fuses are not automatically resetting devices – once they operate someone must replace them. For this reason we have two conflicting objectives: (a) minimize the number of customers to be interrupted following a fault and (b) minimize the number of fuse operations which will minimize the total interruption time of service to customers (assuming the automatic reclosing of other protective devices may restore service much faster). For this reason "fuse saving" schemes have been developed by appropriate coordination with other protection devices such as reclosers. These schemes are typically applied to distribution systems and therefore they will be covered in Chapter 10.

5.3 Circuit Breaker Technology Review

Circuit breakers are preferred circuit interrupting devices and constitute the workhorse of power system protection. The challenge in circuit breaker technology is the interruption of extremely high current levels that may occur during faults and the ability to perform multiple open/close operations.

Breaker technology evolved over the years. As power systems evolved, interconnected and increased in capacity the fault currents that breakers must interrupt have dramatically increased. Because the power system is mostly inductive (the resistance is relatively small) the interruption of the current encounters the issue of interrupting a large current through an inductor. The basic function of a breaker is to extinguish an electric arc which is normally generated when an opening is generated in a power circuit. The first arc-extinguishing medium in the history of breaker development was air giving genesis to the air circuit breaker (ACB). The air circuit breakers that used compressed air to blast the arc were introduced in the late 1920s [xxx-5-001]. At about the same time experimentation started of interrupting the arc in mineral oils giving birth to the oil circuit breaker [xxx-5-003]. In the 1940s experimentation started of using compressed SF6 (Sulphur Hexafluoride) gas as the insulating medium in breakers. The success of the SF6 gas as an insulating material was the precursor of using SF6 gas as the arc interrupting material. This research was successful resulting in the first SF6 breaker in 1959 [xxx-5-002]. The SF6 breaker technology evolved and improved and today the SF6 puffer type breakers are the breakers of choice for high voltage – high current applications. Another approach to arc interruption was to use vacuum for arc extinguishing. Experiments to interrupt fault currents using vacuum started at the same time as air circuit breakers [xxx-5-004]. A high quality vacuum is almost devoid of gas and therefore of potential ions to sustain the arc. Therefore vacuum circuit breakers are dependent on technology to create high quality vacuum and maintain the vacuum throughout the life of the breaker. The technology has matured and vacuum interrupters are quite popular for medium voltages.

The various breaker technologies have different characteristics from the protection point of view. Specifically, these characteristics may be: maximum current that can be interrupted, ability to withstand transient voltages after interruption (Transient Recovery Voltage - TRV), time required between two successive operations, number of operation before maintenance is required, etc. We will provide a concise review of these characteristics for the various technologies.

Air-Blast Circuit Breakers: In any breaker design the most important objective is to interrupt the arc between the plates once the plates have been separated, i.e. once the breaker is tripped. Air-blast circuit breakers achieve the interruption of the arc by blasting air at the location of the arc. The air is stored in tanks under pressure. When a fault occurs, the breaker plates separate and the air is released from the tanks into the arc area. The arc is blown away into a chamber which is referred to as the arc chute. The area between the plates becomes devoid of carriers of electricity, i.e. an electrical open has been established. The geometry of the arc chute is typically designed to progressively elongate the arc length. The geometry and material of the arc chute is very critical in absorbing and quenching the arc. As the arc length increases it cannot sustained itself. A schematic view is shown in the Figure 5.6 which illustrates the cross section of a blast air circuit breaker design. The advantages of air circuit breakers are: (a) no hazard of fire, (b) fast operation, (c) longer life, and (d) requires less maintenance.

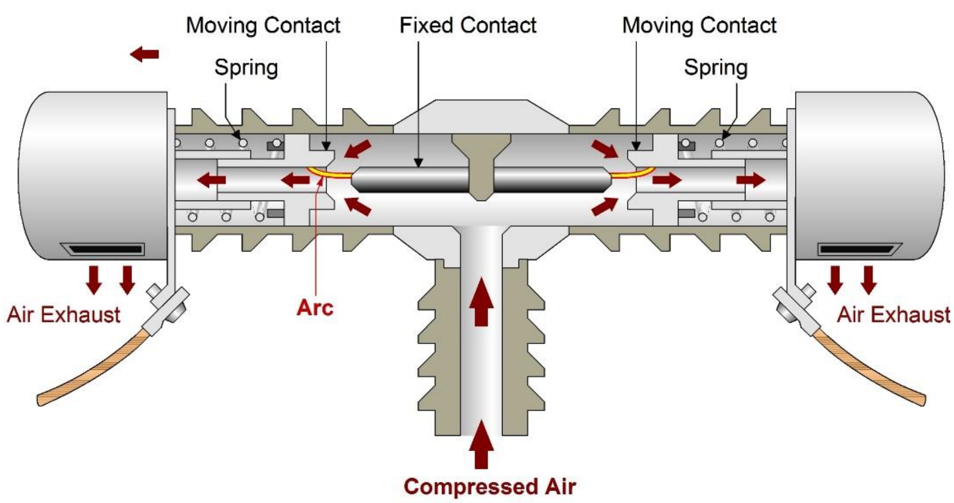


Figure 5.4 Example Air-Blast Breaker
(Photo: Courtesy: Georgia Power Co.)

Oil Circuit Breakers: Oil circuit breakers consist of the main contacts located in a space filled with mineral oil. When the main contacts separate, the arc between the contact plates raises the temperature very rapidly. The high temperature causes the oil to evaporate and the oil vapor may be partially decomposed releasing gases such as ethylene, methane, etc. A gas bubble is formed around the electric arc. The electric arc is extinguished because of three factors: arc elongation as the contacts are moving, cooling due to the gases and oil vapor and as the electric fault current crosses zero. The quenching of the arc is also facilitated with the use of arc chambers. The oil circuit breaker has been the workhorse of the industry for many decades. However, maintenance requirements and the hazard of fire has resulted in using other technologies in new substations or replacing the oil circuit breakers in older substations when the fault duty reaches the limits of the breaker or when maintenance becomes costly.

SF_6 Gas Circuit Breakers: As the name implies, these breakers use SF6 for the interruption of the arc. This technology appeared in 1938. The first industrial application dates to 1953. In 1957, the puffer type SF6 breaker was introduced. In puffer type SF6 breakers the moving part of the breaker is linked to a piston and a cylinder to generate the pressure necessary to blast the arc with SF6 via a nozzle (the nozzle is made up of insulating material). Puffer type breakers require a tank where SF6 is kept under pressure. Many improvements made throughout the years and today SF6 breakers dominate the applications. Some of the characteristics are: (a) relative simplicity, (b) short break time, typically 2 to 2.5 cycles, (c) reliability and (d) low noise levels. On the other hand, SF6 is a known greenhouse gas and care must be provided to avoid any leaks.

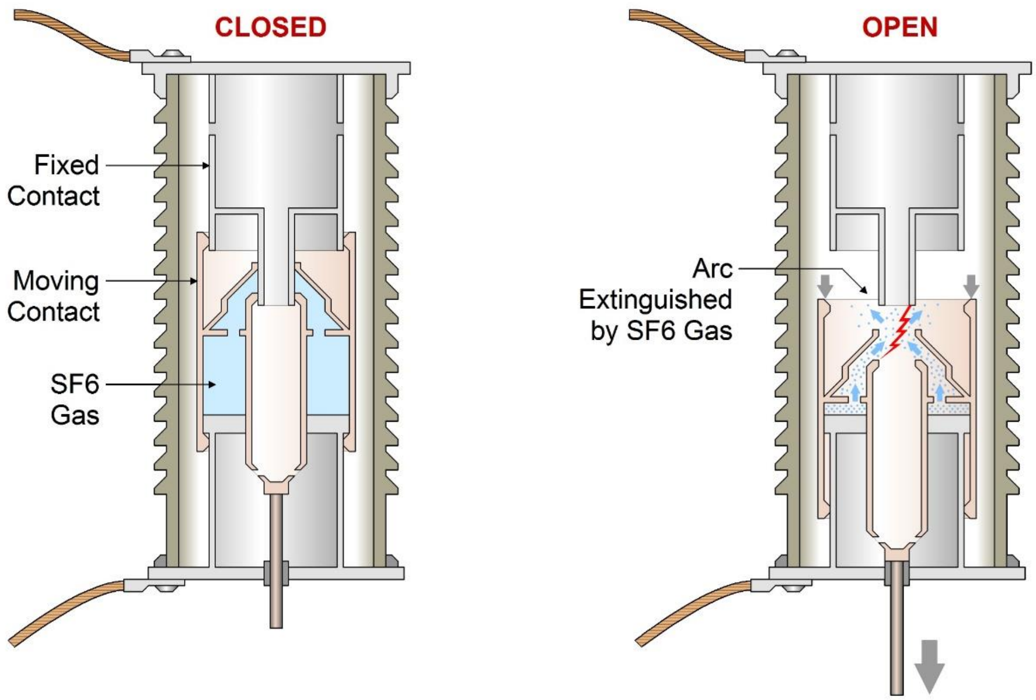


Figure 5.5 Example SF6 Breaker

Vacuum Circuit Breakers: As the name indicates the quenching of the electric arc is done in a vacuum chamber. Specifically, the breaker chamber is kept at a vacuum of about 0.1 Pa (comparatively, the pressure at sea level is about 100,000 Pa). The dielectric strength of vacuum is about 8 times greater than that of air and 4 times greater than that of SF6. The high dielectric strength of vacuum makes it possible to interrupt fault currents with shorter contact gaps. Thus the contacts do not have to travel too much. Upon separation of the contacts, an arc will be sustained to allow fault current flow from one contact plate to another. This arc will be extinguished at the first zero crossing of the current. The physics of the arc during the arcing period is that for low fault current levels the arc diffuses over the area of the plates but for high fault currents, due to magneto-striction, the arc becomes concentrated and may overheat the plate at the point of its contact with the plate. This creates heating and possible local melting of the contact material. For this reason the breaker is designed to create movement of the arc along the plate surface by its own magnetic field. This design results in highly reliable breakers. The advantages of vacuum breakers are (a) reliability and long service life, (b) compactness, (c) no fire hazard as compared to oil circuit breakers for example, and (d) environmentally friendly as compared to other technologies. The technology is suitable for medium voltage levels. Research into higher voltage level vacuum breakers is on-going. One disadvantage of the vacuum circuit breakers is that fault interruption may include many fast restrikes that can generate high transient voltages to nearby transformers if present. It is known that these transients can lead to transformers failures if no protection is provided against the fast transients.

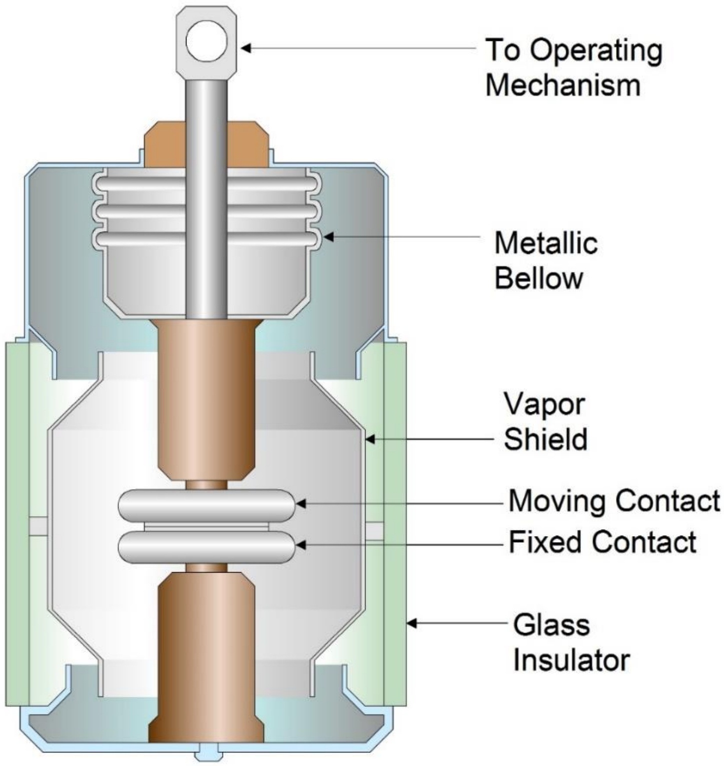


Figure 5.6 Example Vacuum Breaker

The purpose of a breaker is to interrupt fault currents in a reliable manner. Irrespectively of what breaker technology is used, the amount of fault current to be interrupted and the voltage across the breaker depend on the fault transients that were discussed in Chapter 4. The two most important are: (a) the rms value of the fault current at the time of interruption and the (b) the transient recovery voltage. These issues have been discussed in Chapter 4.

5.3 Reclosers and Sectionalizers

Reclosers and Sectionalizers are automatically operated breakers and switches that are typically installed in distribution circuits as part of a fault protection scheme and automatic reconfiguration of distribution feeders.

Reclosers are breakers (designed to interrupt the fault current) with integrated current/voltage sensing, control circuits, relays(logic) and actuators capable of automatically opening and closing the breakers. The name indicates that they are designed for automatic reclosing after a fault has been detected and they have operate to interrupt the faulted circuit. Typical reclosers operate as follows. Once a fault current is detected, the breakers are opened and closed several times with varying open and closed times. The objectives of this procedure are: (a) if the fault is temporary then the reclosing restores service to the entire feeder and minimizes the interruption time to consumers, (b) if the fault is temporary, it may result in “saving a fuse” by not allowing the fuse to operate on the fault and when the circuit is reclosed the fault is extinguished (c) if the fault is permanent allow fuses to isolate the minimum possible circuit of the system, and (d) if the fault is permanent and it cannot be cleared by any other protective devices, the recloser opens permanently until the circuit is repaired. The achievement of the above objectives requires coordination of the recloser triggering and open/close timing with the fuses and sectionalizers of the feeder. Chapter 10 discusses the application of reclosers in greater detail.

The construction of a typical single phase recloser breaker is illustrated in Figure 5.4. The reclosers can be programmed to operate with time overcurrent function and reclose after a user defined delay. These protection functions will be discussed later.

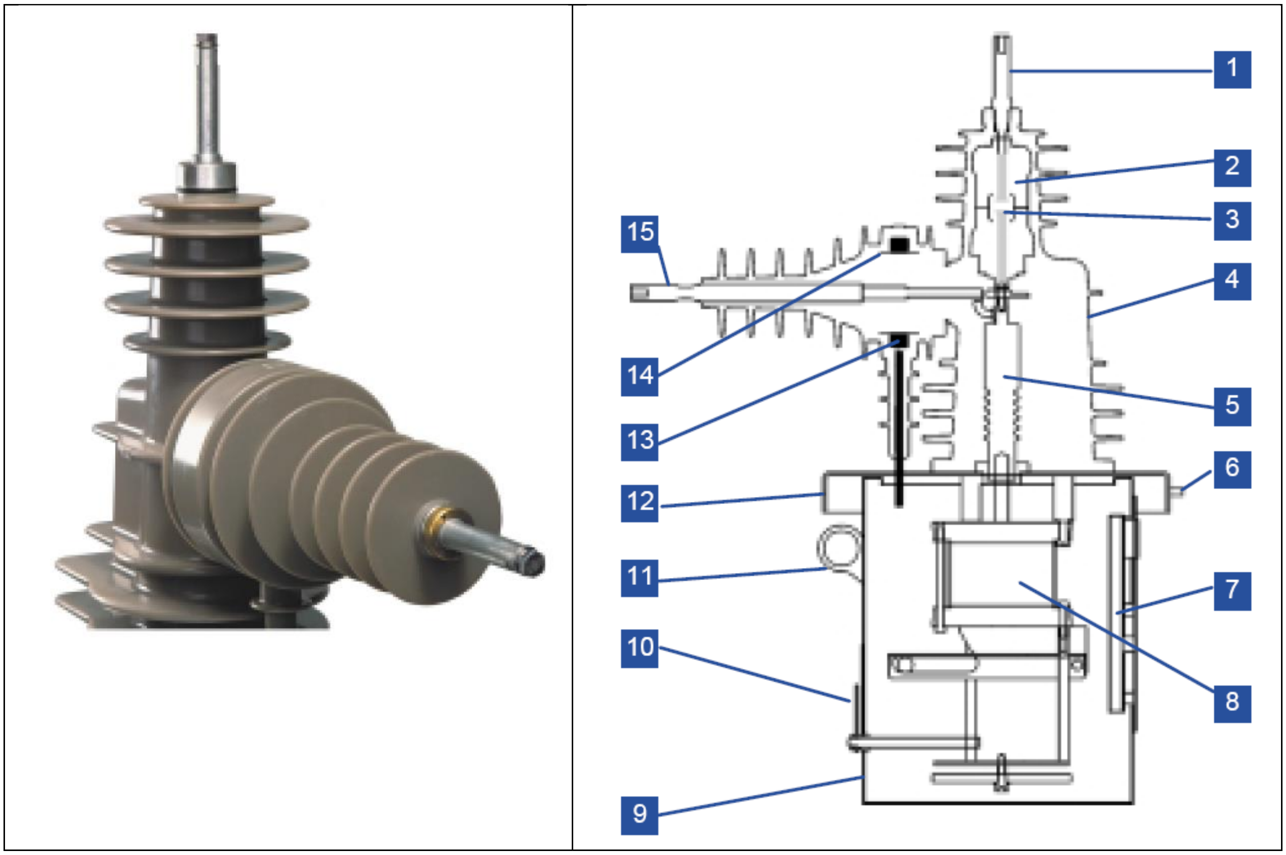


Figure 5.7: Example Recloser – W-series Solid Dielectric Single Phase Recloser: Photograph and Cross Section
(Legend: 1. X-Side terminal, 2. Vacuum interrupter, 3. Contacts, 4. Epoxy housing, 5. Push rod, 6. Earthing point, 7. SCEM Card, 8. Magnetic actuator, 9. Stainless steel tank, 10. Pointer, 11. Manual trip ring, 12. Stainless steel lid, 13. Current transformer, 14. Capacitive voltage transformer, 15. I-Side terminal)

A sectionalizer is a switch design to open a circuit when the electric current is below a design value (comparable to the maximum load current in the circuit, for example if they are designed to interrupt the maximum load current they are called load break switches). A sectionalizer may have a control circuit that monitors the operating condition and it is programmed to control the load break switch. In addition a sectionalizer may be equipped with manual controls, i.e. an electrician may operate it with a “hot stick”. A sectionalizer is shown in Figure 5.5. Thus a sectionalizer consists of load break switches, and current sensing control and actuator circuits capable of operating the load break switches. Note that the design of the load break switches is such that they can interrupt the maximum load current but not the fault current. In the event of a fault, sectionalizers are coordinated with reclosers and open the circuit during the time that the recloser switches are open. Specifically, a sectionalizer monitors the current and once a fault current is detected, it counts the number of times the recloser interrupts a fault level current. It opens its switches after this count reaches a programmed value. The application of sectionalizers and their coordination with other protective devices is covered in detail in Chapter 10.

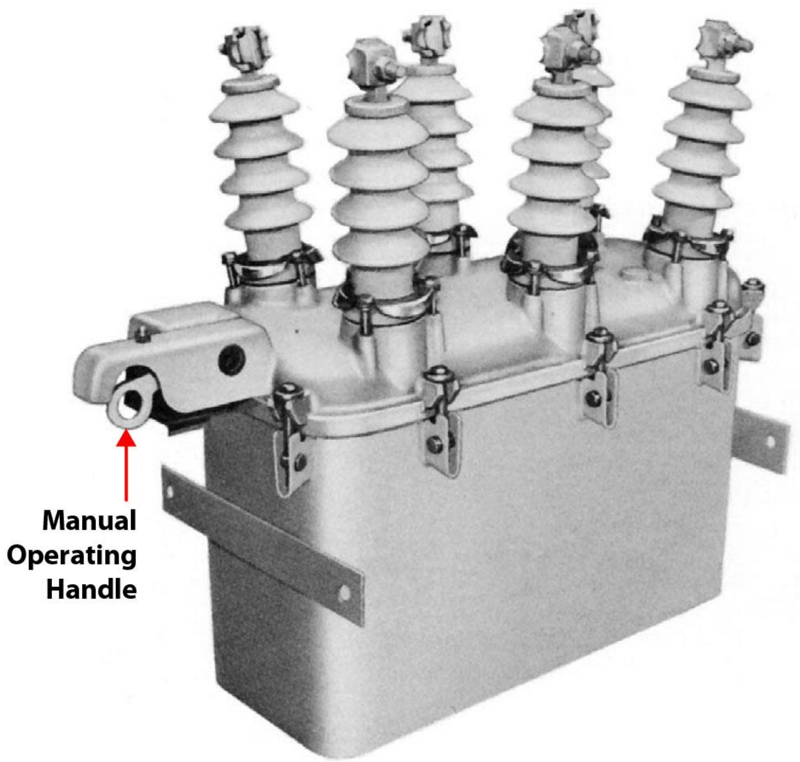


Figure 5.8 Typical 3-Phase Distribution Line Sectionalizer (Courtesy: Cooper Power Systems)

5.5 Protective Relays

A protective relay is a device that monitors certain quantities in the system and upon the occurrence of an abnormal condition it may take an action in the form of tripping a breaker issuing an alarm, etc. Their function is to monitor specific quantities of the system (typically three currents and three voltages) and apply logic to determine when they should issue a control command to an interrupting device, mainly a breaker. Therefore one can think of a relay as the "brain" of the protective system. The functional description of a protective relay is symbolically represented in Figure 5.9. The figure illustrates that the relay is connected to instrumentation transformers (VTs and CTs) that generate input signals that are proportional to the voltages and currents in the power system as well as to breakers switch contacts to monitor the status of the breaker or switch. The protective relay processes the inputs through specific algorithms and decides to trip or not to trip a breaker by operating a contact that is integrated with the tripping system of a breaker as illustrated in Figure 5.9.

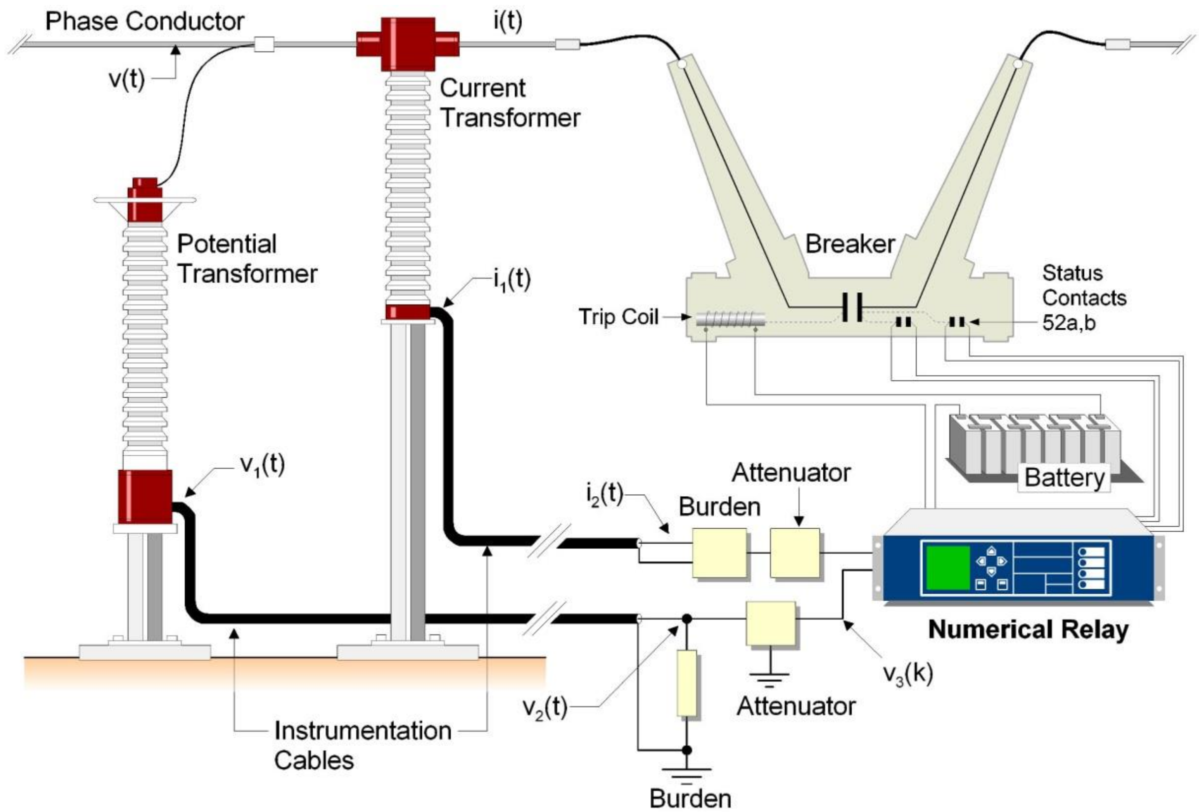


Figure 5.9 Illustration of a Protective Relaying System

Relays evolved over the years from simple electromechanical devices to complex electromechanical devices to solid state devices and finally to full digital systems. It is very interesting to note that over the years many smart engineers developed electromechanical systems that will perform very complex logic required for specific protection functions. The electromechanical systems were replaced/mimicked by solid state devices when the solid state technology became available and eventually with microprocessor based devices when the microprocessors became available. The fundamental principles of computer based relays were introduced in a seminal paper by George Rockefeller [???] in 1966. The paper was followed with the first implementation of a computer relay [???] in 1971. The discovery of the microprocessor in the late 1970s-early 1980 led to the utilization of the microprocessor as the computing engine of the computer relay and the first microprocessor based relay was commercially introduced in 1984 by Ed Schweitzer [???]. The terms digital or numerical relays are applied to microprocessor based relays. Today digital relays are the relays of choice. Digital relays can mimic the operation of electromechanical relays that were introduced in an era before computers but also can perform these functions in a better way and also add additional functions. From a pedagogical point of view, it is interesting to examine the various relay functions as introduced with electromechanical devices and to discuss in parallel their digital implementation. We will follow this path in the remaining of this chapter. We will start with the simple relays and we will continue with the discussion of more complex relays.

As it has been described in Chapter 1 a protective relay is part of a system that consists of four discrete components: (a) the **instrumentation subsystem** that consists of instrument transformers that generate low voltage and low current outputs for input to the relays or logic subsystem. Ideally, these voltages and currents should be scaled replicas of the high voltages and currents of the electric power system. Practically, however, the instrumentation channels introduce errors that slightly distort the waveforms of the high voltages and currents. (b) the **logic subsystem**, i.e. the relay, which processes the voltages and currents (and possibly status inputs) and makes decisions. Present relay systems are microprocessor based and are capable of performing complex computations and logic. The purpose of these computations is to identify and characterize the operating condition of the subsystem that they monitor and protect and through some logic to determine whether action is required to remedy an intolerable condition, (c) the **control subsystem**, consisting of discrete inputs and discrete outputs to activate the relay decisions. For example the status of the interrupting subsystem (breakers, etc.) may be instrumented as an input to the logic subsystem. In this case, the protective relay monitors the status of the interrupting device and the protection logic may take this information into consideration. There are two types of status indicators of interrupting devices (mainly breaker): a 52a contact (normally open switch or open “on the shelf” – it will be closed when the breaker is energized and in closed position) and a 52b contact (normally closed switch or closed “on the shelf” – it will be open when the breaker is energized and in closed position). These contacts are mechanically controlled by the location of the breaker for maximum reliability. The control subsystem invariably controls the trip circuit of a breaker, a motor that controls a switch, alarms, targets, etc.; and (d) the **interrupting subsystem**. The interrupting subsystem consists of a breaker, a motor operated switch, etc.

While in this chapter we will focus on the functions performed by the relay we will occasionally refer to the other parts of this protection and control system if they affect the functions of the relay, for example saturation of the current transformers (instrumentation) may affect the logic of specific functions of the relay.

The logic functions that are performed by relays have been standardized. The logic functions have been developed and evolved over the years as protection engineers try to invent new and better protective schemes and to improve the selectivity, reliability, speed and the overall performance of the relaying schemes. As has been discussed in chapter 1, an IEEE Std defined the various protection functions and devices. A partial list of the protective functions is given in Table 5.1.

Table 5.1 Most Usual Protective Functions

Protective Function Number	Protective Function Description
50	Instantaneous Overcurrent Relay
51	AC Time Overcurrent Relay
67	AC Directional Overcurrent Relay
59	Overvoltage Relay
60	Voltage or Current Balance Relay
62	Time-Delay Stopping or Opening Relay
64	Ground Detector Relay
68	Blocking or “Out-of-Step” Relay
21	Distance Relay
24	Volts per Hertz Relay
25	Synchronizing or Synchronism-Check Relay
27	Undervoltage Relay
81	Frequency Relay
32	Directional Power Relay
87	Differential Protective Relay
40	Field Relay
46	Reverse-Phase or Phase-Balance Current Relay
47	Phase-Sequence or Phase-Balance Voltage Relay
49	Machine or Transformer Thermal Relay

The above listed protective functions are most common. The protective functions were developed oven many years initially as electromechanical relays and later on as solid state relays and finally as numerical relays. In this book we will describe the most notable developments of electromechanical relays and the implementation of these functions by numerical relays. Some of the most notable developments for the protective relay logic unit are listed below. These basic electromechanical systems were cleverly used to implement very complex protection schemes.

1. The plunger relay
2. The induction disk or induction cup relay
3. The cylinder unit relay
4. The balancing beam relay

In subsequent sections we will introduce these electromechanical systems and their utilization for various relaying functions.

5.6 Overcurrent Protective Relays

An overcurrent protective relay is a device that includes a monitoring system of specific electric currents, logic to initiate action and control circuit. The functional description of this relay is symbolically represented in Figure 5.10. The figure illustrates that the relay is connected to a Current Transformer (CT) that generates a current which is proportional to the current in the electric power system but much lower (instrumentation level current). The protective relay monitors the electric current in the secondary of the current transformer and when this current exceeds a certain value a logic activates which eventually may operate a contact that is integrated with the tripping system of the breaker as it is illustrated in the figure.

The overcurrent relay evolved over the years from simple electromechanical devices to complex electromechanical devices and then to solid state devices and finally to full digital systems. The first overcurrent relay was the plunger type relay. More sophisticated overcurrent relays were the induction (or cup) relay, followed with the digital implementation of these relays.

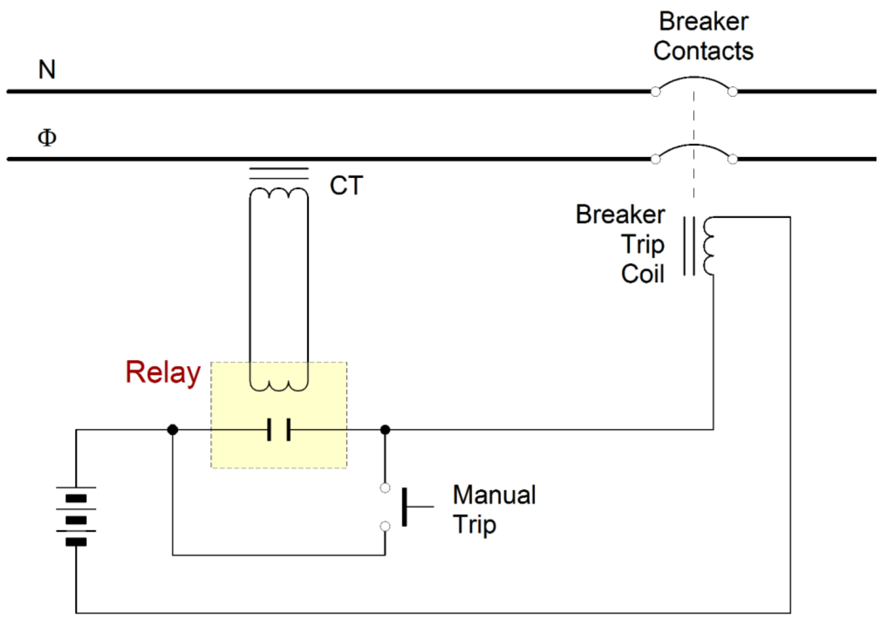


Figure 5.10: Functional Description of a Protective Relay

5.6.1 The Plunger-Type Overcurrent Relay

The first and simplest overcurrent relay design is a plunger type. This relay consists of an electromagnet that develops a force on a plunger when electric current flows through the coil. The plunger is kept at a rest position by a spring or gravity. When the electromagnetic force exceeds the force of the spring the plunger will move and will close a contact.

The construction of a typical plunger type relay is conceptually illustrated in Figure 5.11. The figure also shows the trip circuit of a breaker connected to the contacts of the plunger relay. The moveable element is in the center of the coil and can move vertically. Both movable and stationery parts are constructed with iron (magnetic material). The magnetic circuit involves two air gaps. Assuming that the magnetic resistance of the iron body of the plunger is negligible, the magnetic circuit can be approximately analyzed by considering the magnetic resistance of the air gaps only and the electromotive force of the coil.

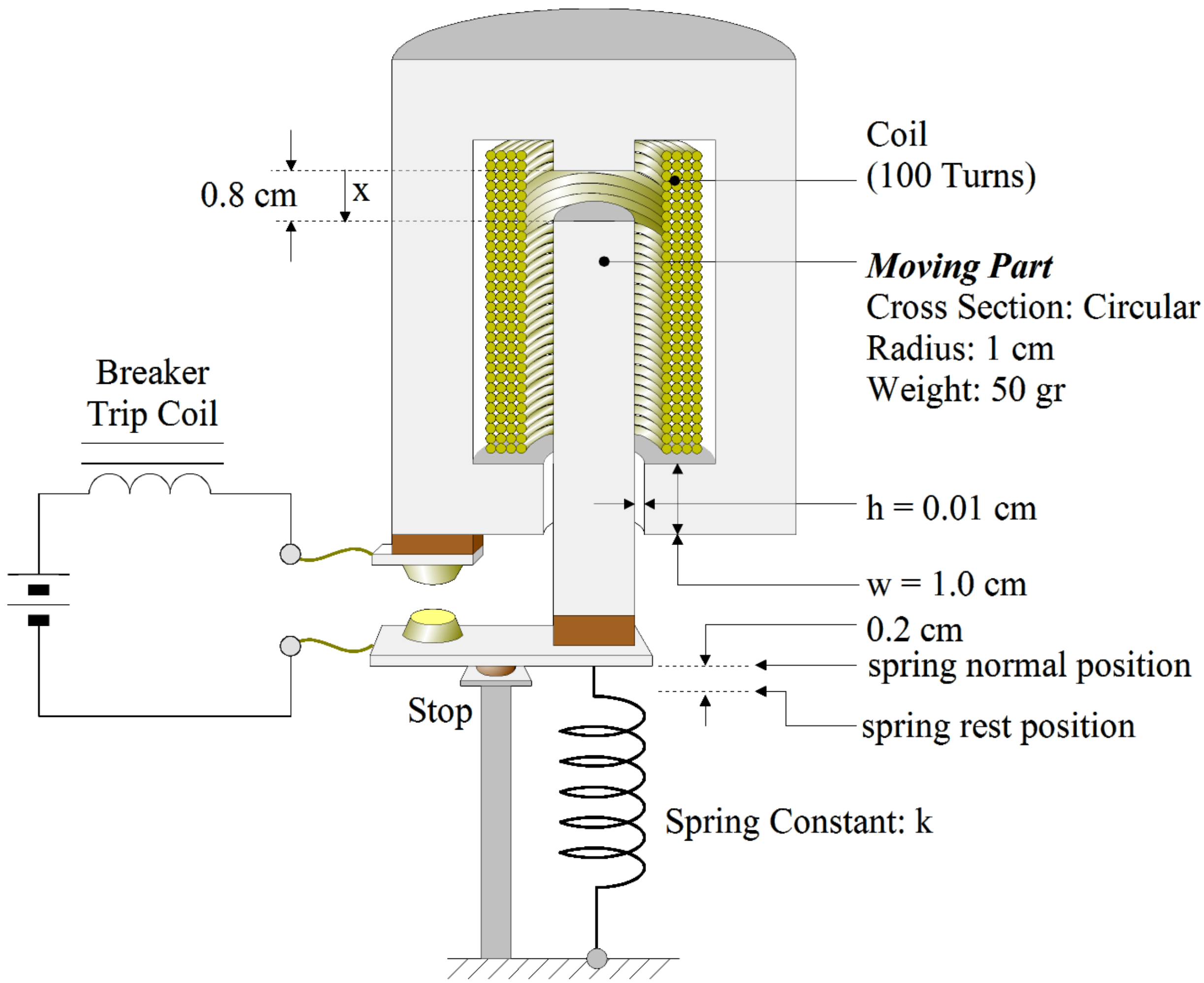


Figure 5.11: Conceptual Illustration of a Plunger Type Relay

The magnetic force exerted on the movable part is presented next. From Amperes law, the following can be derived for the magnetic circuit consisting of the plunger, body and air gaps:

$$\Re\varphi = Ni$$

where N is the number of coil turns, *i* is the coil current, φ is the magnetic flux, and $\Re$ is the total reluctance of air gaps. The reluctance of an air gap of length ℓ and cross-sectional area A is:

$$\Re = \frac{\ell}{\mu A}$$

Applying the above reluctance formula to each gap yields:

$$\Re = \frac{x}{\mu_0 A_1} + \frac{h}{\mu_0 A_2} = \frac{x}{\mu_0 \pi r^2} + \frac{h}{\mu_0 2\pi r w}$$

The flux through the magnetic circuit is:

$$\varphi = \frac{Ni}{\Re} = \frac{Ni}{\dfrac{x}{\mu_0 \pi r^2} + \dfrac{h}{\mu_0 2\pi r w}} = \frac{\mu_0 \pi r^2 Ni}{x + \dfrac{hr}{2w}} = \frac{c}{x+b} i$$, with b and c accordingly defined

Since $\varphi = B_1 A_1 = B_2 A_2$ at the two airgaps:

$$B_1 = \frac{c}{\pi r^2 (x+b)} i, \quad \text{and} \quad B_2 = \frac{c}{2\pi r w (x+b)} i$$

Next the force is computed as the rate of change of the energy stored in the magnetic field with respect to the plunger displacement (x). The magnetic field energy stored in each air gap can be computed given the magnetic flux density (flux over cross-sectional area) and the volume of the airgap as follows:

$$W_{mag} = \frac{1}{2} \frac{V}{\mu} B^2$$

Where φ =BA and V is the gap volume.

Note that there are two airgaps: (a) One of length x. The change of the magnetic energy with respect to x in this gap will create a force that will try to move the plunger upwards. (b) another of length h. The change of the magnetic energy with respect to x in this gap will create a force that will be horizontal and since the plunger is not permissited to move horizontrally this force is not of interest. We will focus on the vertical force

Thus the total magnetic field energy in the airgap of length x can be expressed as follows:

$$W_{mag} = \frac{1}{2} \frac{\pi r^2 x}{\mu_0} \left(\frac{c}{\pi r^2 (x+b)} i \right)^2 = \frac{c_1 x}{c_2 (x+b)^2} i^2$$

The magnetic force is computed as the partial derivative of the above expression with respect to the plunger displacement x:

$$F = \frac{\partial W_{mag}}{\partial x} = \frac{c_1}{c_2 (x+b)^2} i^2 - \frac{2 c_1 x}{c_2 (x+b)^3} i^2 = \frac{c_1 (b-x)}{c_2 (x+b)^3} i^2$$

Note that the force is proportional to the electric current squared. In addition when the airgap x decreases the force increases. When the magnetic force exceeds the force of the spring the plunger will move. As it moves the air gap x decreases and the force increases further causing the quick operation of the relay. The value of the current at which the magnetic force equals the spring force is the minimum current at which the relay will operate. This is the “pick-up” current. Note that there is no **intentional** time delay for the operation of this relay. We refer to this operation as instantaneous operation. In reality, the operation is not instantaneous since the movable part has some inertia and it takes some time to move from the normal position to the closed position. The term “instantaneous” refers to the fact that there is no intentional time delay. Once the relay has operated, it will reset only when the current assumes a small enough value so that the force from the spring will be greater than the magnetic force and will move the plunger to the rest position. The value of current at which this will occur is referred to as the “dropout” current. The value of the dropout current is smaller than the pickup value.

Example E5.1: Consider the plunger relay of Figure 5.11. The dimensions of the relay are: gap=0.8 cm, the cross area of the plunger is circular of 1 cm radius, the height of the plunger is 2 cm, and its weight is 50 gr. The clearance h is 0.01 cm, the width w is 1 cm, the spring constant is 50 N/m and the number of turns is 100. At the “open” position, the spring is stretched by 0.2 cm. For this relay compute the “pickup” current and the “dropout” current. Note: One kilogram-force (kg) equals 9.80665 Newtons (N).

Solution: The pickup value is determined by the condition the electromagnetic force equals the gravity and the force of the spring at the open position:

$$F_{gravity} + F_{spring} + F_{mag} = 0$$

where:

$$F_{gravity} = (0.05)(9.80665) = 0.49\ N$$

$$F_{spring} = (50\ N/m)(0.002\,m) = 0.1\ N$$

$$F_{mag}\left(x = 0.008\right) = \frac{c_1\left(b - x\right)}{c_2\left(x + b\right)^3} i^2 = 0.01523 i^2$$

thus:

$$i_{pickup} = \sqrt{\frac{0.49 + 0.1}{0.01523}} = 6.224\ A$$

To compute the “dropout current, repeat computations with x = 0, which yields:

$$F_{mag}\left(x = 0\right) = \frac{c_1}{c_2\left(b\right)^2} i^2 = 394.8 i^2$$

thus:

$$i_{dropout} = \sqrt{\frac{0.49 + 0.1}{394.8}} = 0.03866\ A$$

Note the very small dropout current value.

Limitations of Instantaneous Overcurrent (OC) Protection: Instantaneous overcurrent protective relays operate on the current through a circuit. While this operation is very simple it also has the disadvantage that they do not differentiate on the basis of the fault location or direction and they cannot be coordinated with other relays.

5.6.2 Time Overcurrent Relays

The plunger type relay described in the previous section will interrupt a fault immediately after the fault current becomes higher than the trip value, i.e. there is no delay other than the response of the tripping circuit. This type of operation does not allow for coordination and discrimination of what relay should respond to what fault condition. To provide flexibility in protection coordination it is important to control the time to trip once a fault current has been detected. Controlling the time to trip has been achieved with the introduction of the induction disk relay in the 1920s. Figure 5.12 illustrates the typical construction of an induction disk electromechanical time delay overcurrent relay.

Note to myself: delete this or move it elsewhere.

Note: *In practice many times only three such relays are used for a four wire system. The three relays monitor either (a) the current in the three phases of the system, or (b) the current in two phases and in the neutral wire. The rational for this practice is that any fault involves at least two wires, thus all possible faults will be detected. This thinking is faulty for the simple reason that there may be a fault from the phase without the relay to the ground in which case the fault will not be detected by any of the three relays. Such cases have actually been documented where a fault on the unprotected phase remained undetected resulting in dangerous conditions and in some cases loss of human life.*

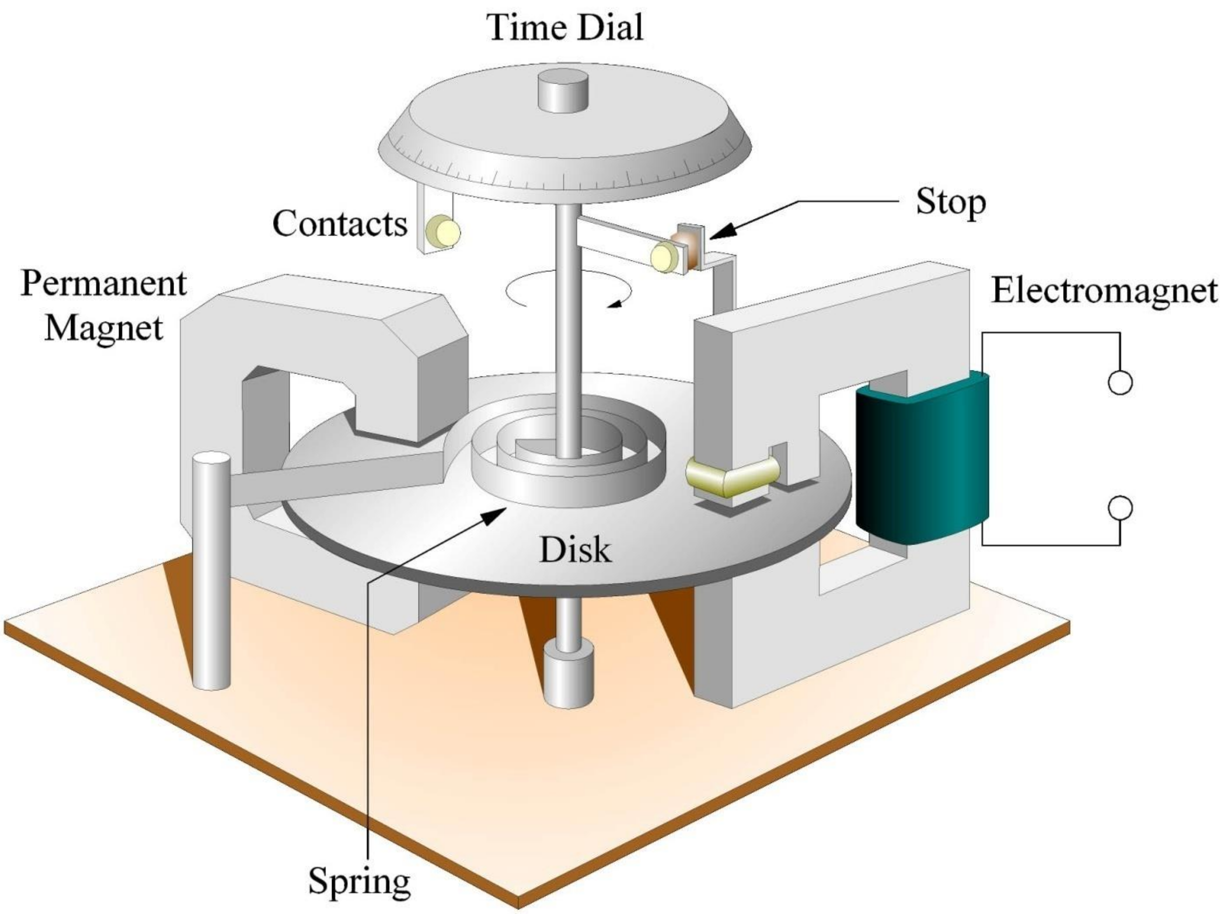


Figure 5.12: Electromechanical Time Delay Overcurrent Relay

This relay comprises a conducting non-magnetic disk (usually made of aluminum) that can rotate about a spindle. An electromagnet generates magnetic flux perpendicular to the disk surface. The electromagnet has two sets of poles one of which is encircled by copper rings. This results in a phase difference between the fluxes of the two-pole pair. The interaction between the two out of phase magnetic fluxes generates a net torque on the disk proportional to the square of the electromagnet coil current. Furthermore, torque in the opposite direction is applied by a helical spring, and a permanent magnet. Note that the permanent magnet generates a torque only when the disk starts moving and it is opposing the motion. The spring torque is proportional to the rotation angle of the disk, while the permanent magnet torque is proportional to the rotation speed, and in the direction opposing the disk rotation.

The torque generated by the electromagnet is derived as follows: Consider that the magnetic fluxes generated by the two poles and passing through the disk are φ_1 and φ_2. The fluxes are time varying thus they induce eddy currents in the disk. The induced currents are proportional to the rate of change of the fluxes:

$$i_{d1} = \alpha \frac{d\varphi_1}{dt}, \qquad i_{d2} = \alpha \frac{d\varphi_2}{dt}$$

where α is a constant depending on the geometry and material characteristics. The torque developed by the interaction of the magnetic flux and induced currents is:

$$T = \varphi_2 i_{d1} - \varphi_1 i_{d2}$$

Assuming the fluxes are sinusoidal:

$$\varphi_1 = \Phi_1 \sin(\omega t + \theta_1), \quad \varphi_2 = \Phi_2 \sin(\omega t + \theta_2)$$

Then the developed torque is:

$$\begin{aligned} T &= \varphi_2 i_{d1} - \varphi_1 i_{d2} = \varphi_2 \alpha \frac{d\varphi_1}{dt} - \varphi_1 \alpha \frac{d\varphi_2}{dt} \\ &= \alpha\omega\Phi_1\Phi_2 \sin(\omega t + \theta_2)\cos(\omega t + \theta_1) - \alpha\omega\Phi_1\Phi_2 \sin(\omega t + \theta_1)\cos(\omega t + \theta_2) \\ &= \alpha\omega\Phi_1\Phi_2 \sin(\theta_2 - \theta_1) \end{aligned}$$

Since the flux generated by the electromagnet is proportional to the coil current the generated torque will be:

$$T = k\omega I^2$$

Where k is a constant and I is the RMS value of the coil current. The equation of motion for the disk is:

$$J\frac{d^2\theta_d}{dt^2} = T_{total}$$

where J is the moment of inertia of the disk and T is the net torque applied on the disk (developed by the electromagnet, magnet, and spring).
Since there is friction (as a matter of fact the permanent magnet generates substantial torque that is proportional to the speed), the equation of motion will be:

$$J\frac{d^2\theta_d(t)}{dt^2} + k_d\frac{d\theta_d(t)}{dt} = T - T_{spring} = k\omega I^2 - T_0 - \left(\frac{T_{\theta_{\max}} - T_0}{\theta_{\max}}\right)\theta_d(t)$$

Where: k_d is the damping factor, θ_{max} is the maximum travel angle of the induction disk, and $T_{\theta max}$ and T_0 are the spring torques at maximum displacement and at the initial position, respectively.

The solution of above equation is complex. Of course numerical techniques can be used to obtain the solution. We will consider here two alternative approximate solutions.

Approximate Solution 1: This solution involves the following approximations and assumptions: the damping factor is negligible (this means the permanent magnet shown in Figure 5.12 is not present) and the spring torque is approximately constant. Then, if the disk must travel θ_{max} until making contact, the time to operate will be (for a constant current):

$$t_o = \sqrt{\frac{2J\theta_{\max}}{kI^2 - T_{spring}}}$$

The above approximation is valid for the case that the permanent magnet does not exist and thus the damping torque is negligible. For a device with a permanent magnet the damping torque is significant leading to the second approximate solution:

Approximate Solution 2. This solution involves the following approximations and assumptions: the inertia term is neglected as well as the additional torque from the elongation of the spring. This is a reasonable approximation for a device with an intentional damping provided by the permanent magnet, making the k_d term dominant. Then the time to operate is given by the equation:

$$t_0 = \frac{\dfrac{k_d \theta_{\max}}{T_0}}{\left(I_r^2 - 1\right)},$$

where:

I_r is the normalized electric current or $\left(I_r = \dfrac{I}{I_{pickUp}} \right)$, and

$$I_{PickUp} = \sqrt{\frac{T_0}{k}}\,.$$

Note that the form of above equation is:

$$t_0 = \frac{A}{I_r^2 - 1}$$

For large values of current the time to operate is inversely proportional to the square of the input current. By designing the electromagnet so that the magnetic core saturates, and adding a secondary mechanism to provide a constant time delay the time-current characteristic becomes:

$$t_0 = \frac{A}{I_r^p - 1} + B$$

where the parameters p, A, and B depend on physical design parameters. Thus, by adjusting the physical design parameters different time-current characteristic can be implemented. The time-current characteristics are classified according to the steepness of the curves and the overall time delay as:

- Short Time
- Long Time
- Definite Minimum Time
- Moderately Inverse
- Inverse
- Very Inverse, and
- Extremely Inverse

A typical family of curves of belonging to the *Inverse* category is shown in Figure 5.13.

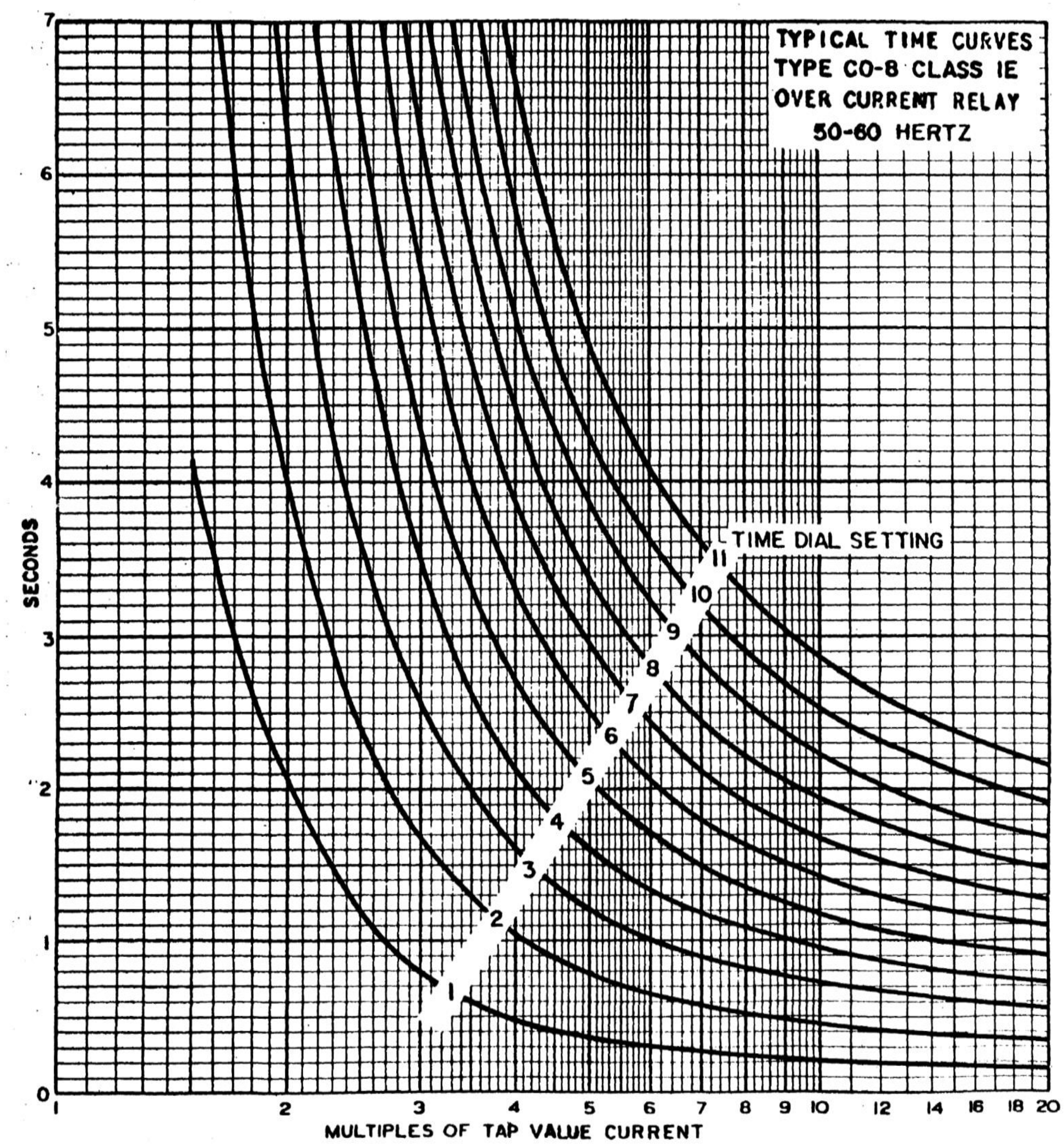


Figure 5.13: Typical Overcurrent Relay Time-Current Curves
Curve Classification: Inverse Time
Curve Index is Time Dial Setting
(Courtesy ABB)

Example E5.2: Consider an induction disk type relay with the following parameters:

$$\theta_{\max} = 0.1\, rads, \quad T_0 = 0.0005\, N.m, \quad k_d = 0.1\, N.m.\sec, \quad k = 0.00002\, N.m / A^2$$

(a) Compute the pickup current for this relay.
(b) Compute the time versus current trip curve of this relay.

Solution: We will assume the approximate solution of the previous paragraph.

(a) The pickup current is:

$$I_{PickUp} = \sqrt{\frac{T_0}{k}} = 5\, A$$

(b) The time versus current curve is computed point by point:

$$t_0 = \frac{\frac{k_d \theta_{\max}}{T_0}}{\left(I_r^2 - 1\right)} = \frac{20}{0.04 I^2 - 1} \quad \sec$$

The results are tabulated below, and plotted in Figure E5.2 in log-log scale.

Current (A)	Time (sec)
5	infinity
10	6.666
20	1.333
40	0.317
80	0.078

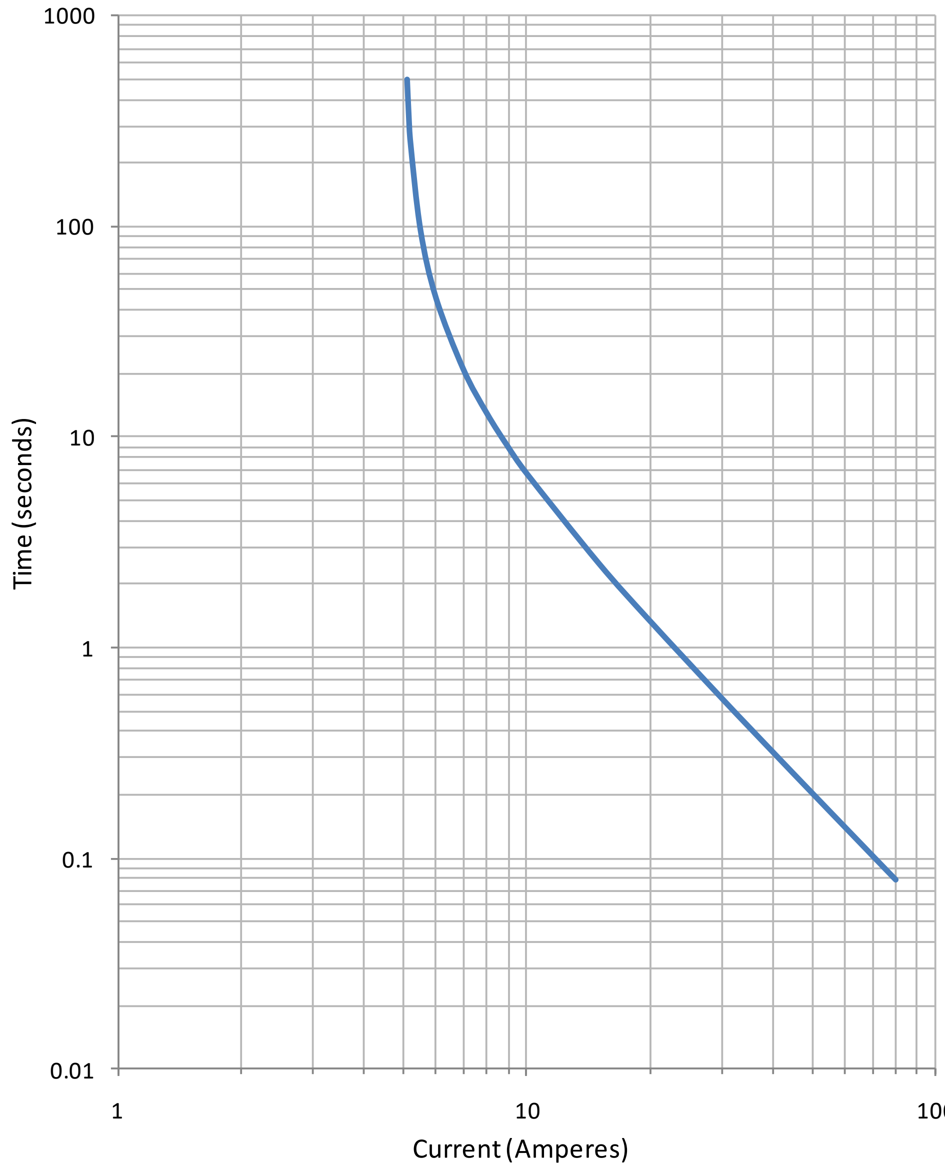


Figure E5.2: Time-Current Curve of Induction Disk Relay of Example E5.2

Example E5.3: Consider a 600 A circuit, 13.8 kV, two miles long. The source impedance (on a 100 MVA basis) is:

$$z_1 = z_2 = j0.30\ pu, \quad z_0 = j0.28\ pu$$

The CT is rated 1200:5A. The impedance of the circuit is

$$z_1 = z_2 = j0.70\, ohms/mile, \quad z_0 = j2.10\, ohms/mile$$

Select the relay pickup current so that the relay will open the breaker for any type of fault anywhere along the distribution circuit in less than 30 cycles. The engineer has a numerical relay with the following characteristics:

Moderately Inverse: $$t_0 = \frac{0.0515}{I_r^{0.02} - 1} t_d + (0.114)\, t_d$$

Very Inverse: $$t_0 = \frac{19.61}{I_r^2 - 1} t_d + (0.491)\, t_d$$

Extremely Inverse: $$t_0 = \frac{28.2}{I_r^2 - 1} t_d + (0.1217)\, t_d$$

Where t_d is the time dial that can be set to the following discrete values: 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, 7.0 and 10.0.

Solution: First it should be observed that the least fault current will occur for a fault at the end of the circuit. For completeness we compute the currents at the source end and the remote end. The positive, negative and zero sequence equivalent circuits, with all parameters in pu on a 100 MVA, 13.8 kV system, are:

The three phase fault current and the single phase to ground fault current at the source end of the line is:

$$I_{3\Phi,s} = \frac{1.0}{j0.30} = -j3.333\, pu \quad or\, 13.944\, kA$$

$$I_{1\Phi,s} = (3)\frac{1.0}{j0.30 + j0.30 + j0.28} = -j3.409\, pu \quad or \quad 14.262\, kA$$

The three phase fault current and the single phase to ground fault current at the remote end of the line is:

$$I_{3\Phi,r} = \frac{1.0}{j0.30 + j0.7351} = -j0.966\, pu \quad or \quad 4.04\, kA$$

$$I_{1\Phi,r} = (3)\frac{1.0}{j0.30 + j0.30 + j0.28 + j0.7351 + j0.7351 + j2.2054} = -j0.6585\, pu \quad or \quad 2.754\, kA$$

Next select a CT with a ratio such that the secondary current will not exceed 100 A under maximum fault current and 5 A at maximum load the current. This will ensure that the CT will operate reliably and within its accuracy specifications. For this system we select 1200:5. In this case the secondary current at maximum load will be 2.5 Amperes and at the maximum fault current is be 59.42 Amperes. Next we select the pickup current to be at about twice the maximum load current or 1200 A primary, or 5 Amperes at the secondary of the CT. Next, we can select the relay characteristic that will interrupt any fault at time less than 30 cycles.

For a fault at the end of the circuit (fault current 2,754 A, therefore I_r=2.295) the trip times will be:

The moderately inverse relay with time dial set at the minimum value of 0.1 yields:

$$t_0 = \frac{0.0515}{I_r^{0.02}-1}t_d + (0.114)t_d = \frac{0.0515}{(2.295)^{0.02}-1}t_d + (0.114)t_d = (3.1879)t_d = 0.3188\,\text{sec}$$

The very inverse relay with time dial set at the minimum value of 0.5 yields:

$$t_0 = \frac{19.61}{I_r^2-1}t_d + (0.491)t_d = \frac{19.61}{(2.295)^2-1}t_d + (0.491)t_d = (5.0867)t_d = 0.5086\,\text{sec}$$

The extremely inverse relay with time dial set at the minimum value of 0.5 yields:

$$t_0 = \frac{28.2}{I_r^2-1}t_d + (0.1217)t_d = \frac{28.2}{(2.295)^2-1}t_d + (0.1217)t_d = (6.7305)t_d = 0.6730\,\text{sec}$$

For a close in fault the trip times will be:

Moderately inverse:

$$t_0 = \frac{0.0515}{I_r^{0.02}-1}t_d + (0.114)t_d = \frac{0.0515}{(11.885)^{0.02}-1}t_d + (0.114)t_d = (1.12875)t_d = 0.1129\,\text{sec}$$

Very inverse:

$$t_0 = \frac{19.61}{I_r^2-1}t_d + (0.491)t_d = \frac{19.61}{(11.885)^2-1}t_d + (0.491)t_d = (0.6308)t_d = 0.0631\,\text{sec}$$

Extremely inverse:

$$t_0 = \frac{28.2}{I_r^2-1}t_d + (0.1217)\,t_d = \frac{28.2}{(11.885)^2-1}t_d + (0.1217)t_d = (0.3227)t_d = 0.0323\,\text{sec}$$

Note that we can meet all the requirements of the protective scheme by selecting the moderately inverse time overcurrent relay.

Applications of Time-Delay Induction Disk Relays: Induction disk or cup-type electromechanical relays can be made to operate on overvoltage, undervoltage, differential and directional. For example, for overvoltage operation, one should simply apply the voltage on the main coil of the relay. Then the current through the coil will be proportional to the voltage. Thus, the relay will respond when the voltage exceeds a specific value, i.e. an overvoltage relay. The specifics will be discussed later in the chapter.

Limitations of Time Overcurrent (OC) Protection: Overcurrent relays operate on the current through a circuit. While their operation is simple they also have the disadvantage that they do not differentiate on the basis of the fault location or direction. On the other hand they have some limited capability for coordination by controlling the time delay to trip. Additional selectivity can be provided by supervising the operation of a time overcurrent relay as well as instantaneous with a directional function. This will be discussed in a later section.

Transient Overreach: The operation of time overcurrent relays is influenced by fault current transients. These transients may result in fault currents that vary as time progresses and therefore they affect the time to trip. Transient current waveforms may be of varying amplitude, contain DC offset, contain harmonics, etc. We normally lump the total impact of all these factors on the time to trip of the relay as “transient overreach”. In other words, "transient overreach" is the premature operation of a relay because of transients in the monitored current waveform which generate additional torque to the relay and speed up its operation. Specifically, the term transient overreach is defined as follows: Let t_1 be the time to trip assuming that the fault current is constant amplitude and symmetric (pure sinusoidal waveform). Let also t_2 be the time to trip during a fault for the actual fault current seen by the relay that includes DC offset, harmonics, etc. but the fundamental component of the current is identical to the sinusoidal current resulting to trip time t_1. Then the transient overreach is defined with:

$$TransientOverreach = \frac{t_1 - t_2}{t_1} \quad (100)$$

The transient overreach phenomenon is illustrated by an example.

Example E5.4: Consider an induction disk type time overcurrent relay. Consider the following two cases:

(1) The fault current is a pure sinusoid of rms value of I.
(2) The fault current contains a fundamental of rms value of I and a third harmonic of 0.25I. Note that the rms value of the total current is 1.0308I. Therefore, the rms values of the two cases are practically identical.

Compute the transient overreach in the case of the fault current with harmonics.

Solution: Note that for the two cases the following two equations will apply

$$J\frac{d^2\theta_d(t)}{dt^2} + k_d\frac{d\theta_d(t)}{dt} = T - T_{spring} = kI^2 - T_0 - \left(\frac{T_{\theta_{\max}} - T_0}{\theta_{\max}}\right)\theta_d(t)$$

$$J\frac{d^2\theta_d(t)}{dt^2} + k_d\frac{d\theta_d(t)}{dt} = T - T_{spring} = kI^2 + 3k(0.25I)^2 - T_0 - \left(\frac{T_{\theta_{\max}} - T_0}{\theta_{\max}}\right)\theta_d(t)$$

Note that we assume that in case of the presence of the third harmonic we will have an additional torque that will be proportional to the harmonic order and proportional to the magnitude of the harmonic current squared. This is a reasonable approximation which ignores some nonlinearities.

Using approximate solution 2, the solution of the first equation yields:

$$t_1 = \frac{\frac{k_d \theta_{\max}}{T_0}}{\left(I_r^2 - 1\right)},$$

The solution from the second equation is:

$$t_2 = \frac{\frac{k_d \theta_{\max}}{T_0}}{\left(1.1875 I_r^2 - 1\right)},$$

The transient overreach is:

$$\frac{t_1 - t_2}{t_1}(100) = \left(1 - \frac{I_r^2 - 1}{1.1875 I_r^2 - 1}\right) 100$$

Using above formula, for a current equal to twice the pickup current, the transient overreach will be 20%.

5.6.3 Digital Instantaneous and Time Delay Overcurrent Relays

The electromechanical time-overcurrent relays have been replicated with digital relays and over the years the digital time-overcurrent relay has become the relay of choice as compared to the electromechanical relays. Specifically, the response of the time overcurrent electromechanical relays has been approximated with analytical expressions (equations). Numerical relays use these equations to determine time to trip etc. An IEEE standard summarizes this approach: “IEEE Std C37.112-1996: IEEE Standard Inverse-Time Characteristic Equations for Overcurrent Relays”. The standard provides the equations as a function of the normalized current M (actual current divided by the pickup current). It is also important to note other international standards (such as IEC 255-03, Electrical Relays, Part 3) define similar equations. Today we have an array of such equations (sometimes referred to as US and European). These equations represent close approximations to electromechanical relay operating characteristics.

As an example, the equations for normalized current M above 1.0 (trip characteristic), are:

For M > 1

$$t(I) = \frac{A}{M^p - 1} + B$$

Where:

t(I) is the trip time (function of I)

M is I / I_{pickup} (I_{pickup} is the relay current set point)

A, B, p are constants determining time-current curve characteristics, as follows

Characteristic	A	B	p	t_r (seconds)
Moderately Inverse	0.0515	0.1140	0.02000	4.85

Very Inverse	19.61	0.4910	2.0000	21.6
Extremely Inverse	28.2	0.1217	2.0000	29.1

Where t_r is the reset time.

The standards also provide the equations for normalized current M below 1.0. These equations provide the reset characteristic of these relays.
The equations make the application of these relays better fitted to computerized approaches as we can easily graph the response of the relays and visualize their coordination with other relays. An example will illustrate the points.

Example E5.5: Consider the distribution feeder of Figure E5.5. The feeder consists of three 3-phase sections of lengths 1.8 miles, 0.6 miles and 0.2 miles respectively; and a lateral (single-phase) distribution circuit of 0.3 miles length. The WinIGS model is provided. The feeder is protected by a breaker at the substation, a recloser at the indicated location and fuse at the indicated lateral (a lateral is a single phase distribution line). Assume that both the breaker and recloser are equipped with numerical relays that have instantaneous overcurrent protection as well as time-overcurrent protection with the following selections:

Moderately Inverse: $t_0 = \dfrac{0.0515}{I_r^{0.02} - 1} t_d + (0.114) t_d$

Very Inverse: $t_0 = \dfrac{19.61}{I_r^2 - 1} t_d + (0.491) t_d$

Extremely Inverse: $t_0 = \dfrac{28.2}{I_r^2 - 1} t_d + (0.1217) t_d$

Where t_d is the time dial that can be set from 0.1 to 15.0 (any numerical value in this interval).

(a) It is desired to protect the feeder by isolating the minimum possible circuit for any fault anywhere in the feeder and laterals and clearing the fault in time of less than 30 cycles. No recloser operation is allowed. Select the CT size for the breaker, the settings of the breaker relay (instantaneous pickup, time overcurrent pickup, and time dial), the CT size for the recloser and the settings of the recloser (instantaneous pickup, time overcurrent pickup, and time dial) and the size of the fuse (use a fuse from Figure 5.3, Chapter 5).

(b) It is desired to protect the feeder by maximizing the service to customers by allowing one reclosing operation and “fuse saving” tactics. Select the CT size for the breaker, the settings of the breaker relay (instantaneous pickup, time overcurrent pickup, and time dial), the CT size for the recloser and the settings of the recloser for the first operation (instantaneous pickup, time overcurrent pickup, and time dial) as well as for the second operation and the size of the fuse (use a fuse from Figure 5.3). It is given that a minimum time of 45 cycles is required between recloser operations.

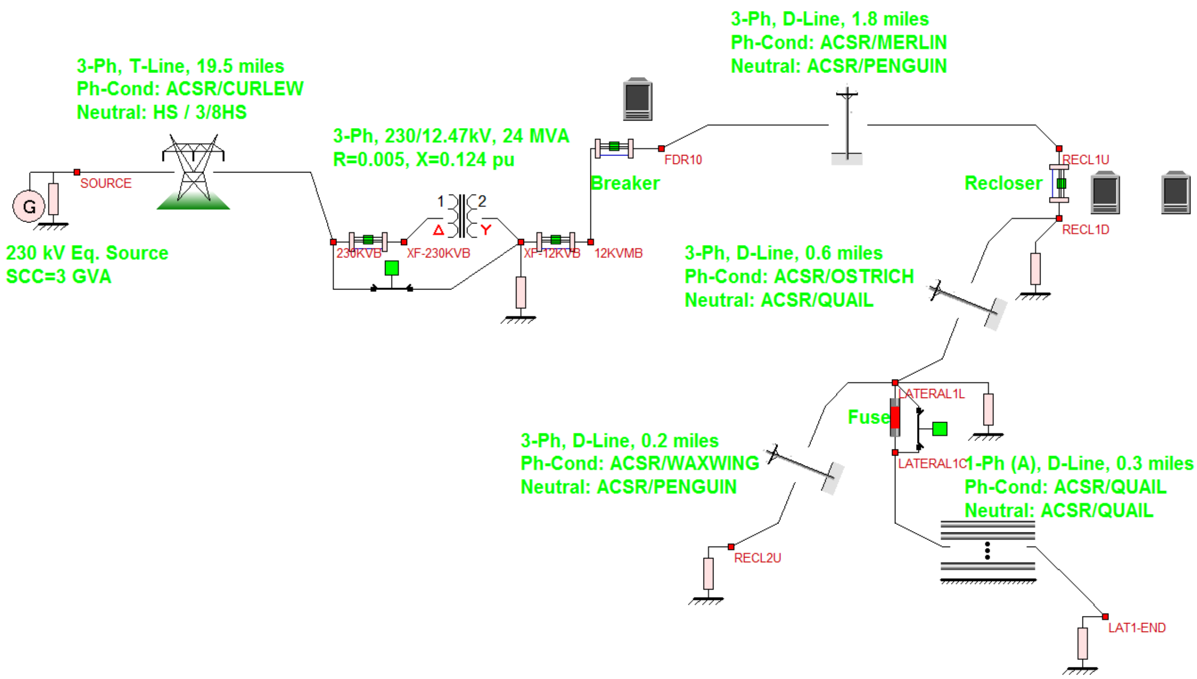


Figure E5.5: Example Distribution System

Solution: The first step is to determine the fault currents along the circuit.
to be completed

5.7 Differential Relays

Differential relays detect internal faults in devices (or more general in a protection zone) such as transformers, generators, motors, reactors, capacitors etc., by monitoring the electric currents on all terminals of a device (or protection zone) and forming the sum of all currents. The sum of all the currents should be identical to zero by Kirchhoff's current law. Thus, differential relays are so designed as to "see" zero current under normal operating conditions or external faults. In case of an internal fault the relay will "see" a substantial current and it will trip the device. Differential relays can detect internal device bolt faults as well as internal high impedance faults (with some limitations) without any additional information or coordination with other protective devices. This section presents the fundamentals of differential protection. In other chapters we present application of differential relaying to several types of devices/protection zones.

The principle of operation of differential relays is illustrated in Figure 5.14. Note that we have a simple device to be protected (assume for example a bus with three circuits). The figure illustrates the differential relay connection for one phase only. Note that the system uses three identical CTs (i.e. same transformation ratio) and an overcurrent relay R. Let $\tilde{I}_1, \tilde{I}_2, and\ \tilde{I}_3$ be the currents in phase A of the three circuits respectively. The direction of the current is selected to flow into the protected device, in this case the bus, as it is illustrated in Figure 5.14 (note that we will be using this convention for convenience). An analysis of the current flow reveals that the current through the overcurrent relay is:

$$\tilde{I}_R = k(\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3)$$

where k is the transformation ratio of the current transformers.

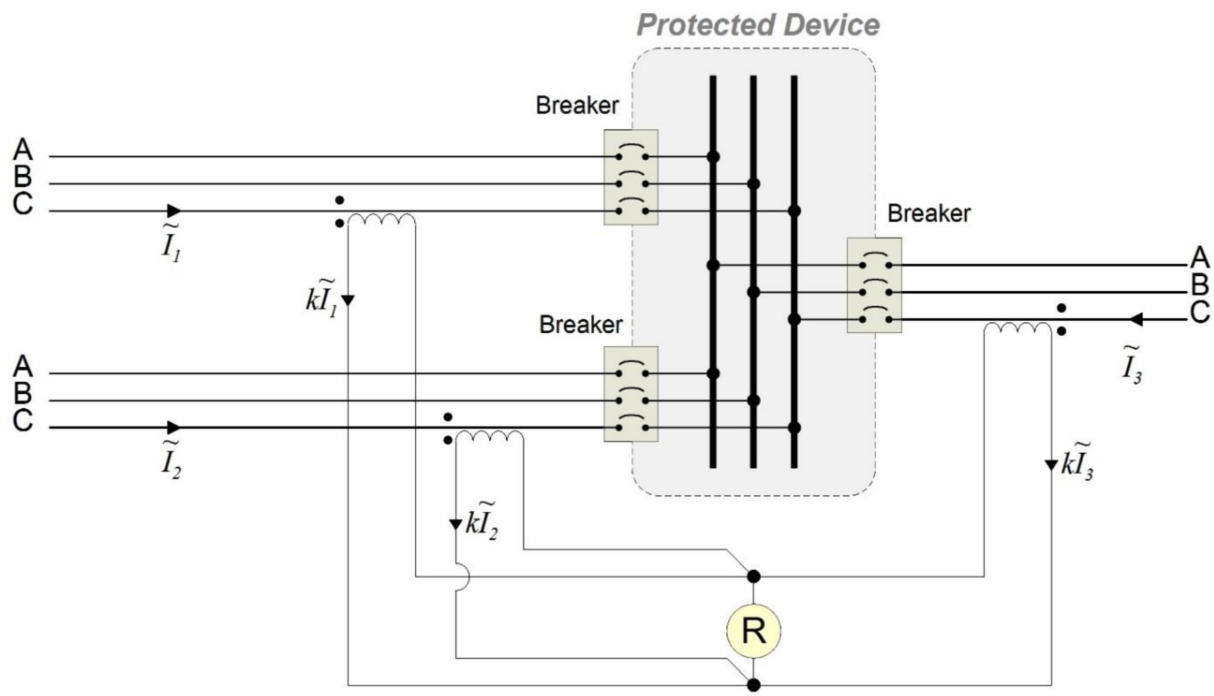


Figure 5.14: Differential Relaying Principle Using Time Overcurrent Relay

Modify to show only two currents

It is important to note that when there is no fault in the system or the fault is outside the area marked by the current transformers, $\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 0$ by virtue of Kirchhoff's current law. Therefore under normal operating conditions or external faults the overcurrent relay will "see" zero current and it will not operate. The overcurrent relay can be set to respond when the current through the relay is relatively small, for example 0.5 Amperes. In general a time overcurrent characteristic can be selected (or instantaneous) with a pickup current of 0.5 Amperes.

It is important to note that under ideal conditions, i.e. perfect (ideal) current transformers the electric current through the sensing device (overcurrent relay in Figure 5.14) will be exactly zero during normal operating conditions or during external faults. Unfortunately current transformers are not ideal (there is error introduced by the impedance of the current transformers, mismatches in transformation ratios, saturation of the magnetic core, especially in case of external faults with large currents, etc.). The end result is that some electric current will be flowing at the location of the sensing device (relay R) under all conditions and especially in cases of an external fault with high fault current. If the "differential current" is due to CT ratio mismatches and other linearly behaving phenomena, it will be proportional to the current through the CTs. On the other hand it is not desirable to operate the relay for these conditions (normal or external faults). To overcome this shortcoming, the concept of a *restraining quantity* is introduced, defined as the weighted sum of the current magnitudes in the secondary of the current transformers, i.e. $I_R = \sum_i n_i k_i |I_i|$.

In this equation, k_i are the transformation ratios of the current transformers. The relay is designed to respond when the ratio I_O / I_R exceeds a certain threshold, where I_O is the operating current through the relay and I_R is the restraining current. We refer to this function as percentage differential protection.

Figure 5.15 shows a percentage differential relay implementation based on the above criterion. Note that the Figure shows that the relay has four coils. Three coils carry the secondary current of the CTs (restraining currents) and one coil carries the sum of these currents (operating current which is supposed to be zero under ideal conditions). The actual implementation of the percentage differential relay is discussed next.

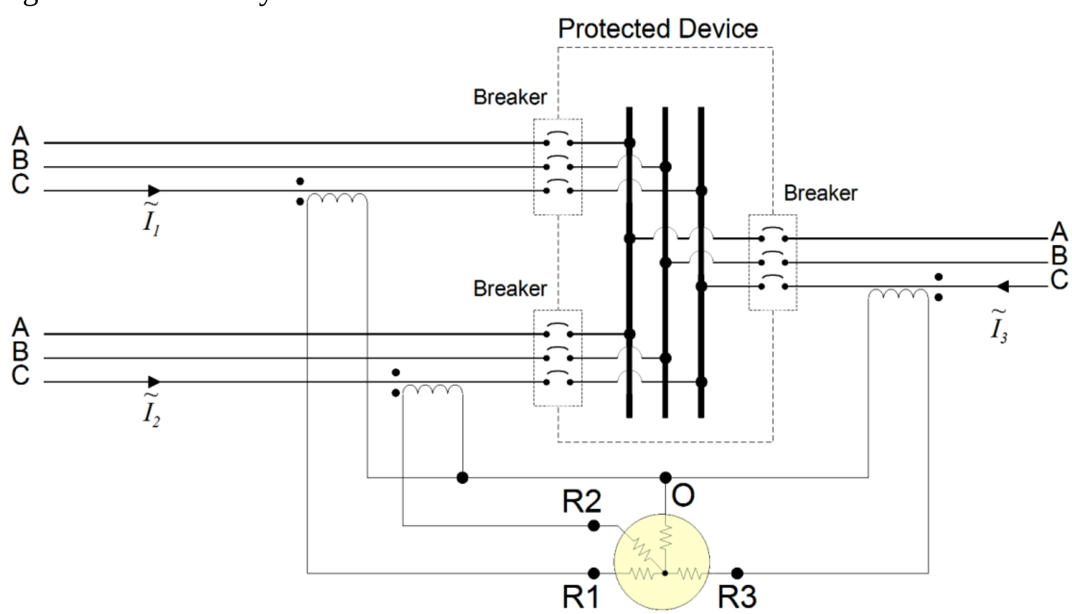


Figure 5.15: Connections for a Percentage Differential Relay with Restraining Currents

Modify to show only two currents

The percentage differential protection has been implemented in the past with electromechanical systems, for example, the balancing beam or the cylinder relay. By far the balancing beam implementation is the most compact and clever. The balancing beam implementation is shown in Figure 5.16. The current transformers (CTs) secondary windings are connected to the coils of the balanced beam electromechanical relay illustrated in Figure 5.16. The connections are shown in Figure 5.15. The currents in the restraining coils generate the force F_R while the currents in the operating coil generate the force F_0. Note that the relay will trip (i.e. the contacts will close) if the force F_0 is greater than the force F_R. As a first approximation the forces F_0 *and* F_R are proportional to the electromotive force of the coils, i.e.

$$F_O = a(N_0 I_0)^2$$
$$F_R = a(N_R I_{R1} + N_R I_{R2} + N_R I_{R3})^2$$

where N_0 and N_R are the number of turns in the operating and restraining coils respectively. The parameter α is a constant depending on the geometry of the electromagnet and the air gap. The contacts of the balancing beam will close when the force F_0 will overcome the force F_R and the force of the spring, i.e.

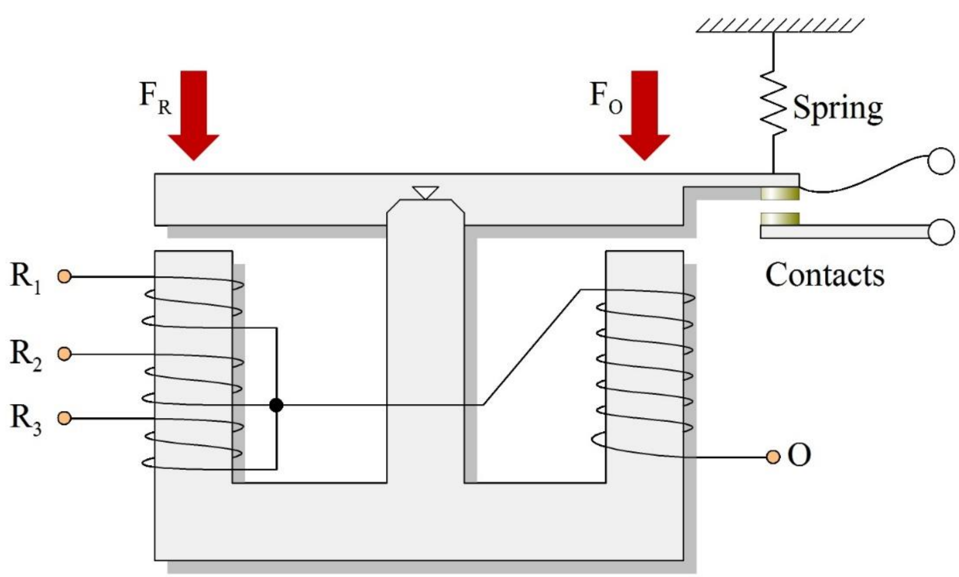
$$F_0 > F_R + F_{spring}$$

Or

$$a(N_0 I_0)^2 > a(N_R I_{R1} + N_R I_{R2} + N_R I_{R3})^2 + F_{spring}$$

By introducing: $I_R = \frac{1}{3}(I_{R1} + I_{R2} + I_{R3})$ above equation becomes:

$$\frac{I_0}{I_R} > \sqrt{\frac{9N_R^2}{N_0^2} + \frac{F_{spring}}{aN_0^2 I_R^2}} = m$$



Wrong polarity, needs correction – just remove / modify example

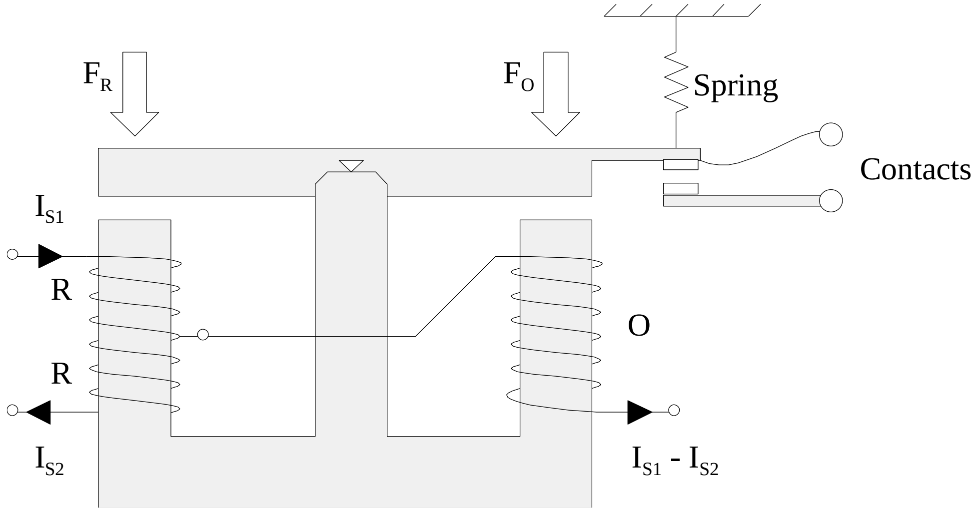


Figure 5.16: Balancing Beam Percentage Differential Relay Configuration

Note the parameter m is approximately constant since in general the spring force will be very small as compared to the electromagnetic force. The balancing beam can be designed to provide constants, 0.05, 0.10, 0.20, 0.40, 0.60, etc.

Many times percentage differential is applied to protection zones with more than two terminals. For example, it is common that percentage differential for a bus zone may have many input currents as it is illustrated in Figure 5.17. In this case, the restraining force must be generated for each restraining current. Figure 5.18 shows an electromechanical implementation of a percentage differential relay where each one of the currents, operating current and restraining currents are passing through a shaded electromagnet that creates a torque on an induction disk. The design of each shaded electromagnet is so selected as to provide the desired torque to enable a desired percentage restraint.

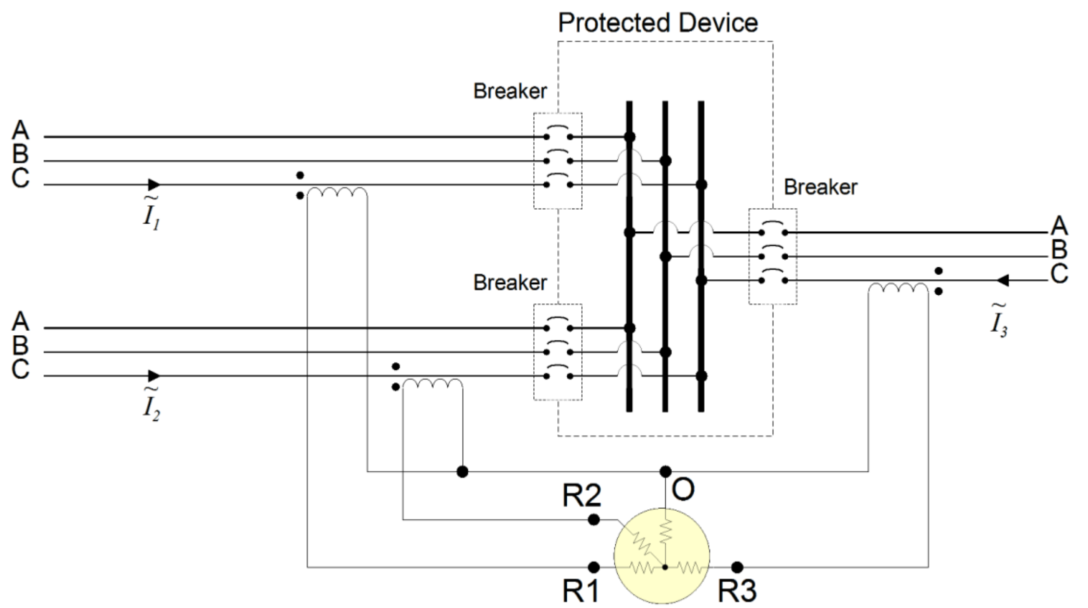


Figure 5.15: Connections for a Percentage Differential Relay with Restraining Currents

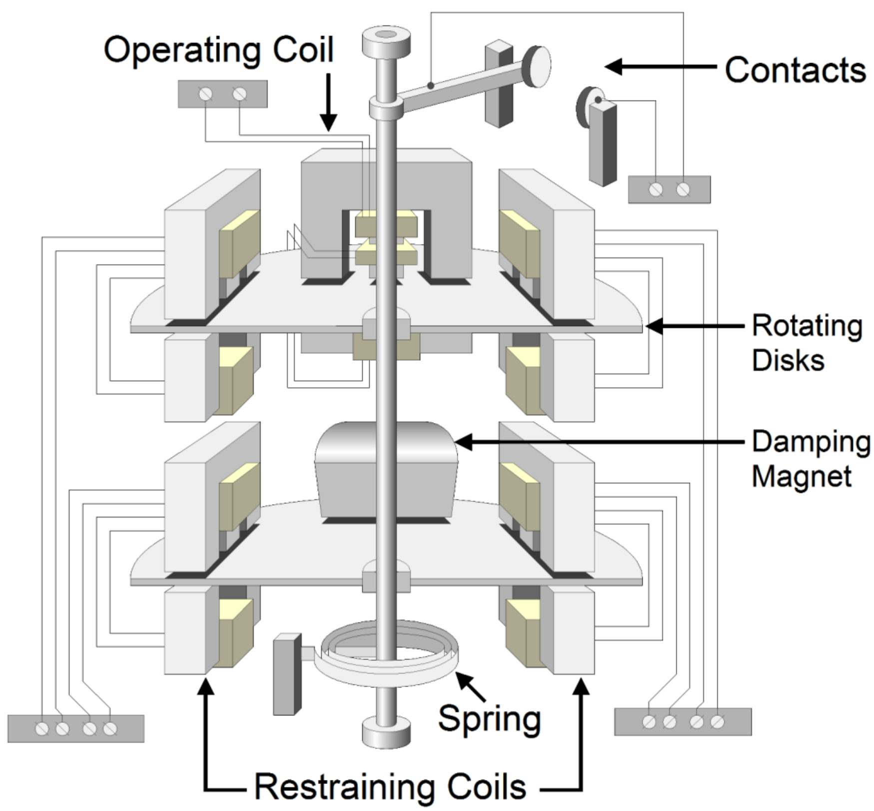


Figure 5.17: Rotating Disk Differential Relay Configuration for Bus Protection Applications

The percentage differential protection scheme is one of the simplest and most secure protection schemes. Presently, when we talk about differential protection we almost exclusively mean percentage differential protection. The advent of digital relays made it possible to implement percentage differential functions with multiple settings (multiple percentage or multiple slopes). These implementations will be discussed later. In the following paragraphs we will discuss some general application issues of differential protection. Coverage of differential protection for specific devices in greater depth will be provided in subsequent chapters.

5.7.1 Differential Bus Protection

Bus protection is of critical importance because faults on buses have the potential to result in multiple outages. False bus tripping is detrimental to power system operation since buses are usually connected to multiple lines, transformers, etc. and therefore any bus fault will result in an outage of multiple components. Differential protection is a very effective scheme to detect and act on only bus faults and to discriminate all other faults.

Differential bus protection is a simple, secure and popular method. The scheme uses identical CTs and the output of the CTs is connected as to provide the sum of the currents through all CTs The differential bus protection scheme is illustrated in Figure 5.17.

The most challenging problem in differential bus protection is CT saturation. High current external faults can saturate CT's, and cause false tripping. Since fault current levels can be very high at buses CT saturation is a real issue.

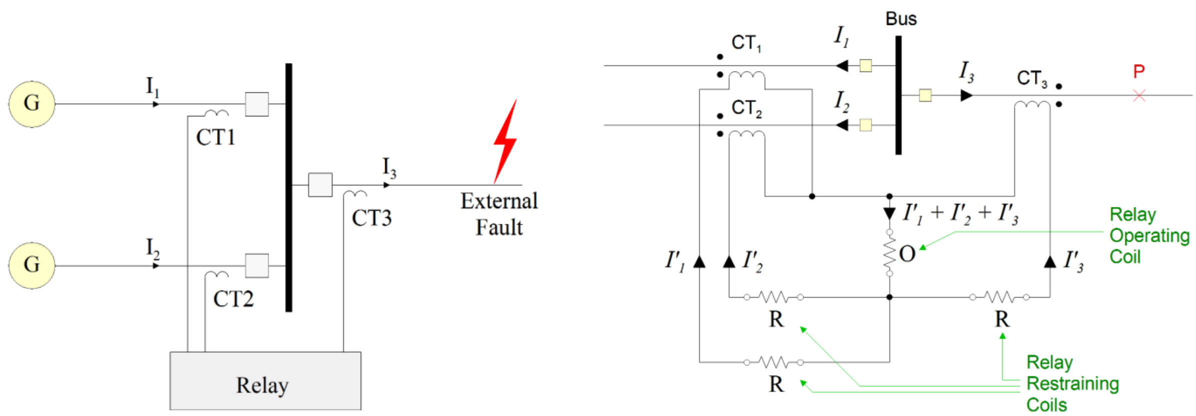


Figure 5.18: Percentage Differential Bus Protection

Methods for false tripping reduction:

- Differential Overcurrent with Variable Percentage Restraint
- High Impedance Voltage Relays
- Moderately High Impedance Voltage Relays
- Directional Overcurrent Relays
- Linear Couplers

- Ground Fault Protection

The above methods are described next.

Differential Overcurrent with Variable Percentage Restraint: This scheme uses increasing percentage as the fault current level increases. The operating region of this scheme is illustrated in Figure 5.18. The scheme has limited success when the fault currents are very high.

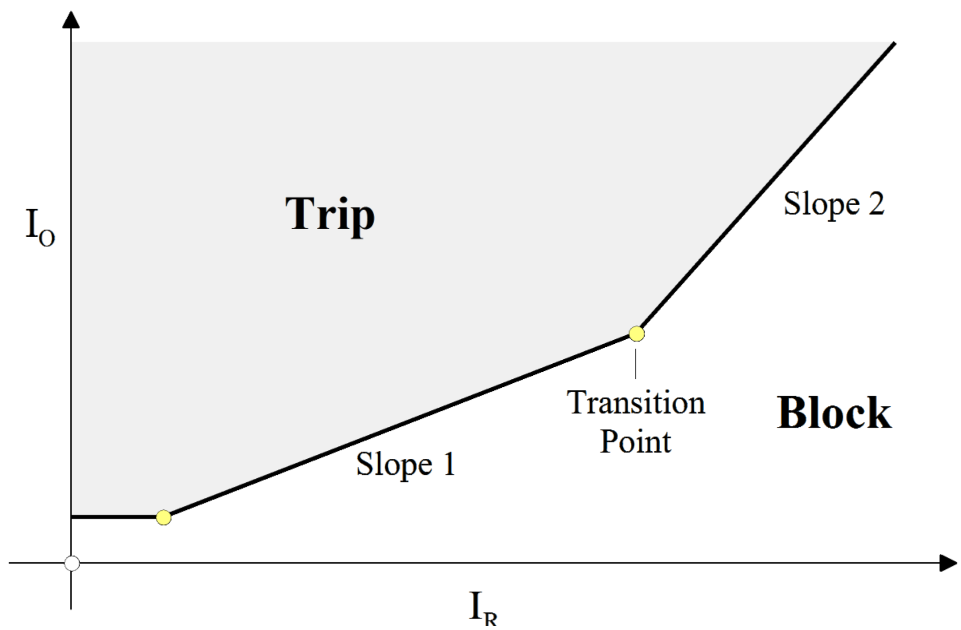


Figure 5.19 Variable Percentage Differential Scheme

High Impedance Differential Overcurrent Bus Protection: This scheme uses high impedance relays. It avoids issues with CT saturation. The scheme is describe in section 5.7.4.

5.7.2 Differential Transformer Protection

Transformers present several unique challenges in the application of differential relaying protection due to (a) current transformation among the various sides of the transformer, (b) saturation characteristics and inrush current phenomena during energization or disturbances, (c) phase shifts in case of delta/wye transformation between sides of the transformer. At the same time, differential protection is always a basic protection function for transformers. It is relatively rare that a transformer may not be protected with a differential scheme.

Single Phase Transformers: For simplicity, the application of differential relaying to transformers is introduced by a single-phase two-winding transformer. However this protection scheme can be easily generalized to three-phase multi-winding transformers. Consider a single phase transformer with N_1/N_2 turns ratio. Let I_1 and I_2 be the primary and secondary currents. The differential transformer protection is based on the observation that under normal operating conditions, the ratio of the primary and secondary currents is constant, and approximately equal to the inverse of the transformer turns ratio. Thus the quantity $I_O = N_1 I_1 - N_2 I_2$ will remain nearly equal to zero, unless an internal fault occurs. However, variable tap transformers to a large extent and instrumentation errors to a lesser extent make this simple criterion inappropriate for practical applications. Specifically, ratio errors of the current transformers result in I_O being proportional to the transformer load current. This shortcoming can be overcome by use of percentage differential relays. Figure 5.19 shows a percentage differential relay protection system for a single phase transformer. Two current transformers (CT's) monitor the transformer primary and secondary currents. The CT secondary windings are connected to the coils of the percentage differential relay. The relay will trip (i.e. the contacts will close) if:

$$\frac{I_O}{I_R} > K$$

and the constant K depends on the relay construction (number of winding turns, beam arm lengths etc). The constant K can be selected among several values (such as 10%, 25% and 50%).

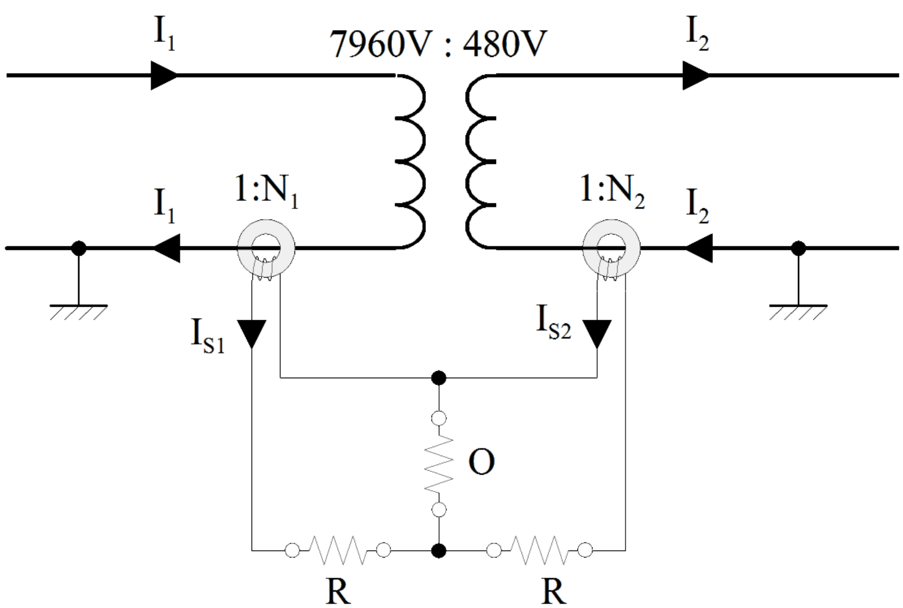


Figure 5.20: Single Phase Transformer Differential Protection

Figure 5.20 illustrates the trip and block regions of the above system. The slope of the line separating the Trip and Block regions is equal to the factor K. Note that for low current values, the trip/block line curves away from the graph origin due to the spring action. This is a desirable property since it prevents false tripping due to the magnetizing current, when the transformer is unloaded.

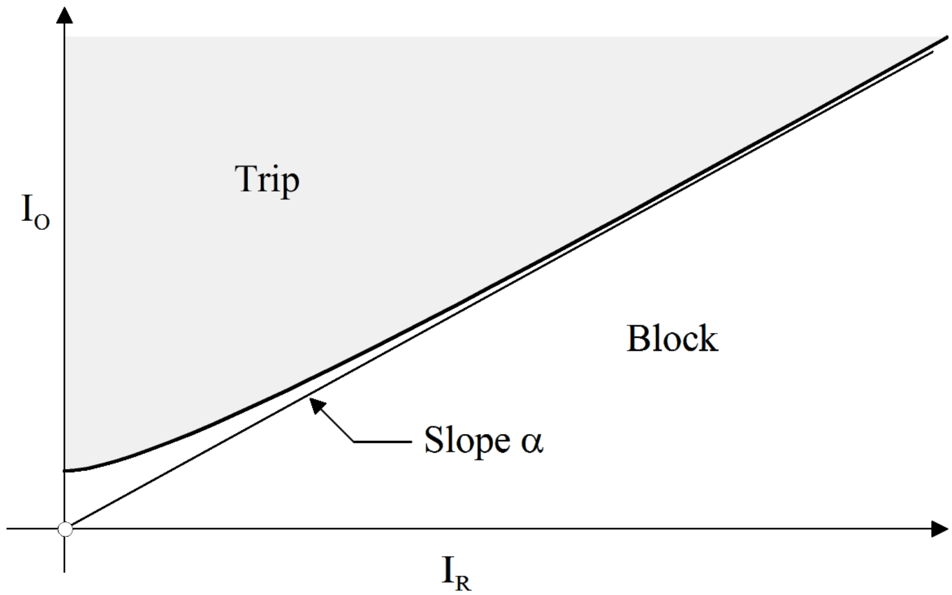


Figure 5.21: Electromechanical Differential Relay Response

Note that the CT ratios must be appropriately selected so that the operating current quantity $I_O = I_{S1} - I_{S2}$ is nearly zero under normal operating conditions. The selection will be illustrated with an example.

Example E5.6: Consider a single phase, 1.2 MVA, 7.96kV:277V transformer with a 5.4% impedance. The transformer is connected to a 7.9 kV system with a fault current capability of 18.3 kA. Select the percentage differential relay scheme for this transformer.

Solution: For this example, the following CT ratio relationship will satisfy this requirement:

$$\frac{N_2}{N_1} = \frac{7960}{277} = 18.736$$

In practice, CT's must be selected from commercially available standard ratios. The following selection from standard CT ratios approximates the above requirement:

Primary CT - 150:5
Secondary CT - 3000:5

Note that the above selection represents a ratio mismatch of:

100 x (1.0 – (150)(18.736)/(3000)) = 6.32%.

Most electromechanical differential relays provide additional adjustments by means of operating and restraining coil taps, usually in 1% increments. Digital relays provide even finer adjustments based on numerical manipulations.

In addition to the ratio mismatch, the CT ratio error should be considered for the maximum fault currents. This error is determined from CT manufacturer data. Note that the CT error is a function of the measured current and burden impedance. If the CT core saturation occurs, the CT error increases by a large factor.

Finally, if the monitored transformer is equipped with a load tap changer, the maximum deviation from the nominal turns ratio should also be considered. For example, assume that the CT ratio errors are 5% and that the tap changer maximum setting is 10%. Then the maximum total ratio mismatch will be 6.32% + 5% + 10% = 21.32%. In this case setting the percentage restraint factor k at 25% will provide adequate restraint under all loading conditions and external faults to prevent false tripping. This simplistic approach will be further refined in section 7 (transformer protection).

Three-Phase Transformers: The presented differential protection scheme is generalized to three phase transformers by monitoring all transformer terminal currents as illustrated in Figure 5.21. The principle of operating and restraining currents are derived in a similar manner as for the single phase case, i.e. we construct an operating current that it is as close as possible to zero for normal operating conditions or through faults (faults external to the transformer). In doing so we need to pay attention to the phase shifts introduced in case of a Delta-Wye or Wye Delta transformations. One simple approach that cancels this phase shift is to connect the delta side CT secondary windings in Wye configuration, and the Wye side CT secondary windings in Delta configuration, as illustrated in Figure 5.21. It is important to note that there are many types of three phase transformers: two winding transformers (delta-wye, wye-wye, etc.), autotransformers with or without tertiary, as well as three winding transformers. In each case the phase relationships must be accounted for. Note that most digital relay implementations can remove the delta-wye phase shift computationally, thus being able to work with any CT arrangement.

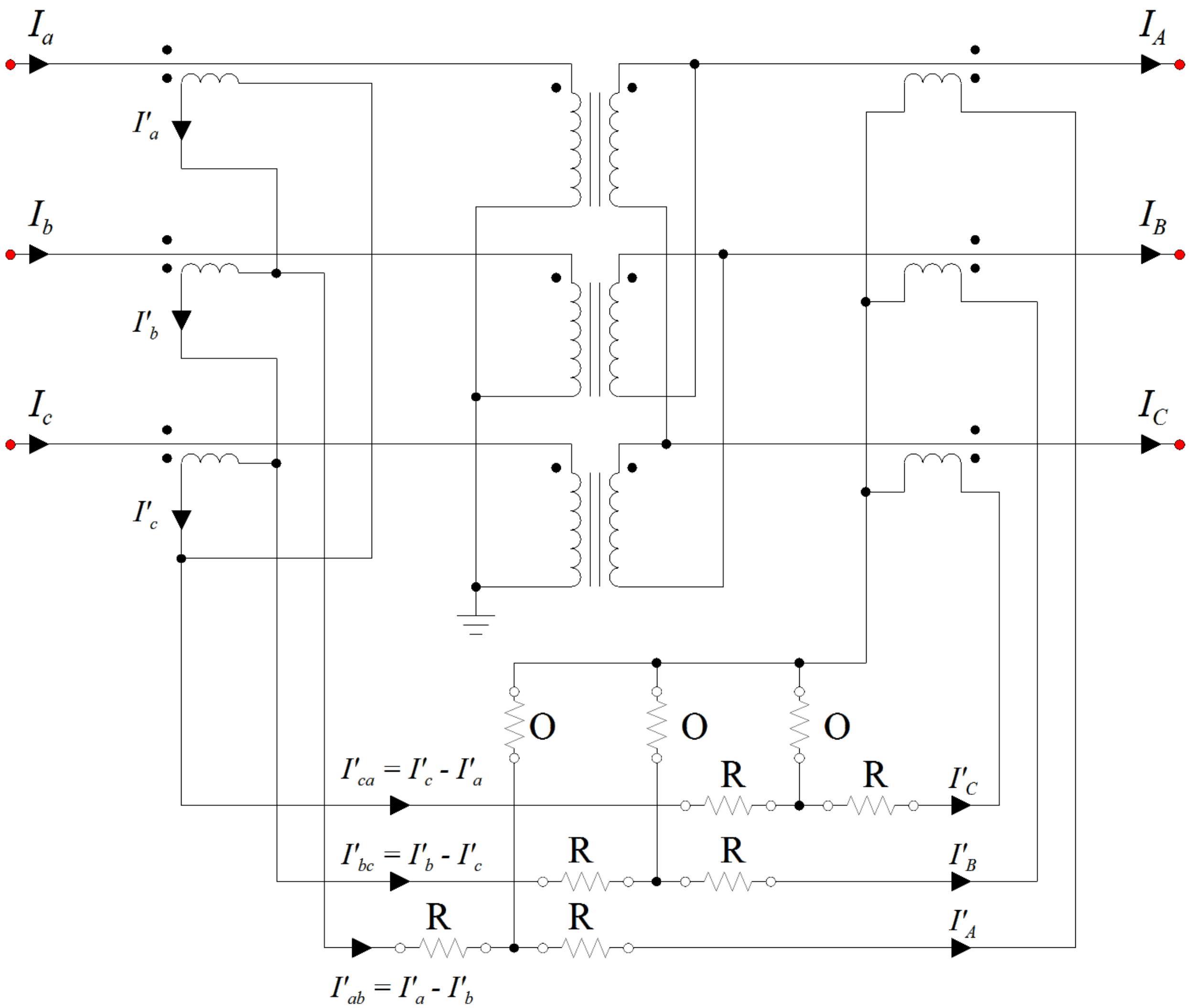


Figure 5.22: Three-Phase Wye-Delta Connected Transformer Differential Protection
(Delta Side is High Voltage and Wye Side is Low Voltage, Standard Connection)

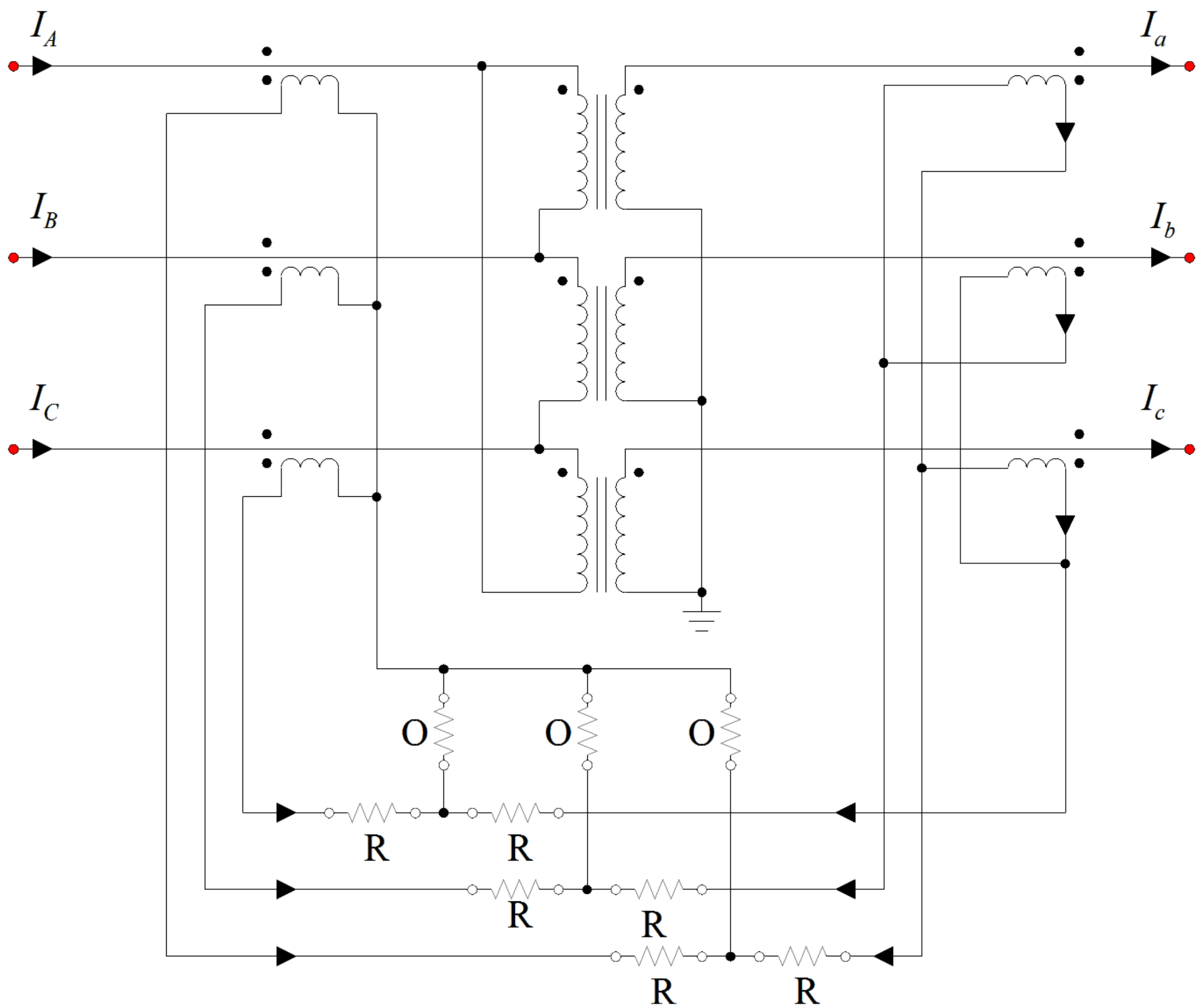


Figure 5.23: Three-Phase Delta-Wye Connected Transformer Differential Protection
(Delta Side is Low Voltage and Wye Side is High Voltage, Standard Connection)

Note that in selecting the CT ratios in the above configuration the factors arising from Delta versus Wye connection ($\sqrt{3}$) must be taken into account. The procedure is illustrated next by an example.

Another advantage of the above CT connection is that the zero sequence current in the event of an external ground fault on the Wye transformer side is blocked by the delta connection of the CT's on that side. This prevents false tripping since the delta side zero sequence currents are also zero. In the case of the computational phase shift correction afforded by digital relay implementations the zero sequence currents should also be removed computationally. Modern digital differential relays perform these tasks by appropriate numerical transformations. The drawback of this approach is that the percentage differential scheme becomes insensitive to ground faults on the wye side especially for ground faults near the neutral of the transformer. These faults can be detected with another differential scheme that we refer to it as sensitive earth fault scheme. We shall discuss this scheme in section 7.

Transformer Inrush Currents: When transformers are energized a large magnetizing current may occur, lasting up to several seconds. The phenomenon of inrush current generation is well known and it is illustrated in Figure 5.22. A typical inrush current waveform is shown in Figure 5.23. The magnetizing current appears only on the source side of the transformer and therefore it appears on its entirety on the operating coil of the differential relay. Thus, large magnetizing currents can cause false tripping of the differential protective relay scheme presented above.

The nature of the magnetizing currents in power transformers is illustrated by an example problem, next.

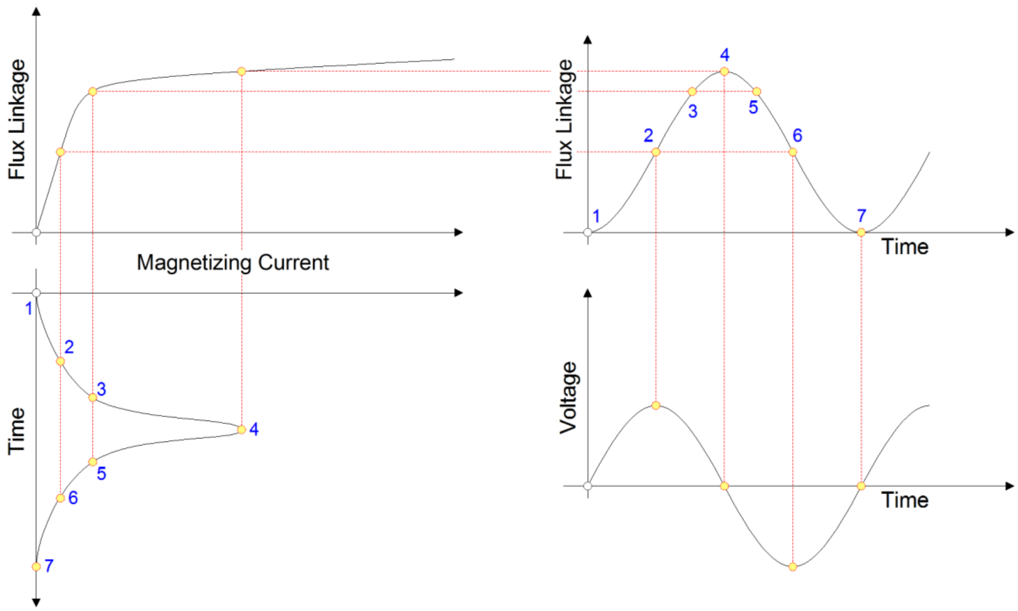


Figure 5.x: Inrush Current Generation Phenomena

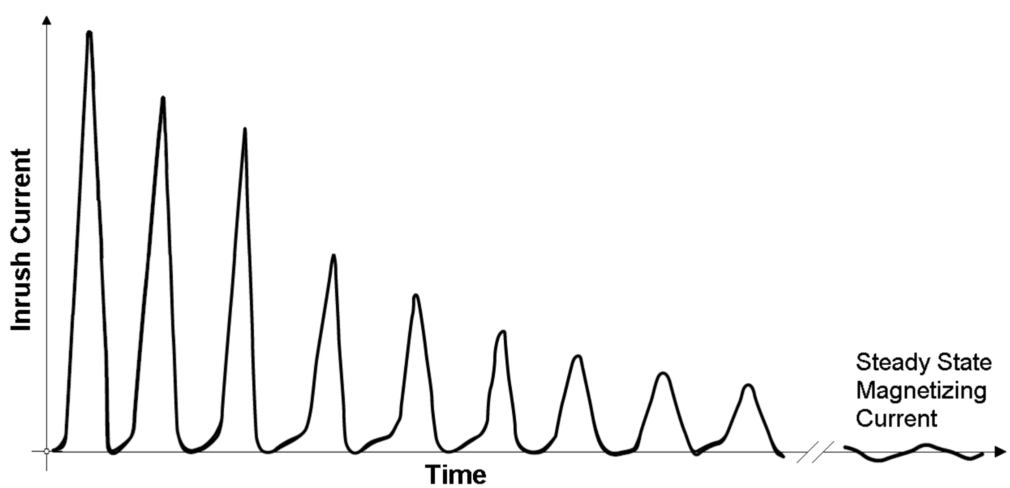


Figure 5.24: Typical Waveform of Inrush Current

Example E5.7: An iron core 14.4 kV/240V, 30 kVA, 60 Hz transformer is energized from an ideal voltage source. The voltage source produces a voltage equal to:

$$e(t) = \sqrt{2}E\cos(\omega t + 1.00) \quad volts,$$

where: E=14.44 kV, ω=377 sec^{-1}

The transformer has the following magnetic flux linkage versus magnetizing current relationship:

$$i_m(t) = i_0\left(\frac{|\lambda(t)|}{\lambda_0}\right)^8 sign(\lambda(t))$$

where, i_0 equals 0.01 pu and λ_0 equals 1.0 pu on the transformer ratings.

The transformer is energized at time t=0. Compute the maximum value of the magnetizing current in Amperes. What is the second harmonic current value of the inrush current for this transformer and for the specified condition?

Solution: The differential equation governing the relationship of flux and voltage is:

$$\frac{d\lambda(t)}{dt} = e(t)$$

Initial condition of the flux linkage of the transformer is:

$$\lambda(0) = 0$$

$$\lambda(t) = \int_0^t e(\tau)\,d\tau = \frac{\sqrt{2}E}{\omega}\sin(\omega t + 1.0) - \frac{\sqrt{2}E}{\omega}\sin(1.0) = 54.1677\sin(\omega t + 0.1) - 45.5806 \quad Wb$$

The maximum magnetizing current $i_m(t)$ is obtained at maximum flux linkage $\lambda_{\max}(t)$:

$$\lambda_{\max}(t) = (54.1677)(-1) - 45.5806 = -99.748\,Wb$$

$$i_{m,\max}(t) = 0.01\left(\frac{30\times 10^3}{14.4\times 10^3}\right)\left(\frac{99.748}{54.1677}\right)^8(-1) = -2.7547\ A$$

The second harmonic current value of the inrush current for this transformer is:

$$\frac{1}{\sqrt{2}}\sqrt{a_2^2 + b_2^2}$$

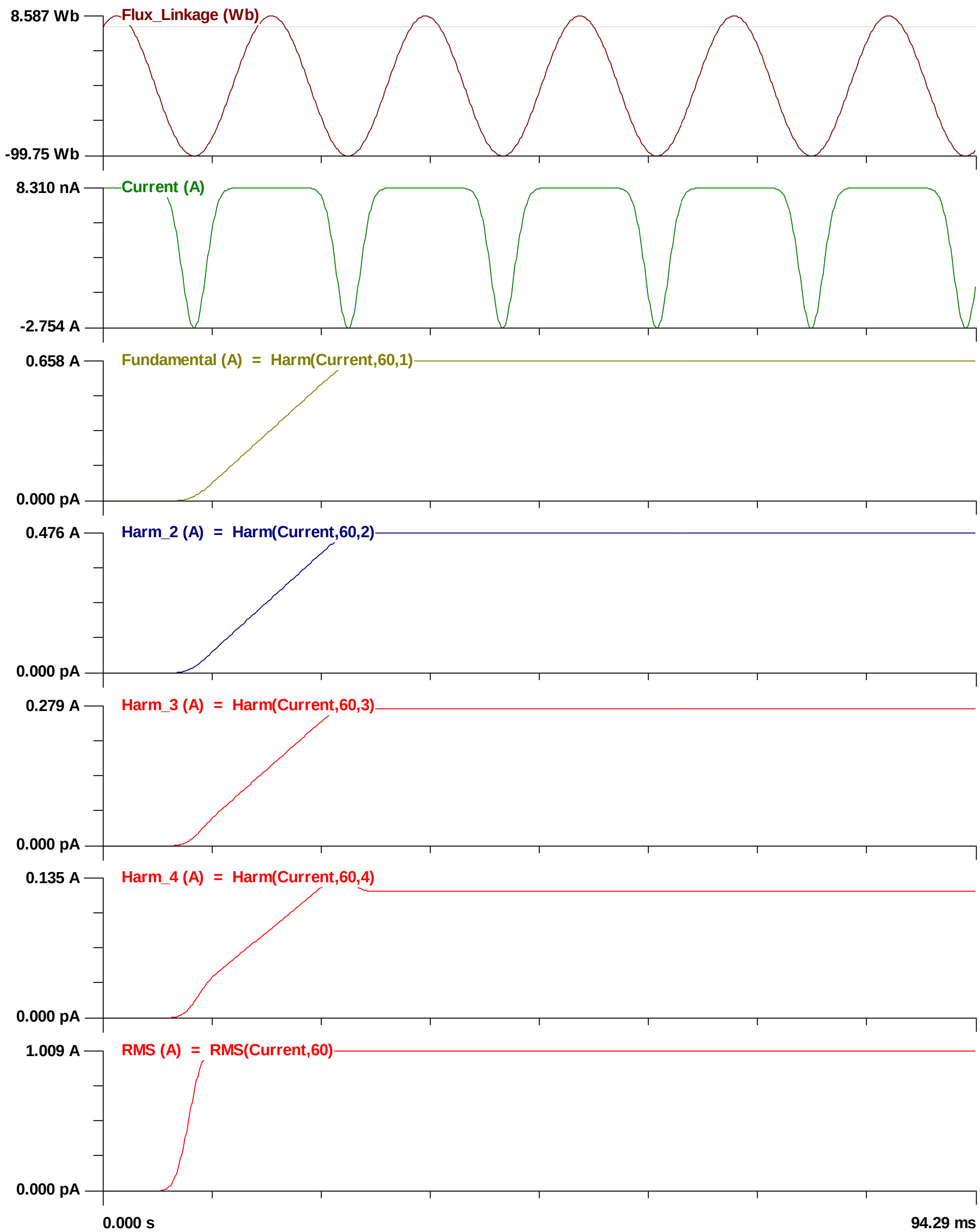
where: $a_2 = \int_0^T i_m(t)\sin(n\omega t)\,dt$ $\qquad$ $b_2 = \int_0^T i_m(t)\cos(n\omega t)\,dt$

The above integrals were computed numerically for n = 0, 1, 2, 3, and 4 yielding the corresponding harmonics. The results are listed in the Table E5.7:

Table E5.7 Excitation Current Harmonics

Harmonic Order	RMS Value (A)
DC	-0.525
Fundamental	0.658
2nd	0.475
3d	0.273
4th	0.123
RMS Value	1.009

From the above example problem it can be seen that magnetizing currents are characterized by large harmonic distortion. Both even and odd harmonics are present due to the asymmetry of the current waveform (positive and negative pulses have different magnitudes). This characteristic can be used to identify magnetizing currents and prevent differential relay false tripping.


8.587 Wb
Flux_Linkage (Wb)
-99.75 Wb
8.310 nA
Current (A)
-2.754 A
0.658 A
Fundamental (A) = Harm(Current,60,1)
0.000 pA
0.476 A
Harm_2 (A) = Harm(Current,60,2)
0.000 pA
0.279 A
Harm_3 (A) = Harm(Current,60,3)
0.000 pA
0.135 A
Harm_4 (A) = Harm(Current,60,4)
0.000 pA
1.009 A
RMS (A) = RMS(Current,60)
0.000 pA
0.000 s
94.29 ms

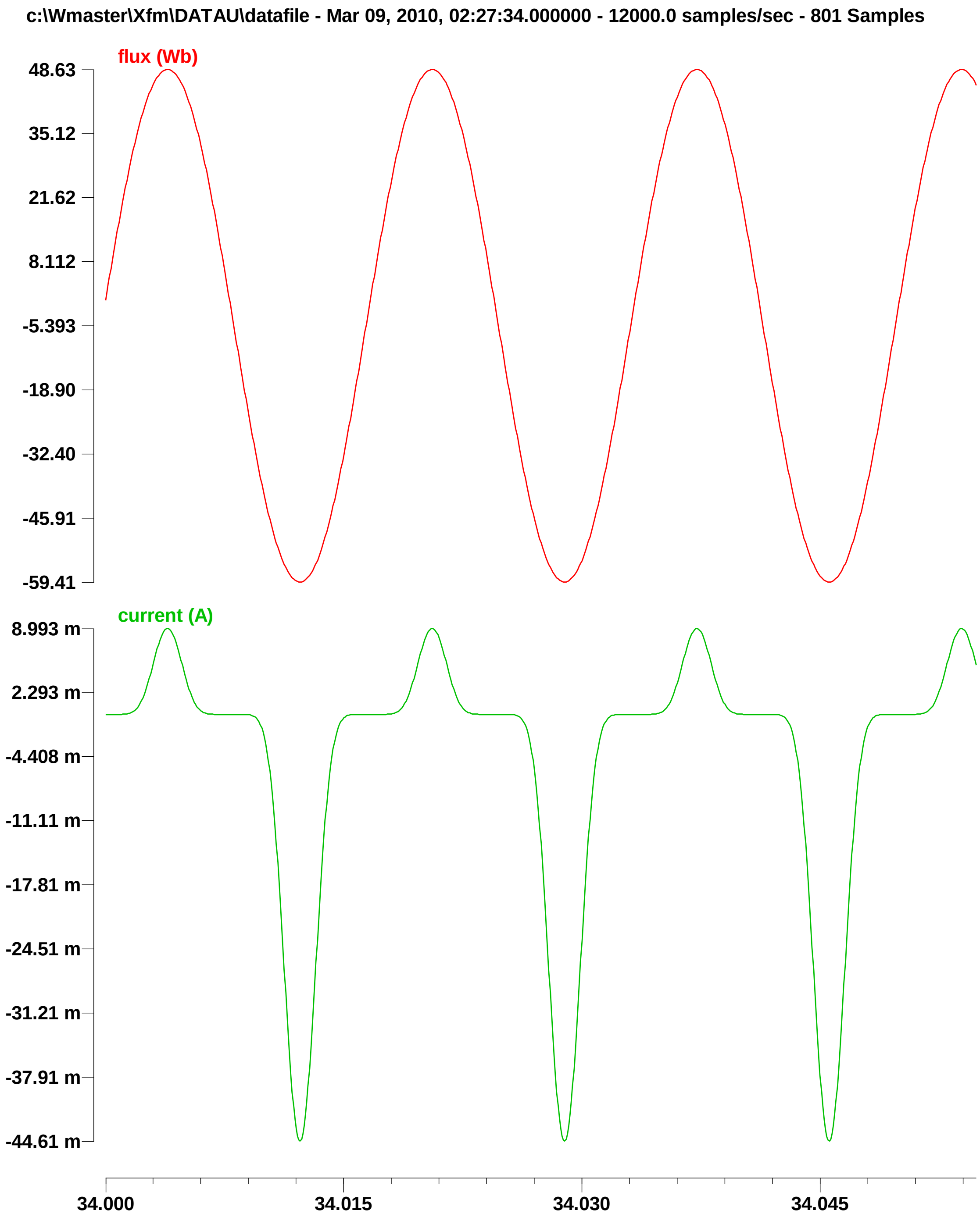


Figure E5.7a: Flux Linkage and Excitation Current for θ = 0.1

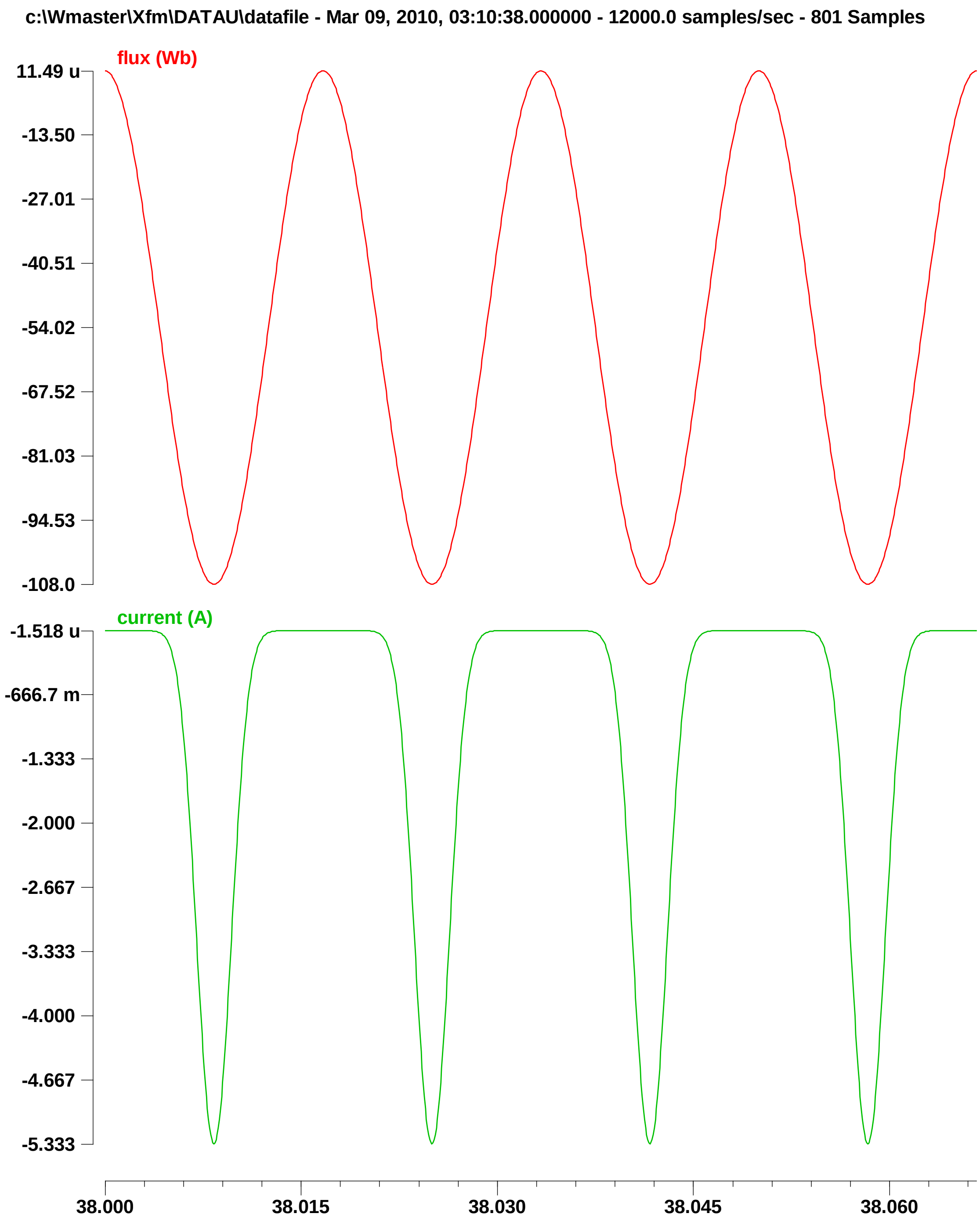


Figure E5.7b: Flux Linkage and Excitation Current for θ = π/2

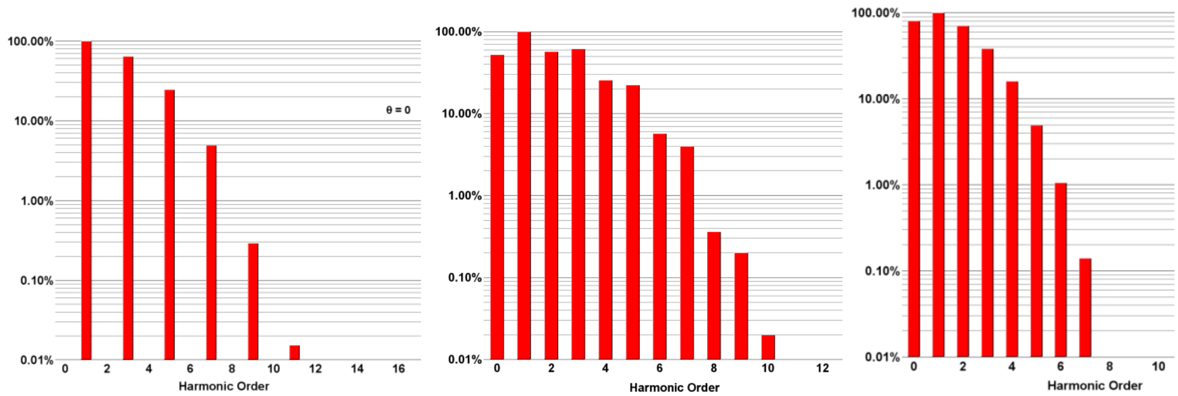


Figure E5.7c: Normalized Excitation Current Harmonics for θ = 0, θ = 0.1 rad, and θ = π/2 rad

Figure 5.25 illustrates a circuit providing "harmonic restraint" to a differential relay. The circuit elements L_1 and C_1 form a band pass filter. L_2 and C_2 form a band reject filter. Both filters are tuned to the power frequency fundamental. Thus the operating coil responds mostly to the fundamental while the harmonic restrain coil responds to the harmonic components. An additional restraint coil provides the RMS current restraint function.

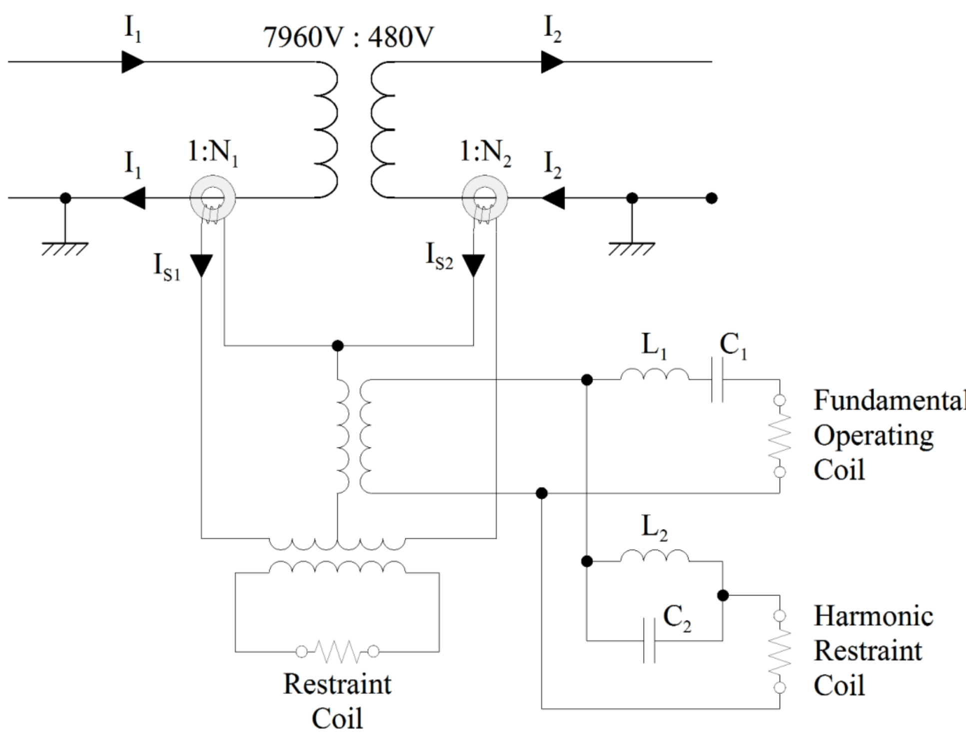


Figure 5.25: Electromechanical Differential Relay Response

The above passive filter based approach has several limitations. The filter selectivity (quality factor) is limited by the passive component losses. The required capacitors and inductors are typically large and are subject to drift with aging and temperature. Active analog filters based on differential amplifiers can provide improved performance, although are still subject to component parameter drift. Digital relay implementations mitigate all such limitations, and in addition provide practically unlimited flexibility in designing advanced detection schemes.

5.7.3 Effects of CT Saturation

CT saturation occurs when the electric current through the CT results in CT magnetic flux linkage above the level for which the CT has been designed. When saturation occurs, the secondary current is not a scaled replica of the primary current. In case of internal faults this is not a problem as the current in the secondary will cause the differential to operate as expected. However when saturation occurs during external faults then the differential protection is operate but this operation results in a false tripping. Figure 5.25 illustrates typical saturated CT primary and secondary current waveforms. Note that saturation results in a distorted waveform. Furthermore the RMS value of a saturated CT is lower than the one expected by the CT ratio.

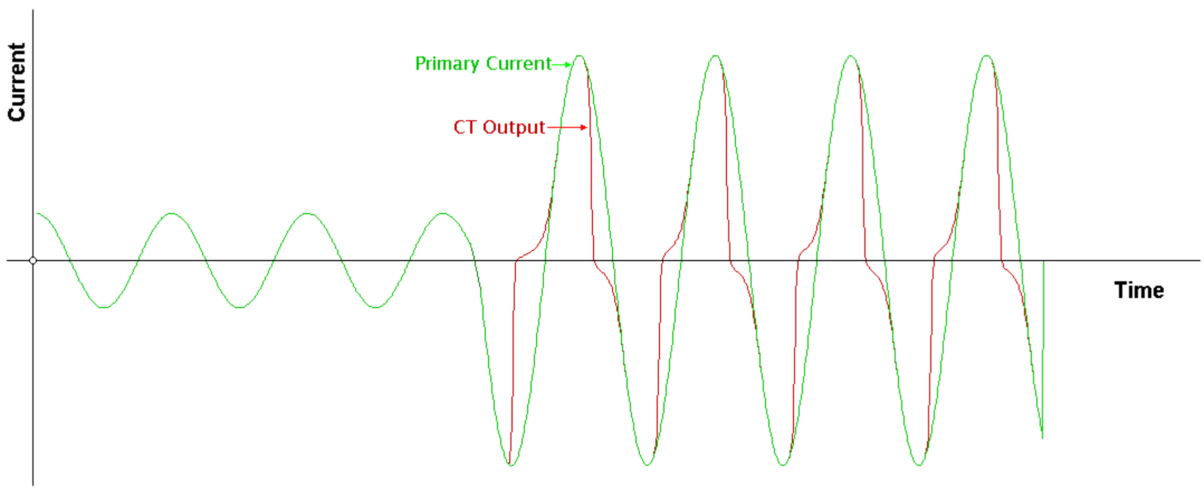


Figure 5.26: Typical Saturated CT Current Waveform

One technique that reduces the possibility of false tripping in the event of CT saturation, while retaining the relay sensitivity for lower current internal faults is the use of Dual Slope Trip/Block curve. Figure 5.26 illustrates a dual slope Trip/Block characteristic. Note that the slope is lower for low current values and higher past a transition point. This allows for a high sensitivity to low current internal faults and lower sensitivity in regions where the CT's may saturate.

In digital relay implementations the trip/block characteristic curve is user defined. Typically the user specifies the two slopes (in %) and the restraint current value at the transition point. These parameters are selected by considering the accuracy and saturation characteristics of the CT's.

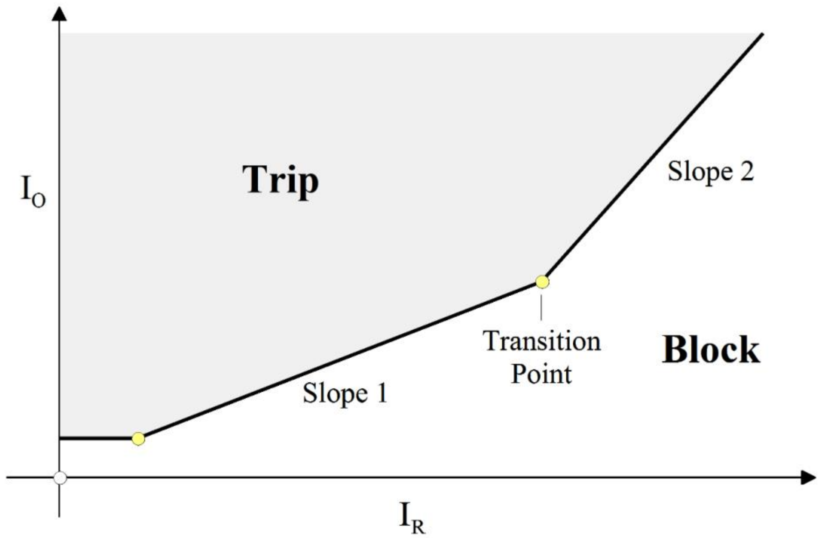


Figure 5.27: Dual Slope Differential Protection

An approach to limit the effects of CT saturation is to use higher voltage rating CTs and control the total burden of the CT (see chapter 6). This is an engineering problem which leads to the selection of CTs that will not saturate for any expected fault current at the location of application of the CT.

Another approach to avoid CT saturation is to use non-saturable core current transformers. For example, Rogowski coils are CTs with non-saturable core. This approach may be used in generator terminals where the fault currents can be potentially very high. The disadvantage of non-saturable core CTs is the low accuracy of these CTs and the fact that their transformation ratio is sensitive to the positioning of the CTs. For this reason, such CT's should be calibrated after installation, or any modification of their positioning.

Additional Topic: Digital relay handling of saturated CT's by comparing RMS output with peak output.

5.7.4 High Impedance Differential Relays

While the differential relaying scheme discussed earlier has tremendous advantages, it also has some disadvantages. For example in case of substantial differences in fault current contribution from different circuits in a differential scheme it is possible that one CT may saturate but others will not. In this case mis-operation can occur. One way to avoid the effect of different saturation by the various CTs in a differential scheme is to use the so called high impedance differential scheme.

The high impedance deferential relaying scheme uses relays with very high input impedance and CTs that are connected in parallel. An example high impedance differential relaying scheme for bus protection is shown in Figure 5.28. Ideally, when the net current is zero and the CTs are identical, the total current through the "burden" of the relay will be zero. In reality there may be some mismatch and some current will flow through this circuit.

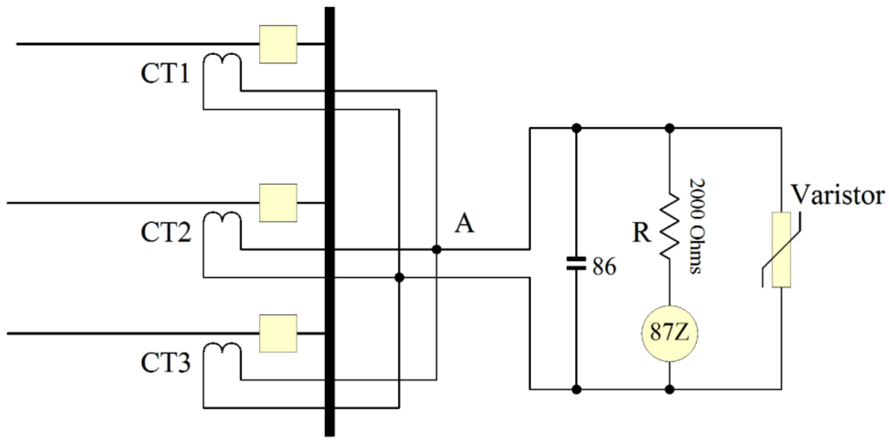


Figure 5.28. High Impedance Voltage Relay Bus Protection

Under normal operating conditions this current will be small and therefore the voltage seen by the relay will be small.

In the case of a high current external fault, one or more of the CT's may saturate while other CTs may not. This will result in a substantial voltage across the relay terminals. The high impedance relay should be designed so that it will not trip the breakers under this condition. In order to determine the relay setting we must determine the worst case (highest voltage that can develop) for external faults. For this purpose, we consider the worst case will occur when the CT monitoring the faulted line is driven into complete saturation, while all other CTs remain in the linear operating region. The voltage across the relay terminals in this case is computed by considering the equivalent circuit shown in Figure 5.29

Each CT branch in this equivalent circuit consists of a current source injecting a current I_i/N, where N is the CT turns ratio, in series with the CT winding resistance R_W and the instrumentation cable resistance R_C. Applying KCL at the common node at the + terminal of the relay, and neglecting the current through the relay, the current flowing through the saturated CT branch is the sum of all the unsaturated CT currents, which is also equal to the fault current divided by the turns ratio N. Since the completely saturated CT acts as a short circuit, the voltage seen by the relay (V_{max}) can be expressed as:

$$V_{\max} = (R_W + R_C)\frac{I_F}{N}$$

where: R_W is the CT winding resistance.

R_C is the max cable resistance from each CT to junction point A.

I_F is the maximum fault current.

N is the CT ratio.

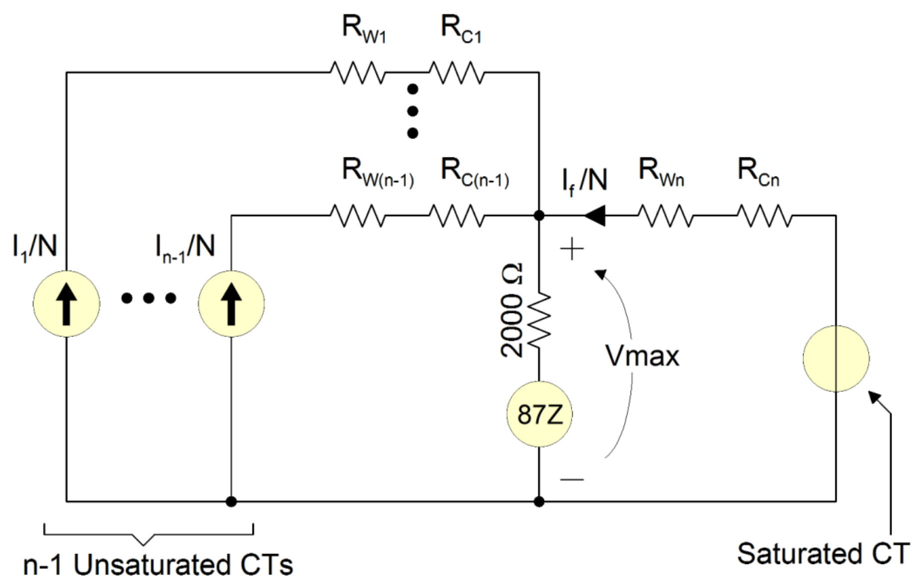


Figure 5.29. Equivalent Circuit of High Impedance Relay Under Worst Case External Fault

The voltage element is set to trip above V_{max} with a safety factor K:

$$V_R = K(R_W + R_C)\frac{I_F}{N}$$

In the case of an internal fault, the net current from all the CT's will not be zero even before any of the CT's saturate resulting in a much higher voltage developing across the relay terminals. In fact, it is necessary to limit the voltage in order to protect the relay and CTs from insulation failure. The varistor (or MOV) shown in Figure 5.28 provides this function. The clamping voltage rating of the varistor is selected to be several times above the pickup voltage V_R of the relay. The energy absorption capability of the varistor should be selected so that it can withstand the maximum current provided by the CT's for a period at least equal to the breaker trip time, typically a few 60 Hz cycles. However, in case of breaker failure the high current flow through the varistor may continue for a much longer time period, possibly destroying the varistor. In order to protect the varistor, a lockout relay (86) is added in parallel with the varistor (see Figure 5.28). The lockout relay contacts are closed by the high impedance relay when an internal fault is detected, thus revering the current from the varistor.

The selection of high impedance differential versus low impedance differential is dependent upon the particulars of a specific system.

5.8 Volts per Hertz Protection

Power apparatus that involve a saturable magnetic core (transformers, motors, generators, etc.) are vulnerable to the saturation of their magnetic core. When the magnetic core saturates it requires large electric currents (magnetizing current) to sustain the magnetic flux for the operation of the device. Depending on the level of saturation, the effects may be increased ohmic losses and gradual overheating of the device. The overheating may result in temperatures above the spontaneous combustion level of the device oil (if it is an oil filled device). Overexcitation of transformers has resulted in melting of transformer coils.

The onset of saturation depends on both the applied voltage on the device and the frequency of the applied voltage. Consider for example a saturable magnetic core inductor. The relationship between the magnetic flux linkage and the voltage is:

$$v(t) = \frac{d\lambda(t)}{dt}$$

In terms of phasors the above equation becomes:

$$\tilde{V} = j\omega\tilde{\Lambda}$$

On absolute value the voltage is proportional to the product of the frequency and the magnetic flux linkage in the device. For a specific device, saturation occurs when the magnetic flux linkage exceeds a certain value. Thus a quantity that determines whether the device will be driven into saturation is the voltage divided by frequency:

$2\pi\Lambda > 2\pi\Lambda_0$, which yields: $\frac{V}{f} > 2\pi\Lambda_0$

This means that an effective way to monitor whether the saturable magnetic core device will be driven into saturation is by monitoring the ratio of the voltage over the frequency.

The effects of magnetic core saturation are increased magnetizing current and associated harmonic currents with the end result being the overheating of the device. Overheating occurs gradually because any device will have certain thermal capacitance. Therefore the speed by which a device will be overheating depends on the level of saturation (level of magnetizing current) and the thermal capacitance of the device. The device should be protected against high temperatures. Specifically, when the temperature exceeds a certain value, then the device should be tripped. It should be understood that the threshold depends on the design of specific devices. One can generate for each specific device a chart that provides the time at which the maximum permissible temperature of the device would be reached versus the saturation level or equivalently the voltage over the frequency ratio. Such a curve for a specific device is provided in Figure 5.30.

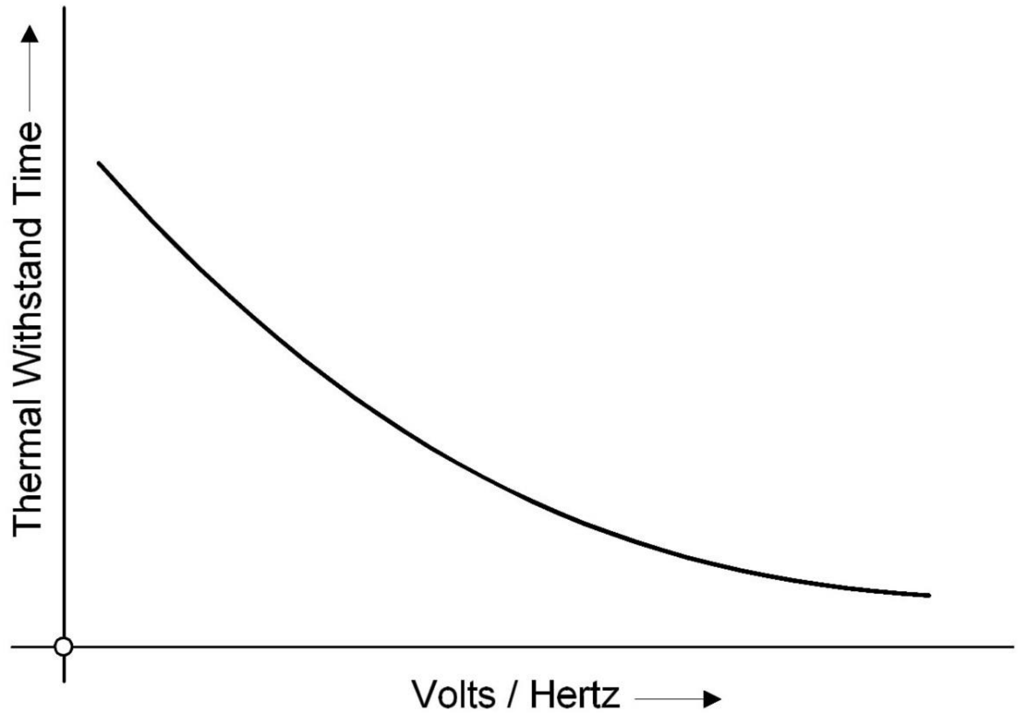


Figure 5.30: Thermal Withstand Capability of a Saturable Core Device as a Function of Volts per Hertz

Given the “withstand” curve for a device, one can design a relay with trip characteristics to match the “withstand” curve of the device to be protected. This task can be achieved with electromechanical relays and “volts per Hertz” relays have been developed. Digital relays provide much better flexibility to define the trip characteristics to better match the “withstand” curve of a device. Figure 5.31 illustrates the trip characteristic of a relay that can be entered into the relay point by point.

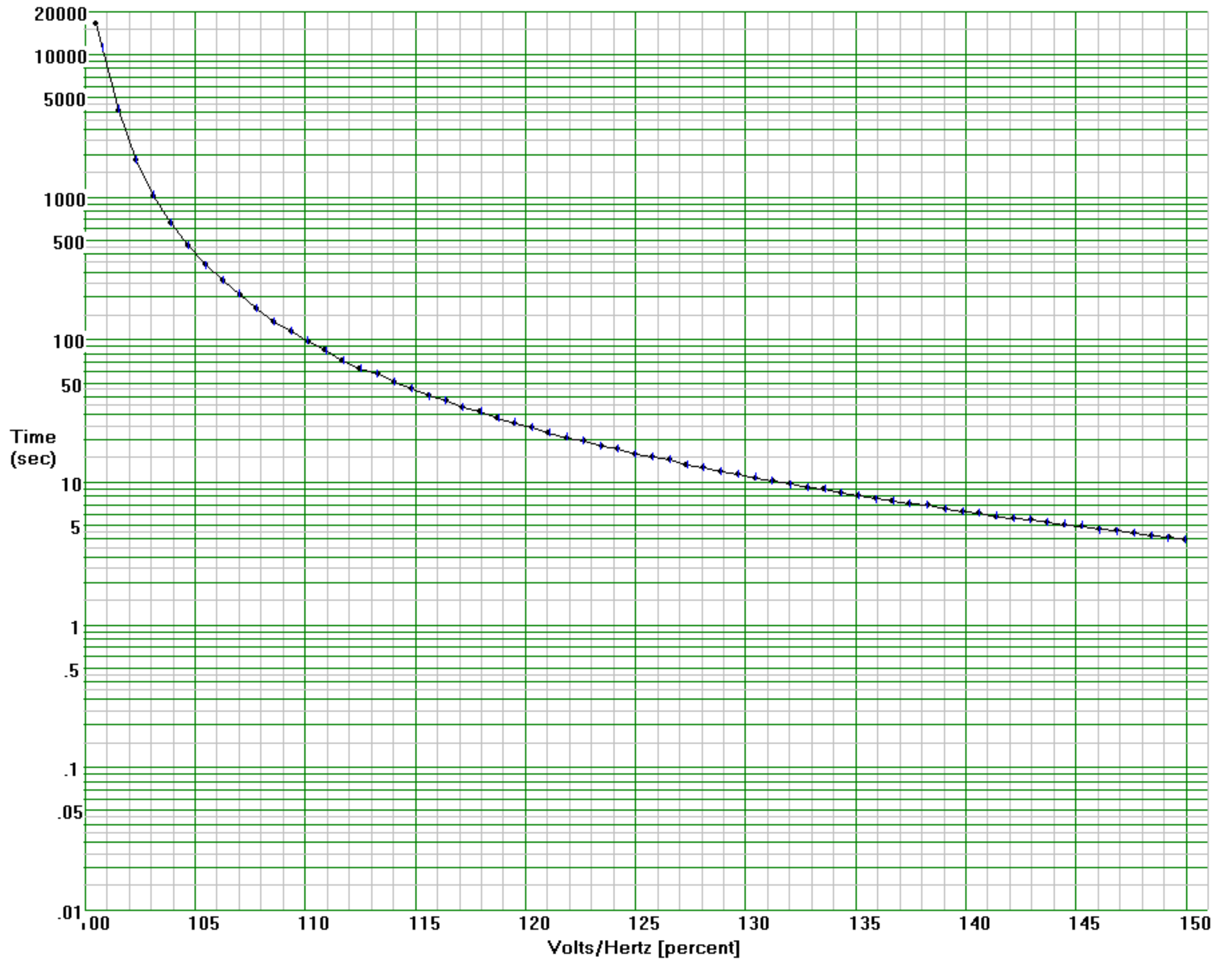


Figure 5.31: Example of PC Based Volts-per-Hertz Entry user interface for digital relays (Courtesy SEL Inc.)

A volts per Hertz relay uses a sensor for the ratio of the voltage over the frequency. In analog relays a sensor for voltage over frequency can be realized by a simple circuit such as the one shown in Figure 5.32. The output of this simple circuit is:

$$V_0 = \frac{V_1}{t}\frac{1}{\sqrt{1+\omega^2C^2R^2}}$$

If the parameters of the circuits are so selected that $\omega^2C^2R^2 \gg 1$, then:

$$V_0 = \frac{V_1}{f}\frac{1}{2\pi tRC}$$

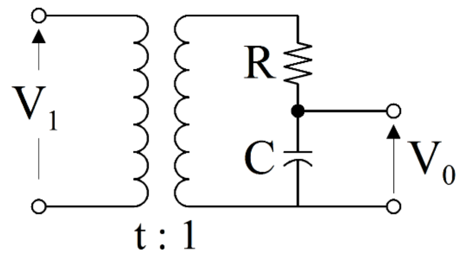


Figure 5.32 An Analog Implementation of a Volts Over Hertz Relay

5.9 Directional Overcurrent Relays

The basic overcurrent relay has a major disadvantage that it cannot discriminate for the direction of the fault since the fault current is alternating and its operational characteristic depends on the magnitude of the current. This disadvantage limits the ability of this relay to coordinate with other protective devices in a networked system. It is possible however to add the ability to detect the direction of the fault. For this purpose a polarizing voltage is needed to determine the fault current direction. The resulting relay is a directional overcurrent relay. The directional overcurrent relay can be constructed from induction disk (or induction cup) relays energized by the fault current and a polarizing voltage. A conceptual illustration of the overcurrent relay is shown in Figure 5.33 using an induction disk.

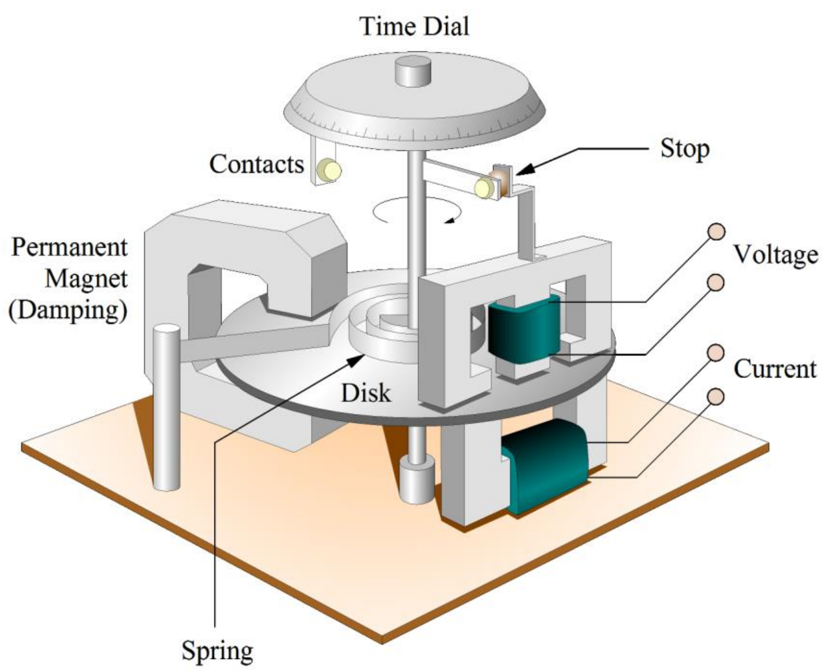


Figure 5.33 Directional OverCurrent Relay

The torque developed on the disk is proportional to the applied voltage and current as well as the phase angle between the voltage and the current. Depending on the phase angle difference it can be in the forward or the reverse rotation of the disk. The relationship is developed as follows: One coil is energized with the current coming from the secondary of the CT and the other coil is energized with the voltage from the secondary of a PT (polarizing voltage). Thus the electric currents in the two coils are:

$$i_1(t) = \sqrt{2} I_1 \cos(\omega t + \varphi)$$
$$i_2(t) = \sqrt{2} \left| \frac{V}{Z} \right| \cos(\omega t + \varphi_z)$$

Then the developed torque is:

$$T = \lambda_2 i_{d1} - \lambda_1 i_{d2} = \frac{2\omega L_m^2}{Z} I_1 V \sin(\varphi_z - \varphi)$$

ALSO: Discuss implementation with induction cup and two 90 degree e-magnets

Thus the developed torque is proportional to the phase angle between the polarizing voltage and current. The relay can be designed so that will operate whenever the torque is positive. Figure 5.34 illustrates the operating region of this relay.

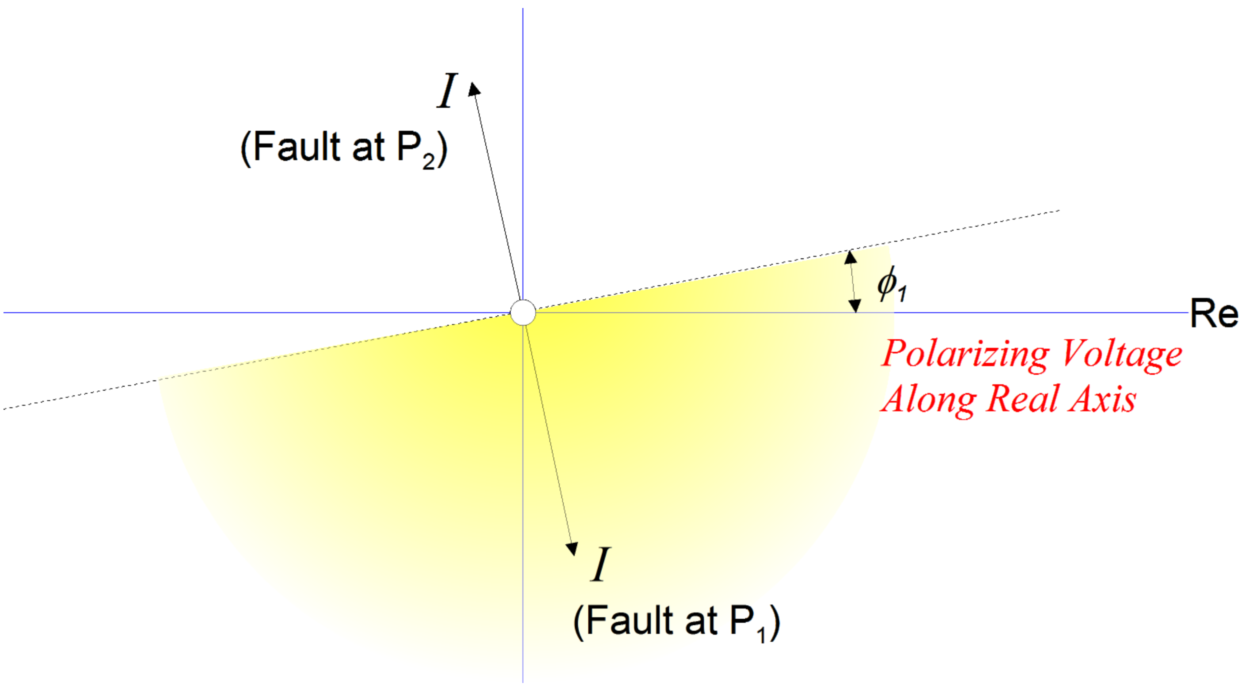


Figure 5.34 Operating Region of a Directional Relay

Note that the directional relay trips when the direction of the fault is at the specified direction. In order to have a relay that trips on the fault current when the fault direction is the selected direction, one normally uses two relays, an overcurrent relay element and a directional relay element as shown in Figure 5.35. Note that the overcurrent element and a directional element are connected in series and therefore the relay trips when both conditions are met, i.e. the fault current causes the overcurrent to trip and the direction of the fault causes the directional relay to trip. Since we have discussed the overcurrent element in previous sections we will focus on the directional element.

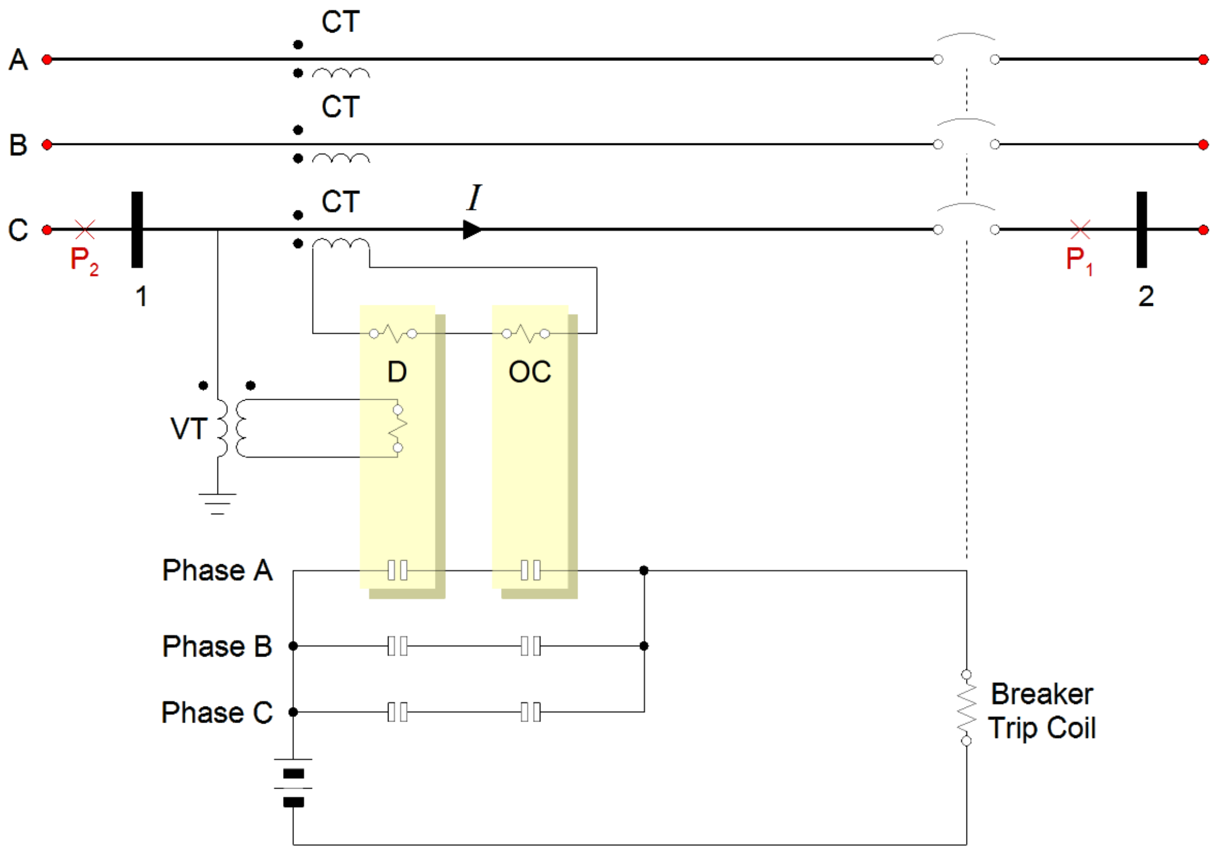


Figure 5.35 Schematic Representation of a Directional Overcurrent Relay

By combining a directional element and an overcurrent element, a relay can be constructed that will respond to fault in a selected direction from the location of the relay. The characteristic of this relay is shown in Figure 5.36.

The disadvantages of directional overcurrent relays are:

1. Prediction of time to trip is dependent upon the location of the fault. Specifically, the torque is dependent upon the voltage magnitude that may depend on the location of the fault.

2. It is very difficult (in general impossible) to coordinate directional overcurrent relays in a network transmission system.

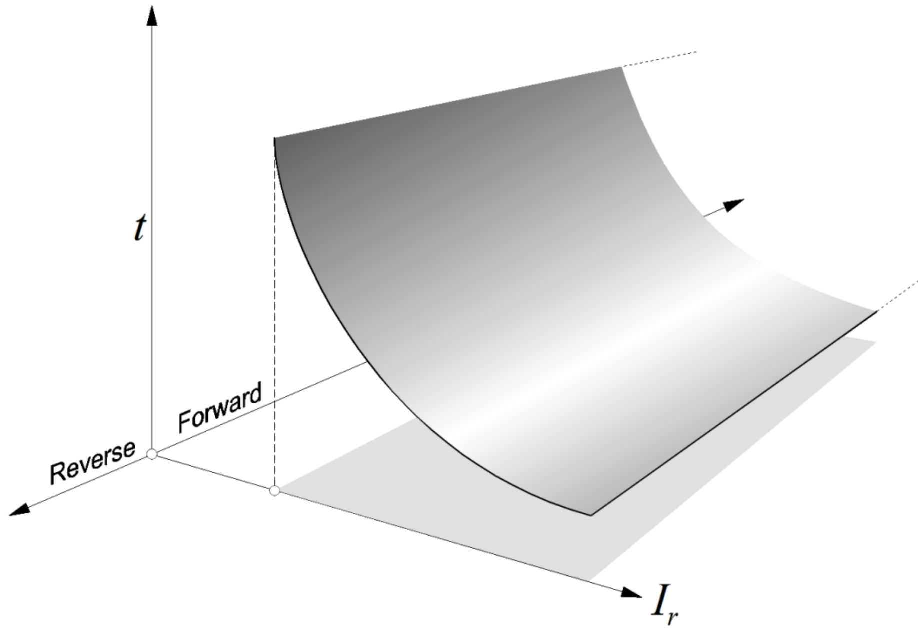


Figure 5.36 Operational Characteristic of a Directional Time Overcurrent Relay

The selection of the polarizing voltage is also critical for the correct operation of directional relays. One basic problem is that close in faults may result in a near voltage collapse at the relay location and cause mis-operation. To avoid this and since most faults are single phase faults, the polarizing voltage is selected from other than the faulted phase: Specific polarizing voltages commonly used are:

(a) a → bc, 90 degree phase shift
(b) b → ca,
(c) c → ab,
(d) zero sequence
(e) neutral current

Example E5.8: The objective of this example is to illustrate that directional relays cannot be coordinated for a general network system. Consider the three bus electric power system of Figure E5.x. Assume that the impedance of each one of the power lines is j23.4 ohms. The short circuit capacity of the three sources is: 2 GVA, 4 GVA and 8 GVA for units G1, G2 and G3 respectively. Assume that each line is protected at each end by a directional overcurrent relay. Determine the settings of these relays in such a way that for a fault in a specific power line, only the relays at the two ends of the line will operate. The system is a 115 kV system.

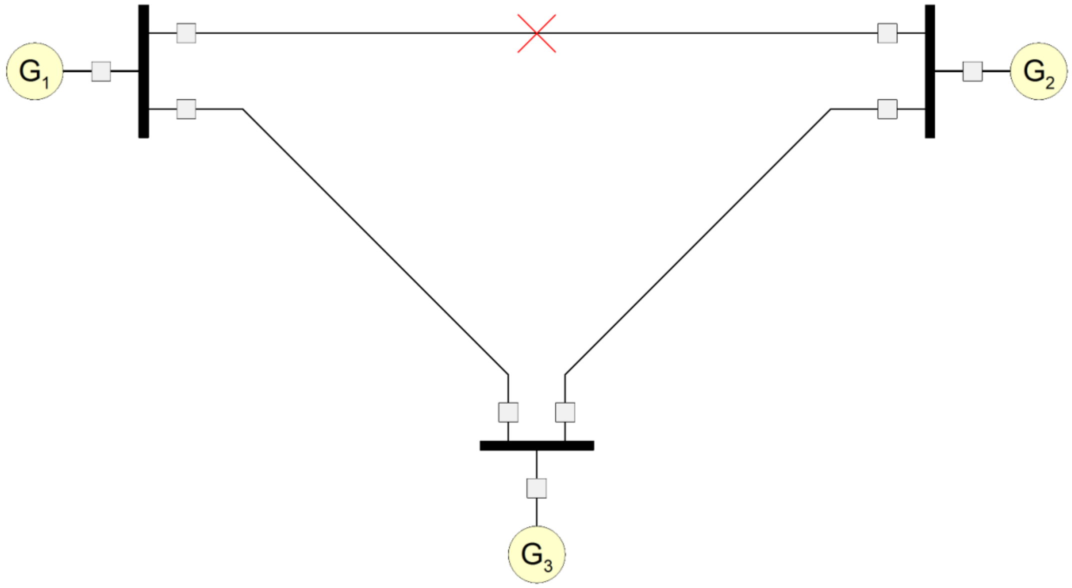


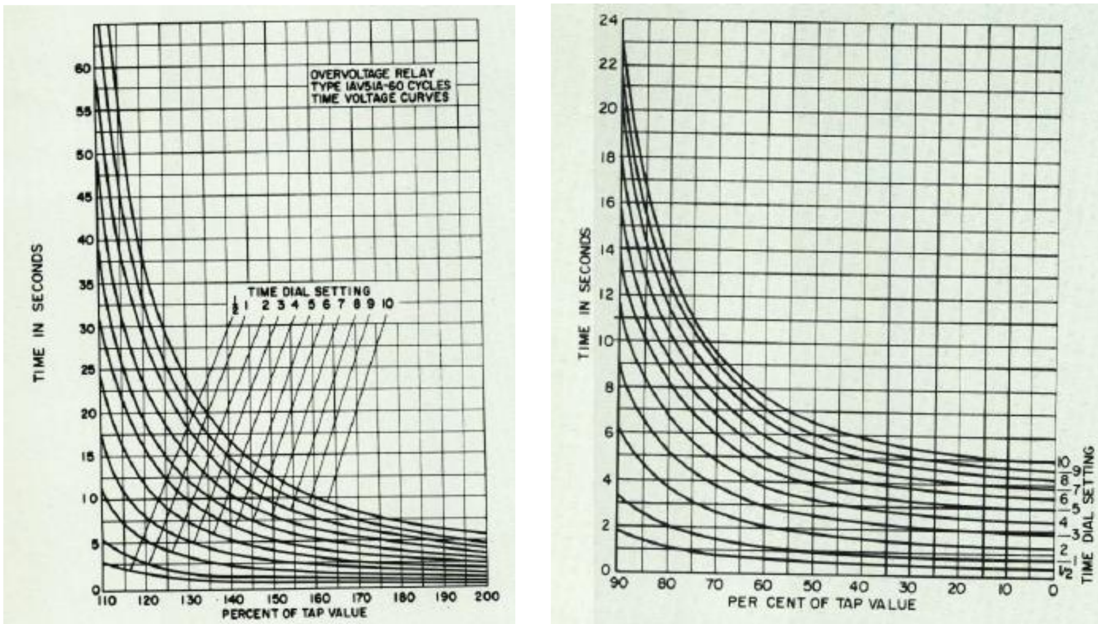
Figure E5.8 Application of Directional Overcurrent Protection in a Network Transmission System

Solution: First the short circuit currents at the location of each relay is computed and illustrated in Figure E5.8a.

To be continued…

5.9 Overvoltage and Undervoltage Protection

In many cases it is necessary to protect equipment from abnormal voltages. For this purpose, the overvoltage and/or undervoltage relay can be used. This relay is a modified time overcurrent relay that operates on voltage. Typical characteristics for this relay are shown in Figure 5.36 for overvoltage (Figure 5.37a) and for undervoltage (Figure 5.37b).



(a) (b)

Figure 5.37 Typical Characteristics of a Voltage Relay
((a) Overvoltage Relay, (b) Undervoltage Relay)

5.10 Directional Power Relays (32)

Many times we need to protect a device against power flow in a direction that is not designed to operate (for example synchronous generators, network transformers, etc.) or if the direction of the power will adversely affect the overall system (for example network transformers (protectors)). In this case we need a relay that will monitor the direction of power flow and it will act if the power flow is reversed.

The implementation of directional power relays can be achieved in the same way as the directional relays.

Digital implementation of directional power relays are also very simple. Specifically, numerical relays sample the voltage and current at a certain location, from which they compute the average power flow as a sum of voltage-current sample product over a certain time window. For a three phase system, (generator, etc.), the net 3-phase real power flow is computed as follows:

$$P = \frac{1}{N} \sum_{i=k-N+1}^{k} v_a(t_i) i_a(t_i) + v_b(t_i) i_b(t_i) + v_c(t_i) i_c(t_i)$$

Where k is the latest sample of the collected data, and N is the number of samples in the time window.

Note that above formula provides the average real power over the selected time window. If the numerical value of the computed power reverses sign, it indicates reversal of the power flow at the monitoring location.

5.11 Impedance Relays

Power system protection can be improved if relays have the capability to determine the fault location in a circuit. For this purpose, impedance relays were developed, initially for the protection of transmission lines. Specifically, an impedance relay tracks the apparent impedance looking into a transmission line. When a fault occurs on this line, assuming that the line construction is uniform, this impedance is proportional to the line length between the fault and the relay location. Thus, this information can be used to identify whether a fault is within the desired zone of protection, and consequently whether or not the breaker should trip the line.

Impedance relays were developed long time ago, before microprocessors and digital relays. As a result the first impedance relays were electromechanical devices. Many features were added to the impedance relays and the sophistication of the impedance relays, even in the days of electromechanical relays reached a great level of complexity and capability. We will examine some of the main features of these developments. It should be clear that the capabilities of the electromechanical impedance relays can be emulated with computer algorithms in numerical relays.

5.11.1 The Basic Impedance Relay

An analog implementation of the impedance relay is shown in Figure 5.38 via a balancing beam. The two coils of the balancing beam are connected to the polarizing voltage and the current in the circuit respectively as is shown in the figure. Note that the beam pivots and it will move in one direction or another depending on the level of forces F1 and F2. The coils are excited with currents that are proportional to the polarizing voltage and current respectively. In this case the forces are:

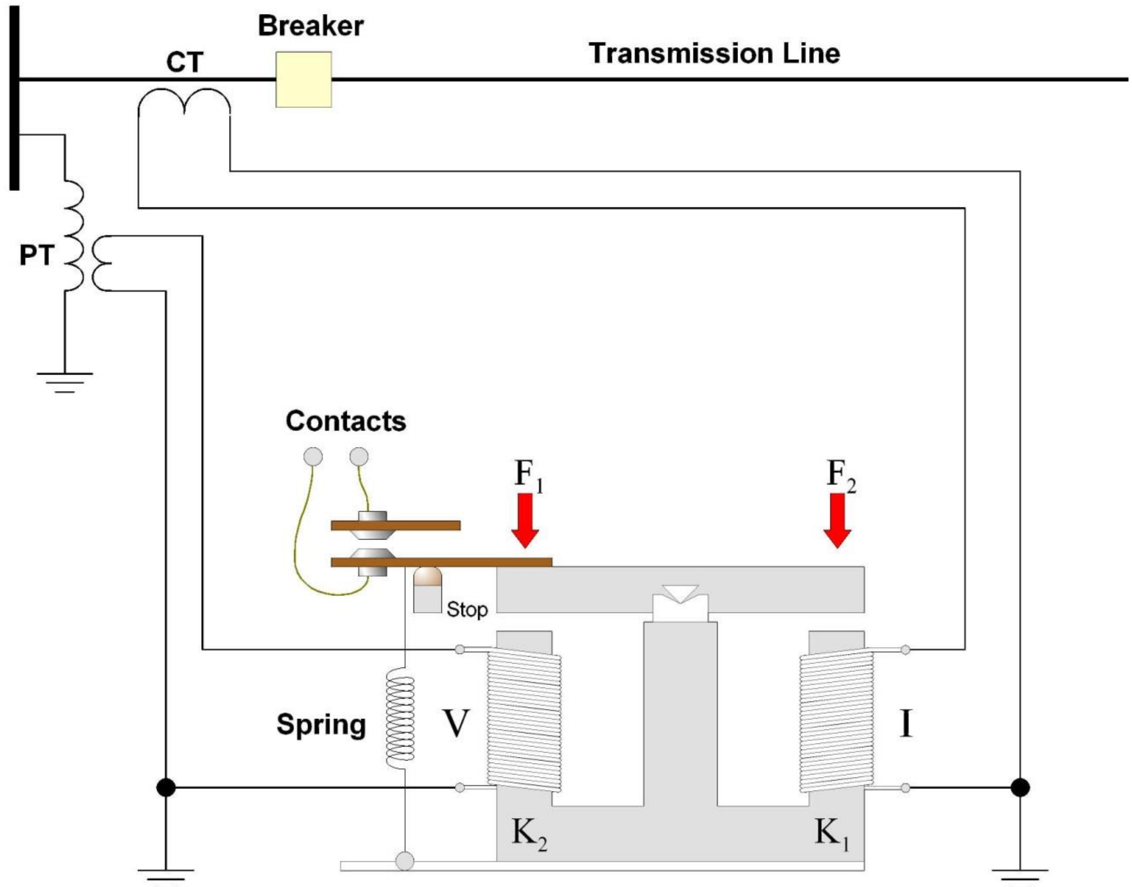


Figure 5.38 Balancing Beam Implementation of an Impedance Relay

$$F_1 = k_1 V^2$$
$$F_2 = k_2 I^2$$

The relay will close the contact when:

$$F_1 \le F_2 \quad \rightarrow \quad \frac{V}{I} \le \sqrt{\frac{k_2}{k_1}}$$

In digital relays the impedance function can be easily implemented by computing the phasor of the voltage and current and taking the ratio of these phasors.

An application of a distance relay is shown in Figure 5.39. The relay monitors the voltage and the current at a certain location of a line. The relay operates whenever the impedance "seen" is below a selected value. The operating region of an impedance relay is shown in Figure 5.40.

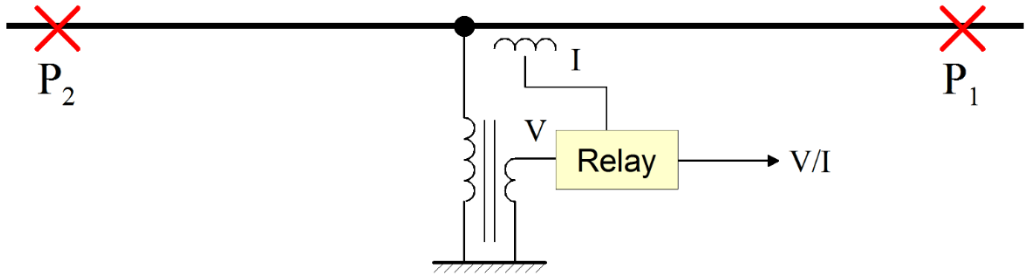


Figure 5.39. Illustration of Application of Impedance (or Distance) Relay

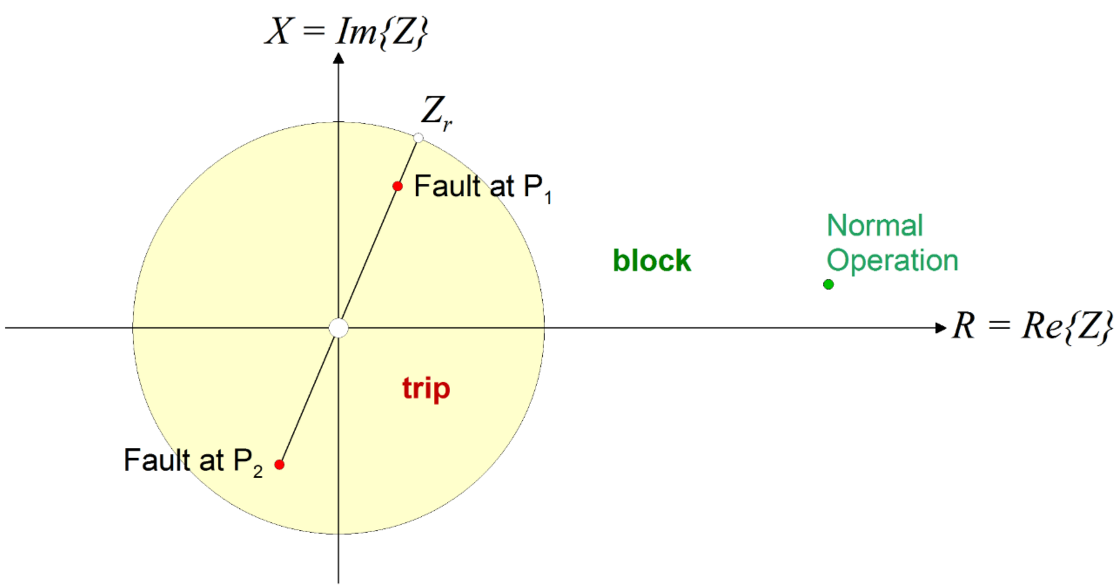


Figure 5.40. Operating Region of an Impedance Relay

There are many modifications of the impedance relay. For example many times it is desirable to trip for a fault within a certain distance but in one direction only. A directional element in series with the impedance relay will achieve this objective. In addition, many times it is desirable to have an operating region shape that will better differentiate between faults at a certain location and other conditions such as transients and heavy load conditions. Some of the relays that achieve these objectives are described next.

5.11.2 Impedance Relay with Directional Element

Many times it is desirable to operate for a fault within a certain distance in a specified direction. In this case an impedance relay with a directional element will do just that. The operating region of such a relay is shown in Figure 5.41.

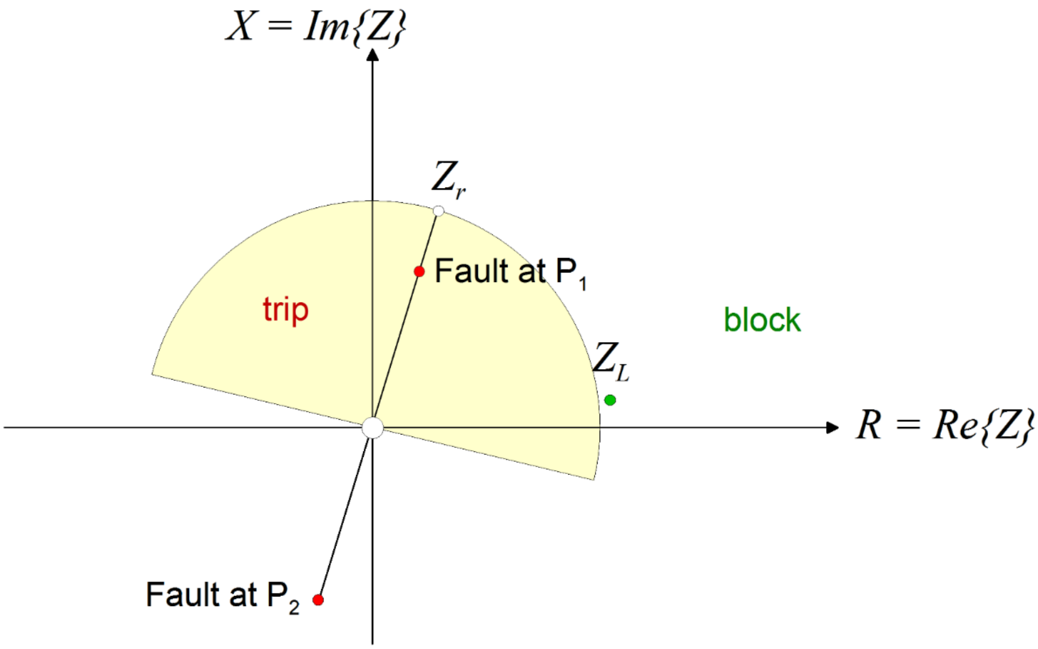


Figure 5.41 Operating Region of a Directional Impedance Relay

Note that the objective for providing a directional element to the impedance relay is to provide direction selectivity, a necessary feature to be able to operate for faults only in one direction. This can be accomplished with a much more sophistication in the modified impedance relays that are described next.

5.11.3 Modified Impedance Relay

Impedance relays that protect transmission lines will "see" and impedance which is proportional to the line impedance per unit length. Typical line impedances have a phase of about 80 degrees. This means that the impedance to the fault "seen" by the relay will be always at about 80 degrees. An operating region that is more selective for the line impedance is the one provided by a modified impedance relay as shown in Figure 5.43. The modified impedance relay can be implemented by simply adding an appropriate signal to the voltage coil as it is illustrated in Figure 5.42.

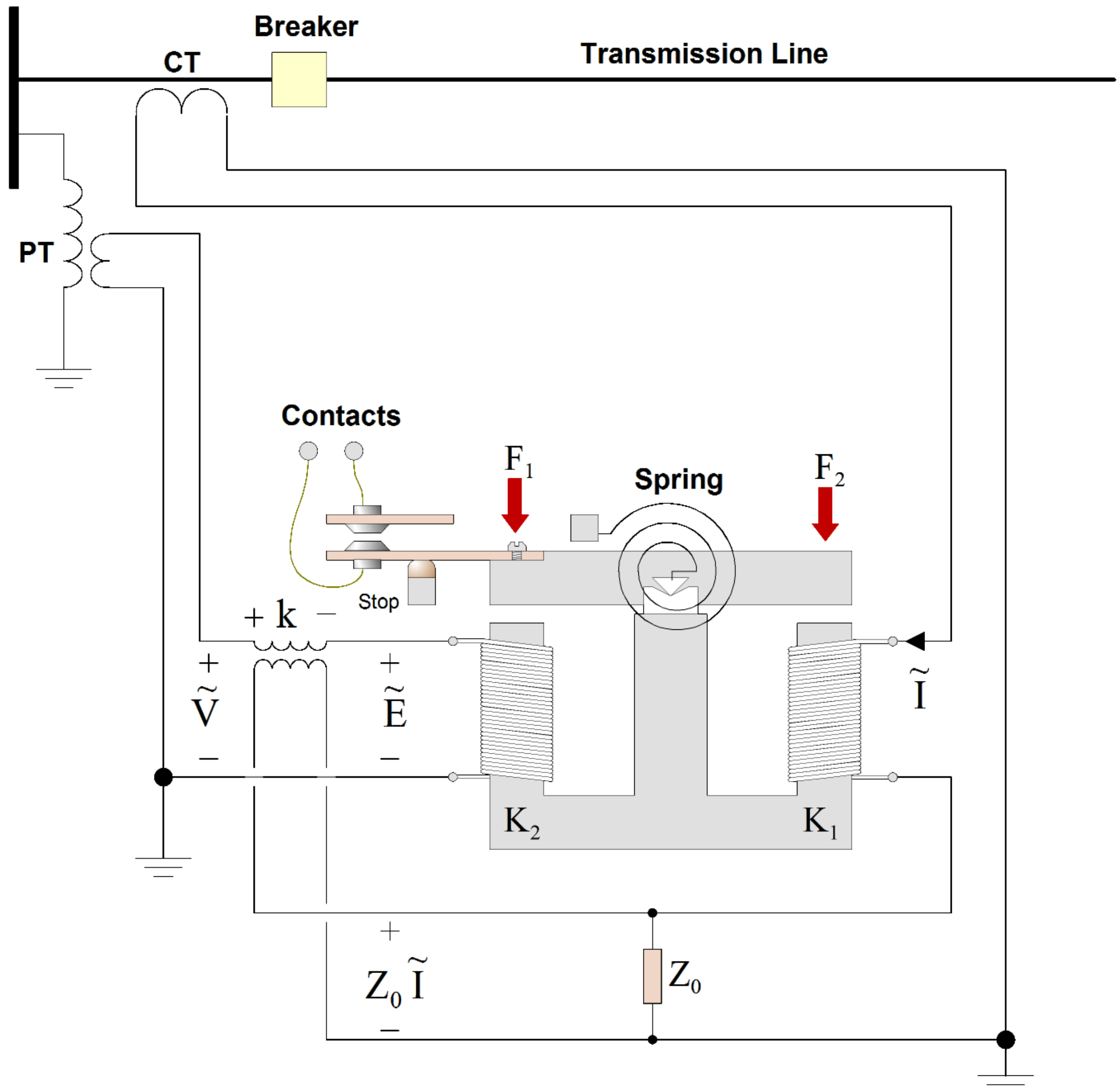


Figure 5.42 Modified Impedance Relay Implementation via balancing Beam

Note that in this case, the voltage applied to the voltage coil will be:

$$\tilde{E} = \tilde{V} - kZ_0\tilde{I} = \tilde{V} - a\tilde{I}$$

in this case, the forces in the balancing beam are:

$$F_1 = k_1\left|\tilde{V} - a\tilde{I}\right|^2$$

$$F_2 = k_2 I^2$$

The relay will close the contact when:

$$F_1 \le F_2 \quad \rightarrow \quad \left| \frac{\tilde{V}}{\tilde{I}} - a \right|^2 \le \frac{k_2}{k_1}$$

This means that the operating characteristic of the relay is a circle of radius $\sqrt{k_2 / k_1}$ and with center at point a (where a is a complex number equal to $k\, Z_0$). If the absolute value of the offset quantity a is equal to the radius of the circle ($\sqrt{k_2 / k_1}$) then the relay operating region circumference passes through the origin. The modified impedance relay operating region is shown in Figure 5.43 (yellow circle).

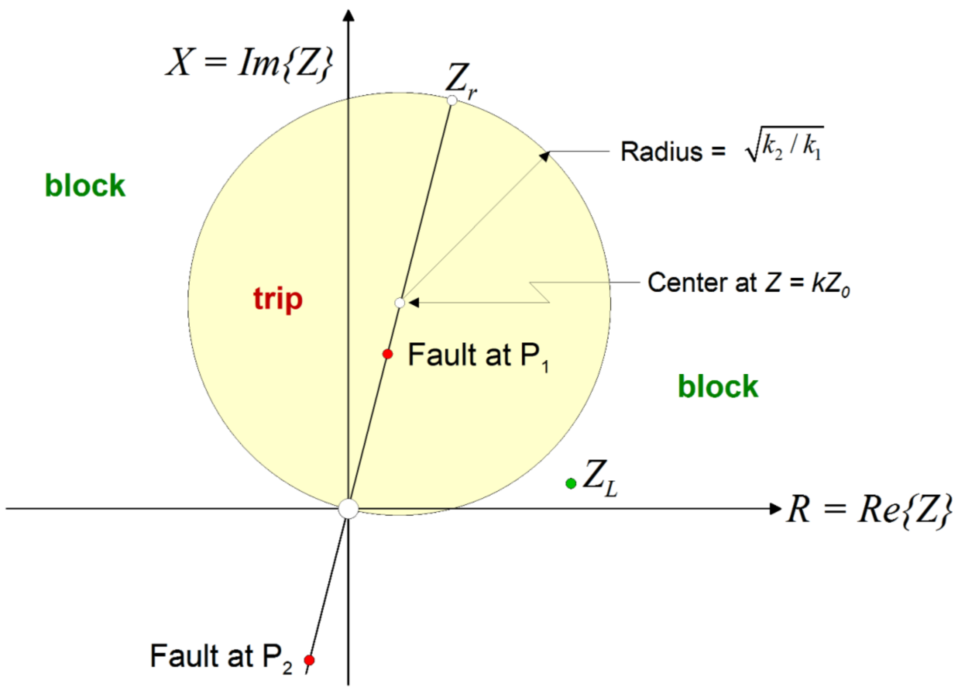


Figure 5.43 Operating Region of a Modified Impedance Relay

The concept has been carried to various levels of shaping the operating characteristic. Modified impedance relays with various operating regions have been developed, for example the "lens" impedance relay, the "tomato" impedance relay, etc. Obviously with numerical relays any shape of the characteristic area can be generated.

5.11.4 Three-Phase Distance Relays

Distance relays permit sophisticated protection schemes. When applied to transmission lines, depending on the fault type, the equivalent per unit length impedance "seen" by the relay may vary. For example for a three phase fault the per unit length impedance of the line equals the positive sequence impedance of the line. For a single line to ground fault the equivalent per-unit impedance is approximately equal to the average of the positive, negative and zero sequence impedance of the line.

For the purpose of standardizing the distance relay design for three phase circuits, the relays should be so designed as to "see" an equivalent impedance that is approximately equal to the positive sequence impedance of the circuit per unit length times the distance to the fault. This is easily achieved with numerical relays by providing appropriate algorithms. For electromechanical relays, one can have multiple relays that will determine the distance to the fault for various fault types and then have logic to select the correct answer. We shall discuss the design of a three phase distance relay that operates on the positive sequence impedance for any type of fault in Chapter 9 in the context of transmission line protection. Here we present an example of the primary algorithm for detecting the fault location.

Example E5.9: Consider the simple three-phase transmission line as it is illustrated in Figure E5.x. The parameters of the line are indicated on the figure. Assume a modified impedance relay at the indicated location. The relay monitors the following quantities: $\frac{\tilde{V}_1}{\tilde{I}_1}$, $\frac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2}$, and $\frac{\tilde{V}_a}{\tilde{I}_a + k\tilde{I}_0}$, where $k = \frac{z_0 - z_1}{z_1}$. Assume various types of faults at the indicated location, i.e. 3-Phase, Line to Ground, and Line to Line, and determine what the relay will "see".

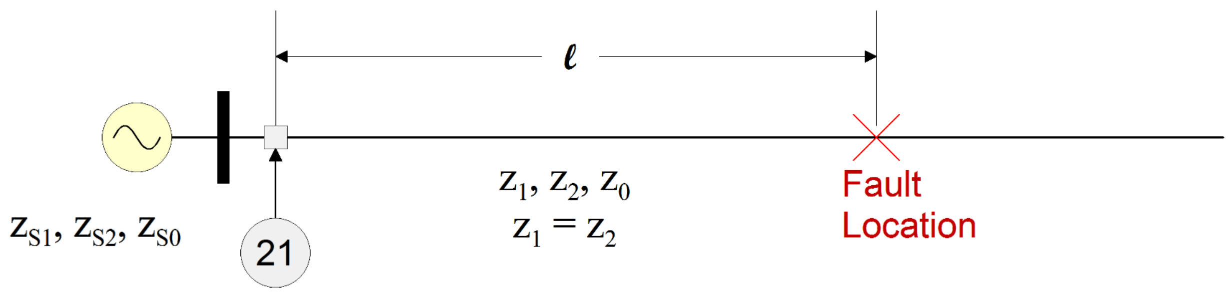


Figure E5.9 Three Phase Transmission Line Protected with a Modified Impedance Relay

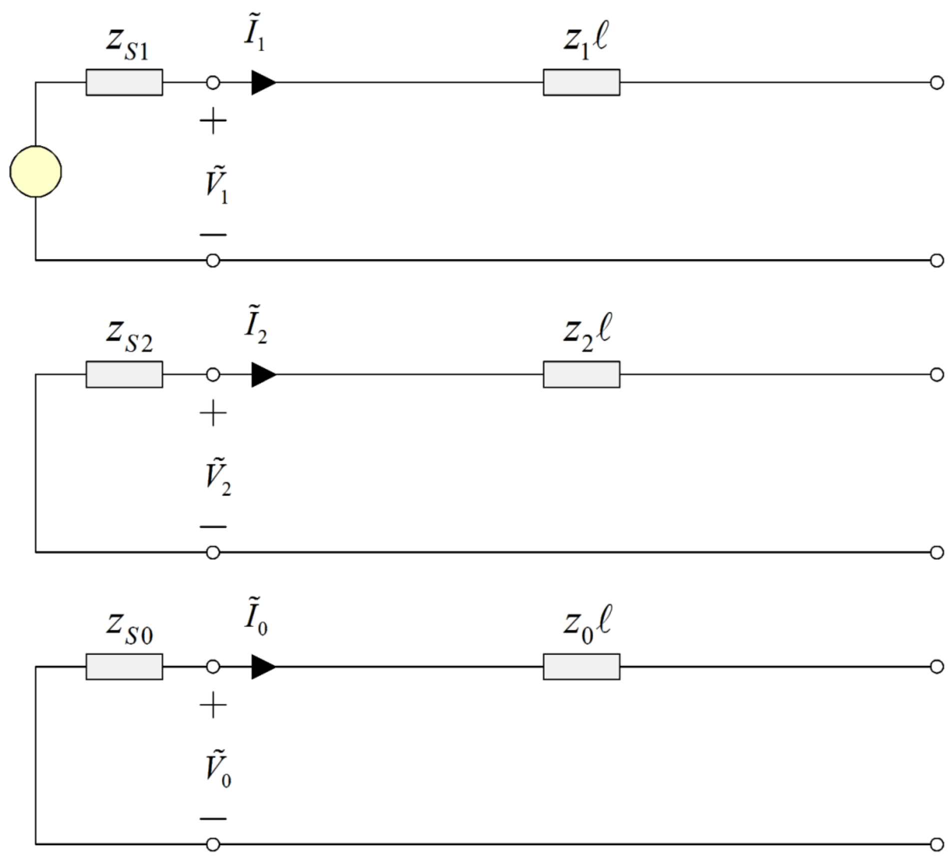
Solution: For the three different faults at the indicated location, these quantities will be as shown in Table E5.9.

Table E5.9: Relay Computations

Case	Monitored Quantity	3Φ Fault	L-L Fault	1LG Fault
1	$\frac{\tilde{V}_1}{\tilde{I}_1}$	$z_1\ell$	$2z_1\ell + z_{S2}$	$2z_1\ell + z_0\ell + z_{S2} + z_{S0}$

2	$\dfrac{\tilde{V}_1-\tilde{V}_2}{\tilde{I}_1-\tilde{I}_2}$	$z_1\ell$	$z_1\ell$	∞
3	$\dfrac{\tilde{V}_a}{\tilde{I}_a+k\tilde{I}_0}$	$z_1\ell$	∞	$z_1\ell$

The above Table was derived from the following computations: The sequence model of the circuit is:



Each one of the three faults is applied to this circuit. Subsequently the faulted circuit analyzed and the results are entered in the monitoring quantity. The end results are sown in the figure.

Note that the for each of the faults one quantity always provides the measurement $z_1\ell$. For example, the first quantity provides the value $z_1\ell$ for a three phase fault, the second quantity provides the value $z_1\ell$ for a line to line fault, and the third quantity provides the value $z_1\ell$ for a single line to ground fault. This value is always the smallest value. Therefore the distance relay monitors all the above values and for any fault event, uses the smallest measurement to determine the fault location.

5.11.5 Fault Distance Computation in Numerical Relays

Impedance relays are based on the computation of the distance of a fault from the relay location. The primary use of this information is to decide on protective action, i.e. to trip or not to trip the appropriate breaker(s). However, this information is also useful for the purpose of finding the fault location and performing the necessary repairs. In particular, microprocessor implementations of distance relays can provide this information on a display or send it to the control center without substantial additional expense.

The problem of fault locating in power circuits has been long ago recognized as an important one for two reasons: (a) minimization of downtime by quick repair and therefore increased system reliability (especially for cable circuits) and (b) improved selectivity of protection schemes by virtue of knowledge of fault location (for example distance relays are based on evaluation of the fault distance). Recent trends towards automation have accentuated the importance of fault locating. Over the years, several technologies for fault locating have been developed. These technologies can be categorized into the following:

a) Methods based on audible noise of fault recreation (thumpers), this method requires an actual crew to go on location and perform the tests,
b) Methods based on strategic placement of faulted circuit indicators, this method requires either visual inspection f the faulted circuit indicator, or if the FCI are equipment with communications, to bring all the information to a central location and determine the location by analysis,
c) Detection and measurement of travel time of first transient, this method required equipment that can record waveforms with very high sampling rates (recall that transients travel with close to the speed of light and in order to measure the travel time it is necessary to capture the transients with sufficient time resolution), and
d) Methods based on estimation of the circuit impedance to the fault and extraction of the fault location from the known impedance values per unit length

The first two methods are extensively used in distribution circuits, especially on URD cable systems. The thumper technology consists of injecting an impulse to the faulted cable. The fault in the cable is reignited under the impulse and the generated noise is utilized to determine the location of the fault. Application of the thumper requires that the cable is out of service and in general it is time consuming. A criticism of the thumper technology is that it subjects the cable to additional surges and therefore may affect the life of the cable.

The faulted circuit indicators are devices which are triggered by the flow of the fault current. Basically, a faulted circuit indicator is a two state device: state one is normal and state two indicates that an electric current above a threshold value has been detected. Application of many faulted circuit indicators at strategic locations along a circuit, i.e. one at each transformer, provides means for determining the location of the fault between two locations. Models with manual or automatic reset are available. Also models with/without communications are available.

The third method is based on specialized hardware that record the travel time to the fault and extract the distance to the fault from the known speed of propagation of EM waves on the circuit. Specifically, the travel time from the monitoring location to the fault is measured with rather sophisticated hardware. For a given circuit (circuit parameters) the speed of propagation of surges along the circuit is known and is utilized to estimate the location of the fault. This technology is complex, requiring sophisticated and expensive hardware.

The forth method requires recording of voltages and currents at any location along the faulted circuit. From the recorded voltages and currents and the known impedance per unit length of the system, the distance to the fault can be estimated. The introduction of numerical relays and digital fault recording equipment have made this method very attractive. Specifically, the mentioned equipment provides recordings of the voltage and current during fault. This data can be processed in the relay or fault recording equipment to provide the fault location. In addition this recordings are typically stored and/or can be transmitted to central locations via a variety of communication media, i.e. telephone, fiber, microwave, etc. The data can be processed at the central location to estimate the distance to the fault. This approach provides the additional benefit that recordings from multiple devices can be used to estimate the fault location thus increasing the accuracy of the fault locating method. There are many ways to estimate the fault location from the recorded voltages and currents. This section reviews the existing methods and provides a discussion of the issues and limitations of the approach.

The basic fault locating method is explained with the aid of Figure 5.44. A relay, disturbance recorder, etc. is placed at one end of the circuit. This device records the voltage and current at that location of the circuit.

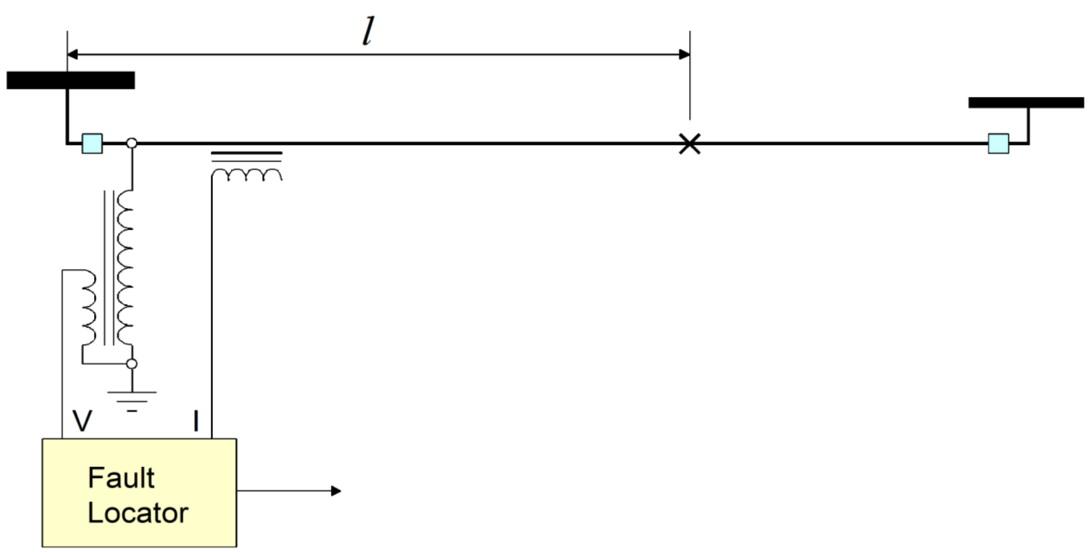


Figure 5.44 Principle of Fault Locating From Circuit Impedance to the Fault

Assume that a fault occurs at a point located ℓ meters away from the recording device. During the fault conditions, the recorded voltages and currents are processed to obtain the phasors $\tilde{V}$ and $\tilde{I}$ of the voltage and current. The impedance "seen" at the location of the recorder is:

$$Z_f = \frac{\tilde{V}}{\tilde{I}} = z\ell$$

where z is the circuit impedance per unit length.

Thus:

$$\ell = \frac{Z_f}{z} = \frac{\tilde{V}}{z\tilde{I}}$$

The above approach is simplistic and it is valid for simple circuits with specific impedance per unit length. In reality, circuits are much more complicated, i.e. three phase circuits with 3, 4 or 5 wires, or single phase circuits with multiple fault current return paths, such as neutrals, ground wires and soil. We will examine the application of above principle to typical power circuits.

Assume that fault data has been recorded at Bus 1 of a line of total length L during a fault at some point of the line (ℓ miles from Bus 1) as it is illustrated in Figure 5.45. The fault recorder or relay (DFR) has captured the voltage and current waveforms at all three phases at the Bus 1 terminal.

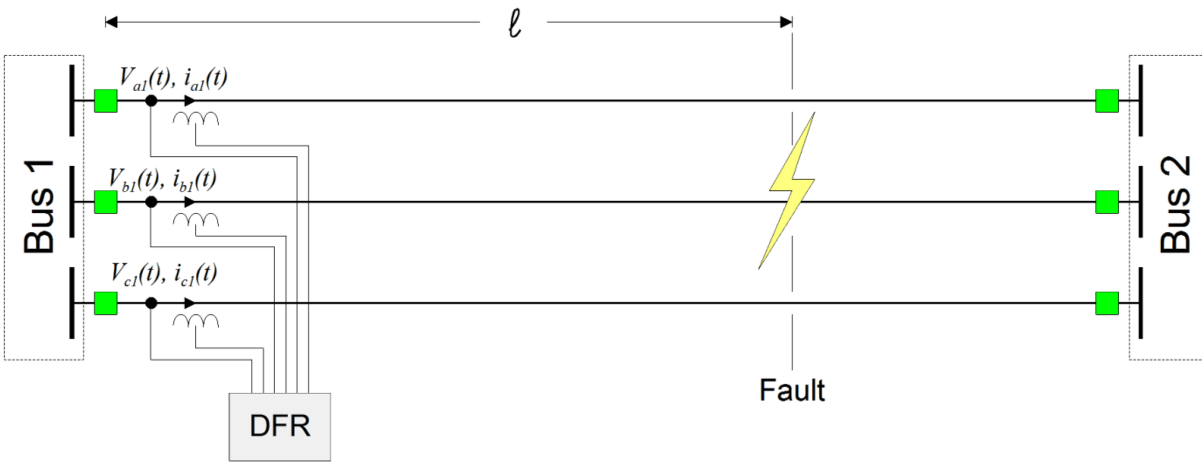


Figure 5.45 Illustration of a Faulted Circuit with Single End Monitoring

The recorded voltage at the faulted phase is:

$$\tilde{V}_f = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)\ell + v_{arc}$$

expressing the arc voltage as a conductance times the fault current yields:

$$\tilde{V}_f = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)\ell + g\tilde{I}_f$$

Splitting real and imaginary parts:

$$V_{fr} = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)_r \ell + g\tilde{I}_{fr}$$
$$V_{fi} = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)_i \ell + g\tilde{I}_{fi}$$

Upon solution of above equations for the distance to the fault:
Note that in this case the fault arc voltage is also computed.

5.12 Pilot Relaying

The various protection schemes that have been described so far have two basic limitations when applied to zones that are geographically dispersed, such as a transmission line, a zone consisting of a circuit and a transformer, etc.: (a) they are not capable to determine with absolute certainty that the fault is within the zone under protection and therefore they must rely on coordination for the proper clearing of the fault, and (b) they lack the ability to clear the fault in zone simultaneously at the two ends of the zone. Because of these limitations, fault detection and clearing cannot be fast. Differential schemes do provide certainty in identifying the location of a fault. However, because of the geographic extent of the cases that we are discussing here (transmission lines, circuits plus transformers, etc.), differential relaying cannot be applied easily (for relatively short zones it is feasible). An alternative to differential schemes is pilot relaying. Pilot relaying can be viewed as the poor man's differential protection scheme. Pilot relaying requires communication between relays at the two ends of the long protection zone. Figure 5.46 illustrates the pilot relaying scheme in a conceptual manner. It is important to note that the various pilot relaying schemes utilize different information between the two ends of the line. In general, any pilot relaying scheme consists of three components: (a) the communications media, (b) the logic, and (c) the trip action. Presently, the utilized options with respect to these three components are shown in Table 5.x.

(a)

(b)

Figure 5.46. Pilot Relaying – Conceptual Arrangement
(a) protection zone consisting of a single transmission line
(b) protection zone consisting of line plus transformer

Table 5.2 Pilot Relaying Media, Logic and Trip Options

Media	Logic	Trip
Pilot Wire Telephone Line Power Line Carrier Microwave Fiber Optic	Directional Phase Differential	Transfer Trip Blocking

The above choices will be described in detail in Chapter 9 that deals with protection of transmission lines.

5.13 Synchronizing Relays

Energization of an active component or in general reclosing a breaker may result in excessive currents and abnormal voltages under certain conditions. For example closing the breaker of a rotating synchronous machine (generator or motor) at a time when the system frequency is not the same as the generated frequency (speed) of the rotating machinery may result in very high currents and fast changing voltages. The same can happen during restoration procedures if the two systems that are to be connected with the breaker are not operating at same frequency or the phase angle between the two is too large. To avoid this potentially catastrophic events, it is necessary to measure in real time the frequency and the phase angle between the two parts that are to be connected by the breaker. This can be achieved by measuring the frequency of the two systems as well as the phase angle between the two systems. Then the permission to close the breaker can be given if the two frequencies are within a specified threshold and the phase angle is within a specified threshold. A device that performs this "permitting function" is called a synchronizing relay. An analog implementation of a synchronizing relay is shown in Figure 5.47. Note that the relay provides the vital quantities on the two sides of the breaker: voltage magnitude, frequency and phase angle.

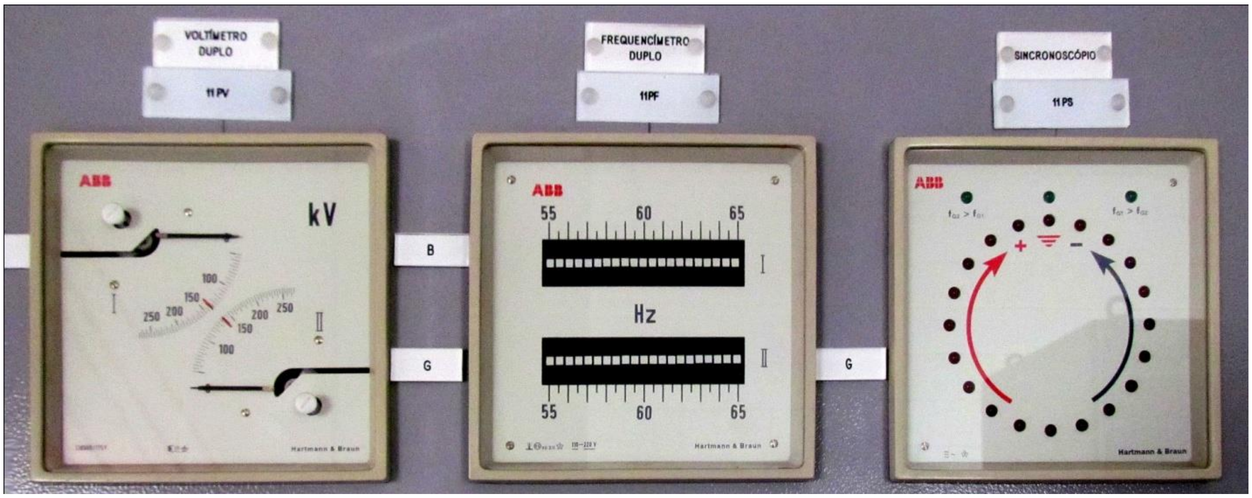


Figure 5.47 Analog Implementation of a Synchronizing Relay
(Photo Courtesy of ArcelorMittal Tubarão, Vitoria, Brazil)

The initial implementations of synchronizing relays were not automatic but rather indicators that an operator will use for closing the breaker. Today a synchronizing relay is a fully automatic relay that provides an inhibiting signal or a permission signal depending on the settings and the actual values of voltage, frequency and phase angle at the two sides of the breaker. It should be clear that the settings of a synchronizing relay will be different for a synchronous generator, a synchronous motor, for synchronizing an island, etc.

5.14 Complexity and Protection Gaps

The current state of art in protective relaying is quite advanced. The numerical relays have capability to perform multiple sophisticated protection functions (multifunctional relays). Yet the complexity of the power system possible fault conditions result in conflicting objectives among the various protection functions and the requirement for coordination. As the number of functions increase so does the complexity of the coordination process. This challenge will keep on becoming severe as the multifunctional numerical relay has only limited information to perform the protection functions, typically three voltage and three currents. Pilot relaying, relays with additional inputs, etc. are efforts to provide more information to the relays for the purpose of increasing their selectivity and coordination. Apparently a more systematic way is needed. In addition, despite the advanced state of modern protection systems, protection gaps still exist. A protection gap is defined as the inability of present protection systems to correctly identify a specific fault condition for which they should operate. The gaps can be classified into two categories: (a) protection problems for which a satisfactory solution does not exist, such as downed conductors, or high impedance faults, faults near neutrals, and (b) protection problems for which present protection schemes leave "compromised protection areas". The latter lead many times to false operations, such as load encroachment, sympathetic tripping, etc. One major challenge of problems in the second category exists in systems with resources that interfaced with power electronics, such as wind farms, PV farms, distributed generation, etc. The main characteristic of these systems are that their fault current capability is limited by the power electronics creating a disparity between the grid side and the resource side. What complicates matters more is the fact that some of the power electronics have complex control functions that the protection system must recognize and distinguish between abnormal operating conditions and legitimate complex response to a disturbance. Another complexity is the fact that for better protection schemes, it is necessary to monitor the DC side of these systems as well and incorporate the conditions of the DC side into the protection schemes. With respect to this issue there is a hardware gap as present day numerical relays have been designed to monitor AC quantities only. For these systems one need numerical relays with capability to measure DC quantities.

Below we provide additional comments on specific issues and challenges.

Wind Farm protection: Wind farms are generating plants with non-conventional generation (induction machines with power electronics, for example type 3 and type 4) that present the following characteristics: (a) the fault current contributions from the power grid may be quite high but the fault current contribution from the wind generator is comparable to the load current. While present protection schemes and numerical relay capability is tweaked to develop a reasonable overall protection scheme for wind farms, the solutions are complex and lack full reliability (security, dependability and speed). Are there better ways to protect these systems? The increased complexity from mandated controls, such as zero voltage ride through capability further makes the protection problem a challenge.

Distribution system with distributed generation: These systems present the same challenges as wind farms with the additional complexity of mixing protection systems that were designed on the basis of radial power flow to a system with bidirectional power flow. Some present standards take the easy way out by suggesting disconnection of distributed resources in case of disturbances and faults. There must be a better way if we want to increase the economic value of distributed resources.

PV Farm protection: PV farms exhibit the same protection challenges as wind farms. It is important to note that while wind farm protection and operational issues are under serious consideration and research activity, for PV farms the activity is very low and under the radar screen. At the same time there is substantial development of utility size PV farm systems and larger activity of residential PV activity. It is important that the protection and operation of these systems be further researched and improved.

Down conductor protection: This problem has been with the industry for a long time with various attempts to solve the downed conductor protection system. While many schemes have been developed, none of the schemes can provide definitive protection against downed conductors.

Above are examples that demonstrate the need for new thinking and new approaches.

5.15 State Estimation Based Approach for Zone Protection

For secure and reliable protection of power components such as a generator, line, transformer, etc. a new approach has emerged based on component dynamic state estimation (state tracking/monitoring). The proposed method uses dynamic state estimation [4-7] to determine whether measurements obtained at various parts of the protection zone (component) fit the dynamic model of the protection zone (component). The dynamic model accurately represents the physical laws that the protection zone (component) must obey. When measurement fit the dynamic model within the accuracy of the meters, it is an indication that the protection zone (component) is free of faults.

The method has been inspired from the fact that differential protection is one of the most secure protection schemes that we have and with two very important characteristics: (a) it does not require coordination with any other protection functions and (b) it requires only a couple and very simple settings. Differential protection simply monitors the validity of Kirchoff's current law in a device, i.e. the weighted sum of the currents going into a device must be equal to zero. Because differential protection monitors only one physical law for the protection zone and ignores other physical laws, it will miss certain faults, for example, a coil to coil fault in a transformer will be missed by differential protection since Kirchoff’s current law will be satisfied by the terminal currents. Note that if one also monitors the satisfaction of Kirchoff’s voltage law, this type of fault will be also detected. Thus, one can generalize the concept into monitoring the validity of all other physical laws that the device must satisfy, such as Kirchoff's voltage law, Faraday's law, etc. This monitoring can be done in a systematic way by the use of dynamic state estimation. Specifically, all the physical laws that a component must obey are expressed by the dynamic model of the protection zone (component). Dynamic state estimation is used to continuously monitor the dynamic model of the component (zone) under protection. If any of the physical laws for the component under protection is violated, the dynamic state estimation will capture this condition. Thus, it is proposed to use dynamic state estimator to extract the dynamic model of the component under protection [2-5] and to determine whether the physical laws for the component are satisfied. The dynamic model of the component accurately reflects the condition of the component and the decision to trip or not to trip the component is based on the condition of the component only irrespectively of the condition (faults, etc.) of other system components. Figure 5.x illustrates this concept. The proposed method requires a monitoring system of the component under protection that continuously measures terminal data (such as the terminal voltage magnitude and angle, the frequency, and the rate of frequency change - this task is identical to present day numerical relays), other variables such as temperature, speed, etc., as appropriate, and component status data (such as the tap setting, breaker status, etc.). The dynamic state estimation processes these measurements and determines whether the measurements "fit" the dynamic model of the protection zone (component). In absence of faults in the protection zone, the process also validates the model of the protection zone and provides the best estimate of the protection zone operating conditions.

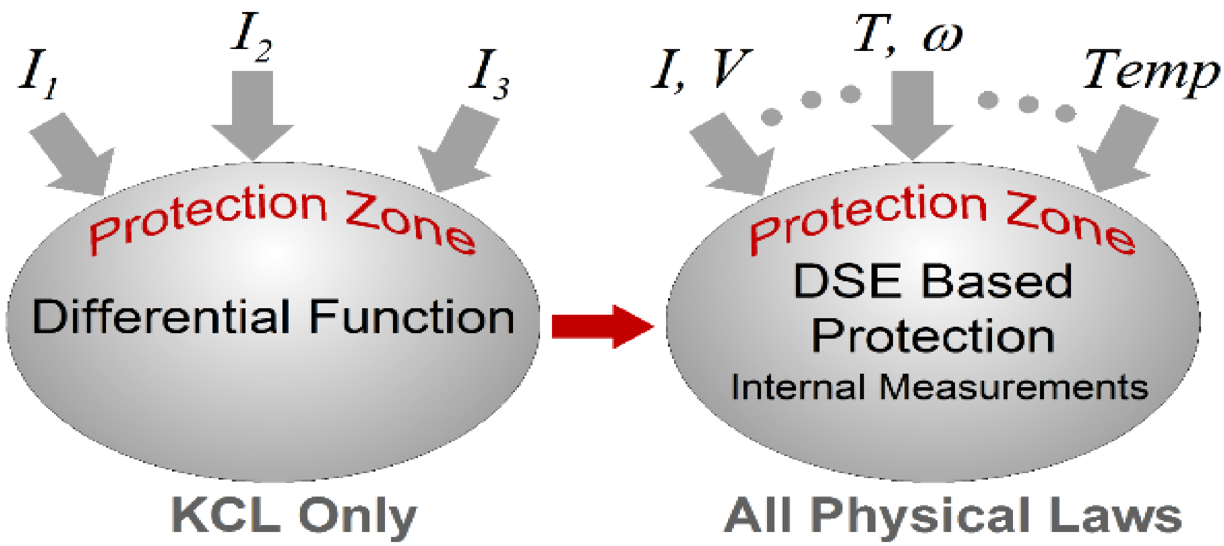


Figure 5.x: The Concept of Dynamic State Estimation Based Protection

After estimating the operating conditions, the well-known chi-square test [6] calculates the probability that the measurement data are consistent with the component model, i.e. the physical laws that govern the operation of the component (see Figure 2). In other words, this probability, which indicates the confidence level of the goodness of fit of the component model to the measurements, can be used to assess the health of the component. The high confidence level indicates a good fit between the measurements and the model, which indicates that the operating condition of the component is normal. However, if the component has internal faults, the confidence level would be almost zero (i.e., the very poor fit between the measurement and the component model).

In general, the proposed method can identify any internal abnormality of the component within a cycle and trip the component immediately. Furthermore, it does not degrade the security because a relay does not trip in the event of normal behavior of the component, for example, in case of transformer protection, inrush currents or over excitation currents, since in these cases, as long as the inrush currents are consistent with the transient behavior of the transformer as dictated by the dynamic model, the method will produce a high confidence level that the transients are consistent with the model of the component. Note also that the method does not require any settings or any coordination with other relays.

It is important to note that the proposed scheme will perform best when: (a) the measurements are as accurate as possible - dependent on the type of instrument transformer used, i.e. VT, CT, etc. and the instrumentation channel, i.e. control cable, etc. and (b) the accuracy of the dynamic model of the component under protection. These issues, while important, are beyond the scope of this paper. These issues will be addressed in a subsequent paper.

The approach is briefly illustrated in Figure 5.48. The method requires a monitoring system of the component under protection that continuously measures terminal data (such as the terminal voltage magnitude and angle, the frequency, and the rate of frequency change) and component status data (such as tap setting (if transformer) and temperature). The dynamic state estimation processes these measurement data with the dynamic model of the component yielding the operating conditions of the component.

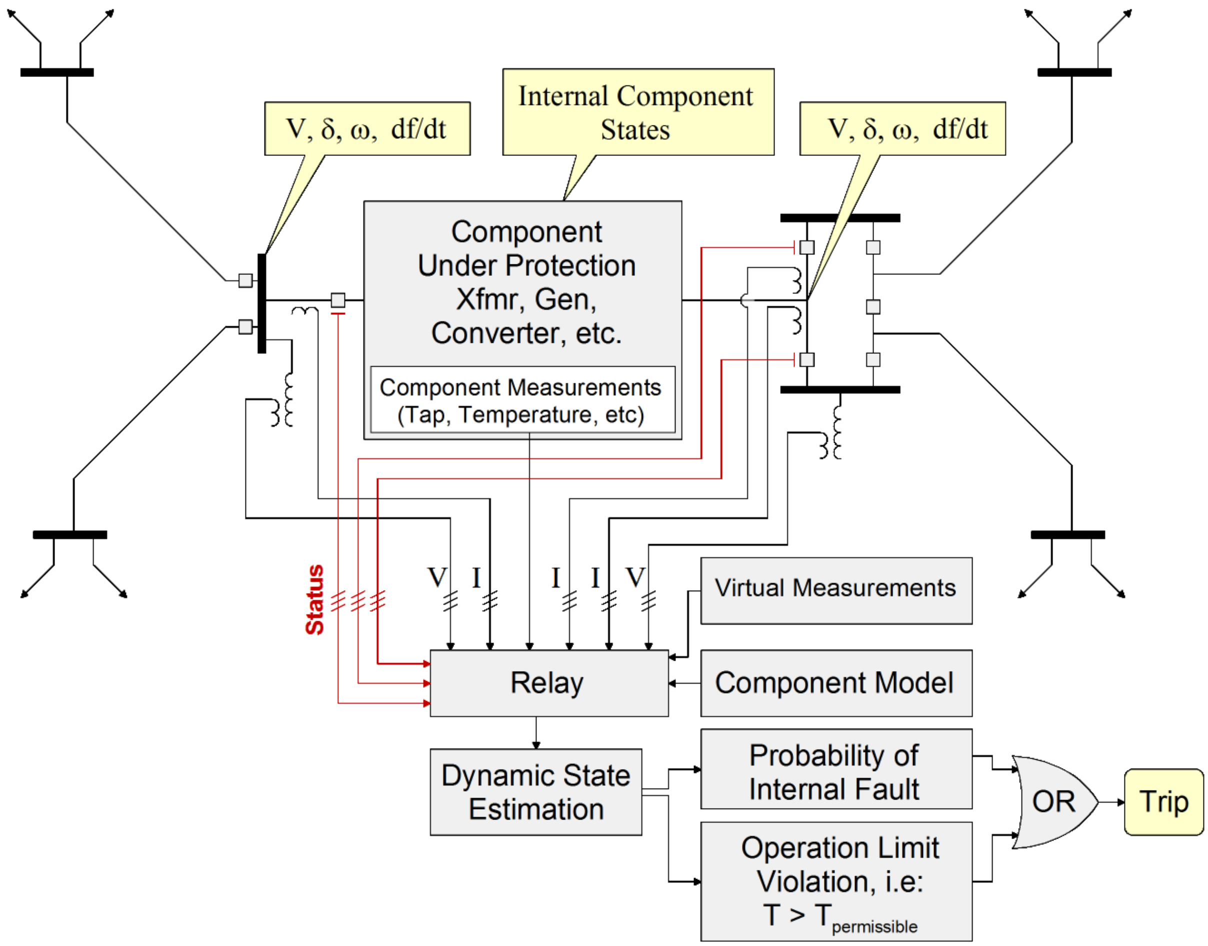


Figure 5.48: State Estimation Based Protection

The operating condition can be compared to the operating limits of the component to develop the protection action. The logic for the protection action is illustrated in Figure 5.49.

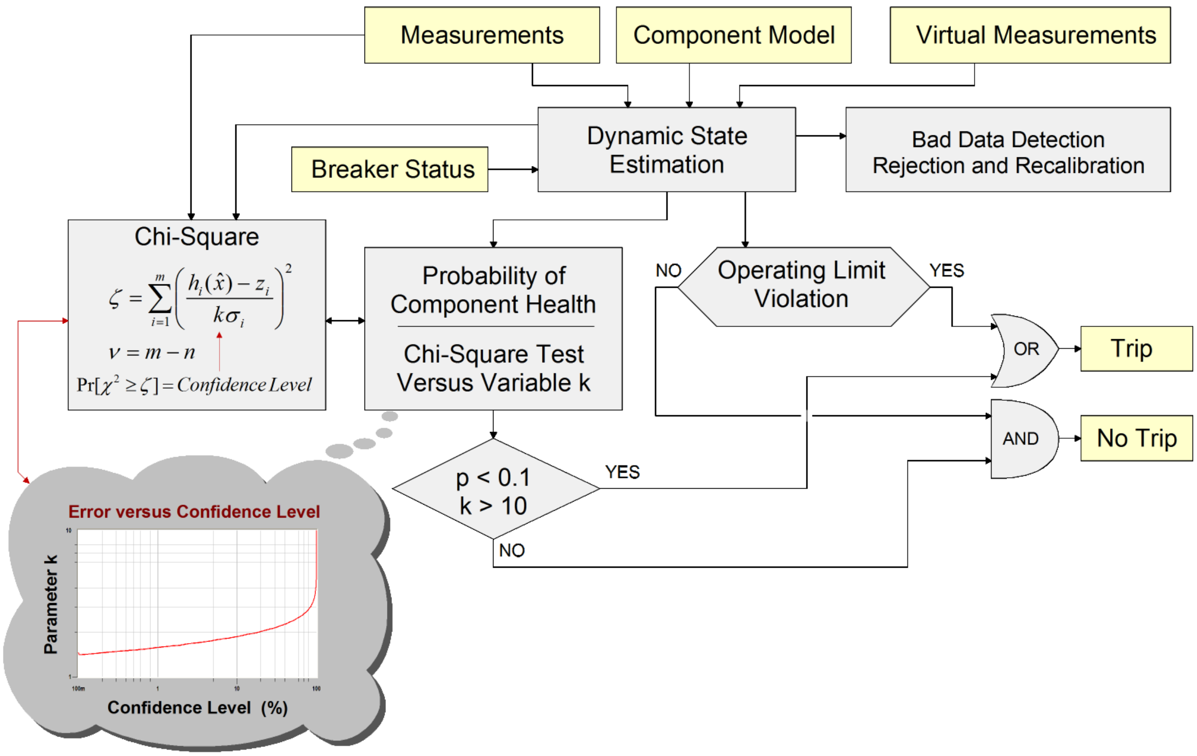


Figure 5.49: Protection Logic in State Estimation Based Protection

This approach faces some challenges which can be overcome with present technology. A partial list of the challenges is given below:

1. Ability to perform the dynamic state estimation in real time
2. Initialization issues
3. Communications in case of a geographically extended component (i.e. lines)
4. New modeling approaches for components - connects well with the topic of modeling
5. Requirement for GPS synchronized measurements in case of multiple independent data acquisition systems.
6. other

The modeling issue is fundamental in this approach. For success the model must be high fidelity so that the component state estimator will reliably determine the operating status (health) of the component. For example consider a transformer during energization. The transformer will experience high in-rush current that represent a tolerable operating condition and therefore no relay action should occur. The component state estimator should be able to "track" the in-rush current and determine that they represent a tolerable operating condition. This requires a transformer model that accurately models saturation and in-rush current in the transformer. We can foresee the possibility that a high fidelity model used for protective relaying can be used as the main depository of the model which can provide the appropriate model for other applications. For example for EMS applications, a positive sequence model can be computed from the high fidelity model and send to the EMS data base. The advantage of this approach will be that the

EMS model will come from a field validated model (the utilization of the model by the relay in real time provide the validation of the model). This overall approach is shown in Figure 5.50. Since protection is ubiquitous, it makes economic sense to use relays for distributed model data base that provides the capability of perpetual model validation.

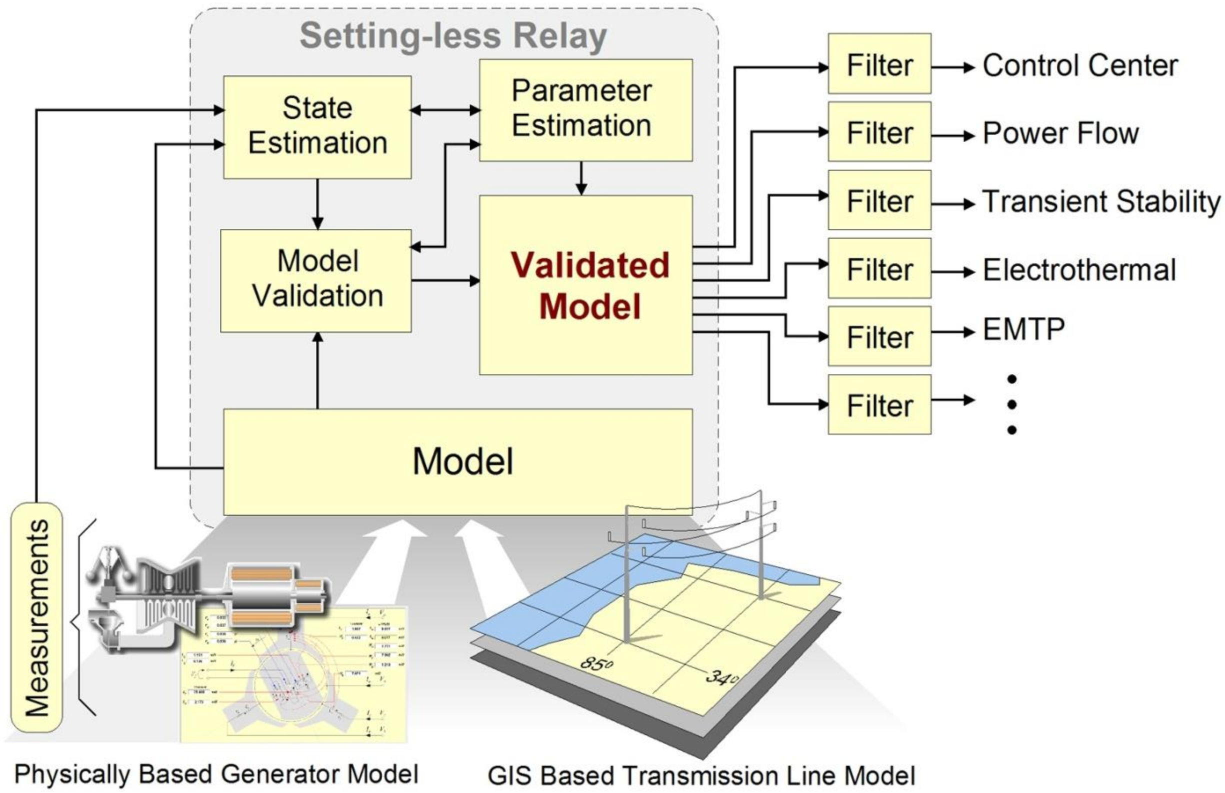


Figure 5.50: Overall Approach for Component Protection

5.15.1 Implementation of State Estimation Based Protection

The implementation of the setting-less protection consists of three steps: (1) Protection zone (component) dynamic model and measurement dynamic model: The component dynamic model consists of a set of algebraic and differential equations in terms of the protection zone states. Using this model, any measurement on the protection zone can be expressed as a function of the protection zone states. (2) Dynamic state estimation: given a set of measurements in the protection zone, all expressed as functions of the protection zone dynamic state, the dynamic state estimation can be performed with well-established dynamic state estimators. We will describe later on three alternate dynamic state estimation algorithms, and (3) Protection logic: when the method detects a fault in the protection zone, the decision to trip the protection zone can be performed with a number of user selected options, such as delay, verification, etc.

The dynamic state estimation can be very complex. The basic problem is to determine how well a set of measurements fits the dynamic model of the protection zone which is in general a set of algebraic and differential equations. A basic method for this is the Extended Kalman Filter

(EKF). An equivalent method has been developed, which uses numerical integration to convert the dynamic model into a set of equivalent algebraic model, refer to by mathematicians as Algebraic Companion Form (ACF). Using the ACF, the dynamic state estimation is converted to an algorithm that resembles a static state estimator. The resulting algorithm has the similar performance as traditional dynamic state estimators such as the EKF. This method is presented next. Because the mathematics tend to be complex, object orientation is used to reduce the complexity and streamline the algorithm.

Consider a protection zone and the dynamic model of the protection zone expressed in terms of a set of differential and algebraic equations. Without loss of generality, we can cast these equations into a set of linear and nonlinear equations of the following syntax.

$$i(t) = Y_{eqx1}\mathbf{x}(t) + D_{eqxd1}\frac{d\mathbf{x}(t)}{dt} + C_{eqc1}$$

$$0 = Y_{eqx2}\mathbf{x}(t) + D_{eqxd2}\frac{d\mathbf{x}(t)}{dt} + C_{eqc2}$$

$$0 = Y_{eqx3}\mathbf{x}(t) + \begin{Bmatrix} \vdots \\ \mathbf{x}(t)^T \left\langle F^i_{eqxx3} \right\rangle \mathbf{x}(t) \\ \vdots \end{Bmatrix} + C_{eqc3}$$

Note that the nonlinear equations have degree not higher than two. We refer to this as the quadratized dynamical model. This can be always achieved, for example if the model has nonlinearities higher than degree two, the additional state variables are introduced until the model becomes quadratic.

The QDM is subsequently integrated to yield the ACF. We use quadratic integration to convert the QDM into an ACF. The resulting ACF will be a Quadratic Algebraic Companion Form (QACF). It will have the following syntax, obtained by quadratically integrating the equations of the QDM:

$$\begin{Bmatrix} i(t) \\ 0 \\ 0 \\ i(t_m) \\ 0 \\ 0 \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \left\{ x^T F^i_{eqxx} x \right\} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - M_{eq} i(t-h) - K_{eq}$$

Where:

$$Y_{eqx} = \begin{bmatrix} \frac{4}{h}D_{eqxd1} + Y_{eqx1} & -\frac{8}{h}D_{eqxd1} \\ D_{eqxd2} + \frac{h}{6}Y_{eqx2} & \frac{2h}{3}Y_{eqx2} \\ Y_{eqx3} & 0 \\ \frac{1}{2h}D_{eqxd1} & \frac{2}{h}D_{eqxd1} + Y_{eqx1} \\ -\frac{h}{24}Y_{eqx2} & D_{eqxd2} + \frac{h}{3}Y_{eqx2} \\ 0 & Y_{eqx3} \end{bmatrix}, \qquad F_{eqx} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ F_{eqxx3} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & F_{eqxx3} \end{bmatrix}$$

$$N_{eqx} = \begin{bmatrix} -Y_{eqx1} + \frac{4}{h}D_{eqxd1} \\ \frac{h}{6}Y_{eqx2} - D_{eqxd2} \\ 0 \\ \frac{1}{2}Y_{eqx1} - \frac{5}{2h}D_{eqxd1} \\ \frac{5h}{24}Y_{eqx2} - D_{eqxd2} \\ 0 \end{bmatrix} \qquad M_{eq} = \begin{bmatrix} I_{size(i(t))} \\ 0 \\ 0 \\ \frac{1}{2}I_{size(i(t))} \\ 0 \\ 0 \end{bmatrix} \qquad K_{eq} = \begin{bmatrix} 0 \\ hC_{eqc2} \\ C_{eqc3} \\ \frac{3}{2}C_{eqc1} \\ \frac{1}{2}hC_{eqc2} \\ C_{eqc3} \end{bmatrix}$$

In summary, the quadratic algebraic companion form of the dynamical model of any protection zone is obtained with two procedures: (a) model quadratization, and (b) quadratic integration. The model quadratization reduces the model nonlinearities so that the dynamic integration method that is applied to the quadratic model assuming that the functions vary quadratically over the integration time step.

Object Oriented Dynamic State Estimation: Any measurement, i.e. current, voltage, temperature, etc. can be viewed as an object that consists of the measured value and a corresponding function that expresses the measurement as a function of the state of the component. This function can be directly obtained (autonomously) from the Quadratic Algebraic Companion Form of the component. Because the algebraic companion form is quadratic at most, the measurement model will be also quadratic at most. Thus, the object-oriented measurement model can be expressed as the following standard equation:

$$\begin{aligned} z_k(t) &= \sum_i a_{i,t}^k \cdot x_i(t) + \sum_i a_{i,t_m}^k \cdot x_i(t_m) \\ &+ \sum_{i,j} b_{i,j,t}^k \cdot x_i(t) \cdot x_j(t) + \sum_{i,j} b_{i,j,t_m}^k \cdot x_i(t_m) \cdot x_j(t_m) + \sum_{i,j} b_{i,j,t,t_m}^k \cdot x_i(t) \cdot x_j(t_m), \\ &+ c_k(t) + \eta_k \ , \end{aligned} \quad (1)$$

where z is the measured value, t the present time, t_m the midpoint between the present and previous time, x the state variables, a the coefficients of linear terms, b the coefficients of nonlinear terms, c the constant term, and η the measurement error.

In general the measurements can be classified as across and through measurements. The across measurements has a simple model as follows:

$$z_j(t) = x_j(t) + \eta_j \ . \quad (2)$$

The through measurement model is extracted from the algebraic companion form, i.e. the measurement model is simply one equation of the ACF model, as follows:

$$z_j\left(t \, or \, t_m\right) = Y_{eqx}^k \mathbf{x} + \left\{x^T F_{eqxx}^k x\right\} - B_{eq}^k \quad (3)$$

where the superscript k means the kth row of the matrix or the vector.

In addition, the model can provide virtual measurements, in the form of equations that must be satisfied. Consider for example the mth QACF model equation below:

$$0 = Y_{eqx}^m \mathbf{x} + \left\{x^T F_{eqxx}^m x\right\} - B_{eq}^m \quad (4)$$

This equation is simply a relationship among the states the component that must be satisfied. Therefore we can state that the zero value is a measurement that we know with certainty. We refer to this as a virtual measurement.

Eventually, all measurement objects form the following measurement set:

$$z = h\left(x,t,t_m\right) + \eta = c + a^T x\left(t\right) + b^T x\left(t_m\right) + \left[x^T\left(t\right) \quad x^T\left(t_m\right)\right] F \begin{bmatrix} x\left(t\right) \\ x\left(t_m\right) \end{bmatrix} + \eta \, , \quad (5)$$

where z is the measurement vector, x the state vector, h the known function of the model, a, b are constant vectors, F are constant matrices, and η the vector of measurement errors.

The proposed dynamic state estimation algorithm is the weighted least squares (WLS). The objective function is formulated as follows:

$$Minimize \quad J(x,t) = \left[z - h(x,t,t_m)\right]^T W \left[z - h(x,t,t_m)\right], \quad (6)$$

where W is the diagonal matrix whose non-zero entries are the inverse of the variance of the measurement errors. The solution is obtained by the iterative method:

$$\hat{x}^{j+1} = \hat{x}^{j} + (H^T W H)^{-1} H^T W (z - h(\hat{x}^{j}, t)), \qquad (7)$$

where $\hat{x}$ is the best estimate of states and H the Jacobian matrix of $h(x,t)$.

It is important to note that for any component, the number of actual measurements and virtual measurements exceed the number of states and they are independent. This makes the system observable.

Component Health Index: The solution of the dynamic state estimation provides the best estimate of the dynamic state of the component. The well-known chi-square test provides the probability that the measurements are consistent with the dynamic model of the component. Thus the chi-square test quantifies the goodness of fit between the model and measurements (i.e., confidence level). The goodness of fit is expressed as the probability that the measurement errors are distributed within their expected range (chi-square distribution). The chi-square test requires two parameters: the degree of freedom (ν) and the chi-square critical value (ζ). In order to quantify the probability with one single variable, we introduce the variable k in the definition of the chi-square variable:

$$\nu = m - n, \qquad \zeta = \sum_{i=1}^{m} \left(\frac{h_i(\hat{x}) - z_i}{k\sigma_i} \right)^2, \qquad (8)$$

where m is the number of measurements, n the number of states, and $\hat{x}$ the best estimate of states. Note that since m is always greater than n, the degrees of freedom are always positive. Note also that if k is equal to 1.0 then the standard deviation of the measurement error corresponds to the meter error specifications. If k equals 2.0 then the standard deviation will be twice as much as the meter specifications, and so on. Using this definition, the results of the chi square test can be expressed as a function of the variable k. Specifically, the goodness of fit (confidence level) can be obtained as follows:

$$\Pr[\chi^2 \geq \zeta(k)] = 1.0 - \Pr[\chi^2 \leq \zeta(k)] = 1.0 - \Pr(\zeta(k), \nu). \qquad (9)$$

A sample report of the confidence level function (horizontal axis) versus the chi-square critical value, k, (vertical axis) is depicted in Figure 6.

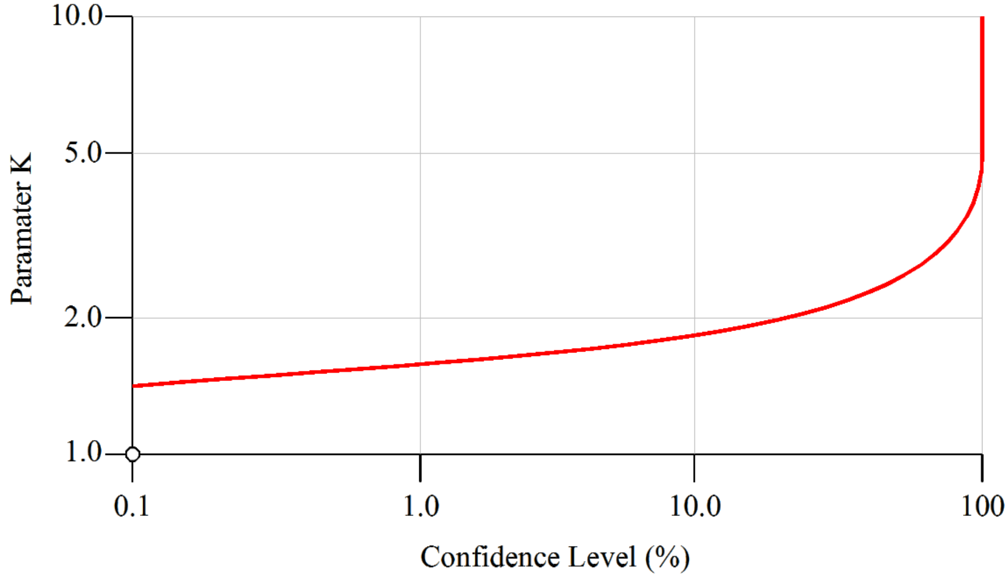


Figure 5.51: Confidence Level (%) vs Parameter k

The proposed method uses the confidence level as the health index of a component. A high confidence level indicates good fit between the measurement and the model, and thus we can conclude that the physical laws of the component are satisfied and the component has no internal fault. A low confidence level, however, implies inconsistency between the measurement and the model; therefore, we can conclude that an abnormality (internal fault) has occurred in the component.

It is important to point out that the component protection relay must not trip circuit breakers except when the component itself is faulty (internal fault). For example, in case of a transformer, inrush currents or overexcitation currents, should be considered normal and the protection system should not trip the component. The proposed protection scheme can adaptively differentiate these phenomena from internal faults. Similarly for start-up currents in a motor, etc.

We present a couple of application examples, one for a transmission line and another for a capacitor bank.

5.15.2 Numerical Examples of State Estimation Based Protection

Consider the transmission line with the characteristics shown in Figure 5.x.

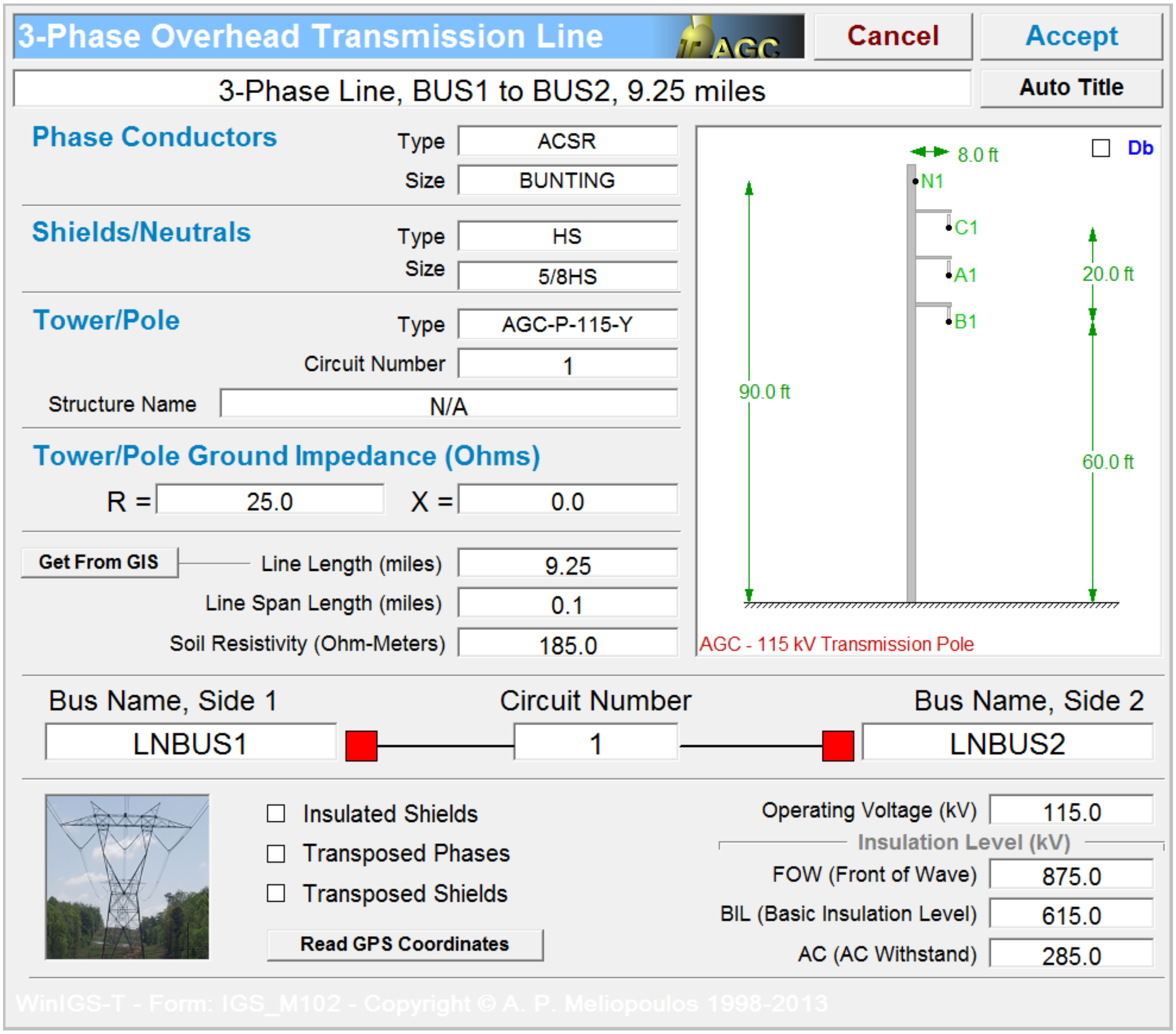


Figure 5.x: Transmission Line Under Protection - Used to Generate the Events in the COMTRADE Files

The circuit model for above transmission line is given in terms of the matrices R, L and C. The matrices are given in Figure 5.y.

Matrix R

	C1	A1	B1	N1
C1	1.618	0.8454	0.8477	0.8408
A1	0.8454	1.622	0.8499	0.8430
B1	0.8477	0.8499	1.627	0.8452
N1	0.8408	0.8430	0.8452	21.83

Matrix omega.L

	C1	A1	B1	N1
C1	+ j12.82	+ j6.702	+ j5.922	+ j6.483
A1	+ j6.702	+ j12.81	+ j6.697	+ j5.862
B1	+ j5.922	+ j6.697	+ j12.81	+ j5.439
N1	+ j6.483	+ j5.862	+ j5.439	+ j19.65

Matrix omega.C (times 0.000001)

	C1	A1	B1	N1
C1	+ j24.05	- j5.955	- j3.012	- j5.263
A1	- j5.955	+ j24.66	- j6.145	- j2.766
B1	- j3.012	- j6.145	+ j23.49	- j1.904
N1	- j5.263	- j2.766	- j1.904	+ j20.20

Figure A.1-2: Transmission Line Parameters (Generalized pi-Equivalent Parameters)

The compact transmission line model is derived from this circuit and it is given with the following equations:

$$i_1(t) = C \cdot \frac{dv_1(t)}{dt} + i_L(t) + G \cdot L \cdot \frac{di_L(t)}{dt}$$
$$i_2(t) = C \cdot \frac{dv_2(t)}{dt} - i_L(t) - G \cdot L \cdot \frac{di_L(t)}{dt}$$

$$0 = -v_1(t) + v_2(t) + R\cdot(i_L(t) + G\cdot L\cdot\frac{di_L(t)}{dt}) + L\cdot\frac{di_L(t)}{dt}$$

where:

R, L, C: are the transmission line resistance, inductance and capacitance matrices
G : is the matrix of stabilizing conductances

Quadratized Model: The transmission line model is linear. Therefore, the quadratized model is identical to the compact model. In compact matrix form, the model is:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = A\cdot\begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + B\cdot\frac{d}{dt}\begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix}$$

where: $i(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$, $v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & I \\ 0 & 0 & -I \\ -I & I & R \end{bmatrix},\quad B = \begin{bmatrix} C & 0 & G\cdot L \\ 0 & C & -G\cdot L \\ 0 & 0 & R\cdot G\cdot L + L \end{bmatrix}$$

Note that the matrices A and B are 12 by 12 matrices; the vectors i(t) and v(t) are 8 by 1 vectors; the vector $i_L(t)$ is a 4 by 1 vector.

The state of the three phase transmission line is defined with:

$$\mathrm{x}(t) = \begin{bmatrix} v_1(t) \\ v_1(t) \\ i_L(t) \end{bmatrix}$$

Algebraic Companion Form Model: the algebraic companion form model is derived after quadratic integration of the quadratized model with a time step h. The result is:

$$\begin{bmatrix} \frac{h}{6}I & 0 & \frac{2h}{3}I & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{h}{24}I & 0 & \frac{h}{3}I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} i(t) \\ 0 \\ i(t_m) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{h}{6}A + B & \frac{2h}{3}A \\ -\frac{h}{24}A & \frac{h}{3}A + B \end{bmatrix}\cdot\begin{bmatrix} v(t) \\ i_L(t) \\ v(t_m) \\ i_L(t_m) \end{bmatrix} - \begin{bmatrix} B - \frac{h}{6}A \\ B - \frac{5h}{24}A \end{bmatrix}\cdot\begin{bmatrix} v(t-h) \\ i_L(t-h) \end{bmatrix} - \begin{bmatrix} \frac{h}{6}I & 0 \\ 0 & 0 \\ \frac{5h}{24}I & 0 \\ 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} i(t-h) \\ 0 \end{bmatrix}$$

Let $E = \begin{bmatrix} \frac{4}{h}I & 0 & -\frac{8}{h}I & 0 \\ 0 & I & 0 & 0 \\ \frac{1}{2h}I & 0 & \frac{2}{h}I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}^{-1}$, $F_1 = \begin{bmatrix} \frac{h}{6}A + B & \frac{2h}{3}A \\ -\frac{h}{24}A & \frac{h}{3}A + B \end{bmatrix}$, $F_2 = \begin{bmatrix} B - \frac{h}{6}A \\ B - \frac{5h}{24}A \end{bmatrix}$, $F_3 = \begin{bmatrix} \frac{h}{6}I & 0 \\ 0 & 0 \\ \frac{5h}{24}I & 0 \\ 0 & 0 \end{bmatrix}$

Then: $\begin{bmatrix} i(t) \\ 0 \\ i(t_m) \\ 0 \end{bmatrix} = E \cdot F_1 \cdot \begin{bmatrix} v(t) \\ i_L(t) \\ v(t_m) \\ i_L(t_m) \end{bmatrix} - b_{eq}$

where $b_{eq} = E \cdot F_2 \cdot \begin{bmatrix} v(t-h) \\ i_L(t-h) \end{bmatrix} + E \cdot F_3 \cdot \begin{bmatrix} i(t-h) \\ 0 \end{bmatrix}$

You should compute the above model (matrices EF1, EF2, and EF3) using the parameters of the specific transmission line, provided in section A.2. Then, using these matrices, write explicitly the model equations (a total of 24 equations).

The measurements will be as follows:

Actual measurements: six currents at time t (phase A, phase B, and phase C; side one and side two of the line); six voltages at time t (phase A-N, phase B-N, and phase C-N; side one and side two of the line); six currents at time tm=t-h/2 (phase A, phase B, and phase C; side one and side two of the line); six voltages at time tm=t-h/2 (phase A-N, phase B-N, and phase C-N; side one and side two of the line); For these measurements assume a measurement error with standard deviation equal to 0.01 pu.

Virtual measurements: these measurements represent the zero value on the left hand side of the equations 9 through 12 and 21 through 24: four measurements with value equal 0.0 at time t (equations 9 through 12); four measurements with value equal zero at time tm=t-h/2 (equations 21 through 24); For these measurements assume a measurement error with standard deviation equal to 0.001 pu (as a matter of fact these measurements have zero error but you cannot use zero as this will generate singularity in the state estimation algorithm).

Pseudo measurements: these measurements represent quantities that are normally not measured, such as current in the neutral, or voltage at the neutral; for these quantities we can assume a certain value and assign a relatively large measurement error; in this case the pseudo measurements will be: two measurements of the neutral voltages at time t (neutral voltage on side one of the line, neutral voltage on side two of the line); two measurements of the neutral voltages at time tm=t-t/h (neutral voltage on side one of the line, neutral voltage on side two of the line); for these measurements assume a measurement error with standard deviation equal to 0.1 pu.

Note that for this line, you will have 24 actual measurements, 8 virtual measurements, and 4 pseudo measurements; a total of 36.
Note that for this line, you will have 24 states.
This provides a redundancy of 50% ((36-24)/24).

Summarizing and substituting the numerical values, the set of measurement models are:

$$z_1(t) = v_{an}(t) = v_a(t) - v_n(t) + \eta_1$$
$$z_2(t) = v_{bn}(t) = v_b(t) - v_n(t) + \eta_2$$
$$z_3(t) = v_{cn}(t) = v_c(t) - v_n(t) + \eta_3$$
$$z_4(t) = v_{AN}(t) = v_A(t) - v_N(t) + \eta_4$$
$$z_5(t) = v_{BN}(t) = v_B(t) - v_N(t) + \eta_5$$
$$z_6(t) = v_{CN}(t) = v_C(t) - v_N(t) + \eta_6$$
$$z_7(t) = i_a(t) = \ldots\ldots\ldots + \eta_7$$
$$z_8(t) = i_b(t) = \ldots\ldots\ldots + \eta_8$$
$$z_9(t) = i_c(t) = \ldots\ldots\ldots + \eta_9$$
$$z_{10}(t) = i_A(t) = \ldots\ldots\ldots + \eta_{10}$$
$$z_{11}(t) = i_B(t) = \ldots\ldots\ldots + \eta_{11}$$
$$z_{12}(t) = i_C(t) = \ldots\ldots\ldots + \eta_{12}$$
$$z_{13}(t) = 0 = \ldots\ldots\ldots + \eta_{13}$$
$$z_{14}(t) = 0 = \ldots\ldots\ldots + \eta_{14}$$
$$z_{15}(t) = 0 = \ldots\ldots\ldots + \eta_{15}$$
$$z_{16}(t) = 0 = \ldots\ldots\ldots + \eta_{16}$$

Plus pseudo measurements at time t

Repeat for measurements at time tm

To be completed.

5.16 Problems

Problem P5.1: Consider the simplified power system of Figure P5.1. The indicated three lines are protected with the following scheme.

Problem P5.2: An engineer considers the underreaching direct trip (UDT) scheme for a three terminal line. Discuss the advantages and disadvantages of UDT scheme in this case.

Problem P5.3: Future.

Problem P5.4: Provide the functional description of the relay 32 (a couple of sentences will suffice). Describe also the settings of a typical 32 relay function.

Solution: Relay 32 is a reverse power relay. It monitors the real power flow on a circuit and trip the circuit if the real power flow is in a certain direction.

Problem P5.5: Consider a 600 A circuit, 13.8 kV, two miles long, as it is illustrated in Figure 5.5a. The equivalent source impedance (on a 100 MVA basis) is:

$z_1 = z_2 = j0.30\ pu, \quad z_0 = j0.28\ pu$

The CT is rated 1200:5A. The impedance of the circuit is

$z_1 = z_2 = j0.70\, ohms/mile, \quad z_0 = j2.10\, ohms/mile$

A time overcurrent relay is located at the indicated location. The trip characteristics of this relay are illustrated in Figure P5.5b as function of multiples of pickup current and time dial setting. The relay settings are: pickup current=6A, time dial=2.0. Compute the time to trip for a line to line fault at the middle of the line.

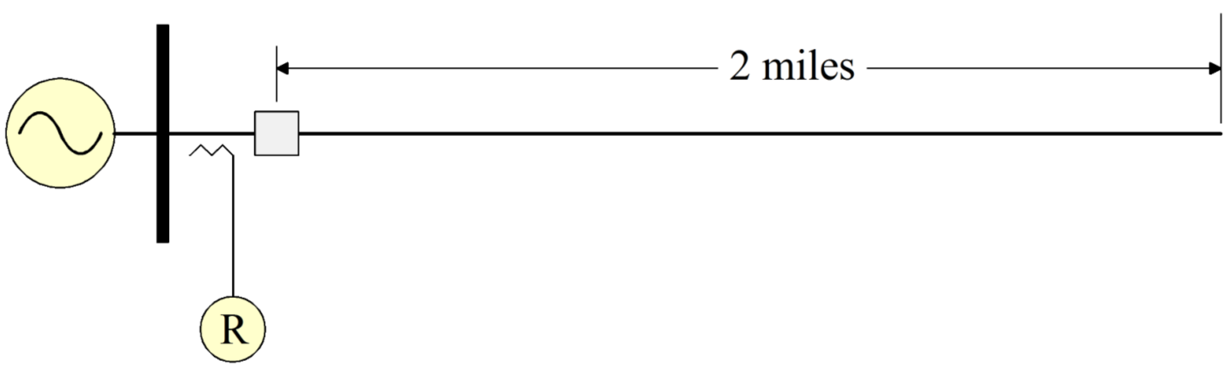


Figure P5.5a

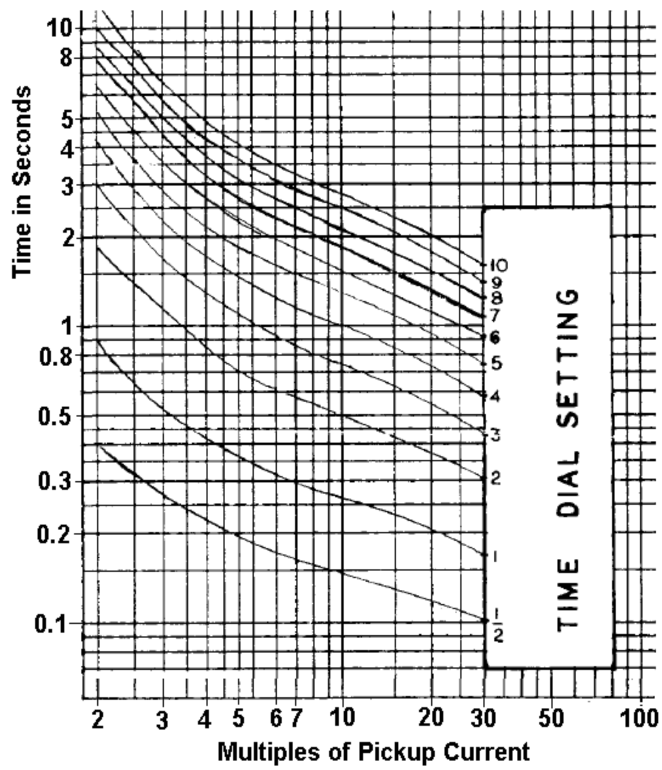
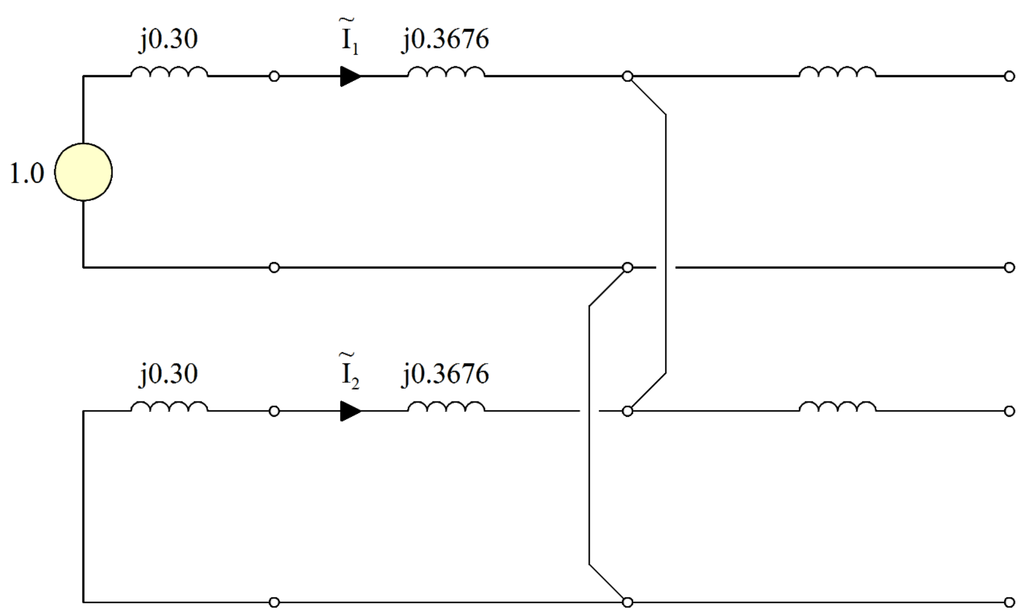


Figure P5.5b

Solution: The equivalent circuit for a line to line fault in the middle of the line is shown in the figure below.



The electric currents are:

$$\tilde{I}_1 = \frac{1.0}{j1.3352} = -j0.7490\,pu, \quad \tilde{I}_2 = j0.7490\,pu$$

Thus:

$$\tilde{I}_a = \tilde{I}_1 + \tilde{I}_2 = 0, \quad \tilde{I}_b = a^2\tilde{I}_1 + a\tilde{I}_2 = -1.2973\,pu, \quad \tilde{I}_b = a\tilde{I}_1 + a^2\tilde{I}_2 = 1.2973\,pu$$

In actual units:

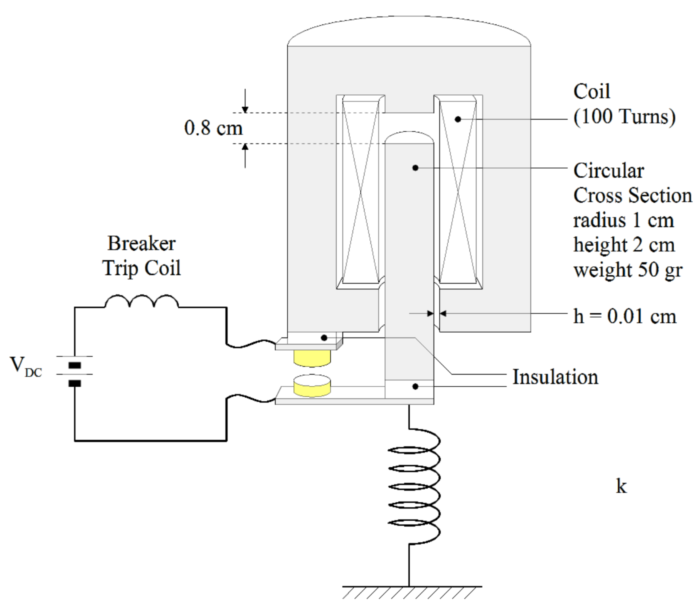
$$\tilde{I}_b = -1.2973\frac{100/3}{13.8/\sqrt{3}} = -5.4275\,kA$$

The current in the secondary of the CT will be:

$$I_R = 5.4275/240\,kA = 22.6\,A, \quad or \quad 3.769\,times\ pickup\,current$$

From graph and for time dial setting of 2, the time to trip is 0.9 seconds.

Problem P5.6: Consider the plunger relay of the Figure. The dimensions of the relay are: gap=0.8 cm, the cross area of the plunger is circular of 1 cm radius, the height of the plunger is 2 cm, its weight is 50 gr and the height of the bottom air gap is 1 cm (w=1 cm). The clearance h is 0.01 cm, the spring constant is 50 N/m and the number of turns is 100. At the "open" position, the spring is stretched by 0.2 cm. For this relay compute the "pickup" current and the "dropout" current. Note: One kilogram-force (kg) equals 9.80665 Newtons (N).



Solution: The pickup current is the current that will generate a electromagnetic force that will be equal the force of the spring plus gravity at the "open" position.

To be continued.

Problem P5.7: Obtain a copy of the IEEE Std C37.2-1996, “IEEE Standard Electrical Power System Device Function Numbers and Contact Designations”.

Provide the functional description of the following devices (a couple of sentences will suffice – you are encouraged to avoid copying the standard and to use your own words):

21
24
27
32
40
46
47
50
51
59
81
87

Problem P5.8: Consider the electric power system illustrated in Figure P5.4a. The construction of the lines in each of the two right-of-ways is illustrated in Figure P5.4b. The phase conductors of the lines have the following parameters:

$$r = 0.08\, ohms/mile, \quad a = 0.5\, inches, \quad GMR = 0.0325\, feet$$

The length of the lines shown horizontally in the Figure is 83 miles, while the length of the lines shown vertically is 12 miles. All lines are 230 kV line. The three equivalent sources are identical with the following parameters:

$$z_1 = z_2 = j12.2\, ohms, \quad z_0 = j8.9\, ohms$$

Prior to the fault the system operates with nominal voltages and unloaded.

(a) Consider a three phase fault at point A. Compute the impedance “seen” by the indicated relay in the figure.
(b) Consider a single phase to ground fault at point A. Compute the impedance “seen” by the relay at the breaker position B.

Use of the computer program WinIGS is encouraged. In case that computer programs are not used, neglect the shield wires and capacitive currents for simplicity.

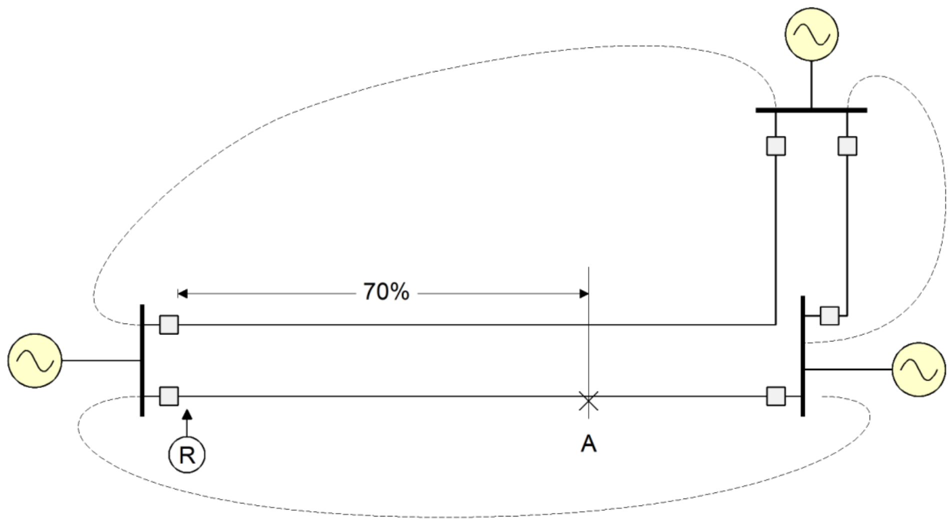


Figure P5.4a

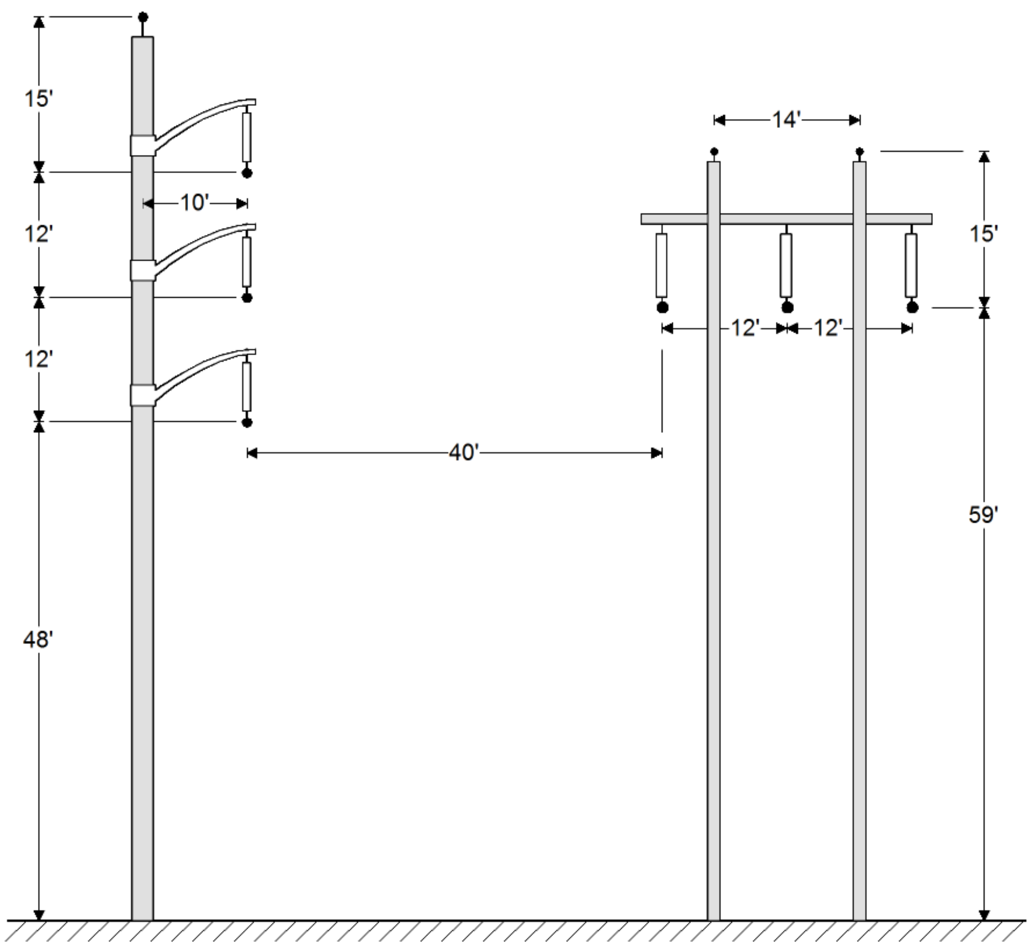


Figure P5.4b

Problem P5.9: Consider the electric power system illustrated in Figure P5.9. The indicated three phase, $36\,MVA$, $138kV/13.8kV$, $z = j0.08\,pu$ delta-wye connected variable tap transformer is protected with a differential relay. The delta side is the 138 kV side. The settings of the relay are: 5% restrain, minimum pickup 0.5 Amperes. The CTs are 200:5 and 1200:5 for the high voltage side and low voltage side respectively. The high voltage side CTs are connected wye and the low voltage side CTs are connected delta. The grounding resistance is $R_g = 25\,ohms$. The parameters of the equivalent source and line are:

Source: $Z_1 = Z_2 = j5.1\,ohms$, $Z_0 = j4.8\,ohms$
Transmission Line: $Z_1 = Z_2 = j13.2\,ohms$, $Z_0 = j32.8\,ohms$

(a) Assume that the transformer tap is set to nominal. Determine whether the relay will trip or not for a single phase to ground fault at the indicated fault location.
(b) Assume that the transformer tap is set to 12.9 kV. Determine whether the relay will trip or not for a single phase to ground fault at the indicated fault location.
(c)

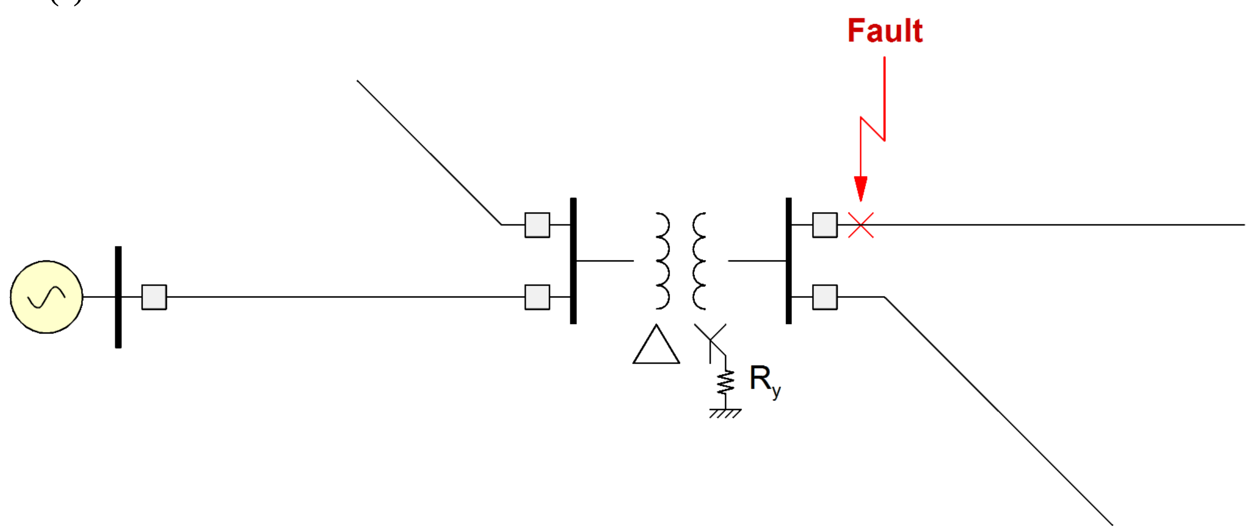
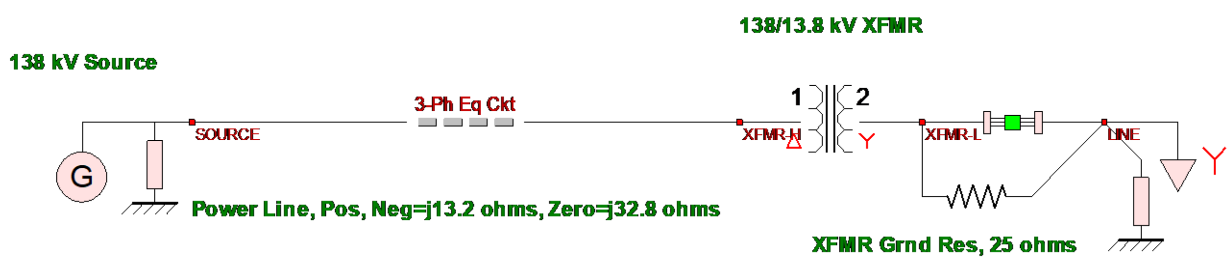
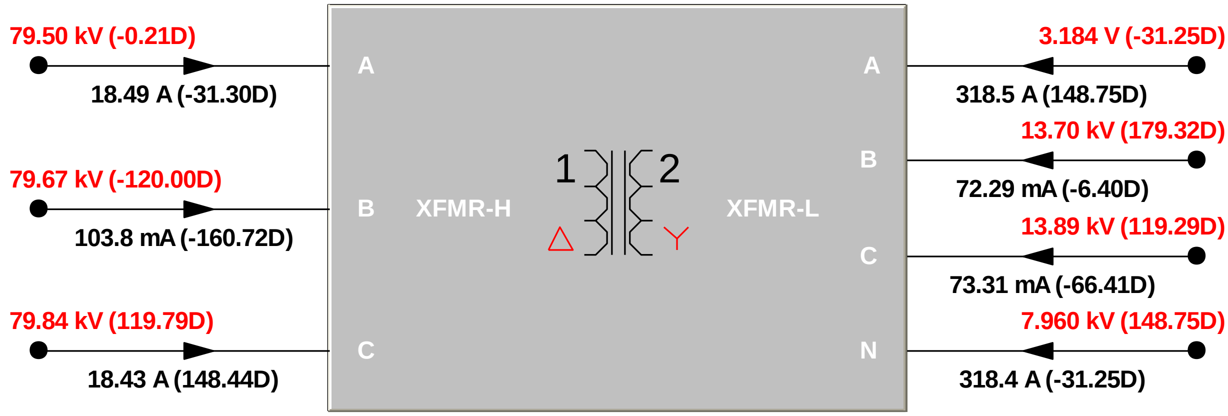


Figure P5.9

Solution: First the system is modeled in WinIGS and a single line to ground fault is performed. The WinIGS model is shown in the figure below.



The results are:

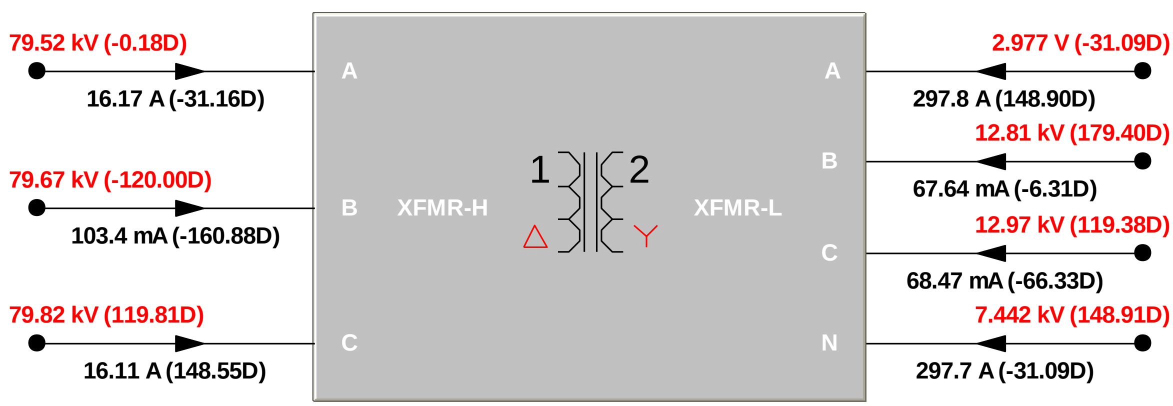


Now considering the connections of the CTs for a delta-wye connected transformer differential scheme, the electric current in the operating coils of the relay are:

Operating Coil A: $0.279e^{-j31.4^0}\ A$
Operating Coil B: negligible
Operating Coil C: $0.278e^{j148.7^0}\ A$

The relay will not operate.

Now by changing the tap and performing a fault analysis the results are:



Considering the connections of the CTs for a delta-wye connected transformer differential scheme, the electric current in the operating coils of the relay are:

Operating Coil A: $0.2848e^{-j31.35^0}\ A$
Operating Coil B: negligible
Operating Coil C: $0.2835e^{j148.8^0}\ A$

Problem P5.10: Consider the electric power system illustrated in Figure P5.6. Line 1 is protected with a POTT scheme. Assume a single phase to ground fault at location A of the line. Determine which breaker will trip first and how. What trips the other breaker?

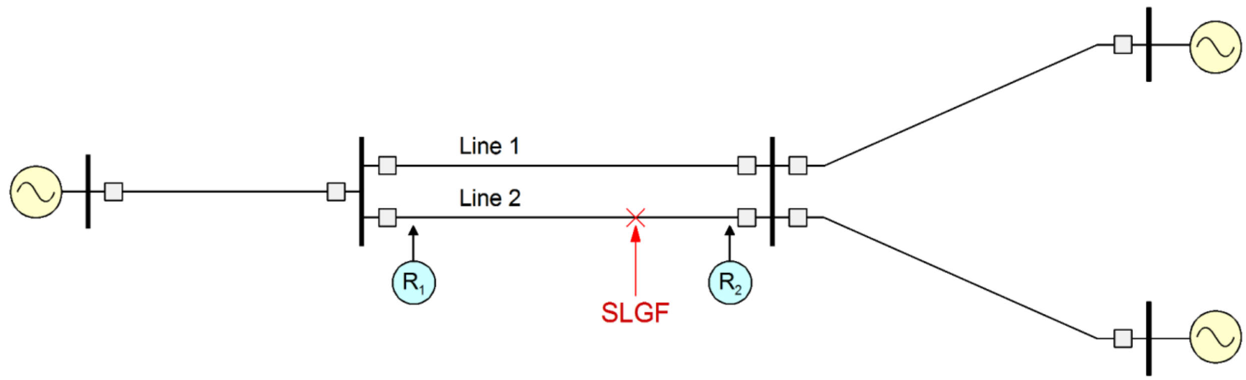


Figure P5.6

Problem P5.11: Consider the electric power system illustrated in Figure P5.7. Line 1 is protected with a distance relay. Assume a single phase to ground fault at location A of the line. Determine the impedance seen by the indicated distance relay.

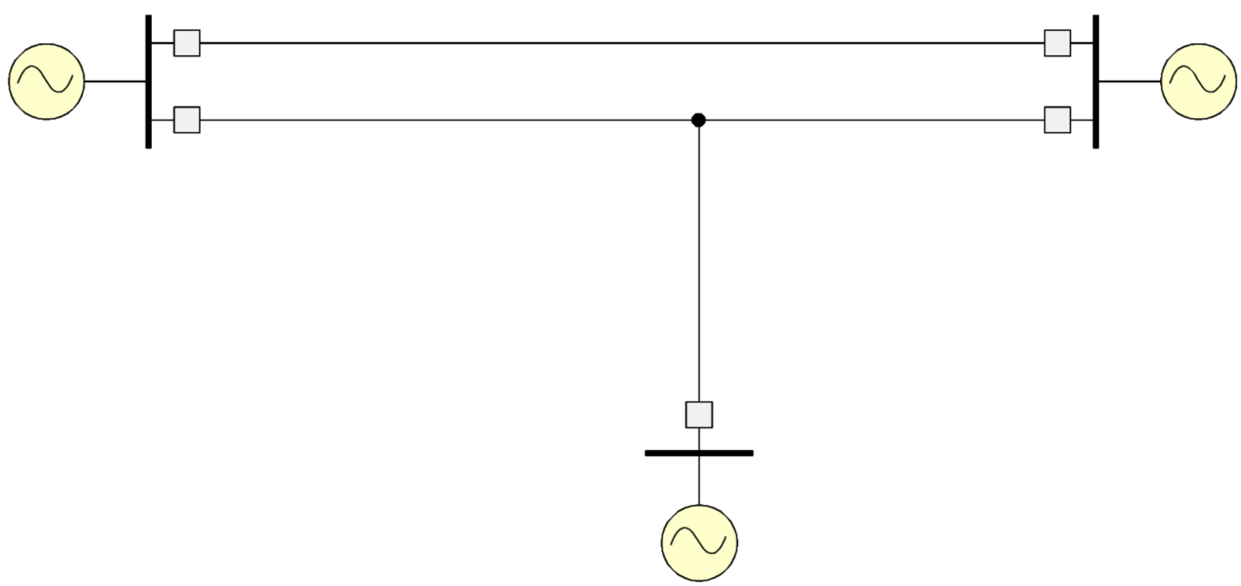


Figure P5.7

Problem P5.12: Consider the 115 kV electric power system illustrated in Figure P5.12. Line 1A is protected with a distance relay. Assume a single phase to ground fault at location A of line 1B. Location A is in the middle of line 1B. Determine the impedance seen by the indicated distance relay.

Each line section has the following parameters:

$z_1 = j0.68\, ohms/mi, \quad z_2 = j0.68\, ohms/mi, \quad z_0 = j2.65\, ohms/mi$
Each equivalent source has the following parameters:
$Z_1 = j0.1\ pu, \quad Z_2 = j0.1\ pu, \quad Z_0 = j0.05\ pu \quad @\,100\, MVA$

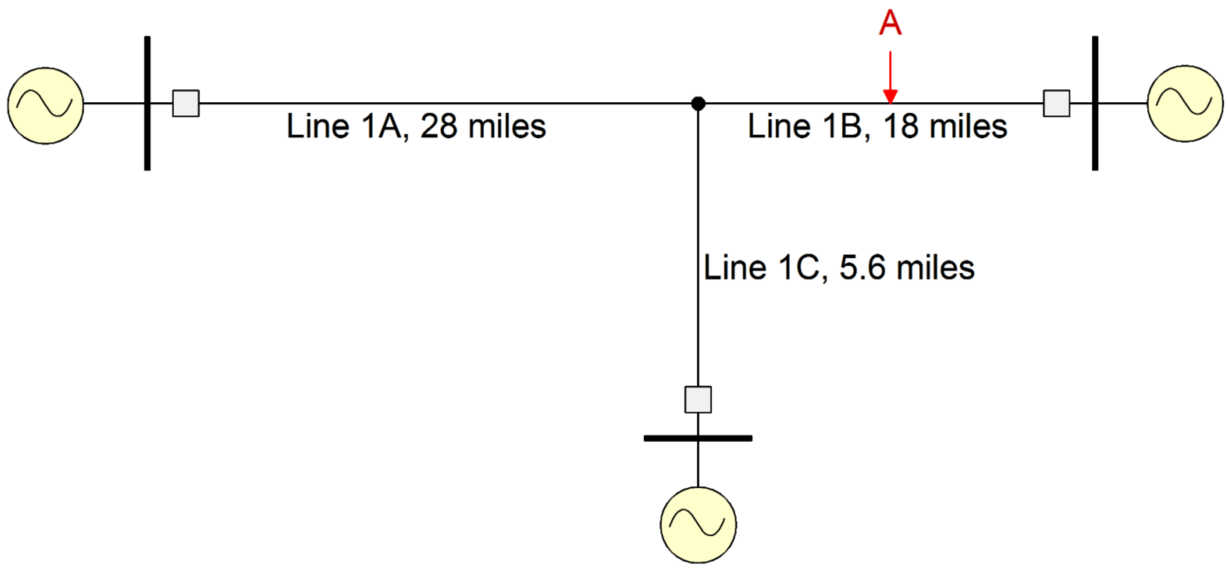


Figure P5.12

Solution: For the specified condition, the voltages and currents at the location of the relay will be determined. The equivalent circuit is shown in Figure P5.12a. In the figure all quantities have been converted into pu. Upon network reduction, the circuit is reduced to the one shown in Figure P5.12b. Solution of this network yields:

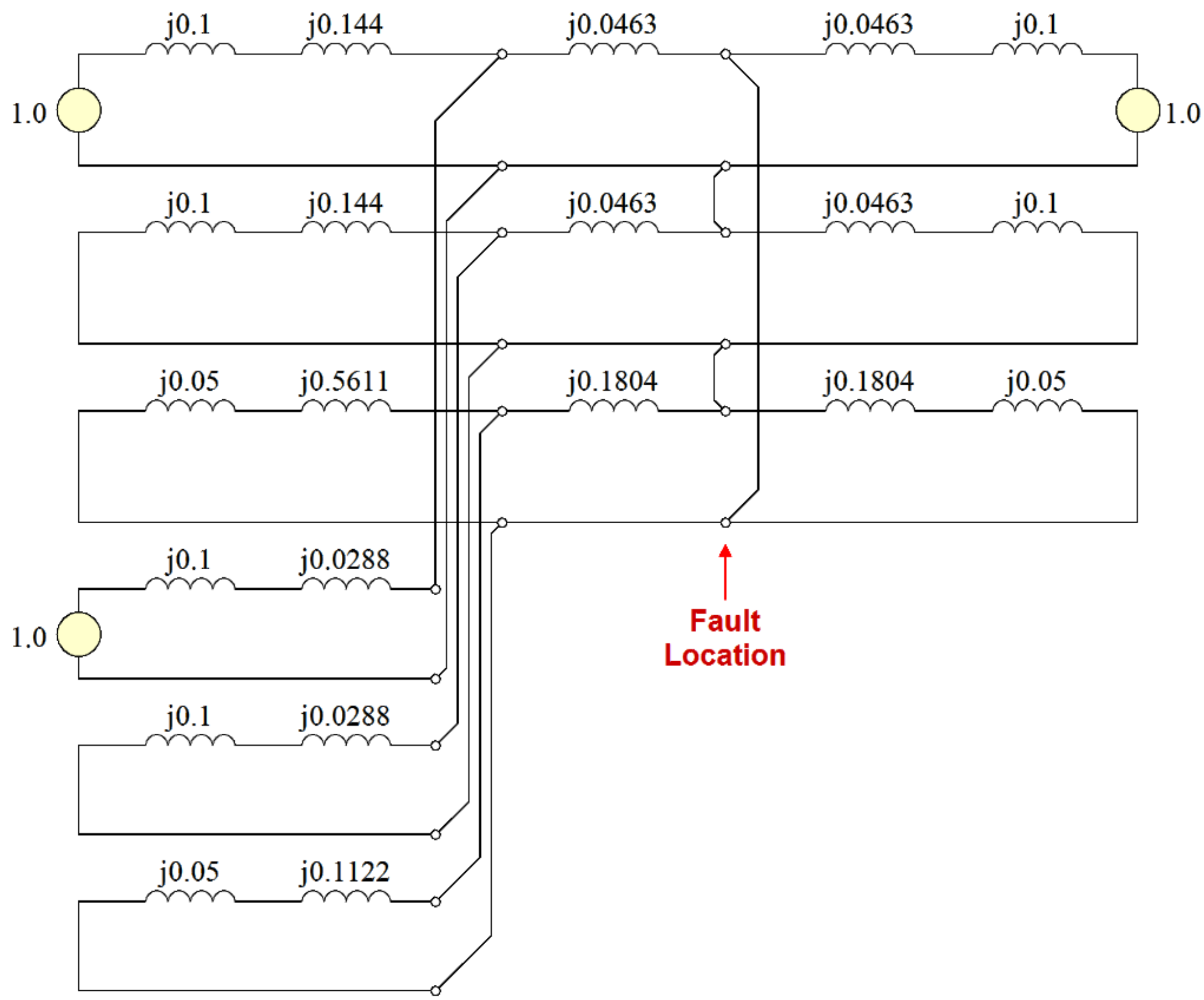


Figure P5.12a

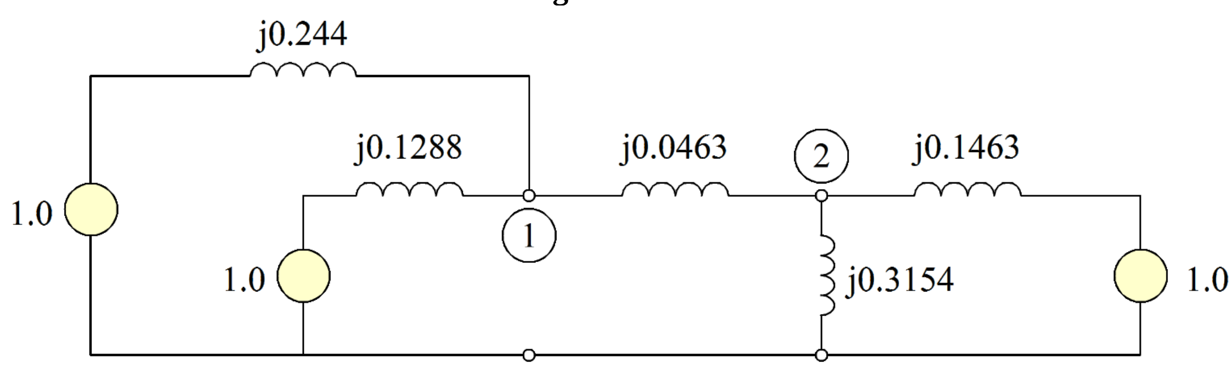


Figure P5.12b

$\tilde{V}_1 = 0.8350\, pu, \quad \tilde{V}_2 = 0.7443\, pu$

The phase A current and zero sequence current at the location of the relay is:

$\tilde{V}_a = 0.8481\,pu, \quad \tilde{I}_a = -j1.6847\,pu, \quad \tilde{I}_0 = -j0.3322\,pu$

The impedance seen by the relay (on the line side) is:

$$Z_{relay(line)} = \frac{0.8481}{-j1.6847 + (2.8)(-j0.3322)} = j0.3243\,pu \quad OR \quad j42.8918\,ohms$$

Problem P5.13: Consider the electric power system of Figure P5.13a. Each transmission line is protected with (a) a directional instantaneous overcurrent relay, (b) a directional time overcurrent relay and (c) a modified impedance relay (mho relay) at both ends of the line. All indicated lines are identical – the design of the lines is shown in Figure P5.13b. The phase conductors are ACSR, BITTERN and the shield wires are ALUMOWELD, 3#7AW. The tower ground resistance is 35 ohms. The soil resistivity is 185 ohms. The two lines 1 and 2 are on the same right of way separated by 75 feet (centerline to centerline). The lengths of the lines are: 32, 58, 27 and 42 miles as indicated in Figure P5.13a. All sources have the following impedances on a 100 MVA, 230 kV basis:

$z_1 = z_2 = 0.001 + j0.01\ pu, \quad z_0 = 0.002 + j0.008\ pu$

The operating voltage of the system is 230 kV. The transformation ratio of the CTs is 2000:5 while the ratio for the potential transformers is 135,000:115.

The settings of the relays are:

Instantaneous overcurrent relay: pickup current: 15A, direction: forward

Time overcurrent relay: pickup current: 5A, very inverse, time dial 0.5, direction: forward.

Modified impedance relay: impedance setting: x ohms, angle: 85 degrees, compensation factor: 2.8, trip delay 0.05 seconds.

Consider a single line to ground fault at the indicate location which is 46 miles from the left terminal of the line.

(a) Generate the model of the system in WinIGS format.
(b) Compute the fault currents at the location of the relays.
(c) Determine which relay will operate or will not operate (note that there are a total of six relays for the line under protection). If a relay will operate determine the time of operation.

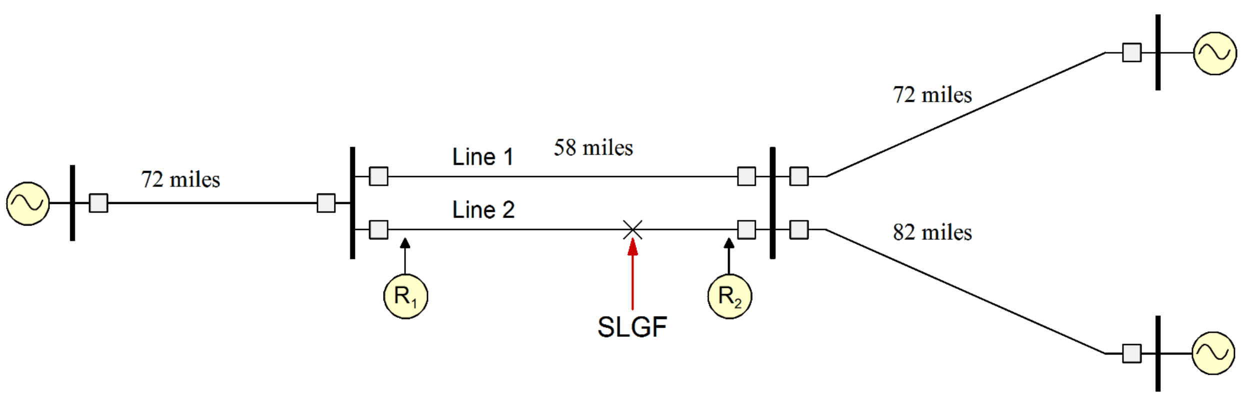


Figure P5.13a

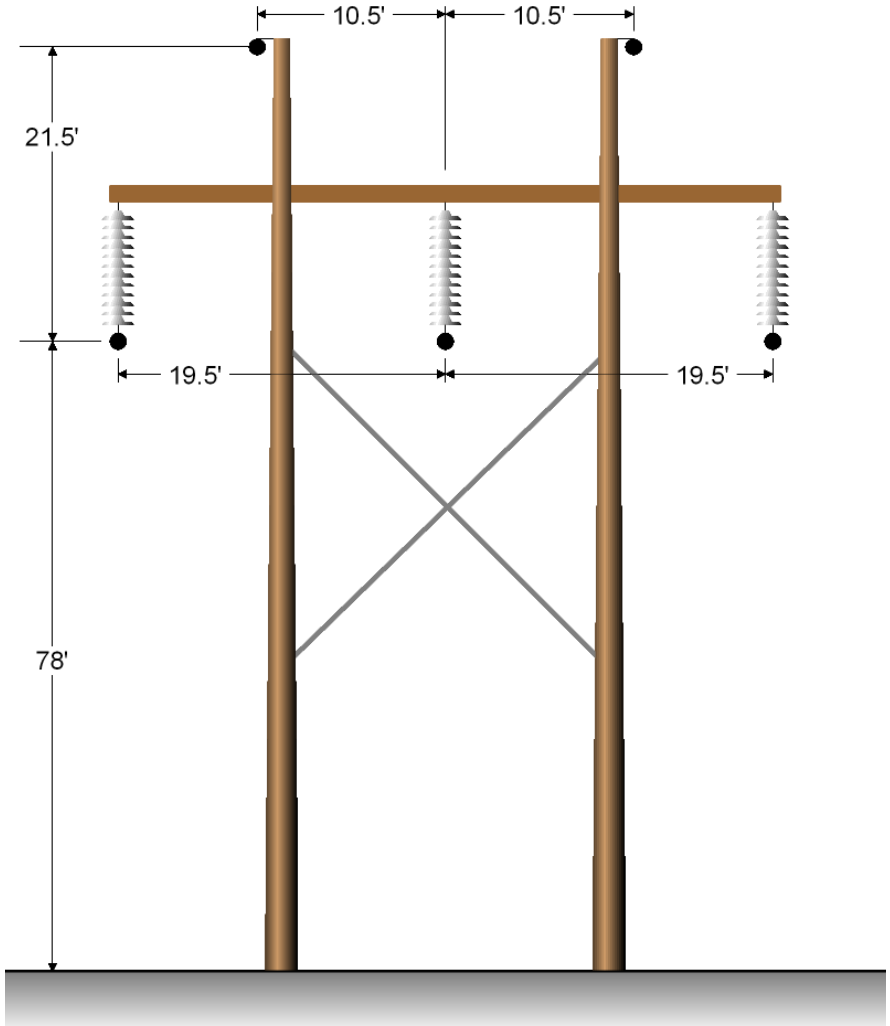


Figure P5.13b

Problem P5.14: Design an outfeed problem. (to be added).

Problem P5.15: A three phase transformer is rated $36\,MVA,\quad 115kV/13.8kV,\quad z = j0.08\,pu$, delta-wye connected (the delta connection is on the high voltage side). It is desired to generate a circuit that will have an output as close as possible to the net current flowing into the transformer. For this purpose, CTs of the following transformation ratios are available: (a) 1200:5, (b) 1500:5, (c) 1600:5, (d) 150:5, and (e) 200:5. Select the proper CTs for this application and compute the operating current when the transformer load at the secondary is nominal, at nominal voltage and power factor 1.0.

Solution: First we select a CT on the high voltage side to provide approximately 5 A on the secondary of the CT under full load conditions. The full load current is:

$$I_{p,\,fullload} = \frac{36}{115\sqrt{3}} = 180.7\ Amperes$$

Select FT ratio: 200:5

The CT ratio that will match this selection is

$$r = \frac{115\sqrt{3}}{13.8}\left(\frac{200}{5}\right) = \frac{2{,}886.75}{5}$$

Select the nearest CT ratio: 1600:5

Under full load conditions, the current in the operating coil will be:

$$I_{op} = (180.7)\frac{5}{200} - (180.7)\left(\frac{115}{13.8}\right)\left(\frac{5}{1600}\right)\sqrt{3} = -3.6331$$

Considering the above mismatch, it will be better to select the following CTs: High voltage side: 150:5 and low voltage side: 1600:5. In this case:

$$I_{op} = (180.7)\frac{5}{150} - (180.7)\left(\frac{115}{13.8}\right)\left(\frac{5}{1600}\right)\sqrt{3} = -2.1272\,A$$

Problem P5.16: Consider the electric power system of Figure P5.16. A three-zone distance relay is to be applied for line 1-2 at the terminal 1. The rated current of the 230 kV line is 1,000 Amperes, the line length is 53.5 miles and the fault currents at this location are: three-phase fault: 23.7 kA, single phase to ground fault: 21.9 kA.

(a) Select the instrument transformer ratio (both PT and CT) and the relay settings.
(b) Select the settings of the relay, all three zones. This is an electromechanical relay with the following settings: impedance in multiples of one ohm, phase in multiples of 2.5 degrees, and compensation factor in multiples of 0.1.
(c) Graph the three zones of operation in a complex plane (horizontal axes: R, vertical axis: X). The graph should be scaled and the axes clearly marked.

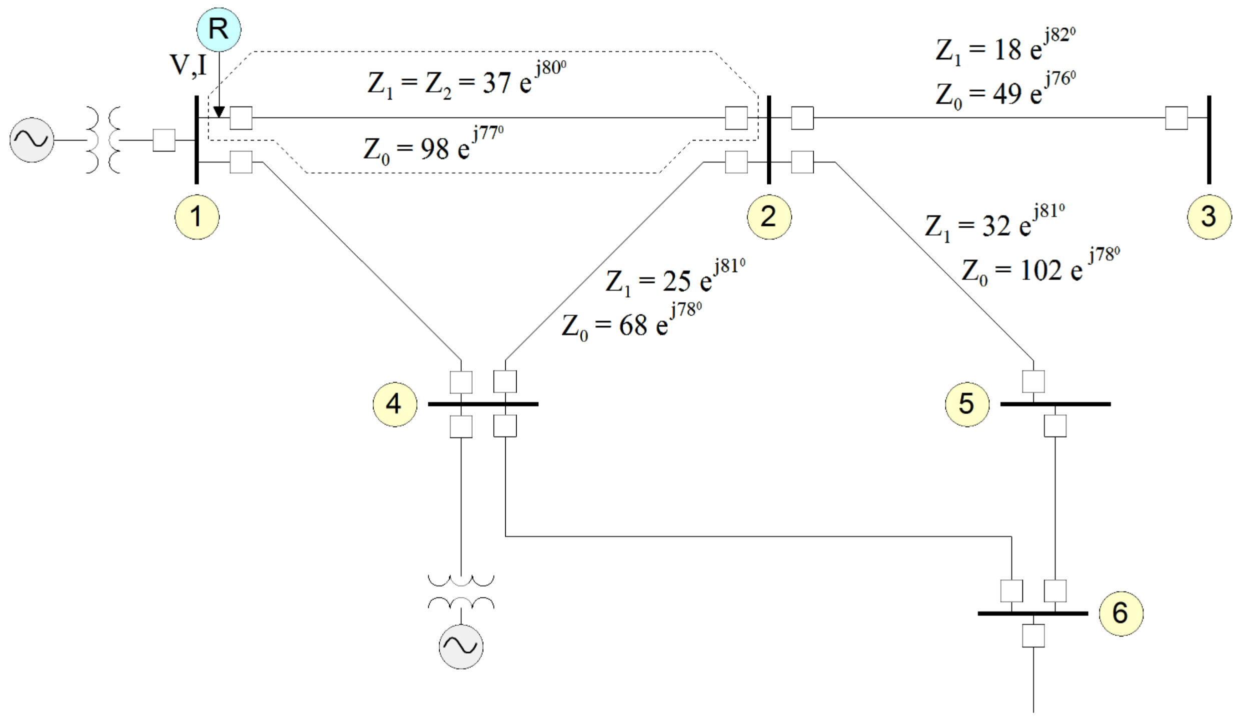


Figure P5.16

Solution: (a) the PTs and CT are selected as follows:

CT: Select 1200:5A, PT Select: 138kV:115V.

(b) The impedance settings of the relay on the line side will be selected as follows:

Zone 1: $Z_{L,zone1} = 29.6e^{j80^0}\ Ohms$, time delay 4 cycles
Zone 2: $Z_{L,zone2} = 46.25e^{j80^0}\ Ohms$, time delay 15 cycles
Zone 3: $Z_{L,zone3} = 77.0e^{j80.5^0}\ Ohms$, time delay 35 cycles

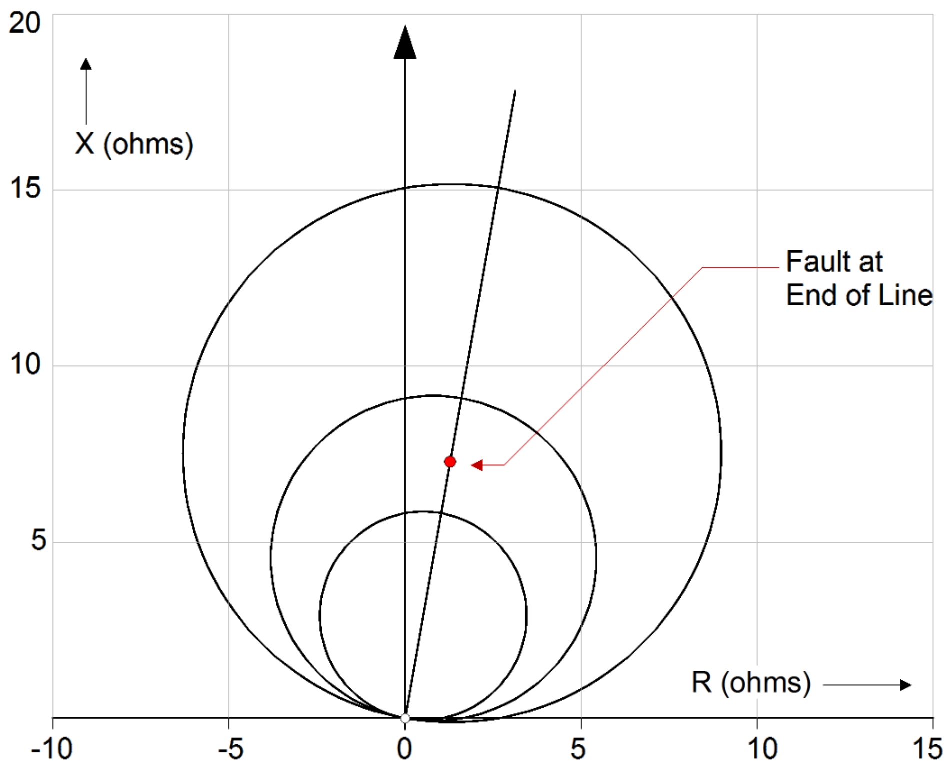
m=1.6486 (z0-z1)/z1

Using the selected PT and CT:

$Z_{Line} = Z_{relay} 5.0$, Thus

Zone 1: $Z_{R,zone1} = 5.92e^{j80^0}\ Ohms$, time delay 4 cycles, Select 6 ohms and 80 degrees, m=1.6
Zone 2: $Z_{R,zone2} = 9.25e^{j80^0}\ Ohms$, time delay 15 cycles, Select 9 ohms and 80 degrees, m=1.6
Zone 3: $Z_{L,zone3} = 15.4e^{j80.5^0}\ Ohms$, time delay 35 cycles, Select 15 ohms and 80 degrees, m=1.6

(d) The graph of the operating regions is given in the Figure. The point under normal operating conditions is outside the graph (16.5 ohms). A three phase fault at the terminal of the line will yield the point indicated in the figure (fault current 3.11 kA, voltage 115.32 kV)



Problem P5.17: Consider the electric power system illustrated in Figure P5.12a. The construction of the lines (all sections) is illustrated in Figure P5.12b. The phase conductors and shield wires of the line are: ACSR, BLUEJAY and HS steel 5/16 respectively. Consider a single phase fault at point A (phase B) of line 2. The indicated relay is a distance relay with a compensation factor of 2.1. Compute the impedance "seen" by this relay. The length of the lines shown horizontally in the Figure is 63 miles, while the length of the lines shown vertically is 8.5 miles. All lines are operating at 230 kV. The three equivalent sources are identical with the following parameters:

$$Z_1 = Z_2 = j0.10\, pu, \quad Z_0 = j0.10\, pu \quad @\,1000\, MVA, 230\, kV$$

Prior to the fault the system operates with nominal voltages and unloaded.

Hint: Perform the short circuit analysis and determine the impedance "seen" by the relay. Use of the program WinIGS is encouraged.

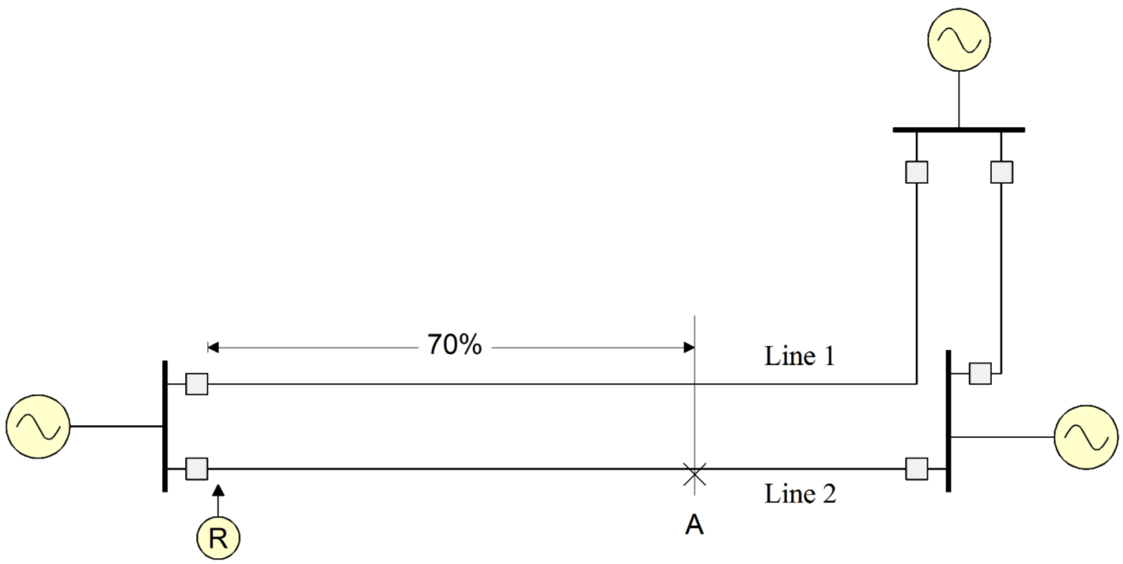


Figure P5.12a

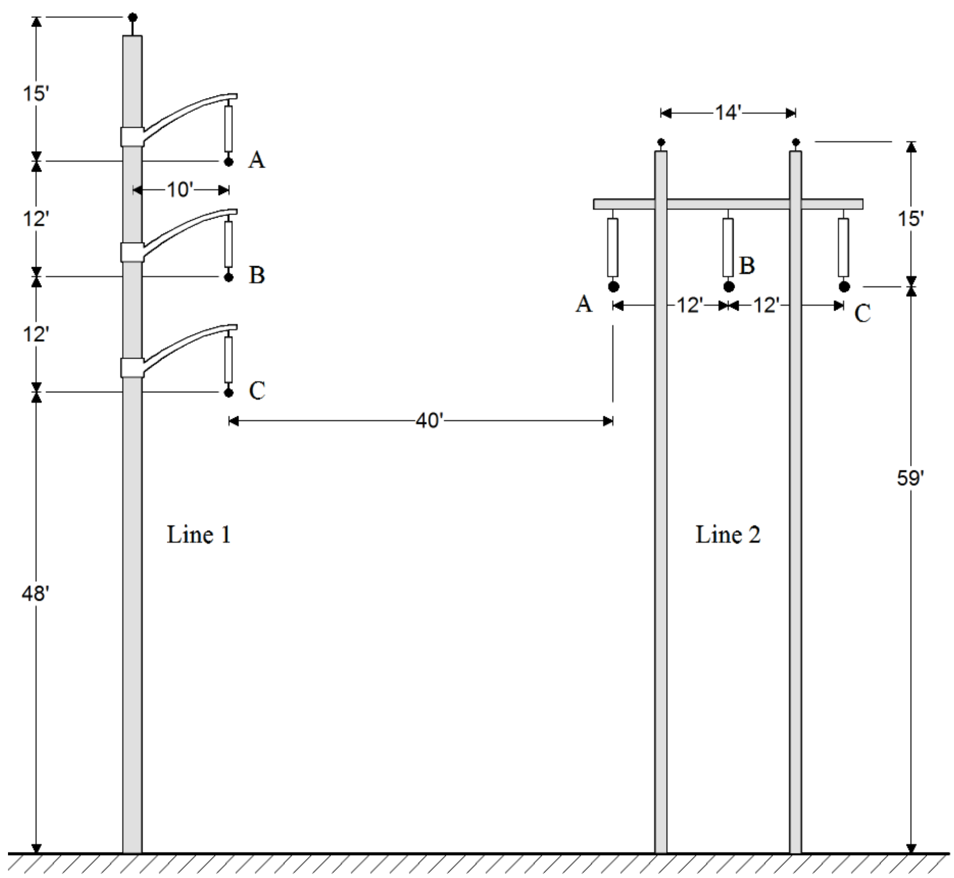
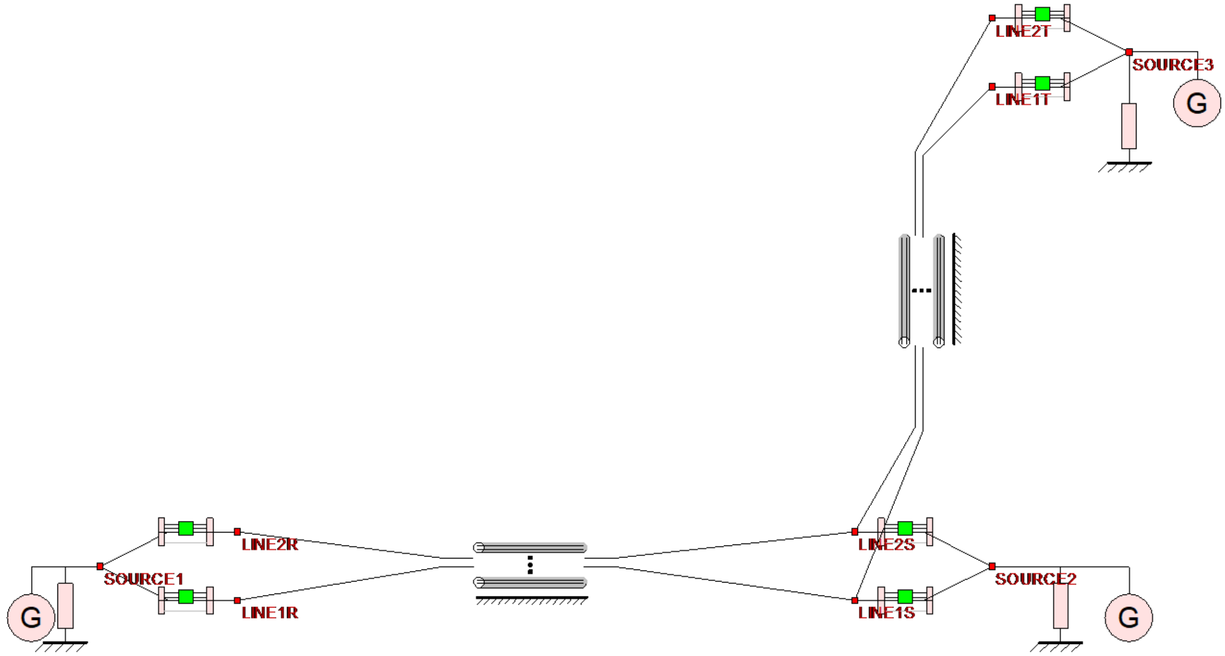
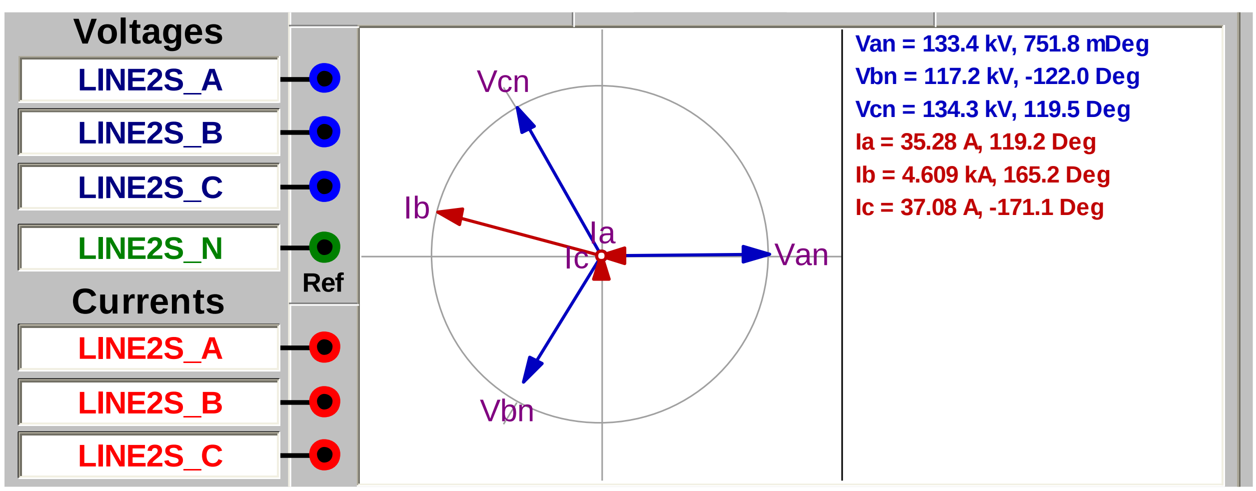


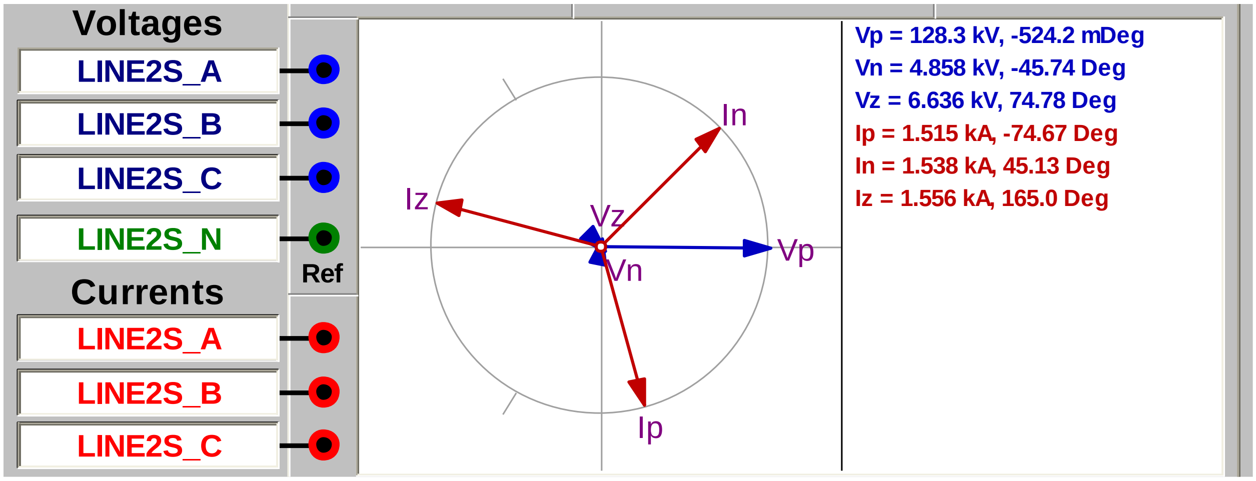
Figure P5.12b

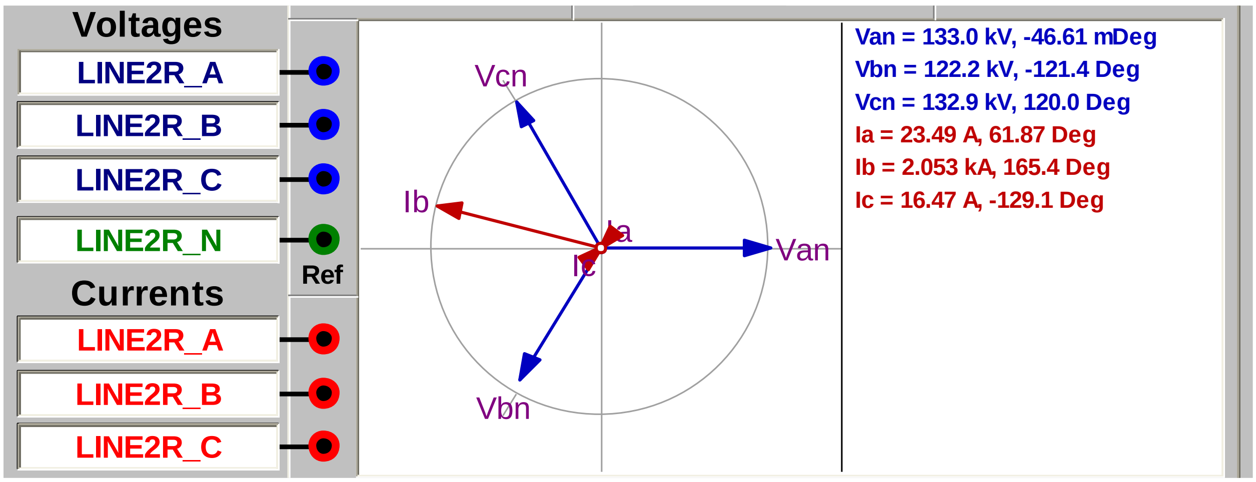
Solution: The system model is developed in the program WinIGS. The WinIGS model is shown in the figure below. A single line to ground fault is performed.

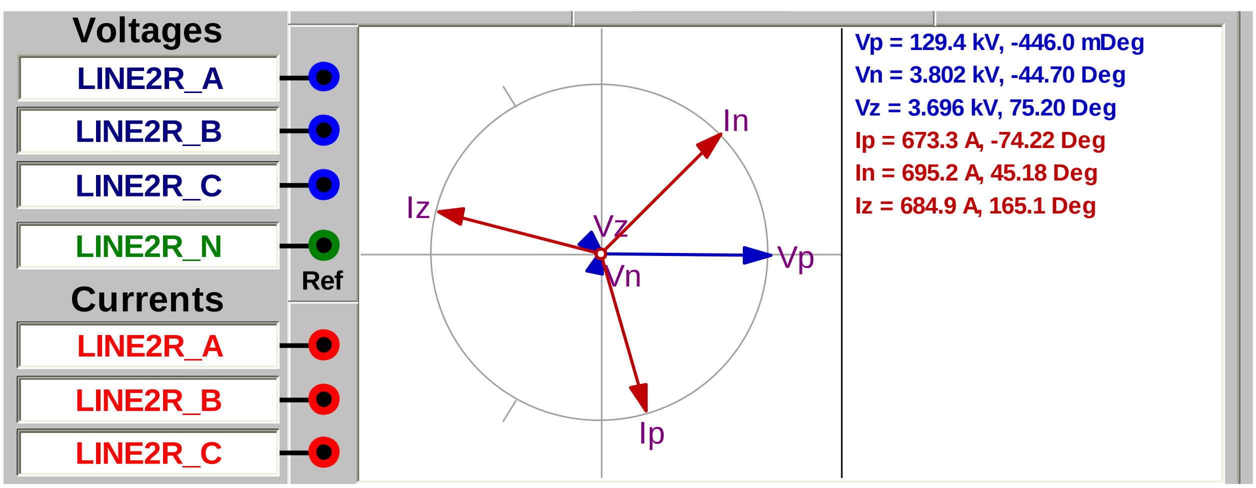


The results are:







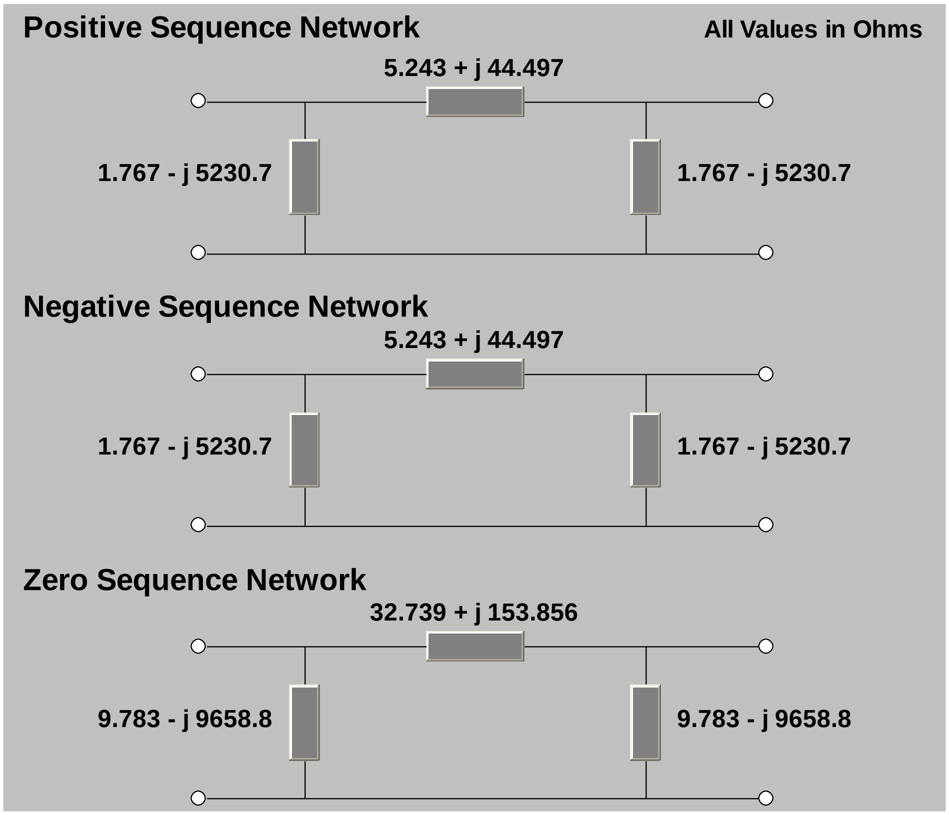


The impedance that the relay will see is:

$$Z_{relay} = \frac{\tilde{V}_b}{\tilde{I}_b + (2.1)\tilde{I}_0} = \frac{122.2e^{-j121.4^0}}{2.054e^{j165.4^0} + (2.1)(0.6849e^{j165.1^0})} = 34.9913e^{j73.324^0} \ ohms$$

The figure below provides the parameters of this line. The distance as a percentage of line length is:

$\ell = \dfrac{34.9913}{44.497} = 0.7864$. This represents an error of 12.34%



Problem P5.20: An impedance relay is used to protect a transmission line with the following parameters:

$$Z_1 = 4.490 + j34.304\,ohms, \quad Z_2 = 4.490 + j34.304\,ohms, \quad Z_0 = 27.022 + j119.343\,ohms$$
$$Z_1^{'} = 0.750 - j148152\,ohms, \quad Z_2^{'} = 0.750 - j148152\,ohms, \quad Z_0^{'} = 4.520 - j24009.3\,ohms$$

The line is 46.5 miles long. The CT ratio of the impedance relay is 2000:5 and the PT ratio is 69kV:115V. The relay engineer decides that the impedance relay will be set as follows: (a) zone 1 will be set to reach 80% of the line, (b) zone 2 will be set to reach 135% of the line and (c) zone 3 will be set to reach 100% of the line plus 125% of the next line. The longest length of the next line is 35 miles and the design is identical to the line under consideration.

The compensation factor settings (m) of the relay are 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8 and 4.0. The impedance settings are: 10 ohms to 30 ohms in increments of 1 ohm.

Select the settings of the impedance relay, i.e. the compensation factor and the impedance setting for each zone in ohms.

Solution: The compensation factor is selected as follows:

$$\frac{z_0 - z_1}{z_1} = \frac{22.532 + j85.039}{4.490 + j34.304} = 2.5428 e^{-j7.38^0}$$

Select: m=2.6

The relay impedance setting will be based on the positive sequence series impedance of the line. The impedance conversion factor from line to relay is:

$$k = \frac{\frac{115}{69000}}{\frac{5}{2000}} = 0.66667$$

Zone 1 setting:

$$Z_1^{relay} = (0.8)(0.66667)(4.490 + j34.304) = 2.395 + j18.295\, ohms = 18.4511 e^{j82.54^0}\ ohms$$

Select Z=18 ohms.

Zone 2 setting:

$$Z_2^{relay} = (1.35)(0.66667)(4.490 + j34.304) = 4.041 + j30.873\, ohms =$$

Zone 3 setting:

$$Z_3^{relay} = \left(1.0 + 1.25\left(\frac{35.0}{45.6}\right)\right)(0.66667)(4.490 + j34.304) = 5.865 + j44.811\, ohms =$$

Problem P5.21: Consider the electric power system of Figure P5.15. The parameters of the various system components are given in the Figure. The sequence parameters of the two lines are given below.

Line 1:
$Z_1 = 7.838 + j53.772\, ohms, \quad Z_2 = 7.838 + j53.772\, ohms, \quad Z_0 = 20.941 + j135.652\, ohms$
Line 2:
$Z_1 = 9.362 + j53.148\, ohms, \quad Z_2 = 9.362 + j53.148\, ohms, \quad Z_0 = 19.062 + j141.123\, ohms$

The mutual zero sequence impedance between the two lines is j27.5 ohms. The mutual positive/negative sequence impedance is zero.

Consider a line to ground fault at location A indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. Assume there is a distance relay located at the infinite bus side of the faulted line. Further assume that the zone one of this relay is set for 54 ohms at an angle of 80 degrees (on the line side) and the compensating factor m of this relay is set to 1.5.

Determine whether the relay will trip on zone 1. Show all your computations.

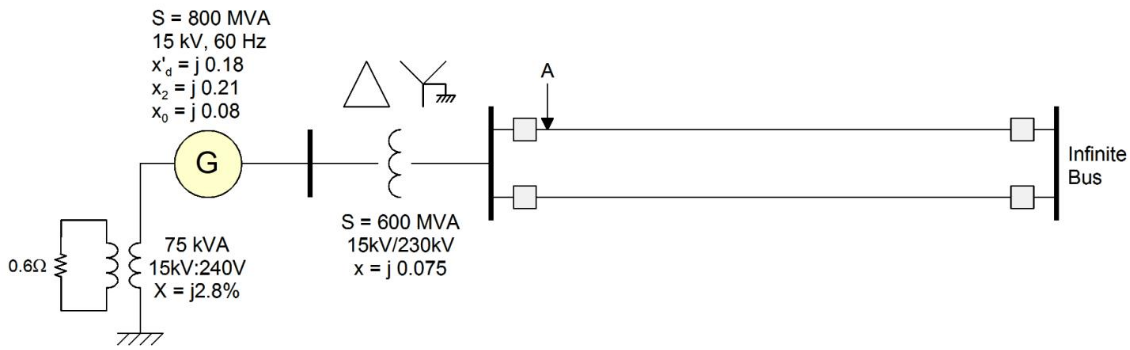


Figure P5.15

$$z = (j0.075)\frac{230^2}{600} = j6.612 ohms$$

$$z_{1,gen} = (j0.21)\frac{15^2}{800}\left(\frac{230}{15}\right)^2 = j13.886 ohms$$

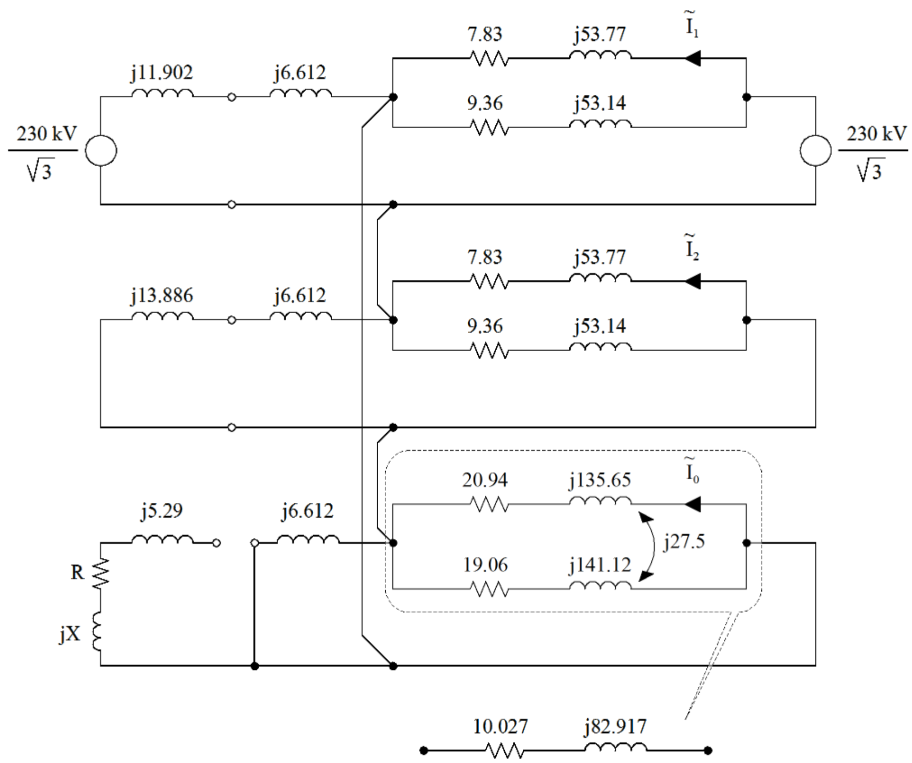
$$V_{0,line} = (20.94 + j135.65)I_0 + (j27.5)I_{0,2}$$

$$(20.94 + j135.65)I_0 + (j27.5)I_{0,2} = (19.06 + j141.12)I_{0,2} + (j27.5)I_0$$

$$Z_{0,bothlines} = \frac{V_{0,line}}{I_0 + I_{0,2}} = 10.027 + j82.917$$

Solution:

The equivalent circuit of the system during the fault is:



Short circuit analysis yields:

$$\tilde{I}_1 = 933.6e^{-j82.31^0}\ A, \quad \tilde{V}_1 = 132.77\,kV$$
$$\tilde{I}_2 = 990.6e^{-j82.53^0}\ A, \quad \tilde{V}_2 \cong 0$$
$$\tilde{I}_0 = 169.1e^{-j80.49^0}\ A, \quad \tilde{V}_0 \cong 0$$

The relay "sees":

$$Z = \frac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0} = \frac{132.77}{\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_0 + 1.5\tilde{I}_0} = 56.6e^{j82.1^0}\ ohms$$

Problem P5.22: Consider the two-unit, two-transformer, and two-transmission line power system of Figure P5.16a. The following data apply to the two units and the two transformers.

Unit G1: Rated Power: 300 MVA, Rated Voltage: 18 kV,
$Z_1 = j0.18\ pu, \quad Z_2 = j0.21\ pu, \quad Z_0 = j0.09\ pu$
Transformer T1: Rated Power: 300 MVA, Rated Voltage: 18 kV/230 kV,
$Z_1 = j0.08\ pu, \quad Z_2 = j0.08\ pu, \quad Z_0 = j0.08\ pu$

Unit G2: Rated Power: 800 MVA, Rated Voltage: 18 kV,
$Z_1 = j0.18\ pu, \quad Z_2 = j0.21\ pu, \quad Z_0 = j0.09\ pu$
Transformer T2: Rated Power: 800 MVA, Rated Voltage: 18 kV/230 kV,

$Z_1 = j0.08\ pu, \quad Z_2 = j0.08\ pu, \quad Z_0 = j0.08\ pu$

All pu values are referred to the corresponding device rated power and voltage.

The two transmission lines have the configuration shown in Figure P5.16b. The two lines parallel for the entire length of the line which is 56 miles. The phase conductors and shield wire sizes are:

Phase Conductors: ACSR, BLUEJAY, Shield Wires: HS steel, 5/16.

Assume a line to line fault on line 1 at a distance of 14.2 miles from transformer 2. Assume also that there is an overcurrent ground fault relay on line 2 at the transformer 2 terminal. The relay is fed via a set of CTs with the transformation ratio of 1200:5. The Overcurrent ground relay is set to trip at 0.5A. Determine whether it will trip during the line to line fault specified above.

Hint: Perform the short circuit analysis and determine the zero sequence current "seen" by the relay. Use of the program WinIGS is encouraged.

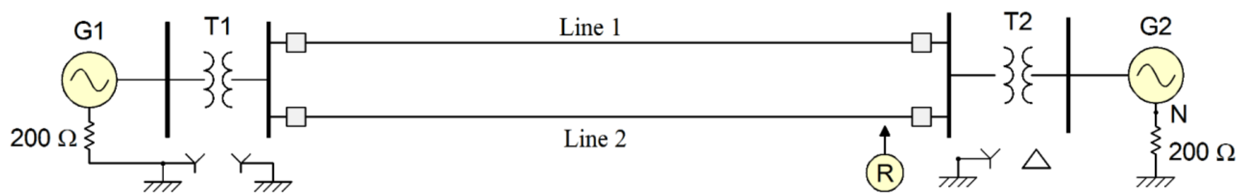


Figure P5.16a

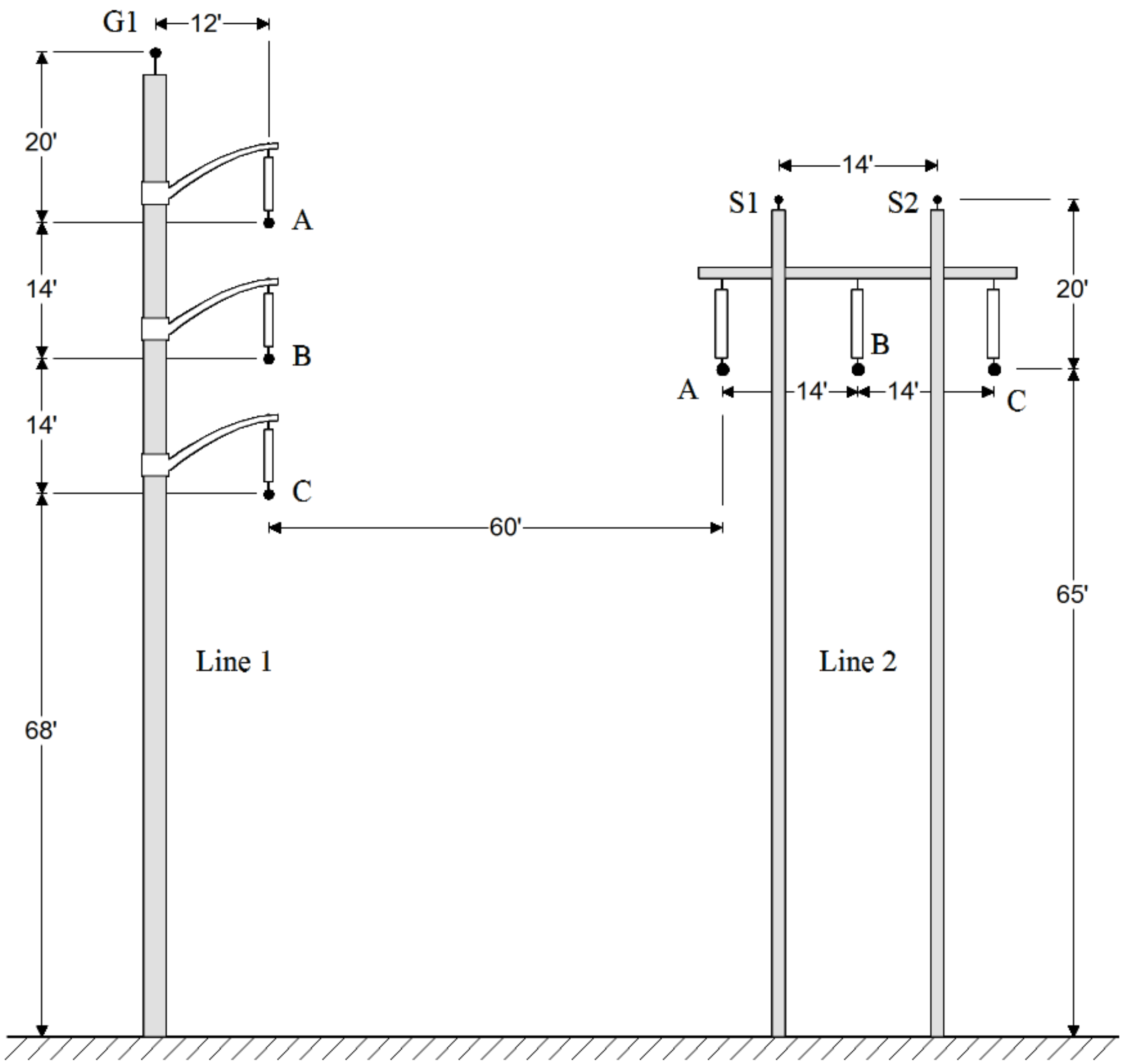
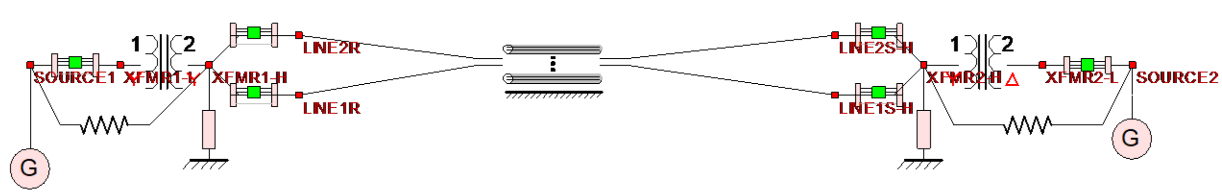


Figure P5.16b

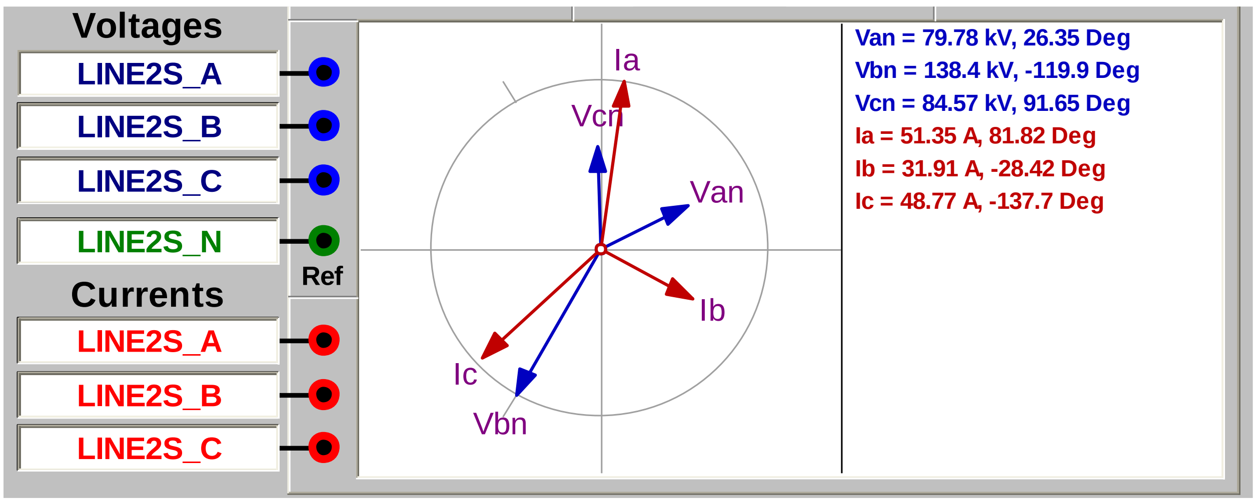
Solution: This problem is solved with the help of the program WinIGS. The system is modeled and the WinIGS model is shown in the figure below.

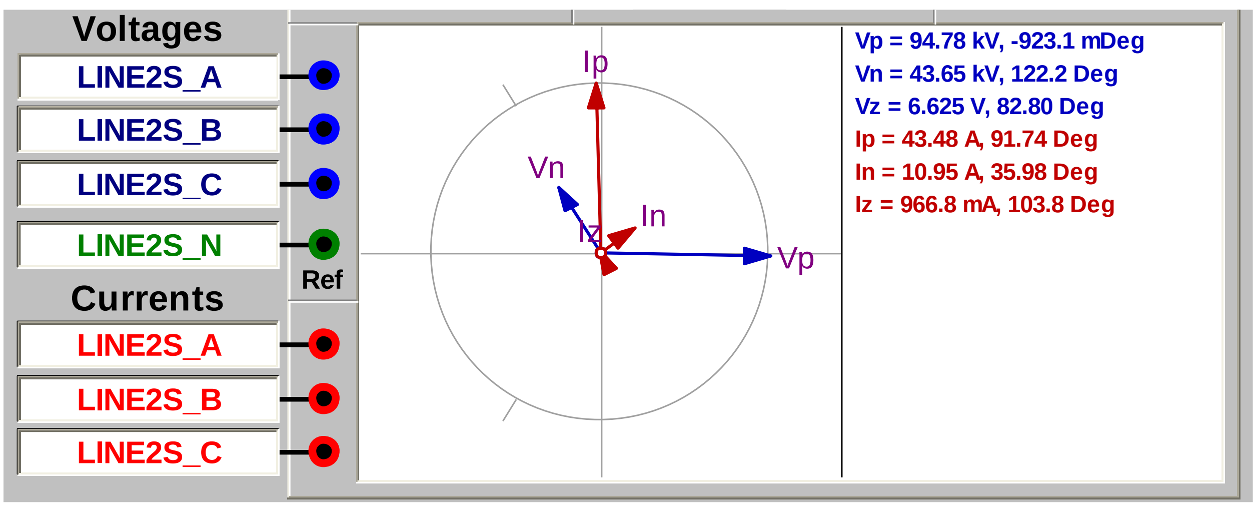


A line to line fault on circuit 1 is simulated. For this condition, the electric currents and voltages on line 2 at the location of the relay are (phase quantities – first figure and sequence components – second figure). The overcurrent ground relay will see the following current:

OC Ground relay current: $0.9668\left(\frac{5}{1200}\right) = 0.004\,A$

It will not trip.





Problem P5.23: A 115 kV, 40 mile long three-phase transmission line connects two electrical power systems as in Figure P4.2. Each of the power systems is represented as an equivalent source with the following sequence impedances:

$$Z_1 = Z_2 = j0.1\,pu, \quad Z_0 = j0.1\,pu \quad @\,100\,MVA, 115\,kV$$

The voltage sources behind the equivalent impedances are in phase. The parameters of the transmission line are

$Z_1 = Z_2 = j0.8\,Ohms/mi\,, \quad Z_0 = j2.1\,Ohms/mi$

(a) Compute the fault current for a single line-to-ground fault at the middle of the line. Assume that a distance relay with a compensation factor of m=1.6 is located on the left terminal of the line. What is the impedance “seen” by this relay?
(b) Compute the fault current for a line-to-line fault at the middle of the line. Assume that a distance relay is located on the left terminal of the line. What is the impedance “seen” by this relay?

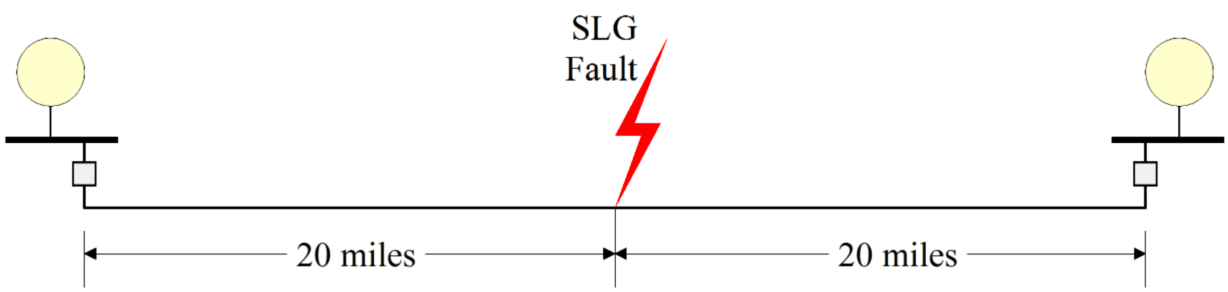


Figure P5.17

Problem P5.24: Consider the electric power system of Figure P5.18. The system model is also given in WinIGS format (the data file has been posted on the course web site. Download the data file and familiarize yourself with the system.
Assume that a distance relay is located at location A of the transmission line 1. The line is 59.5 miles long. The CT ratio of the distance relay is 2000:5 and the PT ratio is 139kV:115V.
The relay should be set to reach 80% of the line. The available compensation factor settings (m) of the relay are in the range 2.4 to 4.0 in increments of 0.2. The impedance settings are: 10 ohms to 30 ohms in increments of 1 ohm. Select the settings of the distance relay, i.e. the compensation factor and the impedance setting in ohms.
Using the selected settings for this relay, compute the distance “seen” by this relay for the following faults on the line:

(a) a line to line fault (phase A to phase B) at a distance of 38.5 miles from the location of the relay.
(b) a line to line fault (phase A to phase C) at a distance of 38.5 miles from the location of the relay.
(c) a line to neutral fault (phase A) at a distance of 38.5 miles from the location of the relay.
(d) a line to neutral fault (phase B) at a distance of 38.5 miles from the location of the relay.

What are your observations about the accuracy of the relay?

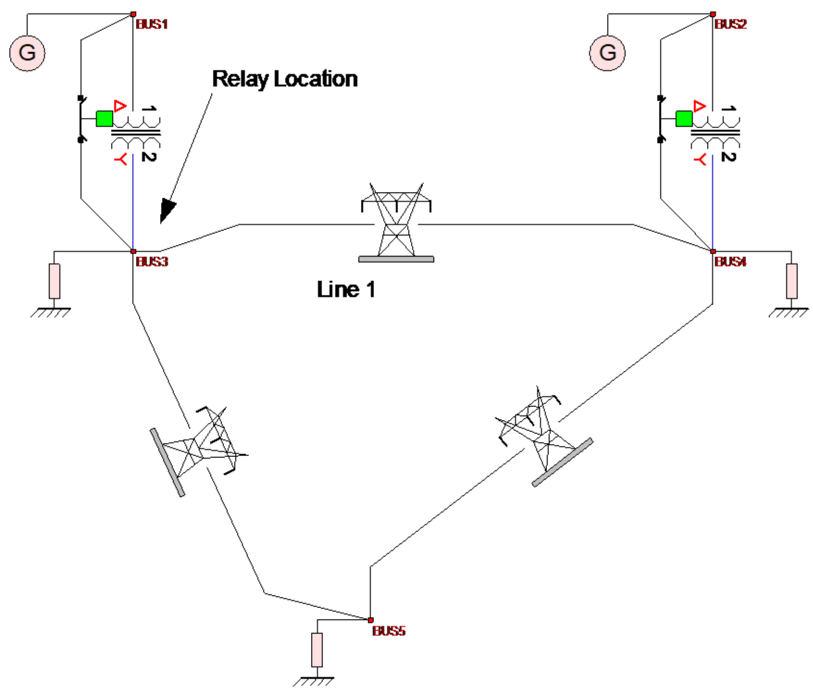


Figure P5.18

Solution: The short circuit analysis is performed with the program WinIGS. First we select the relay settings. For this purpose we compute the sequence parameters of the circuit to be protected. Using the program WinIGS the sequence parameters are:

$$Z_1 = 2.222 + j39.134\, ohms, \quad Z_2 = 2.222 + j39.134\, ohms, \quad Z_0 = 24.564 + j158.775\, ohms$$
$$Z_1^{'} = 0.750 - j5{,}144.6\, ohms, \quad Z_2^{'} = 0.750 - j5{,}144.6\, ohms, \quad Z_0^{'} = 8.331 - j10{,}527.3\, ohms$$

The compensation factor is:

$$m = \frac{z_0 - z_1}{z_1} = 3.0989 e^{ja}, \text{ Select } m = 3.0.$$

The impedance should be set equal to 80% of the positive sequence impedance of the line. This means:

$$(0.8)Z_1 = \frac{\tilde{V}}{\tilde{I}} = \frac{\frac{139{,}000}{115}\tilde{V}_R}{\frac{2{,}000}{5}\tilde{I}_R}$$. Upon solution for the impedance on the relay side:

$$\frac{\tilde{V}_R}{\tilde{I}_R} = 10.3773 e^{j86.75^0}\ ohms, \text{ Select } Z_{zone1} = 10.0\, ohms \text{ and Phase = 85 degrees}$$

(a) The fault currents and voltages at the location of the relay for a line to line fault (phases A and B), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 83.36e^{j3.389^0}\ kV,\quad \tilde{V}_{bn} = 81.77e^{-j63.97^0}\ kV,\quad \tilde{V}_{cn} = 137.4e^{j150.1^0}\ kV$$
$$\tilde{I}_a = 1.858e^{-j27.23^0}\ kA,\quad \tilde{I}_b = 1.857e^{j151.6^0}\ kA,\quad \tilde{I}_c = 0.03798e^{-j98.59^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_a - \tilde{V}_b}{\tilde{I}_a - \tilde{I}_b} \right| = 37.4211\, miles$$

(b) The fault currents and voltages at the location of the relay for a line to line fault (phases A and C), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 84.11e^{j54.05^0}\ kV,\quad \tilde{V}_{bn} = 137.8e^{-j90.07^0}\ kV,\quad \tilde{V}_{cn} = 85.33e^{j125.2^0}\ kV$$
$$\tilde{I}_a = 1.767e^{-j88.63^0}\ kA,\quad \tilde{I}_b = 0.04276e^{-j197.4^0}\ kA,\quad \tilde{I}_c = 1.768e^{j92.74^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_a - \tilde{V}_c}{\tilde{I}_a - \tilde{I}_c} \right| = 42.333\, miles$$

(c) The fault currents and voltages at the location of the relay for a line to ground fault (phase A to ground), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 85.44e^{j25.19^0}\ kV,\quad \tilde{V}_{bn} = 124.9e^{-j82.56^0}\ kV,\quad \tilde{V}_{cn} = 130.9e^{j141.6^0}\ kV$$
$$\tilde{I}_a = 1.708e^{-j49.97^0}\ kA,\quad \tilde{I}_b = 0.1138e^{j113.0^0}\ kA,\quad \tilde{I}_c = 0.1163e^{j147.0^0}\ kA$$

The phase currents, transformed into symmetrical components, are:

$$\tilde{I}_1 = 0.5866e^{-j49.89^0}\ kA,\quad \tilde{I}_2 = 0.6255e^{-j50.03^0}\ kA,\quad \tilde{I}_0 = 0.496e^{-j50.0^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0} \right| = 40.579\, miles$$

(d) The fault currents and voltages at the location of the relay for a line to ground fault (phase B to ground), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 130.5e^{j21.51^0}\ kV,\quad \tilde{V}_{bn} = 85.20e^{-j94.87^0}\ kV,\quad \tilde{V}_{cn} = 124.5e^{j157.40^0}\ kV$$
$$\tilde{I}_a = 0.1261e^{j24.81^0}\ kA,\quad \tilde{I}_b = 1.713e^{-j170.2^0}\ kA,\quad \tilde{I}_c = 0.1151e^{-j3.54^0}\ kA$$

The phase currents, transformed into symmetrical components, are:

$$\tilde{I}_1 = 0.593 e^{-j49.81^0}\ kA, \quad \tilde{I}_2 = 0.6272 e^{j69.65^0}\ kA, \quad \tilde{I}_0 = 0.4932 e^{-j170.4^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_b}{\tilde{I}_b + m\tilde{I}_0} \right| = 40.5081\, miles$$

The accuracy of the relay (on the basis of the above four cases) is:

$$Error = \frac{Maximum\, Difference}{\text{Fault Distance}} = \frac{4.9119}{38.5} = 0.1276\, pu$$

Problem P5.25: A single phase 6.92 kV/277V, 750 kVA, 60 Hz transformer is protected with a differential relay that uses the following current transformers: Primary: 120:5 and secondary: 3000:5. Compute the current in the operating coil of the differential relay as a function of the transformer current in the secondary, assuming ideal CTs.

Solution: Assume the current in secondary is I. Then:

$$I_{relay} = \left(\frac{5}{3000}\right) I - \left(\frac{277}{6920}\right)\left(\frac{5}{120}\right) I = 0.0000012 I$$

Problem P5.26: Consider the $36\, MVA, \quad 138 kV / 13.8 kV, \quad z = j0.08\, pu$ three phase, delta-wye connected variable tap transformer indicated in Figure P5a. This transformer is protected with a differential relay. The delta side is the 138 kV side. The settings of the relay are: 5% restrain, minimum pickup 0.5 Amperes. The CTs are 100:5 and 1800:5 for the high voltage side and low voltage side respectively. The high voltage side CTs are connected wye and the low voltage side CTs are connected delta.

For a specific fault condition and tap setting of the transformer the electric currents and voltages, indicated in Figure 5b, have been recorded at the terminals of the transformer. (a) Compute the electric current in the three operating coils of the relay. (b) Determine whether the differential relay will trip the transformer.

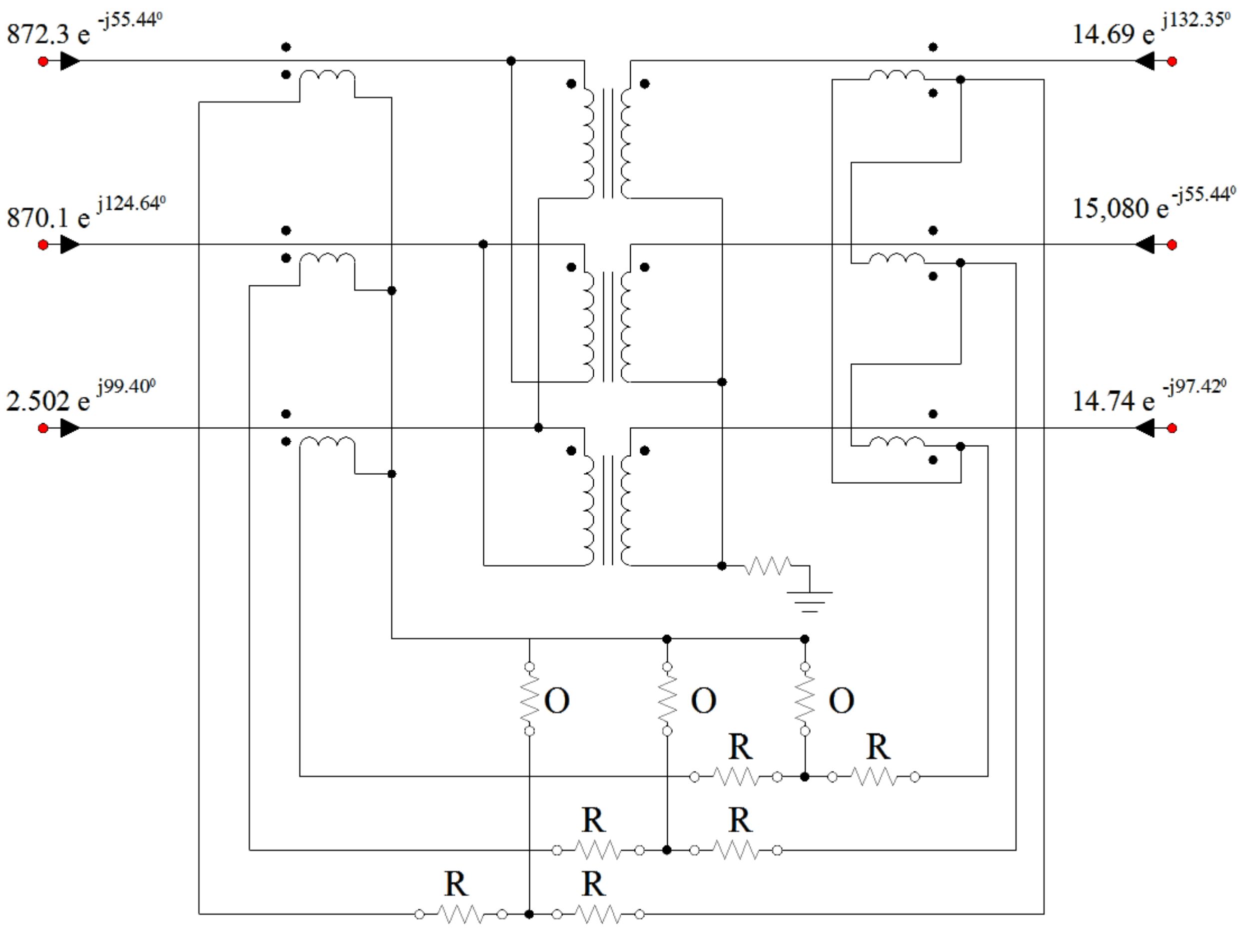


Figure P5a: Three-Phase, Delta-Wye Connected, 138 kV/13.8 kV, Standard Connection Transformer

Problem P5.27: Consider the electric power system of Figure P4. Assume that a distance relay is located at the indicated location the transmission line 1. The line is 59.5 miles long. The CT ratio of the distance relay is 2000:5 and the VT ratio is 139kV:115V.

The zone 1 settings of the relay are as follows: Impedance: 12 ohms, Phase=82 degrees, Compensation factor=2.0, Time delay: 0.

For a specific fault condition the voltages and currents of Figure P4b were recorded at the terminals of the transmission line. Determine whether the zone 1 relay above will trip.

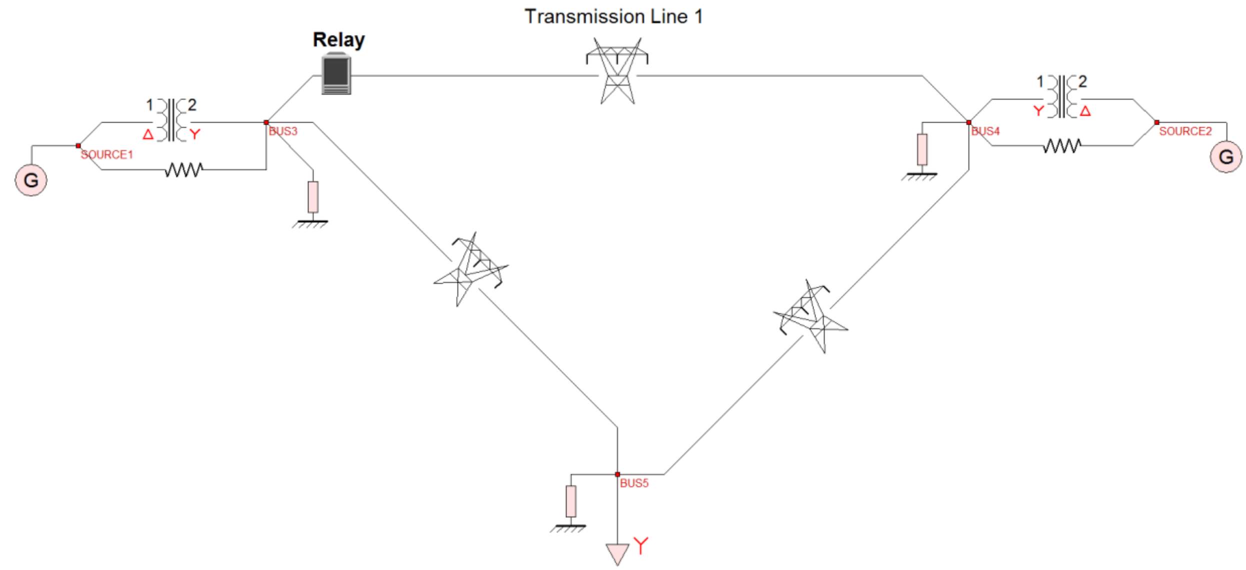


Figure P4a

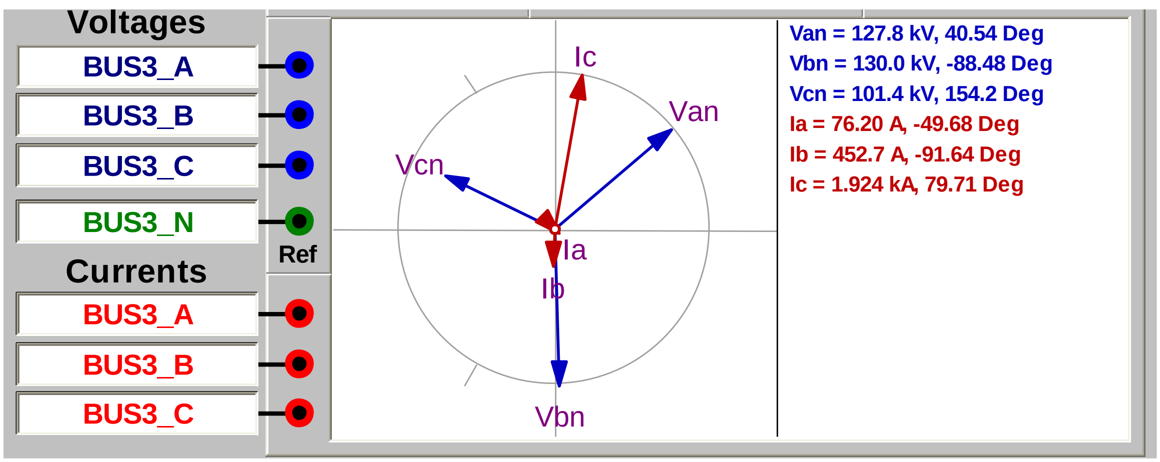


Figure P4b

Solution: First the sequence currents are computed from the phase currents.

$$I_{120} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} I_{abc} = \begin{bmatrix} 734.0e^{-j29.58^0} \\ 732.6e^{-j167.7^0} \\ 477.8e^{j74.63^0} \end{bmatrix} Amperes$$

The relay will "see" the following impedance:

$$z_{relay} = \frac{115/139{,}000}{5/2000} \frac{101.4e^{j154.2^0}}{1.924e^{j79.71^0} + 2.0(0.4778e^{j74.63^0})} = xxx\ ohms$$

Conclusion: to be added.

Problem P5.28: A time overcurrent relay is protecting a distribution line as it is indicated in Figure P1a . The trip characteristics of this relay are illustrated in Figure P1b as function of multiples of pickup current and time dial setting.
The relay setting are: pickup current=8A, time dial=1.0.
The parameters of the equivalent source and line are:
Source: $z_1 = j0.085\,pu$, $z_2 = j0.095\,pu$ and $z_0 = j0.15\,pu$ (@ 13.8 kV (L-L) and 36 MVA)
Line: $z_1 = z_2 = j0.8\,ohms/mi$, $z_0 = j2.1\,ohms/mi$
The current transformer ratio is 1200A:5A. Compute the time to trip for a single line to ground fault at the middle of the line.

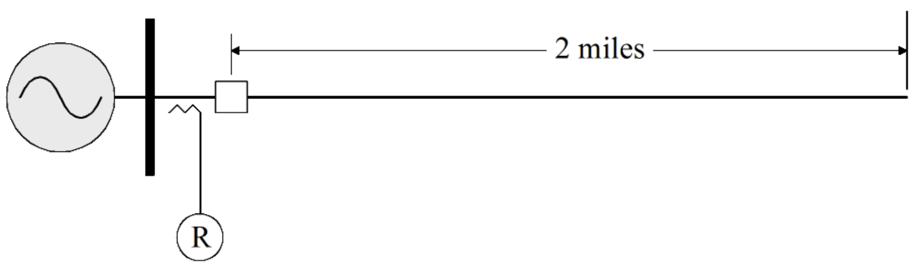


Figure P1a

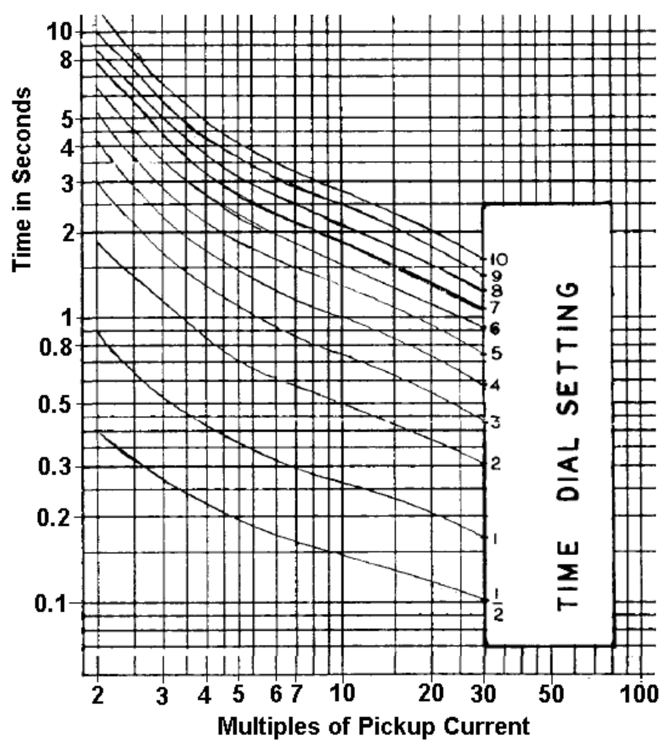
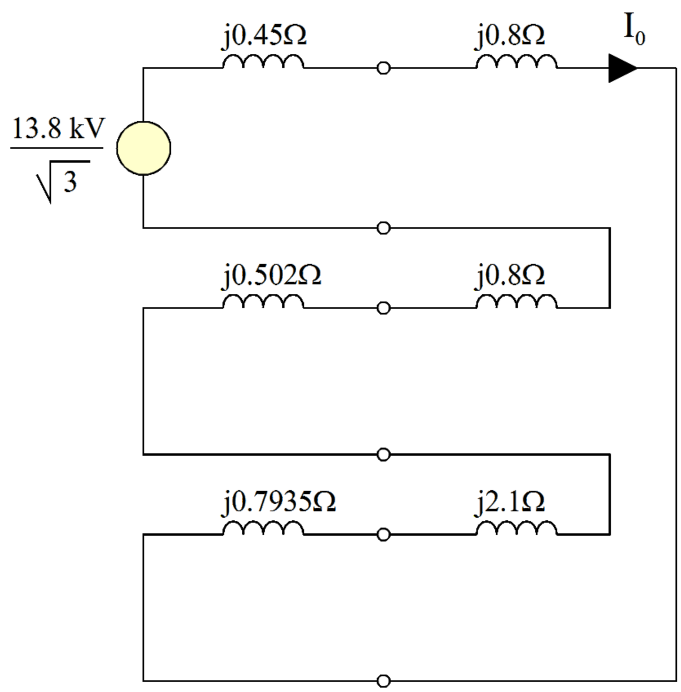


Figure P1b

Solution:

First we compute the fault current:



$$I_0 = \frac{\frac{13.8}{\sqrt{3}} kV}{j5.4457} = -j1.463kA$$

$$\tilde{I}_F = \tilde{I}_A = j4.3892kA$$

$$\tilde{I}_{\text{Re}lay} = j18.28A$$

$$I_r = \frac{I_{\text{Re}lay}}{8.0A} = 2.286$$

Reading the Figure for 2.288 @ time dial 1 ➔ t_{trip} = 0.7 seconds

Problem P5.29: A single phase 6.92 kV/277V, 750 kVA, 60 Hz transformer is protected with a differential relay that uses the following current transformers: Primary: 120:5 and secondary: 3000:5. Compute the current in the operating coil of the differential relay as a function of the transformer current in the secondary, assuming ideal CTs.

Problem P5.30: Consider the $132\,MVA,\ \ 18kV/230kV,\ \ z = j0.08\ pu$ three phase, delta-wye connected variable tap step-up transformer indicated in Figure P5.30. This transformer is protected with a differential relay. The delta side is the 18 kV side. The settings of the relay are: 5% restrain, minimum pickup 0.5 Amperes. The CTs are 200:5 and 1500:5 for the high voltage side and low voltage side respectively. The high voltage side CTs are connected delta and the low voltage side CTs are connected wye.
The transformer is connected to a 220 MVA, 18 kV, 60 Hz, impedance grounded (net ground impedance =j5.0 ohms) generator with the following parameters:

$$z_1 = j0.20\ pu,\quad z_2 = j0.18\ pu,\quad z_0 = j0.11\ pu\quad @18kV\ and\ 220MVA$$

Consider a single line to ground fault on the 230 kV side of the transformer outside the transformer zone of protection. For this condition (a) Compute the electric current in the three operating coils of the relay. (b) Determine whether the differential relay will trip the transformer. Hint: Use the program WinIGS to compute the fault currents.

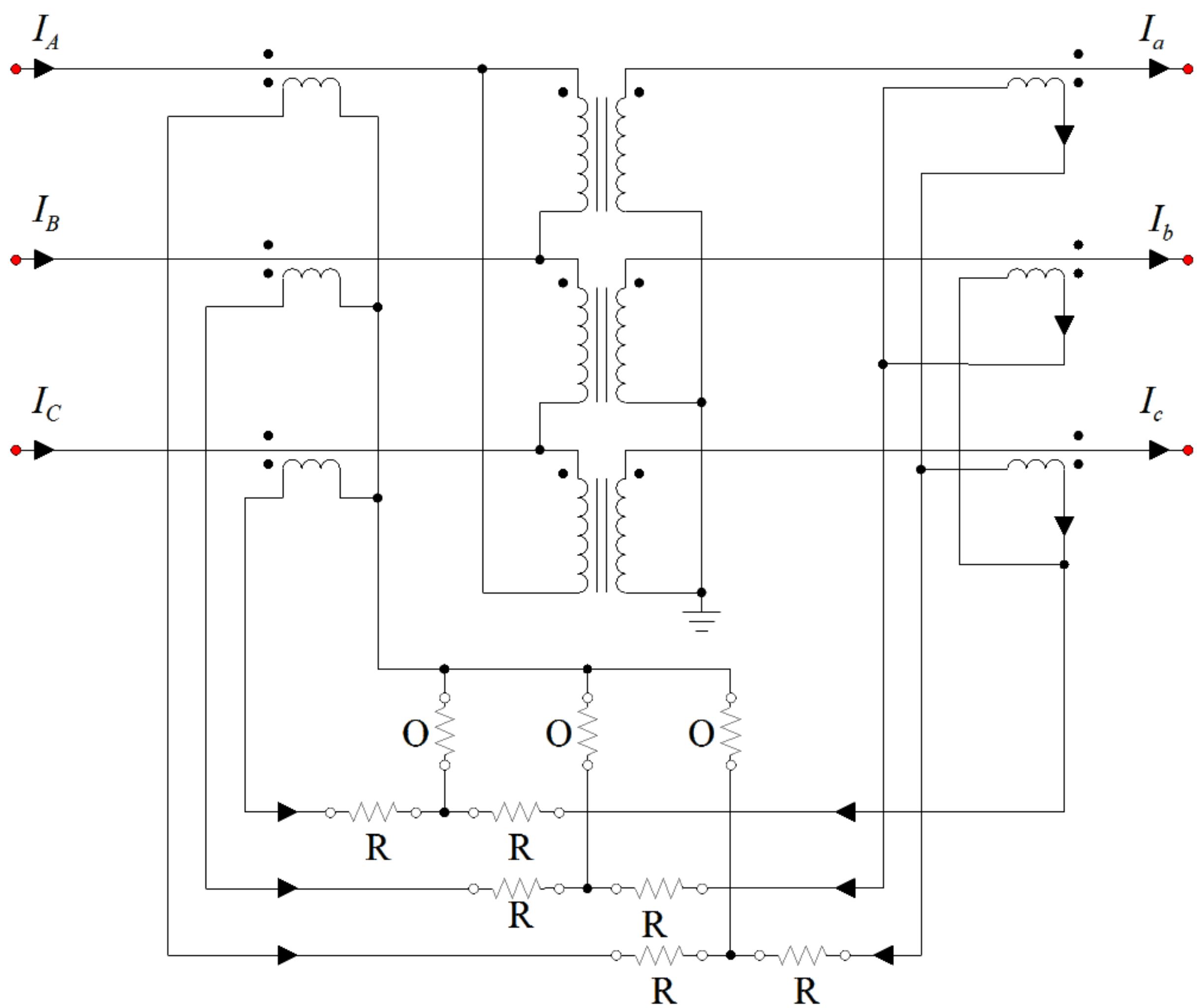


Figure P5.30

Problem P5.31: An impedance relay is used to protect a transmission line. The parameters of the transmission line are:

Rated Voltage (L-L): 138 kV

Rated Current: 1.16 kA

Line length = 56.4 miles

Positive/Negative Sequence Impedance: 5.6+j36.1 ohms

Zero Sequence Impedance: 19.6+j89.64 ohms

The CT ratio of the impedance relay is 2000:5 and the PT ratio is 86kV:115V. The relay engineer decides that the impedance relay will be set as follows: (a) zone 1 will be set to reach 80% of the line, (b) zone 2 will be set to reach 135% of the line and (c) zone 3 will be set to reach 100% of the line plus 125% of the next line. The longest length of the next line is 49 miles and the design is identical to the transmission line under consideration.

1. Compute the settings of the impedance relay in ohms for the three zones.
2. Compute the compensation factor for the distance relay
3. Sketch the operating zones of the relay on a R-X impedance graph.
4. Assume that the line delivers rated current at rated voltage and at power factor 0.92 (current lagging) at the location of the relay. Determine whether the relay will trip on this condition. (Hint: compute the impedance seen by the relay and place the impedance on the sketch of the relay operating zones – part 3).
5. Assume that during an emergency the line delivers 180% rated current at 90% of rated voltage and at power factor 0.92 (current lagging) at the location of the relay. Determine whether the relay will trip on this condition.

Solution:

1. Line Impedance

$$Z_{Line} = \sqrt{5.6^2 + 36.1^2} = 36.53e^{j81.18}\ \Omega$$

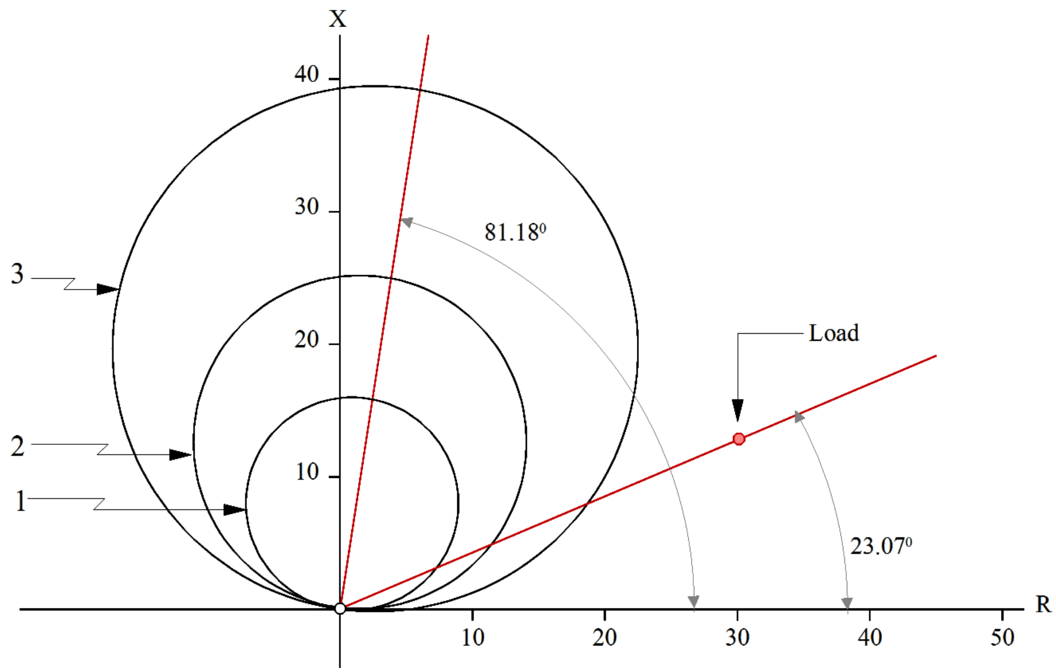
Line impedance on relay side:

$$Z_{\mathrm{Re}lay,l} = \frac{V_{\mathrm{sec}}}{I_{\mathrm{sec}}} = \frac{V_{pri}115/86000}{I_{pri}5/2000} = 0.5349 Z_{line}$$
$$36.53 = 19.54\Omega$$

Zone 1 = (0.80)(19.54) = 15.630 ohms
Zone 2 = (1.35)(19.54) = 26.379 ohms
Zone 3 = ((56.4 + (1.25)(49.0))/56.4)(19.54) = 40.76 ohms

2. $m = \frac{z_0 - z_1}{z_1} = \frac{14 + 53.54}{5.6 + j36.1} = 1.5149e^{-j5.836^0}$

3. Sketch of the relay characteristics



4. Under the specified loading condition:

$$Z = \frac{138/\sqrt{3}}{1.16} e^{j23.07^0} = 68.68 e^{j23.07^0}$$

$$Z_{L,R} = 36.73 e^{j23.07^0}$$

The operating point is outside the relay characteristic but barely.

5. Under the specified loading condition:

$$Z = \frac{(0.90)\left(138/\sqrt{3}\right)}{(1.8)(1.16)} e^{j23.07^0} = 34.34 e^{j23.07^0}$$

$$Z_{L,R} = 18.36 e^{j23.07^0}$$

$$\tilde{D} = (0.5)\left(40.76 e^{j81.18^0}\right) - 18.36 e^{j23.07^0} =$$

$$\left|\tilde{D}\right| < (0.5)(40.17)$$

The operating point is outside the relay characteristic but barely.

Problem P5.32: Consider the electric power system of Figure P4a. Assume that a time overcurrent relay is located at the indicated location of the transmission line 1. The characteristics of the time overcurrent relay are shown in Figure P4b. The relay settings are: Pickup Current = 6 A, Time dial = 1. The CT ratio of the relay is 2000:5.

For a specific near fault condition the electric currents in the transmission line at the location of the relay are: Ia=545 A, Ib=831 A, and Ic=11,589 A. Compute the time to trip of the time overcurrent relay.

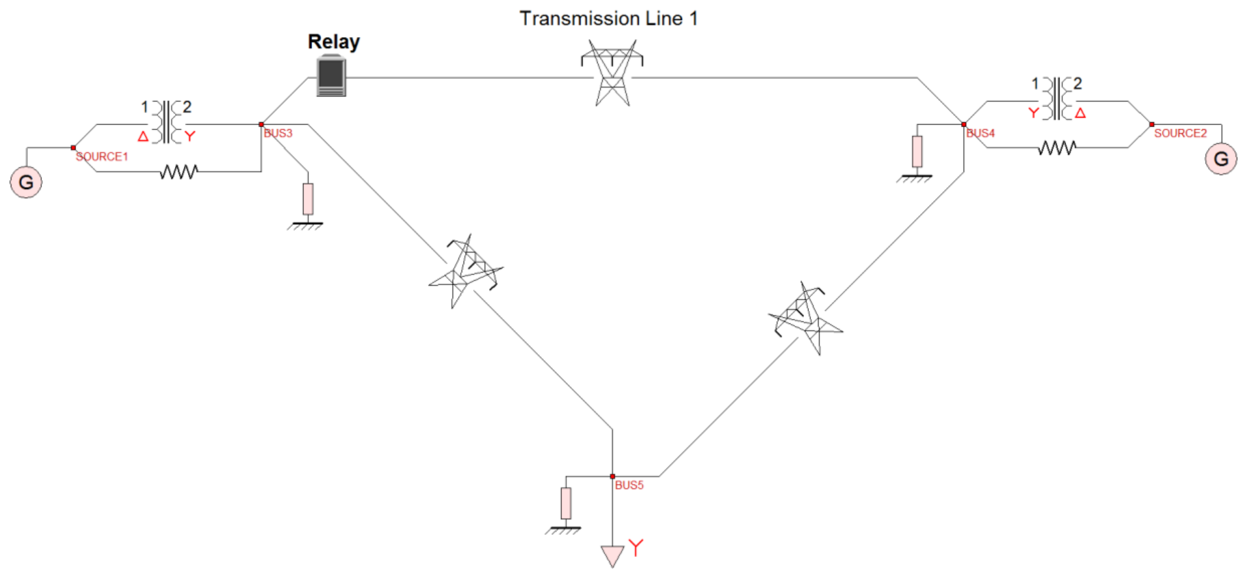


Figure P4a

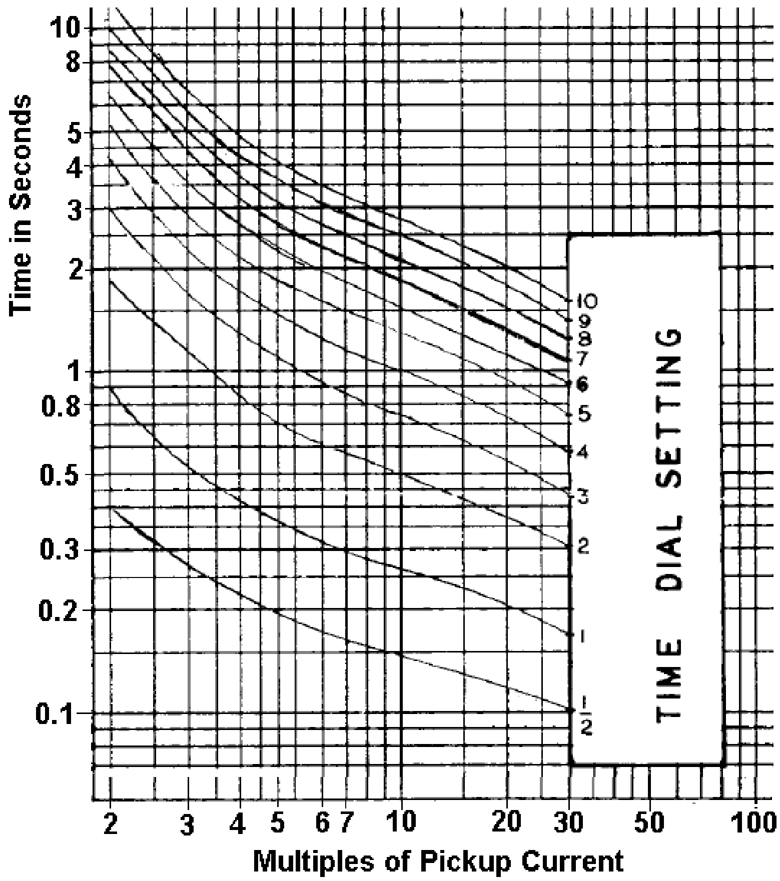


Figure P4b

Solution:

$$I_{Relay} = \frac{5}{2000} 11,589 A = 28.97\,A$$

$$I_{mpu} = \frac{28.97}{6} A = 4.828\,A$$

t = 0.37 seconds or 22.2 cycles

Problem P5.33: Consider the electric power system of Figure P5.33. Assume that a time overcurrent relay is located at the indicated location of the transmission line 1. The CT ratio of the relay is 1500:5. The relay settings are: Characteristic: Very Inverse, Pickup Current = 8 A, Time dial = 1.

For a specific near fault condition the electric currents in the transmission line at the location of the relay are: Ia=845 A, Ib=931 A, and Ic=16,429 A. Compute the time to trip of the time overcurrent relay.

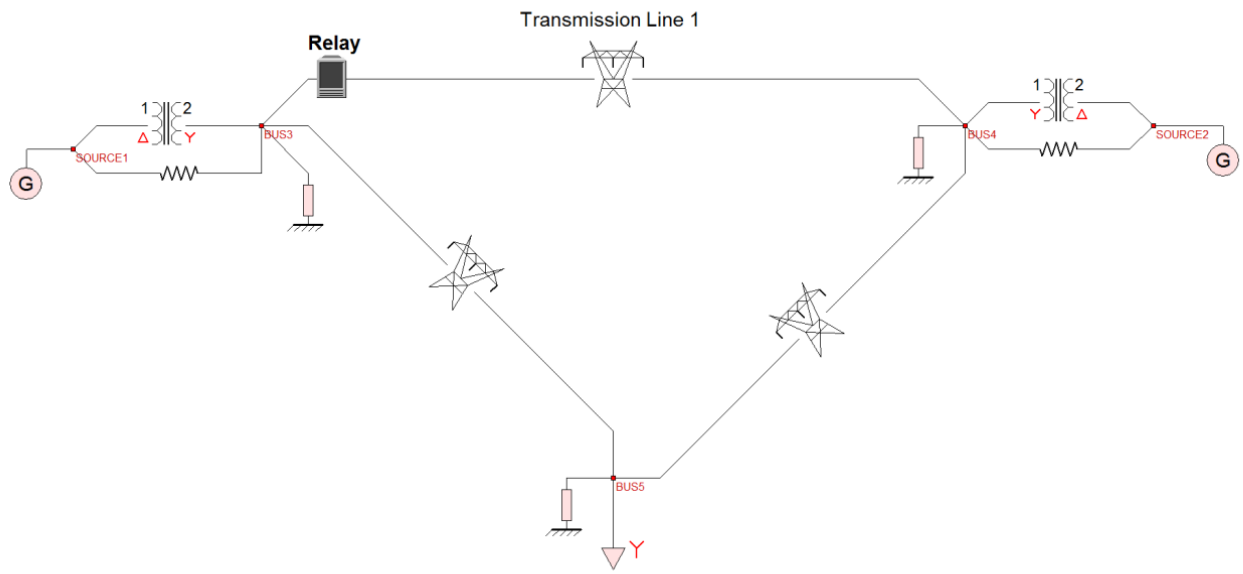


Figure P5.33

Problem P5.34: Consider a three-phase, 600 A circuit, 25 kV, 1.2 mile long, as it is illustrated in Figure P5.34. The equivalent source impedance is:

$$z_1 = z_2 = j0.36\ pu, \quad z_0 = j0.26\ pu \quad \text{(@ 25kV (L-L) and 100 MVA(three-phase) base)}$$

The impedance of the circuit is:

$$z_1 = z_2 = j0.60\ ohms/mile, \quad z_0 = j1.75\ ohms/mile$$

An inverse-time overcurrent relay is located at the indicated location. The CT is rated 800:5A. The relay is a numerical relay with settings: pickup current = 7 Amperes, very inverse, time dial = 0.1. The very inverse function is defined by:

$$t_0 = \frac{19.61}{I_r^2 - 1} t_d + (0.491) t_d$$

(a) Compute the time to trip for a three-phase fault at a point on the line located 0.5 miles to the right side of the breaker.

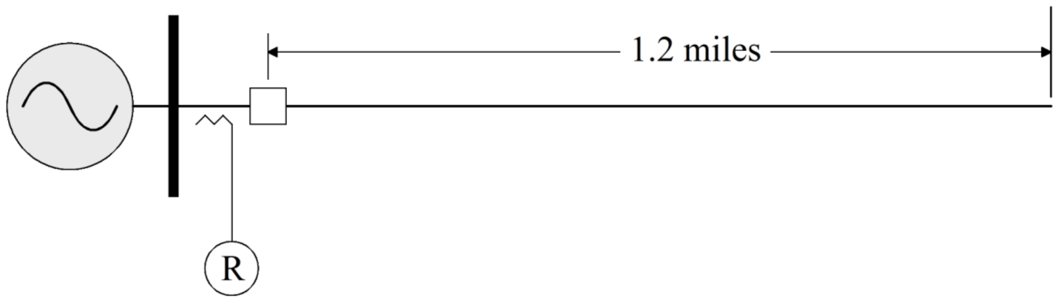
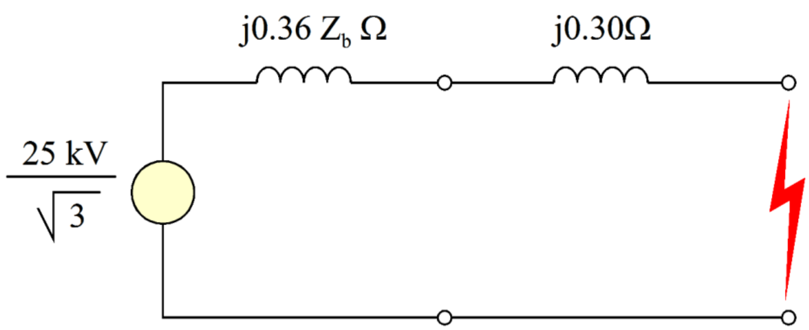


Figure P5.34

(b) Compute the time to trip for a single line to ground fault at a point on the line located 0.5 miles to the right side of the breaker.

Solution (Part a):



$$Z_b = \frac{25^2}{100} = 6.25\Omega$$

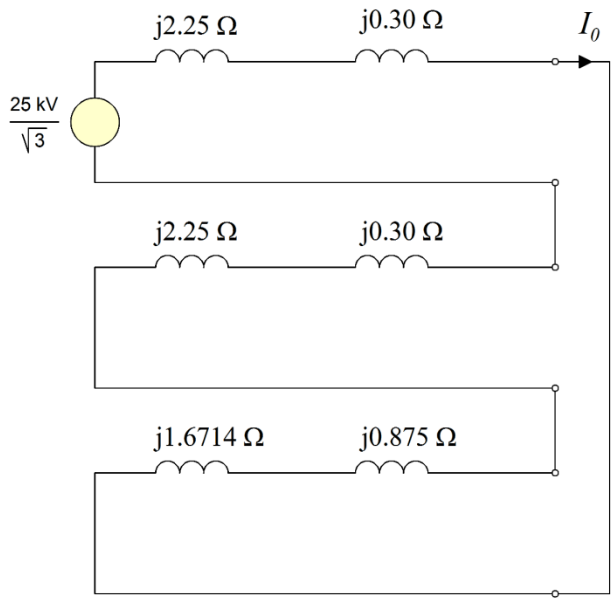
$$I = \frac{25kV/\sqrt{3}}{j2.55\Omega} = 5.66\,kA$$

$$I_{\mathrm{Re}lay} = \frac{5}{800} 5.66\,kA = 35.375\,A$$

$$I_r = \frac{35.375\,A}{7\,A} A = 5.0536$$

Thus: t = 0.129 sec, or 7.74 cycles

Part b:



$$I_0 = \frac{25kV/\sqrt{3}}{j7.6464\Omega} = 1.887\ kAe^{-j90^0}$$

$$\tilde{I}_a = 3\tilde{I}_0 = 5.633\,kAe^{-j90^0}$$

$$I_{relay} = \frac{5}{800}5{,}633A = 35.3938A$$

$$I_r = \frac{35.3938\ A}{7\ A} = 5.0563$$

t = 0.129 sec, or 7.74 cycles

Problem P5.x: Consider the electric power system of Figure P3. Assume that a time overcurrent relay (one for each phase) is located at the indicated location of the transmission line 1. The CT ratio of the relay is 2000:5. The relay settings are: Very Inverse, Pickup Current = 9 A, Time dial = 0.2. The very inverse characteristic is described with the equation:

$$t_0 = \frac{19.61}{I_r^2 - 1}t_d + (0.491)\ t_d$$

(a) For a specific fault condition the electric currents in the transmission line at the location of the relay are:

$$\tilde{I}_a = 18.78e^{j10.8^o}\ kA,\ \ \tilde{I}_b = 4.98e^{-j78.8^o}\ kA,\ \ \tilde{I}_c = 4.17e^{j68.9^o}\ kA$$

Compute the time to trip of the time overcurrent relay (which phase will trip first?).

(b) For another specific condition, the electric currents in the transmission line at the location of the relay are as follows:

For 0<t<5 cycles: $\tilde{I}_a = 19.78e^{j10.8^o}\ kA$, $\tilde{I}_b = 5.77e^{-j78.8^o}\ kA$, $\tilde{I}_c = 3.27e^{j68.9^o}\ kA$

For t>5 cycles: $\tilde{I}_a = 12.89e^{j10.8^o}\ kA$, $\tilde{I}_b = 3.80e^{-j78.8^o}\ kA$, $\tilde{I}_c = 2.67e^{j68.9^o}\ kA$

Compute the time to trip of the time overcurrent relay. Note in this case the time to trip is defined by the equation:

$$\int_0^{T_o} \frac{1}{t(I)} dt = 1$$

, where: $I = I_r$, $t(I_r) = \frac{19.61}{I_r^2 - 1} t_d + (0.491)\ t_d$

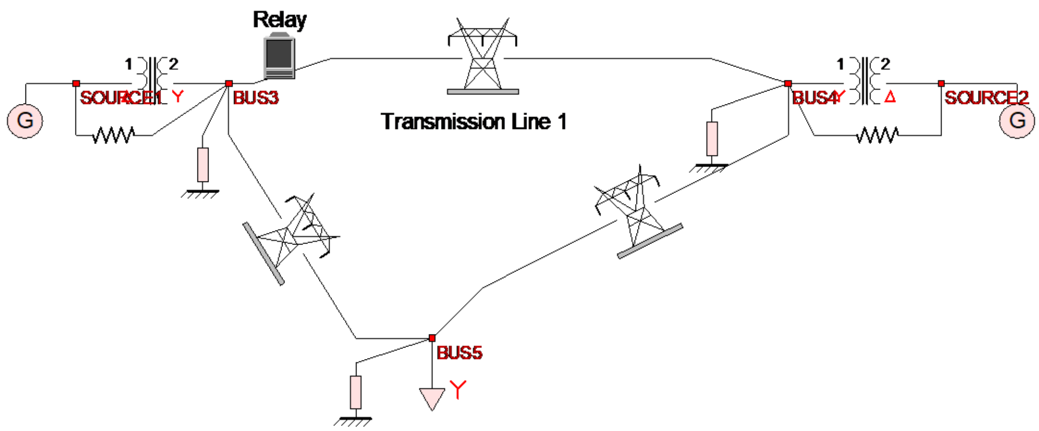


Figure P3

Solution:

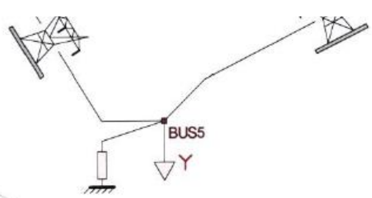


Figure P3

(a) $I_a = 18{,}780A \to I_{rel} = 46.95$

$I_r = 5.2167$

$t_o = 0.2478$ sec

(b) $0 < t < 5$ cycles

$I_a = 19{,}780A \to I_{relay} = 49.45$

$I_r = 5.4944$

$t(I_r) = 0.2326$ sec

$t > 5$ cycles

$I_a = 12{,}890\ A \to I_{relay} = 32.225A$

$I_r = 3.5506$

$t(I_r) = 0.430$ sec

$$\int_0^{T_o} \frac{1}{t(I)} dt = 1 = \int_0^{0.0833} \frac{1}{0.2326} dt + \int_{0.0833}^{T_o} \frac{1}{0.43} dt = 0.3583 + \int_{0.0833}^{T_o} \frac{dt}{0.43}$$

Problem P5.2: Consider a 600 A circuit, 13.8 kV, 2 mile long, as it is illustrated in Figure P2. The equivalent source impedance (on a 100 MVA(three-phase), 13.8 kV (line to Line) basis) is:

$$z_1 = z_2 = j0.36\ pu, \quad z_0 = j0.26\ pu$$

The impedance of the circuit is

$$z_1 = z_2 = j0.60\, ohms/mile, \quad z_0 = j1.75\, ohms/mile$$

An inverse-time overcurrent relay is located at the indicated location. The CT is rated 800:5A. The relay is a numerical relay with settings: pickup current = 6 Amperes, very inverse, time dial = 0.1. The very inverse function is defined with:

$$t_0 = \frac{19.61}{I_r^2 - 1} t_d + (0.491) t_d$$

(a) Compute the time to trip for a single line to ground fault at a point on the line located on the right side of the breaker (zero electrical distance from the source).
(b) Compute the time to trip for a single line to ground fault at the end of the line, i.e. 2 miles from the source.

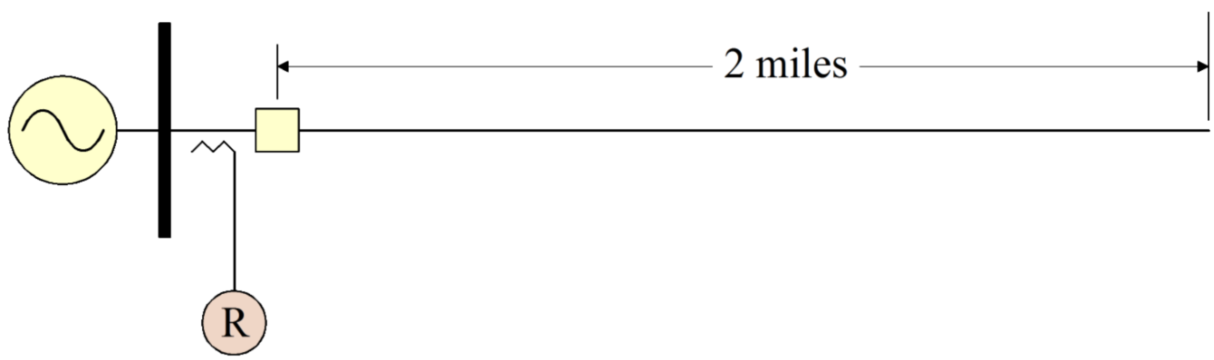


Figure P2

Solution:

(a)

$$I_0 = \frac{1}{j0.36 + j0.36 + j0.26} = -j\,1.0204$$

$$I_{F_{pu}} = 3I_0 = -j3.06122$$

$$I_F = I_{F_{pu}} \cdot \frac{100/3}{13.8/\sqrt{3}} = -j\,12.8072\ \text{kA}$$

$$I_{relay} = 80.04\ \text{A}$$

$$I_p = 13.34$$

$$t_o = 0.06018\ \text{sec}$$

(3.6 cycles)

3

(b)

$$I_0 = \frac{13.8/\sqrt{3}}{(j0.98)\frac{13.8^2}{100} + j\,5.9} = j1.0258\ \text{kA}$$

$$I_F = 3I_0 = -j\,3.0774\ \text{kA}$$

$$I_{relay} = 19.233\ \text{A}$$

$$I_p = 3.205$$

$$t_o = 0.2604\ \text{sec}$$

(15.62 cycles)

DRAFT and INCOMPLETE

Table of Contents

from

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Chapter 6
Protection Instrumentation

6.1 Introduction

This chapter describes measurement instrumentation for power system applications with special attention to the requirements for protective relaying. The chain of measurement instrumentation equipment that starts from the high voltage or current measurement point and ends with an analog or digital signal - (which is a reproduction of the high voltage or current measurement point signal) is called an "instrumentation channel". Figure 6.1 illustrates the devices forming voltage and current instrumentation channels typically found in electric power generating stations, substations and in general electrical installations.

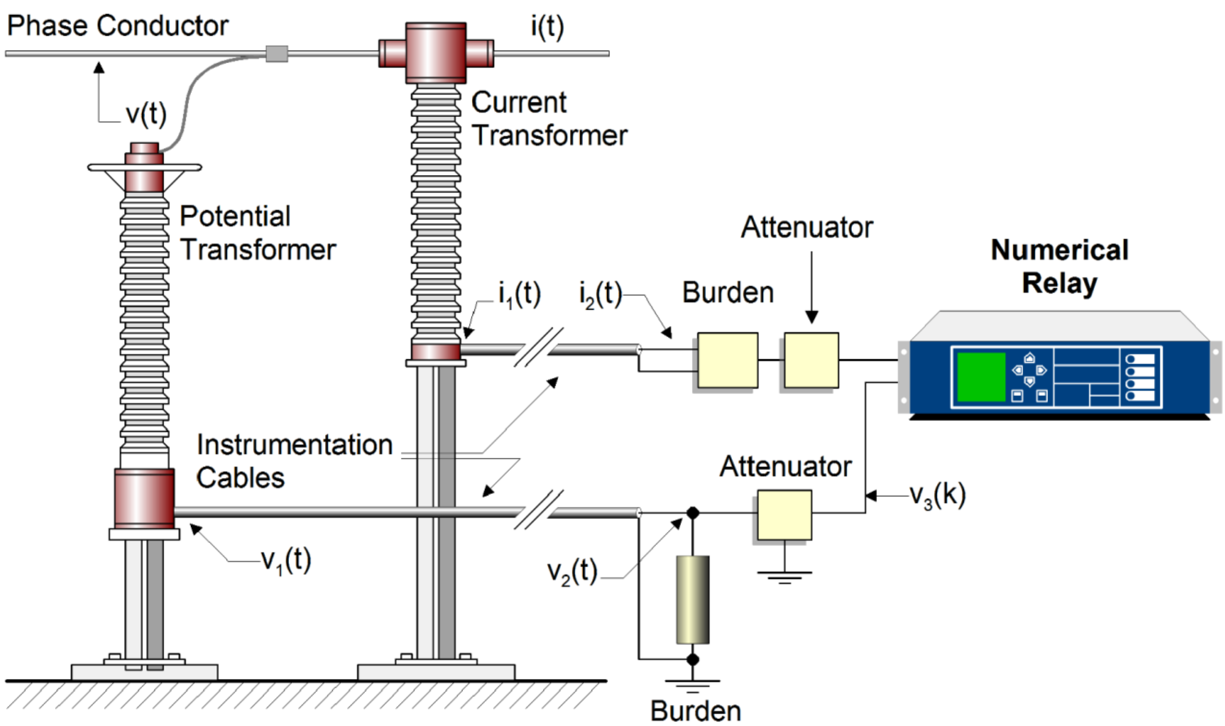


Figure 6.1: Typical Voltage and Current Instrumentation Channels

The purpose of the instrumentation channel is twofold: (a) to provide isolation between the sensitive electronic equipment such as IEDs and the high voltage power system and (b) to reduce the voltages and currents to instrumentation levels or to standard instrumentation level voltage and currents. Ideally, it is expected that the instrumentation channel will produce at the output a waveform that will be an exact replica of the high voltage or current and scaled by a constant factor. In reality, the instrumentation channel introduces an error. Specifically, each device in this chain namely:

- Instrument transformers
- Control cables
- Burdens
- Filters
- A/D converters

may contribute to some degree of signal degradation, i.e. error. Furthermore, the error introduced by one device may be affected by interactions with other devices of the channel. It is thus important to characterize the overall instrumentation channel error.

Another important issue in any measurement is the ability to determine the time at which this measurement has been taken. This can be achieved by properly interfacing a clock to the measurement equipment. The "time tagging" of the measurement becomes very useful if the timing accuracy is such that the phase angle of the voltages and currents can be determined with such precision that the power flows can be determined with precision of standard instrumentation. Consider for example a transmission line operating with a phase angle across of 10 degrees. If it is required to measure the power flow along this line with precision of 0.5%, the required precision of the phase difference measurement is about 0.05 degrees. Since the phase angles at the two ends of the line will be measured with different equipment (the ends of the line may be tens of miles apart) it will be necessary to measure each phase angle with accuracy of better than 0.03 degrees which corresponds to a timing precision of 1.5 microseconds. For this reason, these measurements require a clock that has microsecond precision and proper hardware to measure the timing of the measurement with comparable precision. The development of the GPS satellite constellation by the US government, starting in 1989, provided this clock in an affordable price everywhere on the globe. While there were attempts by few to develop the measurement technology with this precision (see historical note), the first to accomplish this performance was Jay Murphy of Macrodyne in 1992 and he was the first to coin this technology Phasor Measurement Unit (PMU). Specifically, he developed and released in January 1992 the MACRODYNE 1620 PMU capable of measurements with timing precision of 1 microsecond or phase angle precision at 60 Hz of 0.02 degrees. For many years this technology was utilized in specialized applications. Recently, many manufacturers of relays have realized the importance of this technology and started incorporating it in digital relays. It is expected that in the not distant future, all numerical relays will have this capability.

Power system instrumentation channels have many applications, namely:

- Voltage, Current and power flow metering
- Fault Voltage and Current Recording
- Power Quality Monitoring
- Protective Relaying

The tolerable measurement error and channel dynamic range is a function of the application. For example, metering and power quality monitoring requires high accuracy for steady state voltage and current levels, fault recording requires high accuracy as well as wide dynamic range so that both steady state and fault voltage and current levels are accurately monitored. For example, fault currents may reach levels that are several magnitudes of order higher than normal operating currents. In relaying applications, a wide dynamic range is important, but high accuracy is not as important. Standards related to measurements classify the measurements into two distinct categories with the following general characteristics:

(a) P-class (Protection Class): measurements that accuracy for normal operating conditions is not very important but their range accommodates requirements for protection
(b) M-class (Metering Class): measurements that are accurate for normal operating conditions but may not have the range required by protection applications.

The emphasis of this textbook is placed on relaying applications. The traditional approach is to use different instrumentation channels for metering and different for protection. However, as the technology is advancing and instrumentation channels with high accuracy at normal operating conditions and with the range to accommodate repaying applications are becoming available at practical costs, it is cost effective to use the same instrumentation channel equipment for multiple applications. In such cases, both wide dynamic range and high accuracy in both steady state and transient conditions become important requirements.

The first link in the instrumentation channel equipment chain consist of voltage and current transformers, collectively called *instrument transformers* These devices transform power system voltages and currents to levels appropriate for driving meters, relays, fault recorders and other monitoring equipment. Several instrument transformer technologies are presently in use. The most common traditional technology devices are **p**otential and **c**urrent **t**ransformers (PT's and CT's), which are based on magnetic core transformer technology. Another type of commonly used voltage transducers are **c**apacitively **c**oupled **v**oltage **t**ransformers (CCVT's). These are based on a combination of capacitive voltage dividers and magnetic core transformers. Recently, voltage and current transducers have been constructed based on the electro-optical and magneto-optical phenomena. These devices are known as EOVT's (Electro-Optical Voltage Transformers) and MOCT's (Magneto-Optical Current Transformers). Instrument transformers based on these and other alternative technologies are referred to as ***N**on-**C**onventional **I**nstrument **T**ransformers* (NCIT's). The characteristics of these technologies are presented in subsequent sections.

In this chapter we examine instrument transformer technologies and their characteristics especially from the point of view of accuracy. The instrument transformers are not the only devices that may introduce errors. The cables, the burdens, the relays and their A/D conversion sections may introduce errors. Therefore, we examine the effects of control cables and burdens as well as A/D conversion technology used by relays on the overall accuracy of the instrumentation. It will be made clear that for many protective functions highly accurate instrumentation is not necessary. However, the trend today is to use the protective relaying system within automation schemes and utilize the data for a variety of other applications, such as state estimation, stability monitoring, etc., in which high accuracy is required. Therefore, characterization of the accuracy of the instrumentation channels is becoming very important in modern applications.

In addition, we discuss issues of instrument transformer standardization, reliability of instrumentation channels and associated costs.

Before we discuss the above described issues, a historical note on the development of synchronized measurements will be presented.

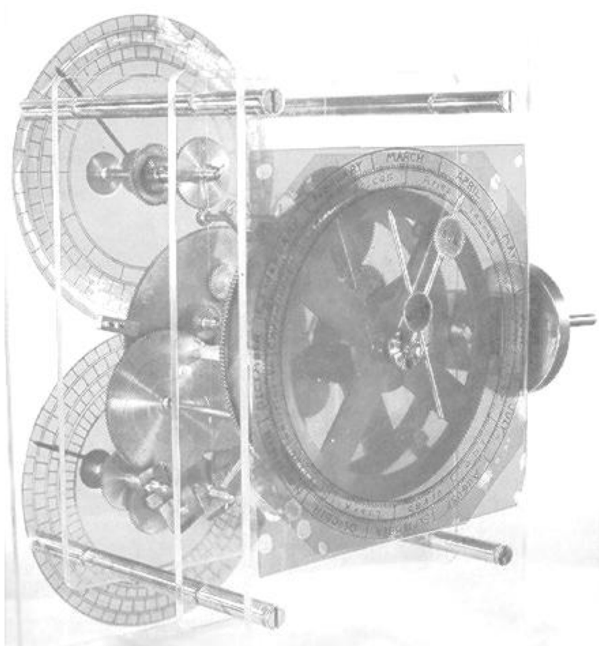
Historical Note:

The value of synchronized measurements has been recognized in many applications. As a matter of fact, synchronization of measurements and observations was sought in ancient times. Key to this capability is the development of accurate clocks. In ancient times the “klepsidra” was typically used for timing. Recently it has been discovered that the ancient Greeks developed a clock mechanism (the antikythera mechanism, dated 89 BC and discovered in 1908) that is illustrated in Figure HN-6.1. More information about the mechanism can be found in http://www.antikythera-mechanism.gr/. While the indented uses of the Antikythera mechanism are under intense research globally, it is certain that this mechanism was the basis of the mechanical watches. As technology evolved, the accuracy of clocks has increased [???]. Table HN-6.1 provides the history of technological developments of clocks.

Table HN-6.1: Clock Technology Development

	Technology	Accuracy
Fifth Century BC	Clepsydra (water clock)	Daily corrections – estimated accuracy “minute”
First Century BC	The Antikythera Mechanism	Unknown – Scientist study the mechanism to determine uses and accuracy
1910-1950	Quartz clocks Error correction via astronomical observations	Varying
1955	First Cesium atomic clock 1 second = 9,192,631,770 cycles of oscillation of Cesium 133	Initially about 1 ns per day. Presently (2014) about 0.02 ns per day.
1990	Single-ion optical atomic clock	0.002 ns per day
2007	Optical-lattice clocks (OLC)	Record accuracy (strontium OLC at JILA): 0.00002 ns per day

Today for most applications the Cesium atomic clock with accuracy of 0.02 ns in one day is used as the time reference. This may change in the future as optical-lattice atomic clocks have accuracy much greater than Cesium atomic clocks (see table above). The cost of the Cesium atomic clock has been prohibitive for many applications (however, recent advances may result in quite inexpensive cesium atomic clocks, for example search for Cesium atomic clocks on a chip). For the purpose of providing a timing and spatial signal to any place on earth, the Federal government has developed and deployed, starting in 1989, a constellation of satellites that transmit the signal of the Cesium clock throughout the globe. A receiver anywhere on earth may receive this signal. If signals from more than three satellites are received, with triangulation/estimation methods the signal of the Cesium atomic clock can be reproduced by the receiver at any point on earth with accuracy of tens of nanoseconds. Note that from the power system applications point of view a timing signal with accuracy of about one microsecond is enough for most applications. With proper metering technology, accuracy of one microsecond translates in measuring the phase angle with accuracy of 0.02 degrees, an accuracy that is adequate for present day power system applications.



The Antikythera Mechanism

GPS Satellite

Figure HN-6.2: Evolution of Time Pieces

Since the introduction of computer relaying (G. Rockefeller, “Fault Protection with a Digital Computer”, IEEE Transactions on Power Apparatus and Systems, Vol PAS-88, No.4, April 1969 [???]) and later microprocessor relays (the first commercial microprocessor relay was introduced by Ed Schweitzer in 1983 [???], See Figure HN=6.1b), efforts were initiated to extract the phasors of an electric power system, using the available time signals at the time. In 1980, Missout and Girard [???] developed a system for measuring the phase angle between Montreal and Sept-Iles using Loran C timing signals. The system is shown in Figure HN-6.2 taken from [???].

Figure HN-6.3: First Commercial Digital Relay, SEL-21 Distance Relay and Fault Locator

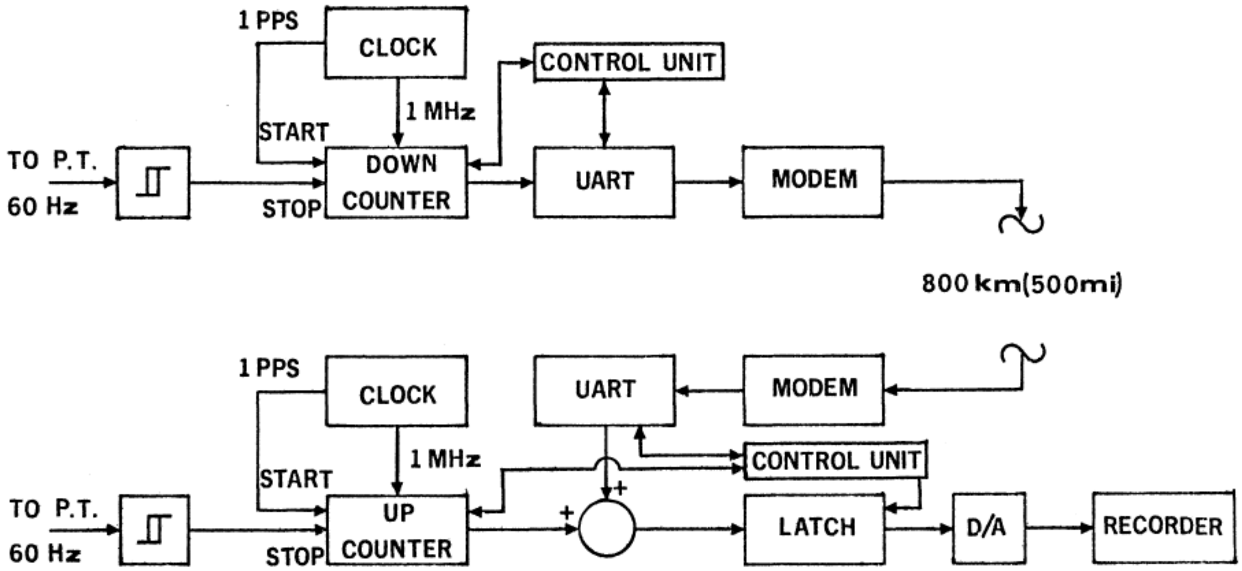


Figure HN-6.4: Block Diagram of Missout and Girard Bus Voltage Angle Measurement System. Based on Loran C (1980) – Reported Precision Better than One Degree [???]

In 1981, Bonanomi [???] described a technique for measuring voltage phase angles and generator shaft angles using synchronized clocks. The principle of these measurements is shown in Figure HN-6.3 taken form [???]. The system was demonstrated on a 220 kV transmission line of the Swiss network. The clocks used were two HBG radio time signals. The accuracy of the phase angle measurements was tested by comparing the power flow computed with the measured phase angles and the power flow recorded by power meters. The errors were below 20 MW which corresponds to a phase angle error of 0.35 degrees. One can say that the overall precision of the system is better than 0.35 degrees.

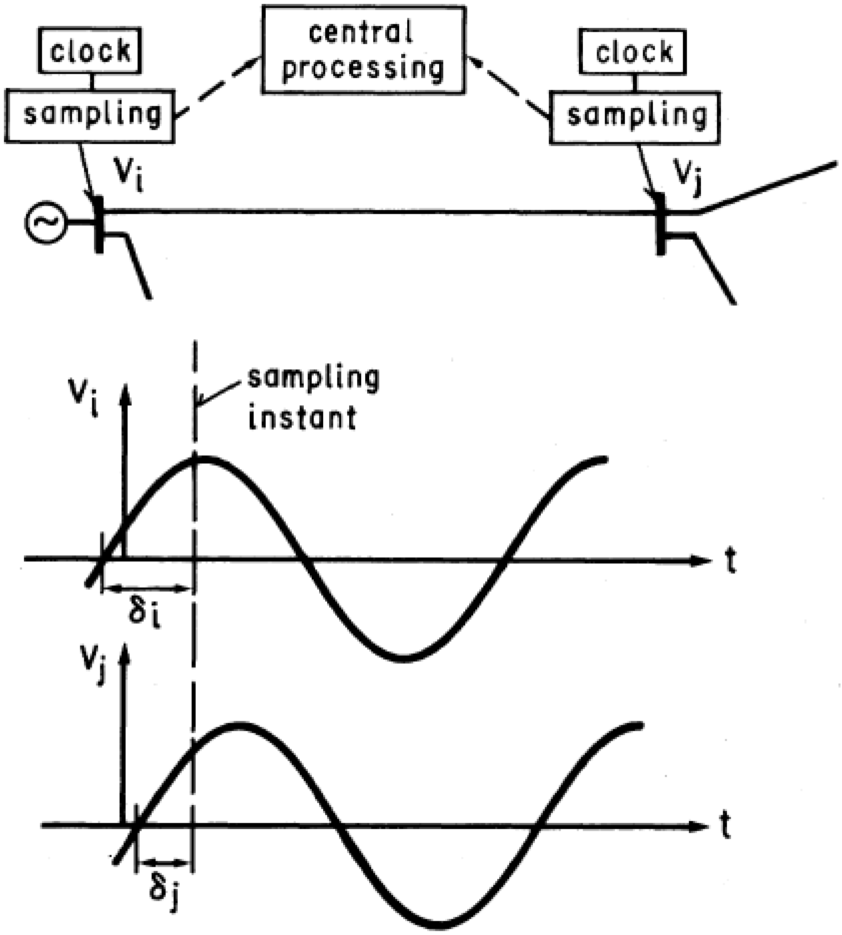


Figure HN-6.5: Principle of Operation of Phase Angle Measurements with Synchronized Clocks for the Simple Case of Two Buses (Bonanomi, 1981)

In 1983, the first author proposed to Utah Power company to develop a system of synchronized measurements using timing signals from GOES satellites for the purpose of monitoring the power loop flow through the state of Utah and the swings of this power flow in response of concern by this utility for the frequent loop flow problems that they were experiencing (EPRI Proposal). The GOES signal has an accuracy of better than 100 microseconds and at that time it was the best available clock (other than a portable Cesium atomic clock). While this proposal was not funded, the authors proposed to NYPA in the spring of 1989 to assess and develop technology of synchronized measurements for the purpose of monitoring harmonic distortion on the NYPA transmission system. This work was funded and in 1991 the authors developed a prototype device that can be added to an existing non-synchronized digital fault recorder to provide time tagged measurements with accuracy better than 2 microseconds. The device block diagram is illustrated in Figure HN-6.4a. It consists of a square wave oscillator that generates a symmetric square waveform with half cycle duration of 6 times the sampling rate of the DFR plus 2 microseconds. The rising edge of the square waveform pulse is synchronized with the UTC second transition (see Figure HN-6.4b). This is achieved using a phase-locked-loop based oscillator driven by the One-Pulse-per-Second signal provided by a GPS receiver.

One of the DFR channels (Channel 1) is dedicated to monitor this square waveform. The DFR sampled output signals are transmitted to a PC where time tagging is performed using the samples of the square wave waveform. The time tagging algorithm is based on the observation that the number of samples corresponding to each half cycle of the square wave is either 6 or 7. The only way that a 7 sample case occurs is that the first sample is at most 2 microseconds from the actual square wave rising or falling edge. Thus, since the times of the square wave transitions are synchronized to the GPS clock, a time tag can be assigned to the first of the 7 samples with maximum error 2 microseconds from the GPS time reference:

$$T = T_1 + n(6T_S + 2\mu s)$$

Where T1 is the time at the last UTC second transition, and n is the number of square wave half cycles between the last second transition and the latest half cycle (2 in the example of Figure HN-6.4b). Assuming that the DFR samples simultaneously all channels, this time tag is valid for all channels. Alternatively, if channels are sampled sequentially, appropriate time tags can be derived for each channel by taking into account the channel to channel sampling latency. The time tags are then used to compute the Fourier transform of the sampled waveform with reference the UTC thus providing the phasor with reference the GPS clock. The advantage of this approach is that this relatively simple device can be used with commercially available non-synchronized digital fault recorders to obtain synchronized measurements.

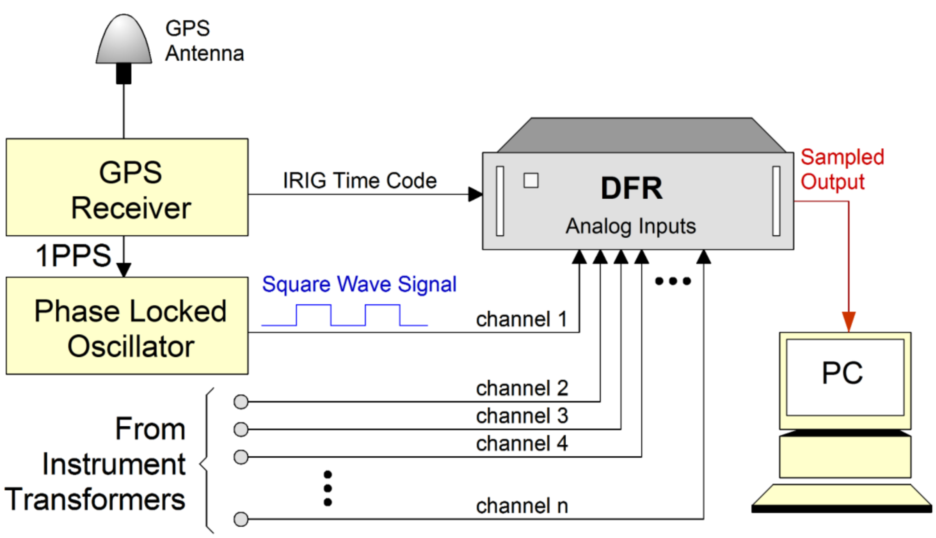


Figure HN-6.6: Block Diagram of Time Vernier System (Meliopoulos, Cokkinides, 1991)

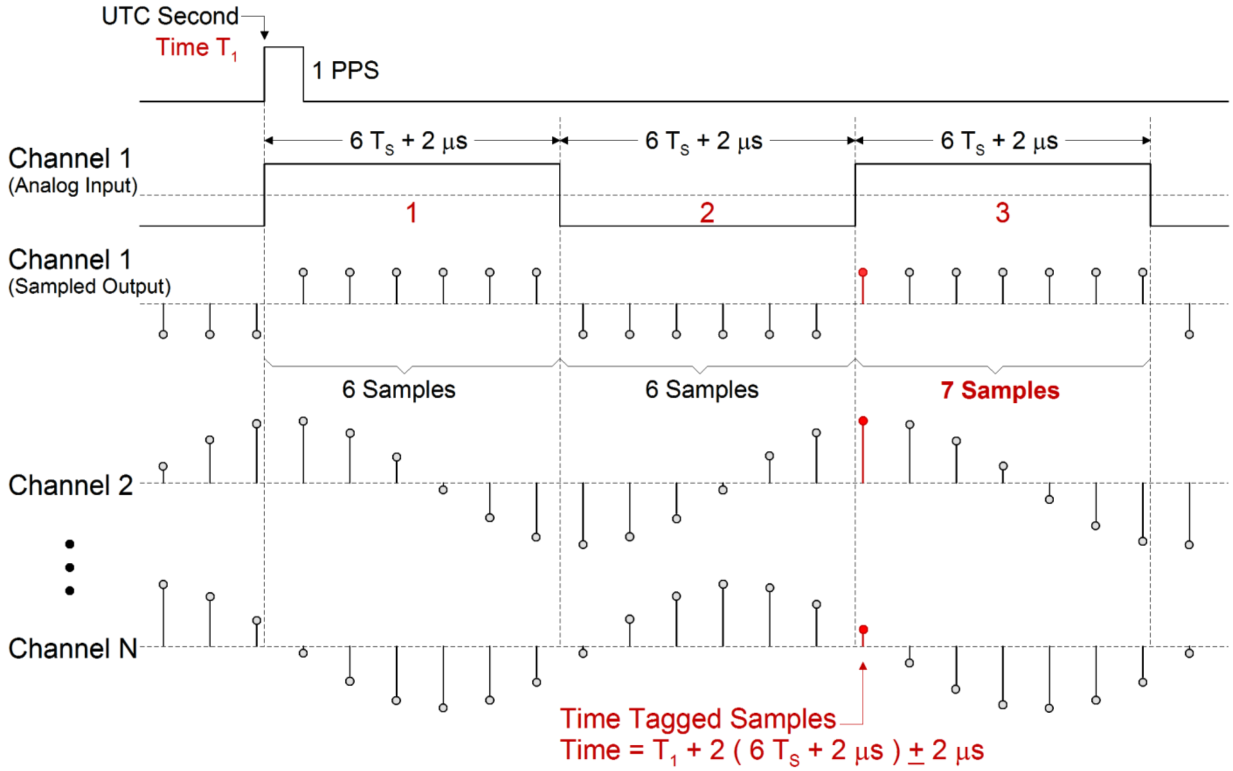


Figure HN-6.7: Operating Waveforms of Time Vernier System
(Meliopoulos, Cokkinides, 1991)

In the period 1990-1992, Arun Phadke developed the PMS which is illustrated in Figure HN-6.2. The PMS used a GPS signal for timing, 720 samples per second sample and hold A/D converter with multiplexing and a front-end anti-aliasing filter with a cutoff frequency of 360 Hz. The combination of the anti-aliasing filter and the multiplexing introduce time delays that are orders of magnitude greater than the accuracy of the GPS clock. Although this device was never tested by independent organizations, the estimated timing errors are more than 50 microseconds. Several PMS were constructed and sold to several utilities (for example three to AEP, one to NYPA, etc.). Despite the use of the GPS clock, the PMS was not capable of performing measurements with comparable accuracy to the GPS clock.

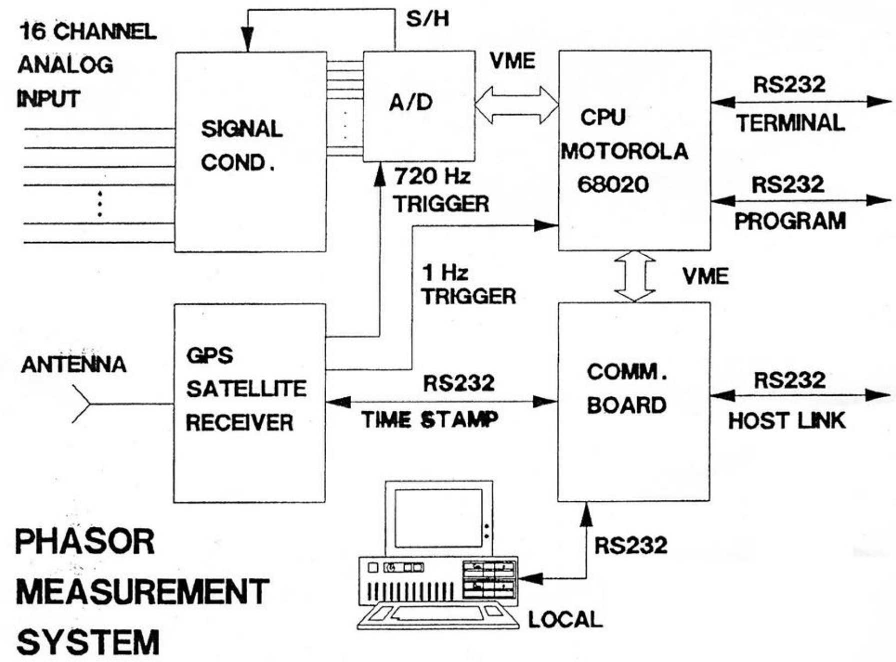
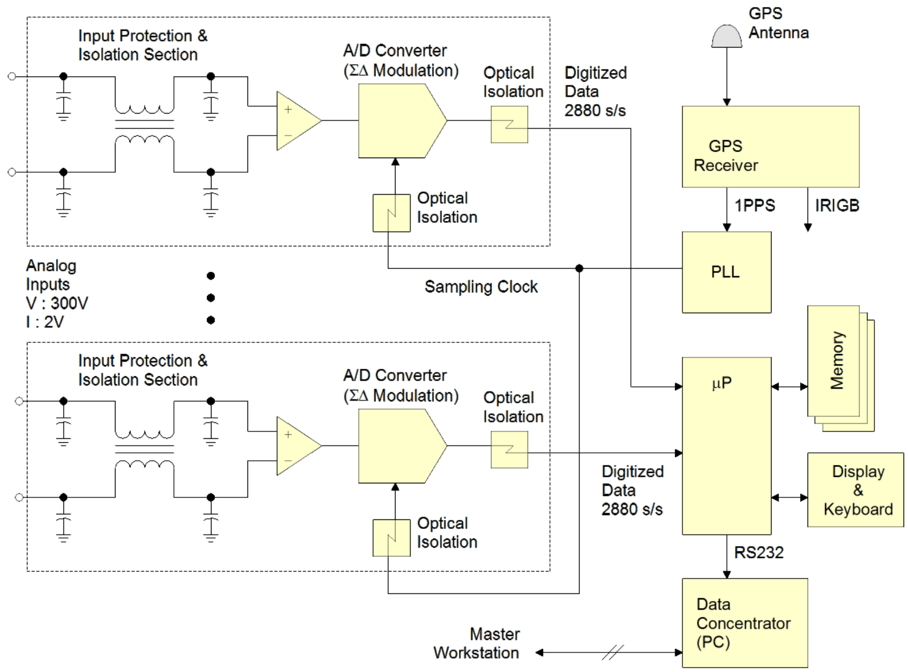


Figure HN-6.8: Block Diagram of Arun Phadke's PMS (as published by Phadke)
(Characteristics: (a) analog anti-aliasing input filters with low cutoff frequency (not specified but must be less than 360 Hz), (b) 12 bit sample and hold A/D technology 720 samples per second with analog multiplexing)

The first device capable of performing synchronized measurements with accuracy comparable to the GPS clock accuracy was developed by Jay Murphy of Macrodyne and was released in the market in January 1992. Jay Murphy named the device the Macrodyne 1620 PMU (Phasor Measurement Unit). Jay Murphy introduced the following innovations to achieve the goal of performing synchronized measurements with accuracy comparable to the GPS clock:

Low distortion front-end analog channel with very high cutoff frequency.
Common Mode Rejection Filter with Optical Isolation.
Individual Channel GPS Synchronization.
16 bit A/D Sigma/Delta Modulation converter operating at several MegaHertz and decimated to 2,880 samples per second.

The Macrodyne 1620 PMU was tested by the authors (in collaboration with Bob Burnett of GPC) who verified that the unit is capable of performing measurements with accuracy one microsecond (or equivalently 0.02 degrees for a 60 Hz waveform). The Macrodyne 1620 PMU is illustrated in Figure HN-6.3.



(a)

(b)

Figure HN-6.9: The Macrodyne 1690 PMU (January 1992)
(a) Block Diagram, (b) Photograph

6.2 Current Transformers

6.2.1 Construction

Current transformers (CT's) are made in a variety of forms and sizes. Figure 6.1 illustrates various CT types. In spite of the varying forms and sizes they share the same basic configuration. Specifically, they consist of a magnetic core with a primary winding (typically one turn), and a secondary winding consisting of several hundred turns (see Figure 6.2). The burden resistor is connected across the secondary winding. The burden resistance is typically very low (0.01 to 0.1 ohms) and thus the net core magnetic flux is low, avoiding core saturation. (Note that it is important to keep the burden resistor connected at all times as long as current flows through the primary winding, otherwise a very high voltage may develop along the secondary winding causing insulation failure).

6.2.2 Terminology

In this section we present the definitions of several terms frequently used with current transformers.

Transformation Ratio: The ratio of the primary current to the secondary current in Amperes.

Ratio Correction Factor (RCF): Factor that is multiplied by the marked nominal ratio to obtain the actual primary to secondary current ratio

The **Composite Error (** ε_C **)** is defined as the RMS value of the difference between the instantaneous values of the primary current minus the instantaneous values of the actual secondary current multiplied by the rated transformation ratio, under steady-state conditions. Specifically the composite error is expressed by the formula:

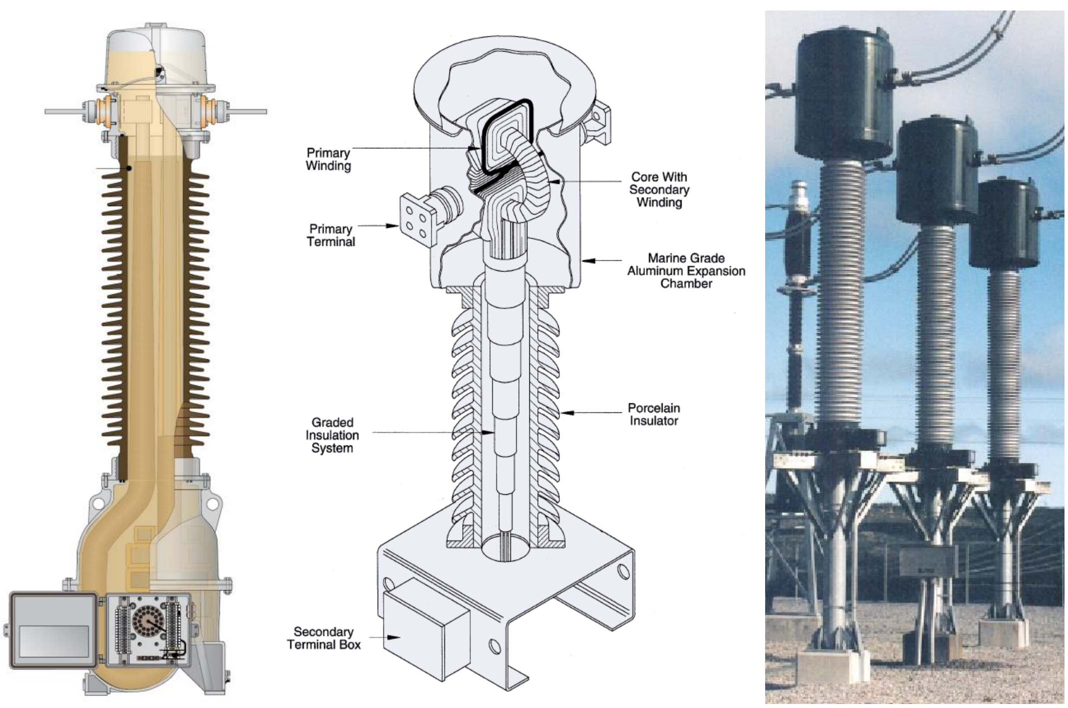
$$\varepsilon_C \equiv \frac{100}{I_P}\sqrt{\frac{1}{T}\int_0^T (k \cdot i_S(t) - i_P(t))^2 \, dt}$$

where:

k rated transformation ratio
I_P RMS value of the primary current
$i_P(t)$ instantaneous value of the primary current
$i_S(t)$ instantaneous value of the secondary current
T duration of one power frequency cycle

Rated Burden: Rated burden is the maximum impedance in ohms that can be connected in the secondary of the CT and for which the CT will maintain the specified accuracy for the maximum specified current (typically 20 times rated current).

(a) (b) (c)



(d) (e) (f)

Figure 6.10: Various types of current transformers
(a) Clamp, (b) Wideband Instrumentation, (c) 600 V Class
(d, e, f) 72.5-800 kV class

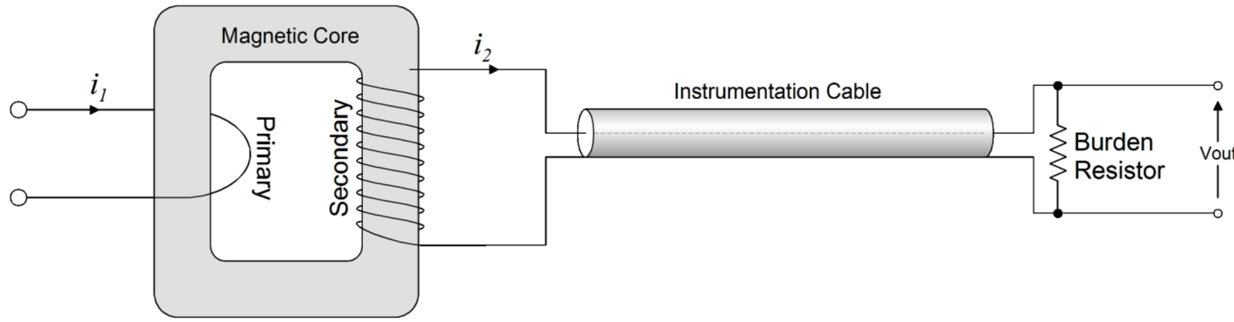


Figure 6.11: Typical CT Configuration

Relaying Accuracy Class. In a relaying class CT, the composite error shall not exceed a specified percentage error at a specified secondary terminal voltage based on a maximum level of secondary current.

Continuous Current Rating Factor. The factor by which the rated current of the CT can be multiplied to obtain the maximum continuous current that the CT can carry without exceeding the temperature rise or accuracy requirements.

Metering Accuracy Class. In a metering class CT, the RCF shall be within specified limits at 10%, 100%, and CCRFx100% at a given power factor with a specified burden.

1 Second Thermal Withstand. The maximum RMS symmetrical primary current that can be carried for one second with the secondary short circuited without exceeding the CT limiting temperature.

Mechanical Withstand. The maximum RMS asymmetrical primary current that a CT can carry with the secondary short circuited without any damage which would render it incapable of meeting other standard accuracy and transformation requirements.

6.2.3 Polarity Markings & Terminal Naming

The polarity of instrument transformer windings is important in many relaying applications where the phase of the transformed waveforms must be preserved. The standard notation indicating winding polarity is illustrated in Figure 6.x. The dots or square blocks marks placed near one terminal of each winding have the following interpretation:

The electric current flowing into the marked terminal of the first winding is in phase with the electric current flowing out of the marked terminal of the second winding.

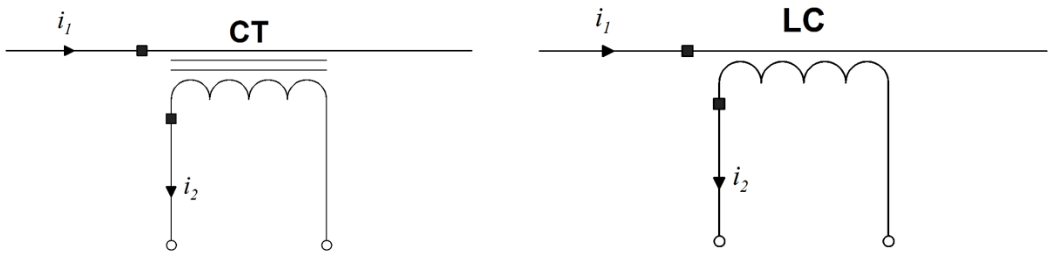


Figure 6.12 – Instrument Transformer Polarity Marks

Instrument transformer terminals are named as H1, H2, H3, etc. for primary windings and X1, X2, X3, etc. for secondary windings. The number following each letter identifies taps of the same winding. If multiple secondary windings exist, the letters Y, Z, W, U are also used. An example of transformer terminal naming with a single primary and two secondary windings with two taps on each secondary windings is illustrated in Figure 6.x.

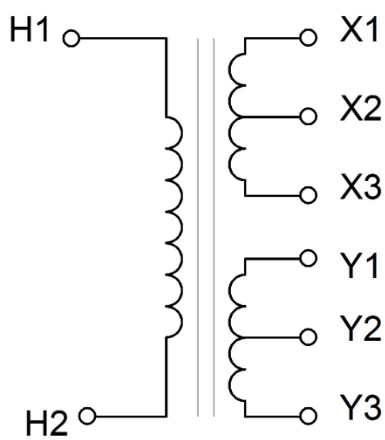


Figure 6.13: Instrument Transformer Terminal Naming

6.2.4 Ratio Standardization

The transformation ratios for CT's for electric power applications have been standardized to the values given in Table 6.1. The first row of the table provides the main CT ratios, while the remaining rows provide the ratios typically available by additional secondary taps. For most relaying applications selecting CT's from these values yields workable design solutions. The availability of many standard transformation ratios was very important in the days of electromechanical relays as it was important to match as close as possible the ratios to the requirements of the applications, such as differential protection, etc. With the introduction of the numerical relay, matching CTs for specific applications is not necessary anymore, thus allowing more flexibility in selecting CTs.

Table 6.2 Current Transformer Standard Ratios

Main Ratios	600:5	1200:5	2000:5	3000:5	4000:5	5000:5
Secondary Taps	50:5 100:5 150:5 200:5 250:5 300:5 400:5 450:5 500:5 600:5	100:5 200:5 300:5 400:5 500:5 600:5 800:5 900:5 1000:5 1200:5	300:5 400:5 500:5 800:5 1100:5 1200:5 1500:5 1600:5 2000:5	300:5 500:5 800:5 1000:5 1200:5 1500:5 2000:5 2200:5 2500:5 3000:5	500:5 1000:5 1500:5 2000:5 2500:5 3000:5 3500:5 4000:5	500:5 1000:5 1500:5 2000:5 2500:5 3000:5 3500:5 4000:5 5000:5

6.2.5 Error Classification

Instrument Current Transformers used for power system applications are classified according to current and voltage level as well as accuracy level. A commonly referred ANSI-IEEE standard (ANSI C57-13 Standard Requirements for Instrument Transformers) defines these classifications for both current and potential transformers. Specifically, the standard provides information on the electrical and mechanical characteristics of instrument transformers and defines techniques for measuring transformation ratio and phase angle, thermal performance, polarity determination. It also provides guidance for the selection of appropriate instrument transformers for specific applications. Furthermore, a naming convention is defined for specifying transformation ratio accuracy as follows:

xxTyy or xxCyy,

where:
- xx is the maximum CT error in % at 20 times the normal current.
- yy is the maximum secondary voltage under which the xx error limit applies
- C indicates that the CT error classification was determined by computation.
- T indicates that the CT error classification was determined by test.

For example, a 1200:5A CT with a rating of 10C400 indicates that the CT error is under 10% given that the secondary voltage is under 400 volts and the secondary current is under 100 Amperes.

The standard classification values for yy are: 10, 20, 50, 100, 200, 400, 800 Volts RMS. The assigned standard value is lower than or equal to the actual maximum secondary voltage, whether it is derived from measurement or computation.

The accuracy classification of a C class CT is determined by considering the CT equivalent circuit and its excitation characteristics. The equivalent circuit used for CT accuracy classification computations is illustrated in Figure 6.3. It consists of the following components:

Z_m Magnetizing Reactance.
R_W Winding wire resistance.
R_B Burden resistance.

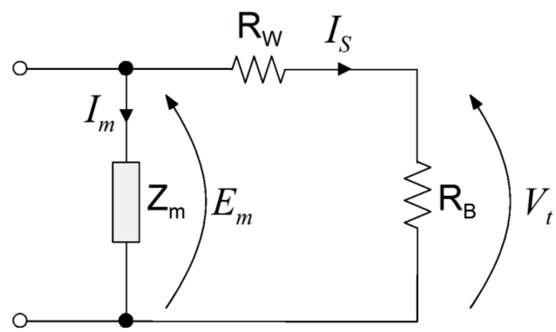


Figure 6.14: CT Equivalent Circuit Referred to the Secondary

Note that the leakage reactance is typically neglected. This is a reasonable assumption for most CT's with a fully distributed winding, i.e. a winding uniformly spread over the entire length of the CT toroidal CT core. The leakage impedance of a CT with a non-distributed winding (i.e. a winding concentrated over one segment of the core) is typically higher and may not be neglected. In this case, the CT is given a T accuracy classification indicating that the accuracy class is determined by test. The accuracy classification computation is illustrated next by an example.

Example E6.x: Consider a 1000:5 CT with the magnetization characteristics given in Figure E6.1. The winding resistance is 1.5 Ohms, and the leakage reactance is negligible. Compute the accuracy classification.

Solution: The solution consists of computing the CT terminal voltage for the following conditions (a) the CT current is 20 times the nominal current, and (b) the ratio error is under 10%. Since the secondary rated current is 5 A, take the secondary current at 20 times 5 A, or 100 A.

Consider the equivalent circuit illustrated in Figure 6.3. Note that the magnetizing branch causes a ratio error. For a 10 % ratio error specification, the magnetizing current must be less than or equal to 10 A. From the excitation characteristics of Figure E6.x, the voltage E_m under these conditions will be 700 Volts. Finally, the terminal voltage is computed as:

$$V_t = E_m - I_S R_W = 700 - 1.5 \times 100 = 550 \text{ V}$$

The computed terminal voltage is compared to the standard classification values. The highest standard value, which is less than or equal to the computed value is selected, i.e. 400 Volts. Thus the accuracy classification code for this CT is 10C400.

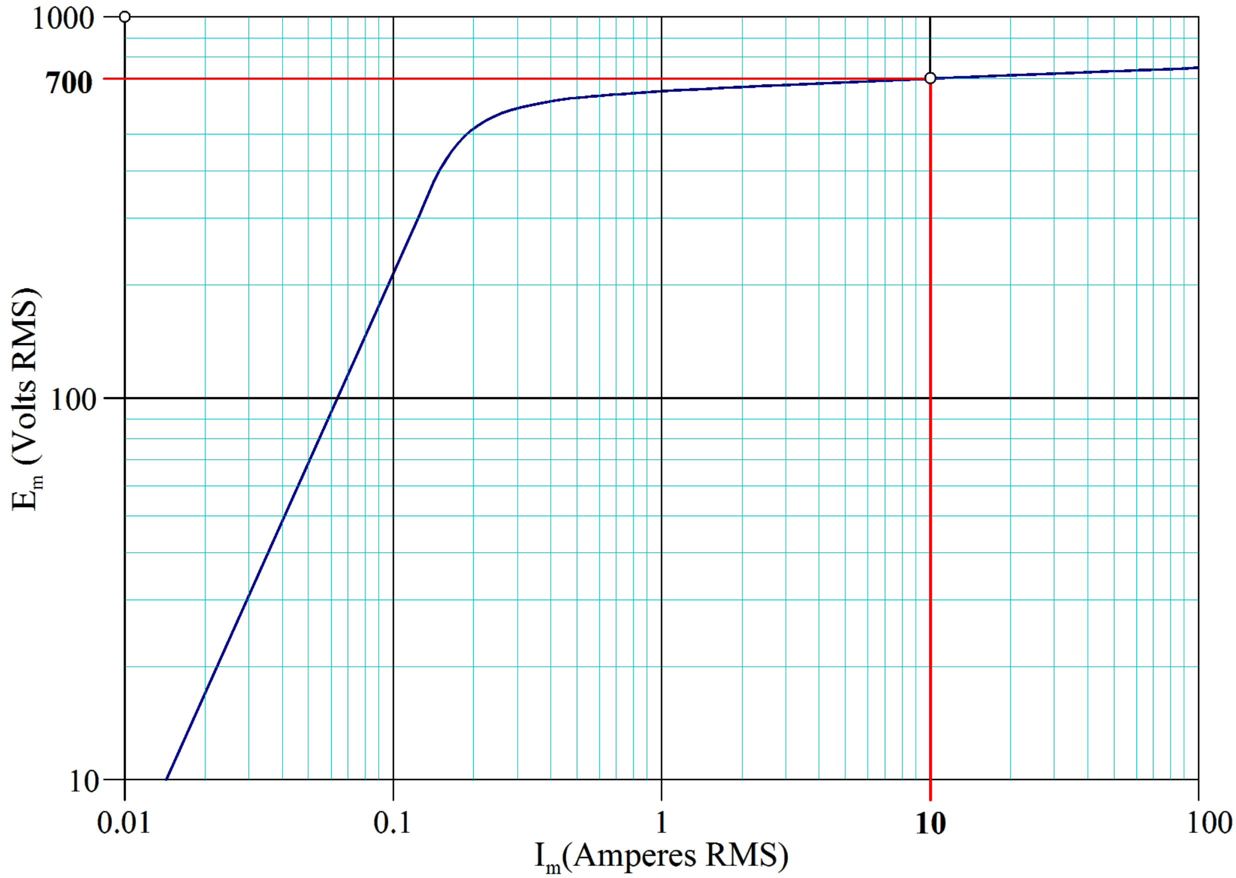


Figure E6.1: CT Magnetization Characteristics

6.2.6 Auxiliary CT

Auxiliary CT's are connected in series with the main CT secondary circuit, as illustrated in Figure E6.xa. The reason for installing an auxiliary CT is to match the ratios of existing (main) CT's for the various relaying applications that require such matching (for example differential bus or transformer protection using electromechanical relays). With the advent of microprocessor based relays the need ratio CT matching has been eliminated and thus new auxiliary CT applications are uncommon.

The performance analysis of auxiliary CT's follows the same principles as main CT's. For example the error of the auxiliary CT output can be evaluated by constructing an equivalent circuit of both main and auxiliary CTs (see Figure E6.xb). Note that adding an auxiliary CT on the secondary of the main CT increases the total burden impedance of the main CT, thus care must be taken to prevent exceeding the main CT maximum permissible burden.

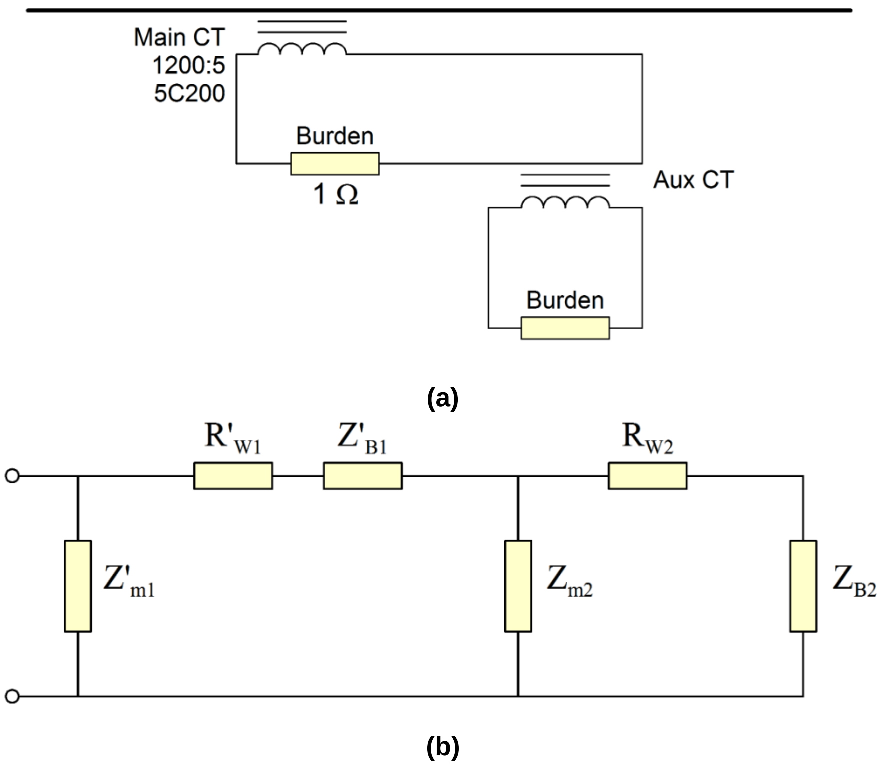


Figure E6.15: Application of Auxiliary CT (a) and Corresponding Equivalent Circuit (b)

6.2.7 Analysis of Steady State Performance at Power Frequency

Fundamental current transformer model equations:

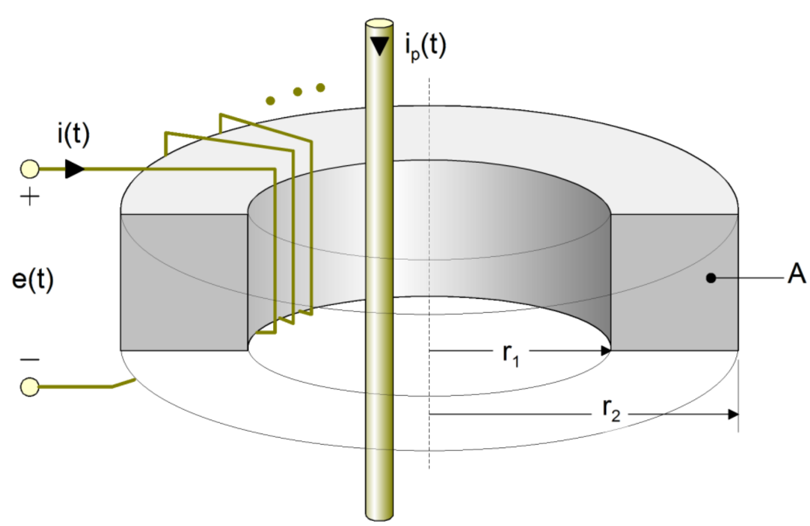


Figure E6.16: Application of Auxiliary CT (a) and Corresponding Equivalent Circuit (b)

Assuming that the CT core has high relative magnetic permeability ($\mu_r >> 1$), the magnetic field intensity H is related to the total current passing through the core hole as follows:

$$\sum_k i_k = lH$$

where: l is the core length. For a rectangular cross-section toroidal core, the core length is approximately $l = \pi(r_1 + r_2)$

The voltage across the CT winding is related to the flux-linkage (λ), total flux (Φ), and flux density (B) as follows:

$$e = \frac{d\lambda}{dt} = n\frac{d\Phi}{dt} = nA\frac{dB}{dt}$$

where n is the number of secondary winding turns. Assuming sinusoidal excitation:

$$E = \omega\Lambda = \omega n\Phi = \omega nAB$$

Note also that:

$$B = \mu H$$

where μ is the magnetic permeability of the core. Using the above equations, the performance of a CT under a variety of conditions can be evaluated. Note that typical magnetic materials used in CT core construction, saturation occurs at about B = 0.5 Tesla. Thus, in order to prevent saturation, the peak winding voltage E must be limited to: $0.5\omega nA$. The analysis procedure is demonstrated next by an example.

Example E6.1: A 2400:5 current transformer consists of a toroidal magnetic core. The cross section of the core is rectangular with dimensions 3.0 x 3.0 cm. The core material saturates at B = 0.5 Tesla. The total impedance on the secondary of the CT, including winding impedance and burdens, is X_b = 2.5 ohms.

(a) Compute the maximum 60 Hz primary current that will not saturate the CT.

(b) Assuming that the relative permeability of the core is 10,000 and the core radii are 9 and 12 cm respectively, compute the magnetizing current at maximum non-saturating primary current, and the CT ratio error for the same primary current.

(c) An engineer desires that this CT should be able to develop a maximum voltage of 400 volts (RMS) on the secondary without saturation. What should be the core cross-section in this case?

Solution – Part (a): First the number of turns in the secondary circuit is computed from:

N1 / N2 = 5/2400, N1=1. Thus N2 = 2400/5 = 480.

Thus the RMS winding voltage E must be limited to:

$$E = 0.5\ \omega\ n\ A / \sqrt{2}\ = 0.5 \times 377 \times 480 \times 9.0 \times 10^{-4}\ / \sqrt{2} = 57.58\ \text{V}$$

Thus the secondary winding current is:

$$I_s = E / X_b = 57.58\ \text{V} / 2.5\ \Omega = 23.03\ \text{A}$$

Neglecting the magnetizing current (reasonable approximation since we avoid saturation), the primary current is:

$$I_p = I_s \times 480\ \text{A} = 11.05\ \text{kA}$$

Part (b) The magnetizing reactance is given by the formula:

$$X = \frac{\omega \mu_0 \mu_r n^2 A}{\ell} = \frac{2\pi 60 \cdot 4\pi 10^{-7} \cdot 10000 \cdot 480^2 \cdot 0.03^2}{(0.09+0.12)\cdot \pi} = 1489\,\Omega$$

at maximum voltage before saturation the magnetizing current will be:

$$I_m = E / X = \frac{57.58V}{1489\Omega} = 0.03867A$$

and the ratio error:

$$\text{error} = 100 \times \frac{0.03867\text{A}}{23.03A} = 0.168\%$$

Part (c) for 400 V secondary voltage

$$400\ \sqrt{2} = 0.5\ \omega\ n\ A, \quad thus\ A = 400\sqrt{2}\ /\ (\ 0.5\ x\ 377\ 480\ ) = 0.00625\ m^2$$

Thus the CT core cross-section (assuming a square form) must be at least 7.9 x 7.9 cm

In an ideal current transformer, the primary to secondary current is independent of primary current magnitude, frequency, and burden resistor value. However actual devices deviate from this behavior due to several causes, namely:

- Winding Resistance
- Leakage Impedance
- Magnetizing Impedance
- Core Hysteresis and Saturation
- Parasitic Capacitance

The nominal CT transformation ratio is given by:

$$i_2 = \frac{1}{N} i_1$$, where N is the number of secondary winding turns

However, the actual transformation ratio differs from the nominal due to the above factors. The normalized difference between nominal and actual transformation ratios is called the “transformation ratio error”. The error magnitude depends on the current magnitude, frequency, burden impedance etc. We first consider the transformation error for steady state 60 Hz operation. The steady state performance of CT’s can be analyzed by considering an equivalent circuit. The equivalent circuit of a current transformer is illustrated in Figure 6.x. It consists of the following components:

L_1, L_2	Primary and secondary Leakage Inductance
R_1, R_2	Primary and secondary Winding Resistance
L_m,	Magnetizing Inductance
G_m	Core Loss Conductance (due to hysteresis and Eddy currents)
Z_B	Burden Impedance
Z_C	Cable Impedance (Cable connecting CT secondary to burden).
N:1	CT nominal (name plate) transformation ratio

The burden impedance Z_B is normally resistive. Typical values of CT burden resistances range from 0.05 ohms to 0.5 ohms.

The components L_m and G_m model the CT magnetizing current. The relationship of this current to the voltage across L_m and G_m is actually nonlinear. However, if this voltage is below a certain threshold this approximation is valid. This topic is discussed in detail in the section **Saturation**.

By referring all impedances to the secondary, a simpler equivalent circuit is obtained, illustrated in Figure 6.x. The actual ratio of primary to secondary current is:

$$x = \frac{I_P}{I_B} = N\frac{I_S}{I_B}$$

Note that unless $I_B = I_S$ the actual transformation ratio differs from the nameplate (nominal) transformation ratio. The factor I_S/I_B is called the *Ratio Correction Factor* (RCF). It can be calculated from the equivalent circuit as follows:
]

$$RCF \equiv \frac{I_S}{I_B} = \frac{Z_M + Z_2 + Z_C + Z_B}{Z_M}$$

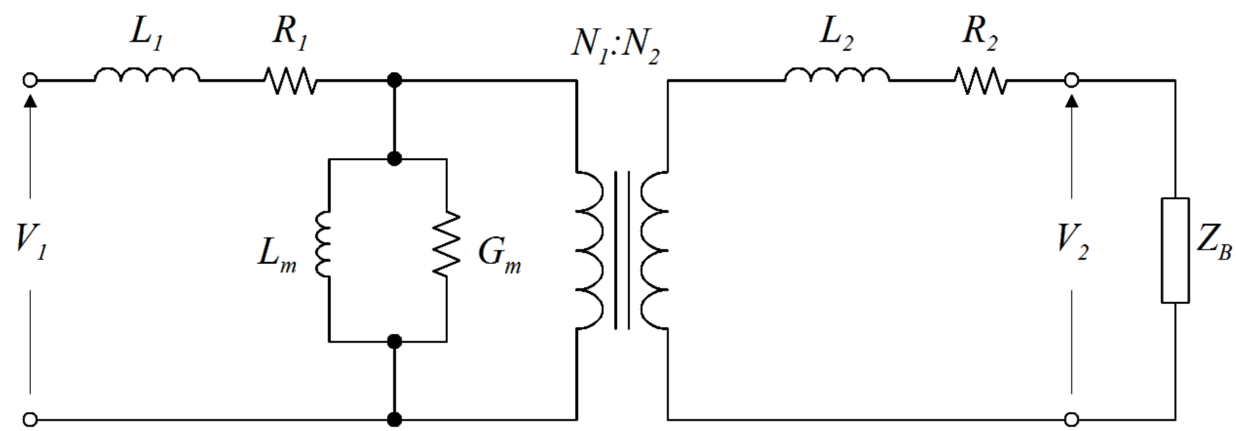


Figure 6.17: Current Transformer Equivalent Circuit

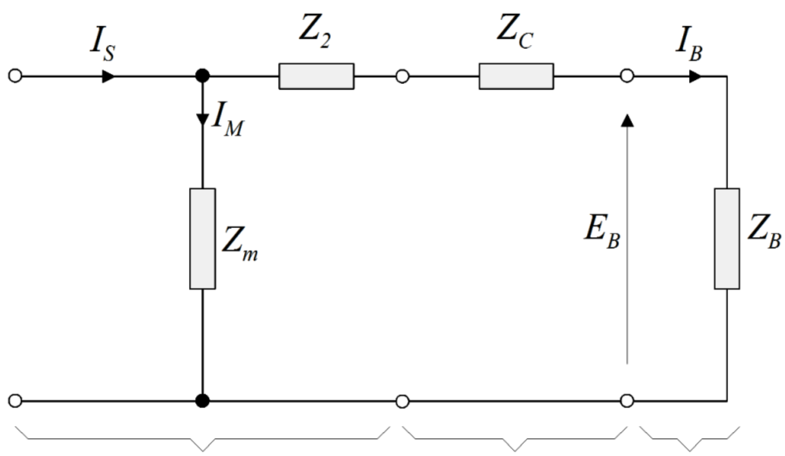


Figure 6.18: Simplified CT Equivalent Circuit

Note that the ratio correction factor is in general a complex quantity. It includes both a magnitude and a phase correction. In determining the ratio correction factor, the following observations are useful:

- The RCF approaches unity as the burden and cable and winding impedances are reduced.
- The burden impedance ZB is normally resistive and typically of small value (below 0.1 ohms).
- The magnetizing impedance is nearly inductive and relatively large unless the CT core is saturated. Core saturation can be determined by the voltage across the magnetizing reactance (see section 6.4.2)

Given the ratio correction factor, the CT error in percent is defined as:

$$\varepsilon = 100\frac{\frac{I_P}{N} - I_S}{\frac{I_P}{N}} = 100\frac{\frac{I_P}{N} - \frac{I_P}{N \cdot RCF}}{\frac{I_P}{N}} = 100\left(1 - \frac{1}{RCF}\right)\%$$

Example E6.2: Compute the maximum ratio error of the CT circuit illustrated in Figure E6.x at 60 Hz.

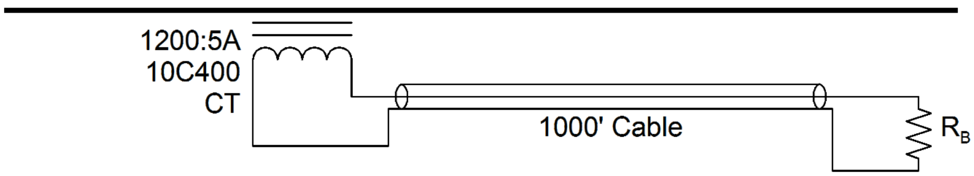


Figure E6.2

The following information is provided:

CT Rating:	1200:5
CT Accuracy Class:	10C400
Winding Resistance:	0.8 Ohms
Burden Resistance:	0.1 Ohms
Cable Series Impedance:	0.00002 + j0.00005 Ohms /foot

Solution: Assuming that at maximum rated secondary voltage of 400 Volts, the ratio error is 10%, the magnetizing impedance is 400V / 10A = 40 Ohms. Thus the equivalent circuit with the CT, instrumentation cable and burden is as shown in Figure xxx.

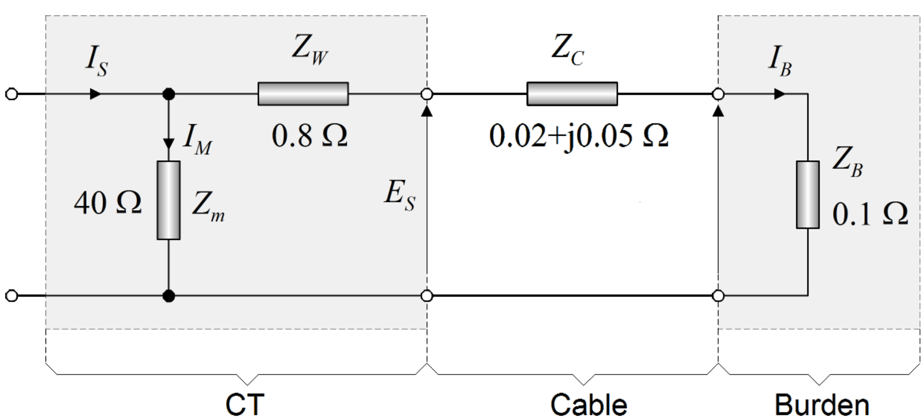


Figure E6.19

At 20 times the rated secondary current, IS = 100 A. Then:

$$I_B = I_S \frac{40}{40 + 0.92 + j0.05} = 97.752 e^{-j0.07^0}$$

Thus the ratio error is:

$$\varepsilon = 100 \frac{|I_B| - |I_S|}{|I_S|} = \frac{97.752 - 100}{100} \% = -2.248\%$$

6.2.8 Frequency Response

The frequency response of CT's deviates from the ideal flat response due to the interaction between the leakage inductance, magnetizing inductance and parasitic capacitance. Note that there is parasitic capacitance between winding turns as well as between each winding and the CT magnetic core (See equivalent circuit illustrated in Figure xxx). Figure 6.x illustrates the frequency response of a 600 V Metering Class CT. Note that this device has a resonance near 4,100 Hz, which causes a gain peak of about 10% and a phase deviation of -40 degrees at 4,500 Hz. The resonance frequency of this device is much higher than the power frequency and thus it does not cause significant error in relaying and metering applications. It is however important to note that the higher the transformation ratio is, the lower the resonant frequency will be.

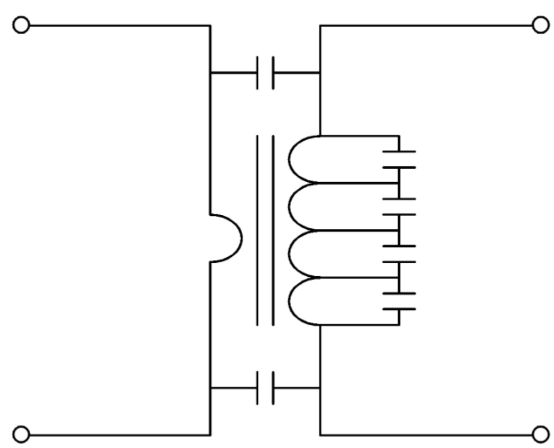


Figure 6.20: CT Parasitic Capacitance

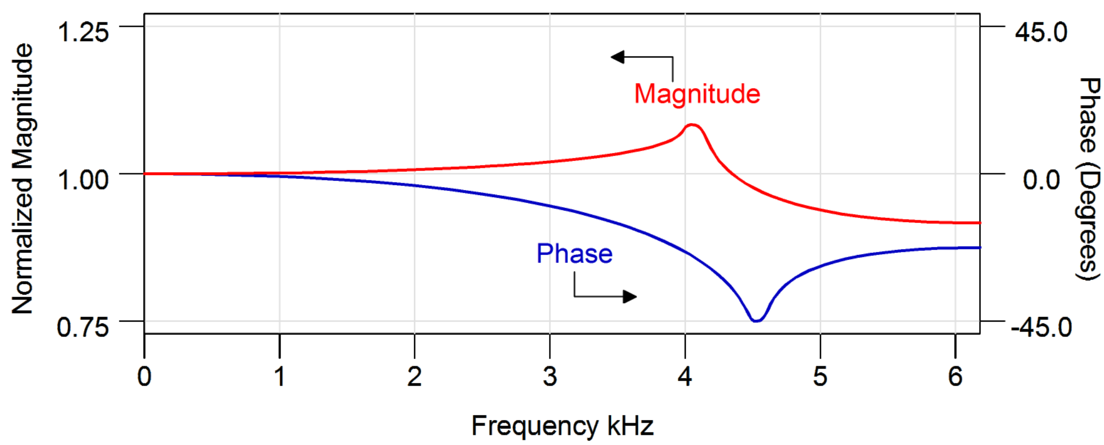


Figure 6.21: Typical 600 V Metering Class CT Frequency Response

6.2.9 Saturation

The CT magnetic core exhibits a nonlinear relationship between the magnetic field intensity and the magnetic flux density. This results into a nonlinear relationship between the magnetizing current and the flux linkage (which is also related to the voltage across the CT winding). Figure 6.x illustrated a typical plot of flux linkage versus magnetizing current.

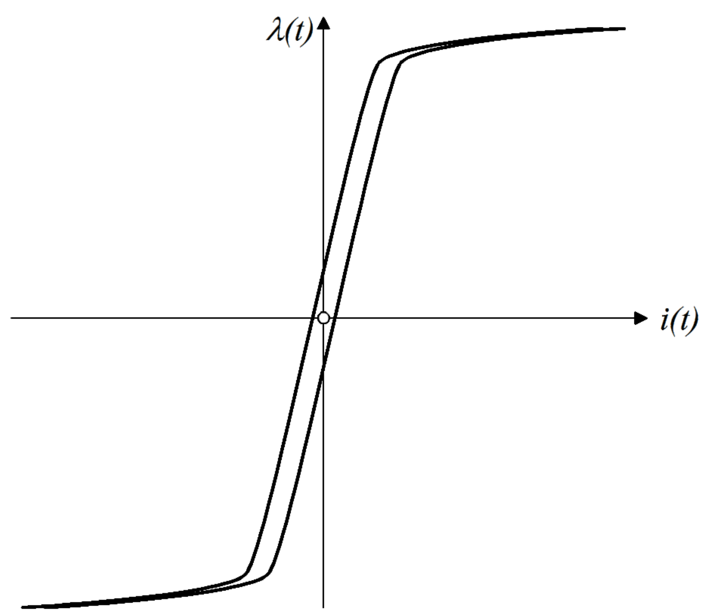


Figure 6.22: Typical flux linkage versus current

Consider the CT equivalent circuit illustrated in Figure 6.x, derived by neglecting the leakage reactance and winding resistance, and referring all quantities to the secondary. The current source represents the CT primary current multiplied by the CT nominal ratio. The element L_M represents the nonlinear magnetizing reactance, and the resistor R_B is the CT burden resistor. By combining the burden and current source into a Thevenin equivalent the circuit in Figure 6.x.b is obtained. Loop analysis of this circuit yields the following equation:

$$R_b i(t) = R_b i_M(t) + \frac{d\lambda\{i_M(t)\}}{dt}$$

Assuming that the operating conditions of the CT are such that saturation is not occurring the following model can be used:

$$i(t) = i_M(t) + \frac{L_M}{R_b}\frac{di_M(t)}{dt}$$

Note that the above model indicates that the unsaturated CT step response is exponential with time constant $\frac{L_M}{R_B}$

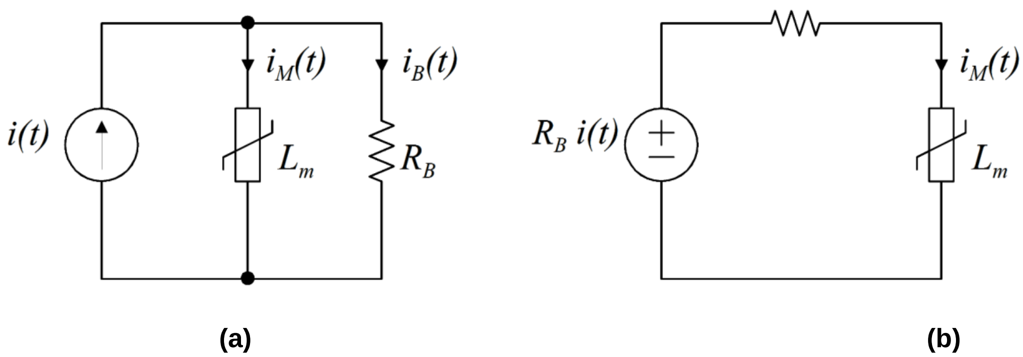


Figure 6.23: Typical Magnetic Core Current Transformers
(a) CT Equivalent Circuit, (b) Thevenin Equivalent

Assuming sinusoidal input the solution to the above equation yields the magnetizing current:

$$i_M(t) = \frac{I_0 / N}{\sqrt{1+(\omega L_M / R_b)^2}}\cos(\omega t + \varphi)$$

where I_0 is the amplitude of the primary current and N is the CT transformation ratio.

Observe that for a given primary current and turns ratio, the amplitude of the magnetization current increases with:

- increasing the burden resistance R_b
- decreasing the frequency ω,

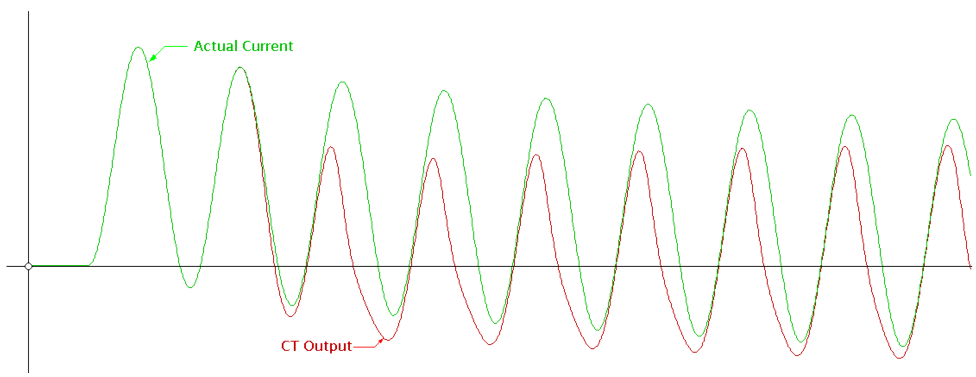
For a given CT, saturation occurs when the magnetization current exceeds a certain value (near the knee of the flux versus magnetizing current curve). Thus, the following conclusions can be drawn:

- A CT is saturated with a much lower DC value than a 60 Hz current.

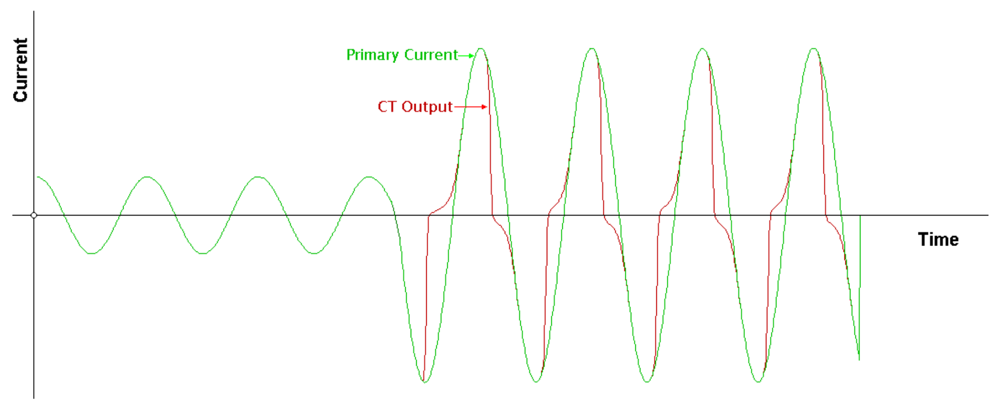
- Decreasing the burden resistance allows the CT to withstand a higher primary current before saturation.

- The CT winding resistance and leakage inductance limits the CT maximum current capability (for unsaturated operation).

When saturation occurs, the CT secondary current is severely distorted. The behavior of a saturated CT cannot be analytically computed. However, numerical techniques can be used to simulate CT behavior during saturation. Figure 6.x illustrates the computed saturating CT response for two cases (a) saturation caused by input current DC offset, (b) saturation caused by large sinusoidal current.



(a)



(b)

Figure 6.24: Typical CT Saturation Waveforms
(a) Cause by DC Offset, (b) Caused by High Fault Current

Figure 6.x illustrates the magnetizing characteristics of a typical 1200:5 CT. These curves are obtained experimentally, and provide the RMS voltage across the secondary winding as a function of the RMS excitation current for each CT secondary tap. The magnetization current is referred to the secondary winding. When the data of Figure 6.x are used, keep in mind that in general the voltage will be approximately sinusoidal but the magnetizing current will be severely distorted.

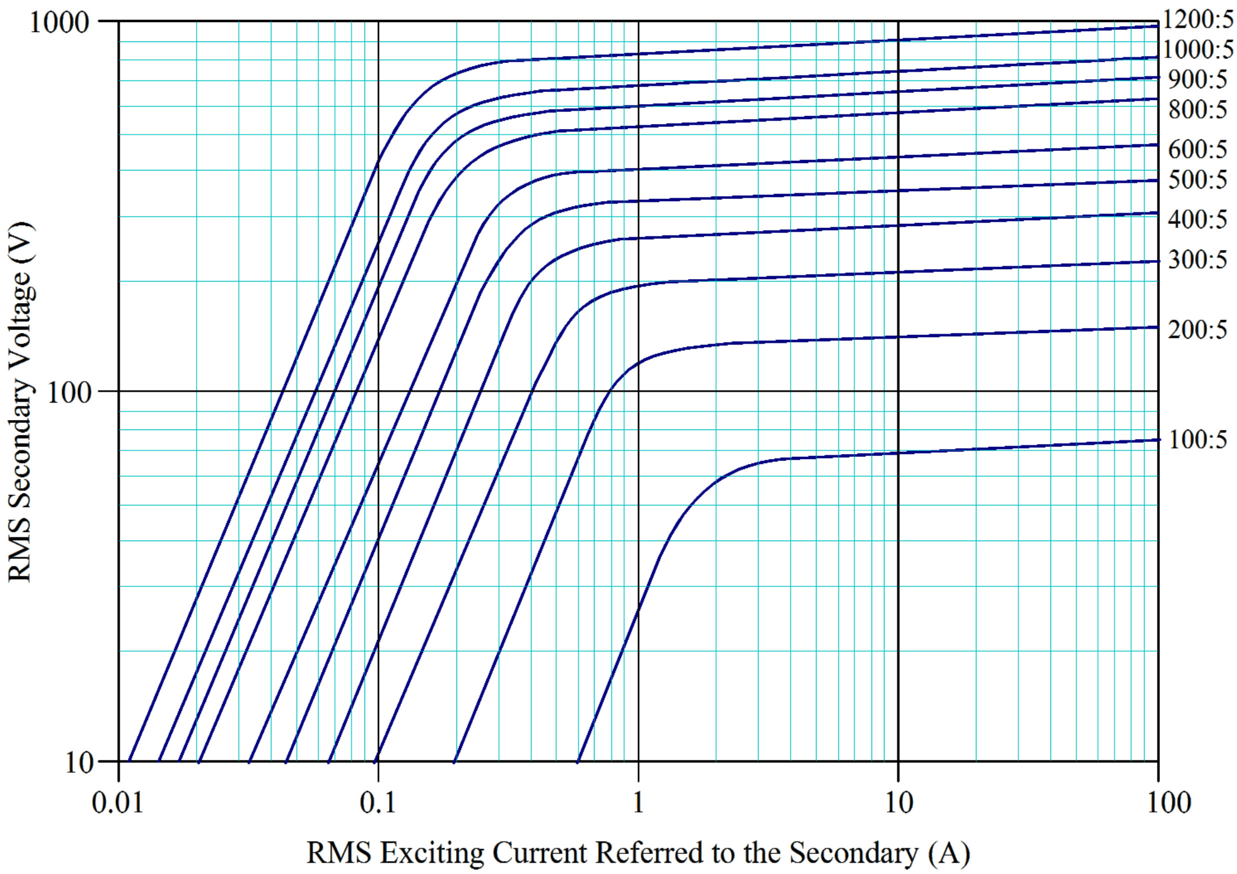


Figure 6.25: Typical CT Magnetizing Characteristics (RMS values)

The magnetization characteristics can be used to compute the CT ratio correction factor, to determine the appropriate burden to avoid saturation etc. The use of these data is demonstrated next by an example.

Example E6.x: A 1200:5 CT must be able to operate without saturation for a primary current of up to 20,000 Amperes. What is the maximum burden resistance that will meet this requirement? (Neglect leakage inductance and winding resistance).

Solution: From the magnetization characteristics, it can be determined that using the 1200:5 tap, the saturation knee begins at the following conditions:

Secondary Voltage: 500 Volts

Magnetization Current: 0.12 Amperes

At 20,000 A the nominal secondary current is 83.3 A. The simplified equivalent circuit of the CT in this case will be as has been described in Figure 6.x. The nominal secondary current is the sum of the magnetizing current and the burden current. Assuming that the magnetizing current is at 90 degrees with the burden current, the burden current is:

$$I_b = \sqrt{83.3^2 - 0.12^2} = 83.3A$$

Thus the maximum burden impedance is:

$$R_b = \frac{500V}{83.3A} = 6.0\Omega$$

Example E6.x: The CT of problem E6.x is used with a burden impedance of 6.0 Ohms. During a fault, the primary current reaches to 40,000 Amperes. Determine the magnetization current, the burden current, and the ratio correction factor.

Solution: Consider the simplified Thevenin equivalent circuit of the CT secondary under the given conditions.

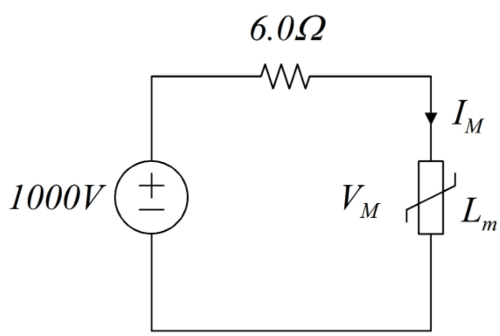


Figure E6.26 Thevenin Equivalent Circuit

The secondary voltage and magnetization current must satisfy the equation:

$$V_M = 1000 - 6I_M$$

In order to find the magnetizing current we plot the above V-I line over the CT magnetization characteristics. Note that since the plot is logarithmic, this line will be curved. The intersection of this curve with the 1200:5 magnetizing characteristic yields the operating point:

V_M = 910 V
I_M = 15 A

Thus the current through the burden is

$$I_b = \sqrt{166.6^2 - 15.0^2} = 165.99A$$

The magnitude of the ratio correction factor is:

$$RCF = \frac{166.66}{165.99} = 1.0041$$

The phase error is:

$$\Delta\phi \ = \ \text{atan}(\ 15.0 \ / \ 165.99\ ) \ = \ 5.16^0$$

Note that the above analysis is approximate. Since the CT is saturated, the magnetization current is not sinusoidal. A more accurate analysis of the saturated CT performance can be achieved using numerical modeling techniques.

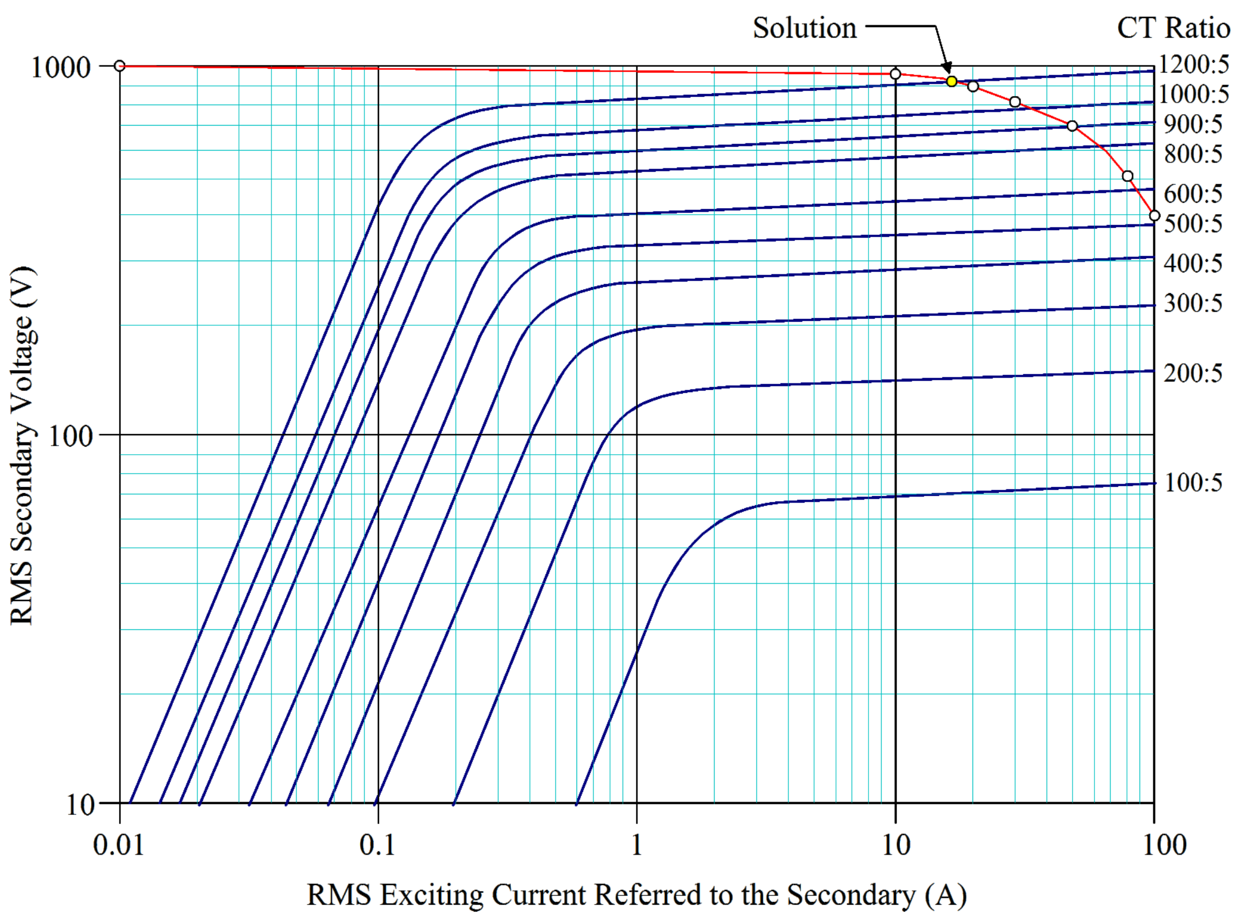


Figure E6.27: Magnetization Characteristics with Load Line Curve

6.2.10 Non-Conventional Instrument Transformers

Magnetic core instrument transformers such as current and voltage transformers have been widely used in power system applications, and thus are collectively called conventional instrument transformers. Recently, instrument transformers have been developed using alternative technologies that provide several advantages over conventional instrument transformers. These devices are referred to as ***N**on-**C**onventional **I**nstrument **T**ransformers* (NCIT's). A number of these devices are described in the following sections.

Air Core Current Transformer (Rogowski Coil)

A current sensor can be constructed by winding a closely wound coil over a toroidal *non-magnetic* form. The voltage developed across the coil terminals is proportional to the rate of change of the current passing through the toroidal form. The proportionality constant is a function of the placement of the structure around the current carrying conductor. This structure is known as a *Rogowski* coil.

In order to obtain an output proportional to the current passing through the toroidal form, the output voltage must be integrated. Figure 6.x illustrates a Rogowski coil current sensor. The output of the coil is connected to integrator to generate a voltage proportional to the measured current *i(t)*.

The main advantage of this device with respect to magnetic core CT's is that it is immune to core saturation effects. The disadvantage is that it requires an integrator and calibration. Analog integrator implementations, are prone to accuracy and stability problems. However, recent advances in digital electronics have resulted in high quality digital integrator implementations. Another disadvantage is that the Rogowski coil generates very low output voltage and thus they are susceptible to errors caused by electromagnetic interference (EMI). The need for calibration comes from the fact that the constant M illustrated in Figure 6.x depends on the relative placement of the structure with respect to the current carrying conductor. Due to these disadvantages, Rogowski coils have not been extensively used in relaying applications, except in specialized applications.

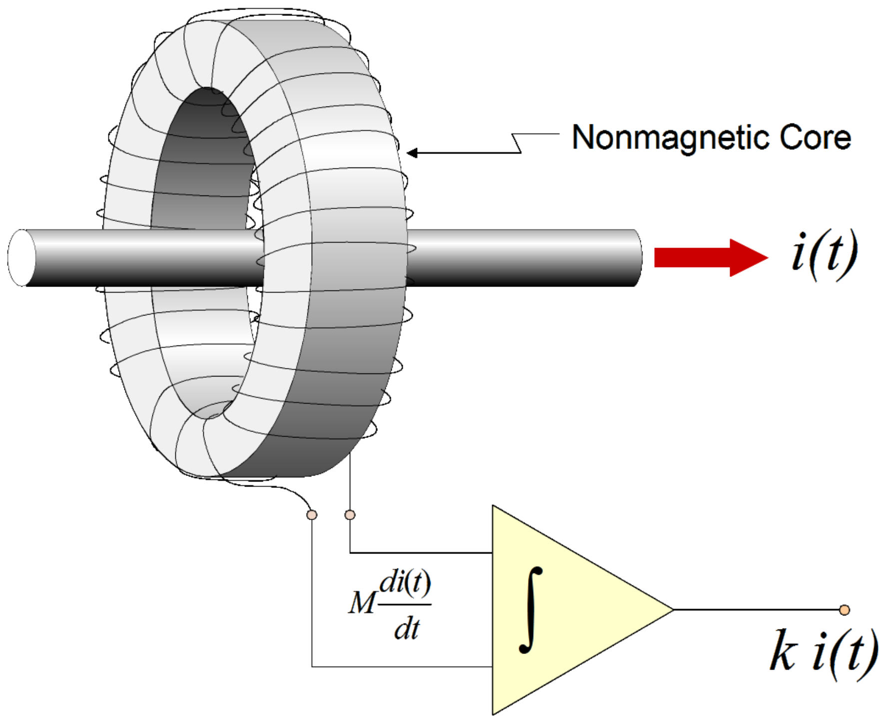


Figure 6.28: Rogowski Coil with Integrator Circuit

6.2.11 Magneto-Optical Current Transformers

Magneto-optical current sensors are based on the Faraday effect. The Faraday effect can be described as follows: Consider a polarized monochromatic light (LASER) beam propagating through a transparent optical medium. If a magnetic field is applied through the optical medium, the polarization angle of the light beam is altered. The angle change is proportional to the magnetic field intensity and the length of the light path through the medium.

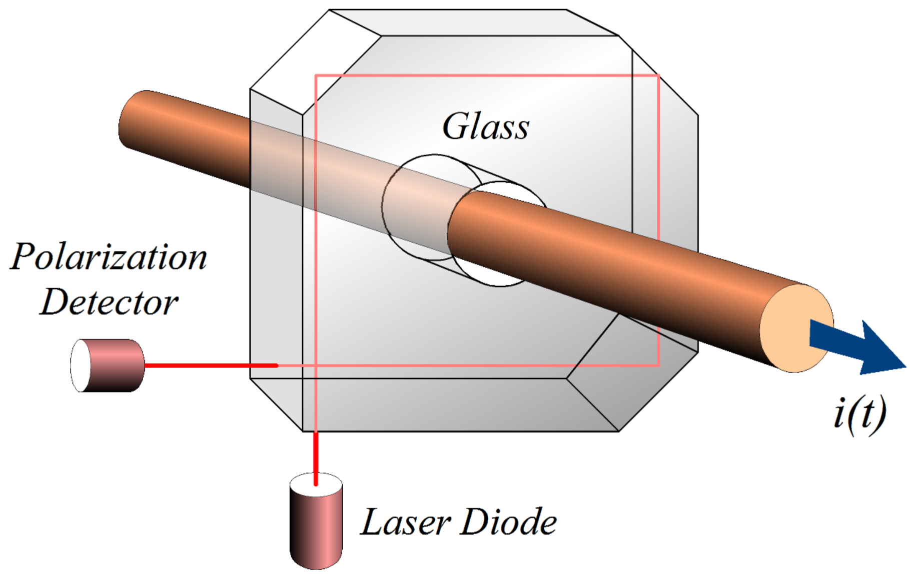


Figure 6.29: Magneto-Optical CT Configuration

Current sensors based on the Faraday effect use a light path that circles around a current carrying conductor as illustrated in Figure 6.x. The light beam is generated by a laser diode, propagates through an optical fiber or a glass block encircling the conductor. The polarization change is detected by a polarimetric system (polarization detector) which converts polarization angle variation to light intensity variation according to the function:

$$P(t) = \frac{P_0}{2}(1 + \sin(k\, i(t)))$$

where P_0 is the input light intensity
$P(t)$ is the output light intensity
k is a constant which depends on the optical material properties and light path geometry.

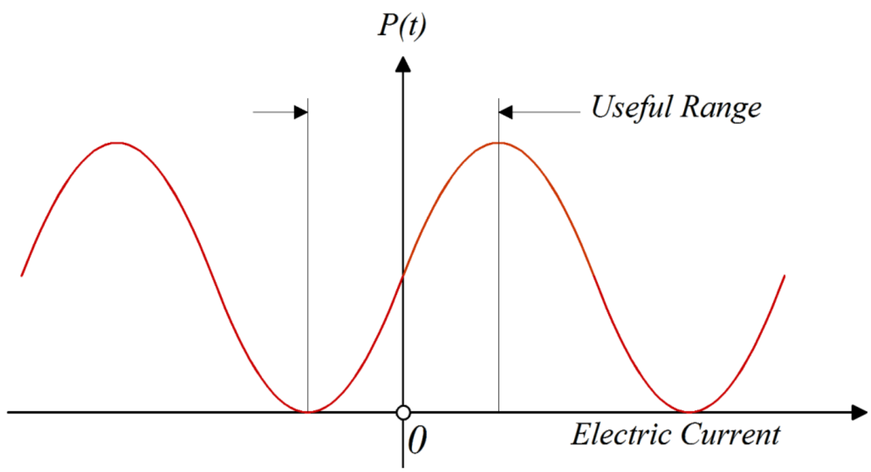


Figure 6.30 - Light Output Intensity versus Electric Current

The advantages of the magneto-optical current transformers are: (a) they provide high measurement accuracy, wide frequency range, and wide dynamic range, (b) the optical fiber implementation naturally provides galvanic isolation to very high voltages.

The disadvantages of the magneto-optical current transformers are: (a) they requires complex support circuitry and power supply, and (b) present implementations also have additional amplification and output circuits to convert the signals into standard relaying voltages and currents (67V/115V and 5A/1A).

6.2.12 Current Transformer Connections

Secondary windings of current transformers can be connected in delta or wye to provide the desired phase shift, or they can be connected as to provide the positive sequence current or the negative sequence current or the zero sequence current. These connections were very important in the era of electromechanical relays, since some of the protection schemes need these currents (sequence components) in the protection logic. Examples of these connections have been discussed in Chapter 2.

Numerical relays accept directly the output of each individual CT and they form the appropriate quantity numerically as required by the particular protection function.

6.3 Voltage Instrument Transformers

High voltage instrument transformer can be classified as conventional instrument transformers, which include:

- Potential Transformers (PT)
- Capacitive Coupled Voltage Transformers (CCVT)

and non-conventional instrument transformers, such as:

- Electro-Optical Voltage Transformer (EOVT)
- Capacitive Voltage Dividers (CVD)

For all the above technologies the umbrella term Voltage Transformers (VT) is used. In high voltage power system applications (outdoor relaying and metering units) PT's and CCVT's are commonly used. These devices are made in a variety of forms and sizes. Figure 6.x illustrates several PT's of different voltage levels for power system protection and metering applications.

In high and extra high voltage power system applications (outdoor relaying and metering units) CCVT's are quite commonly used. These devices contain a capacitive voltage divider and a secondary interface circuit consisting of a magnetic transformer, a neutralizing reactor, and a Ferro-resonance suppression circuit. Figure 6.x illustrates the structure of a typical CCVT.

Electro-optical voltage transformers and capacitive voltage dividers have become available in the last couple of decades and offers many advantages. These technologies are discussed in subsequent sections. Figure 6.x illustrates two commercially available EOVTs.

(a) (b) (c)

Figure 6.31: Metering Potential Transformers - Various Voltage Levels
(a) 2.4 kV, (b) 69 kV, (c) 345 kV

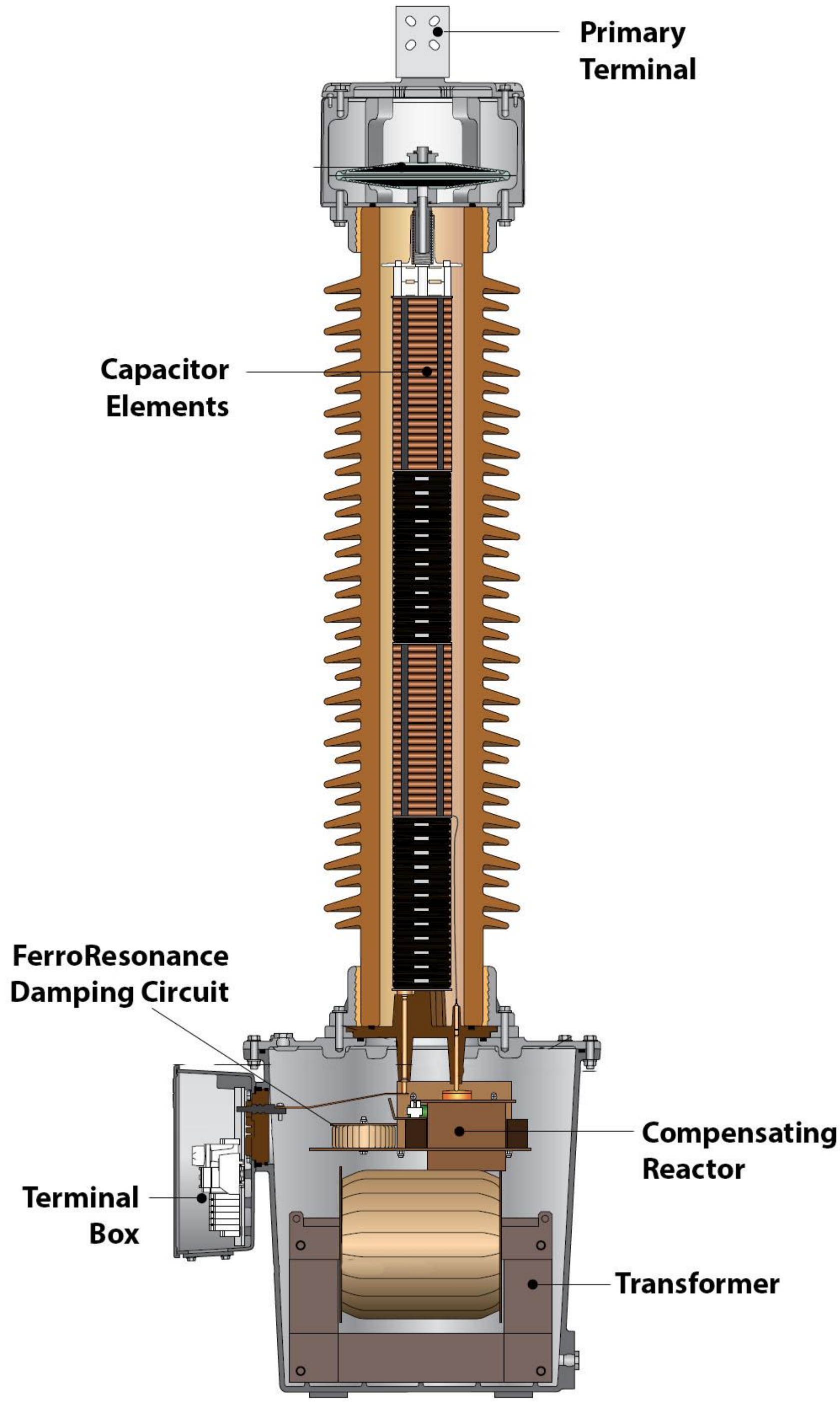
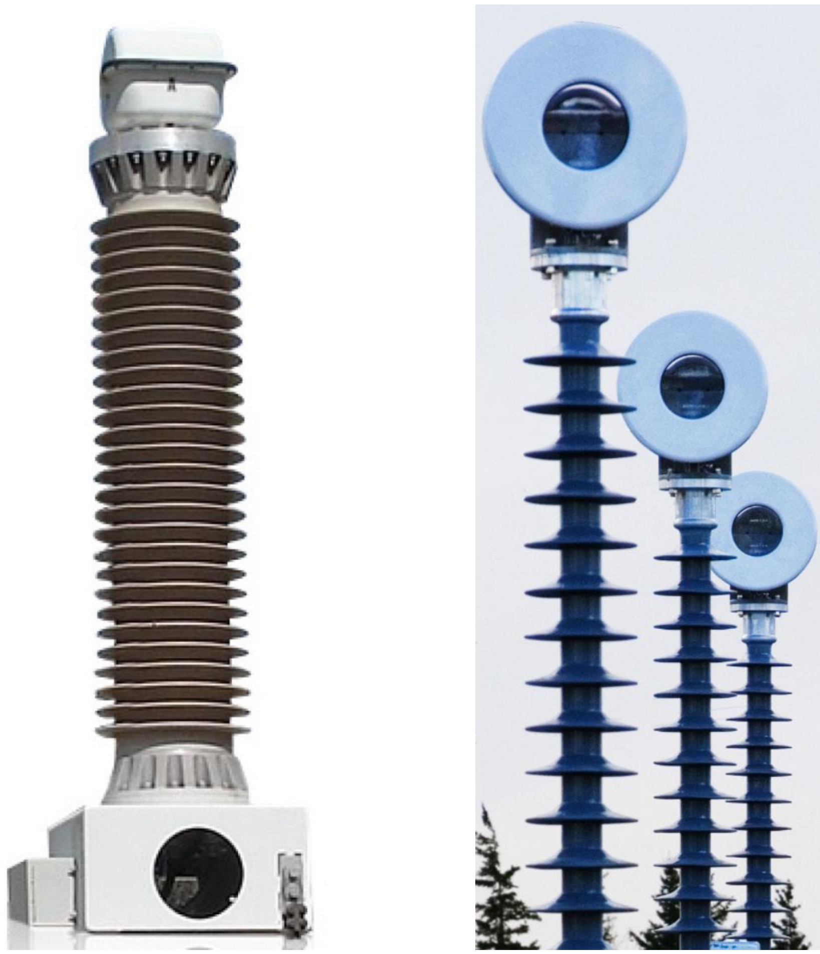


Figure 6.32: Capacitive Coupled Voltage Transformer (ABB)



(a) **(b)**

Figure 6.33: Electro/Magneto-Optical Instrument Transformers
(a) ABB Combined EOVT/MOCT
(a) Alstom/NxtPhase Combined EOVT/MOCTs

Capacitive voltage dividers (CVD) provide accurate wide bandwidth measurements with relatively flat frequency response. In the past, these technologies are limited to laboratory grade instrumentation. Recently, due to advances in electronics, these devices have become practical for power system metering and relaying applications as well.

Resistive voltage dividers (RVD) also provide accurate wide bandwidth measurements with relatively flat frequency response. However, these technologies are not suited for high voltage applications and they are limited to medium voltages and below, due to high power dissipation issues.

The frequency response of voltage instrument transformers varies widely with technology and voltage level. The flattest frequency response is achieved by the electro-optical and capacitive divider technologies.

6.3.1 Voltage Transformers Standardization

The nominal secondary voltage of VT's for power system applications has been standardized to one of the following values:

- 69.3 Volts (for wye connected secondary windings)
- 115 Volts (for delta connected secondary windings)

The error classes have the following standard values:

- 0.3%
- 0.6%
- 1.2%

6.3.2 Voltage Transformer Polarity Markings

Polarity is indicated by a dot placed near one terminal of each winding. The voltages measured from the marked to the unmarked terminals of each winding are in phase.

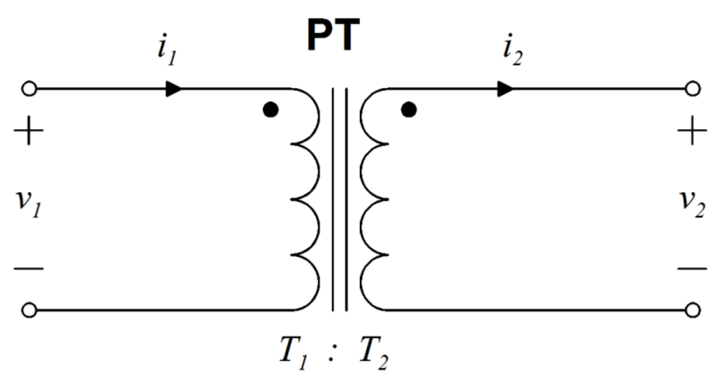


Figure 6.34: Transformer Dot Notation

6.3.3 Potential Transformers (PT's)

Potential transformers consist of a magnetic core with a high voltage (primary) winding, and a low voltage (secondary) winding. Potential transformers are typically enclosed in oil-filled container. The oil provides cooling and insulation.

6.3.4 Analysis of PT Steady State Performance

The PT performance near the power frequency can by evaluated by considering the equivalent circuit illustrated in Figure 6.x. Note that for higher frequencies the inter-winding parasitic capacitance may affect the PT response and thus should be included in the analysis.

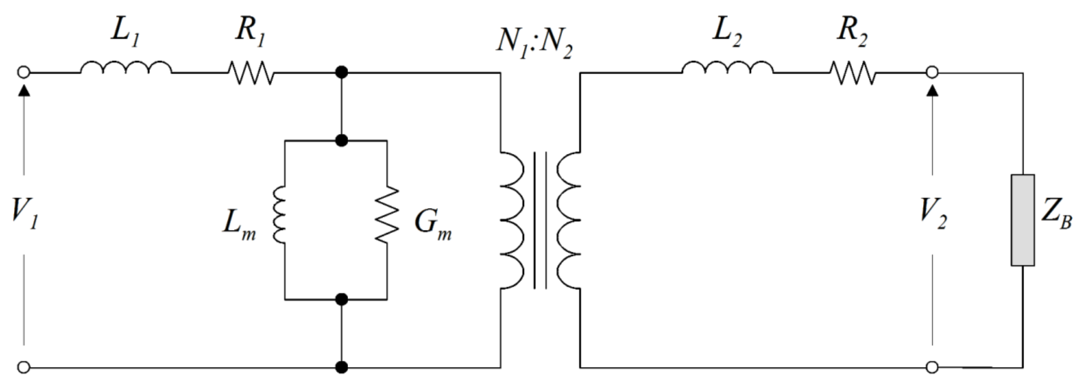


Figure 6.35: Potential Transformer Equivalent Circuit

The PT nominal transformation ratio, t_n, is given by the turns ratio:

$$t_n = \frac{N_2}{N_1}$$

where N_1 is the number of primary winding turns, and N_2 is the number of secondary winding turns. The actual ratio of output over input voltage varies slightly from the nominal ratio due to the interactions of the leakage, magnetization and burden impedances. Given the values of these impedances the ratio error and a correction factor can be computed using circuit analysis techniques. The methodology is illustrated next by an example.

6.3.5 PT Frequency Response

The frequency response of PT’s varies widely with the size, voltage level and burden of the PT. Figure 6.x illustrates the frequency response of a typical 200kV/115V potential transformer, with an 80-ohm resistive burden connected across the secondary winding.

The frequency response irregularities are due to resonance between the leakage and magnetizing inductances and the turn to turn and winding to winding parasitic capacitances. Note that the frequency response resonance peak levels are sensitive to the burden resistance. Specifically, when the burden is removed the magnitude peaks increase by 20%. See reference [???] for additional information of potential transformer frequency response.

PT's can be used for waveform data acquisition provided that their frequency response over the frequency range of interest is known so that appropriate corrections can be made. Generally the resonance occurs well above power frequencies thus for most relaying applications, frequency response correction is not necessary.

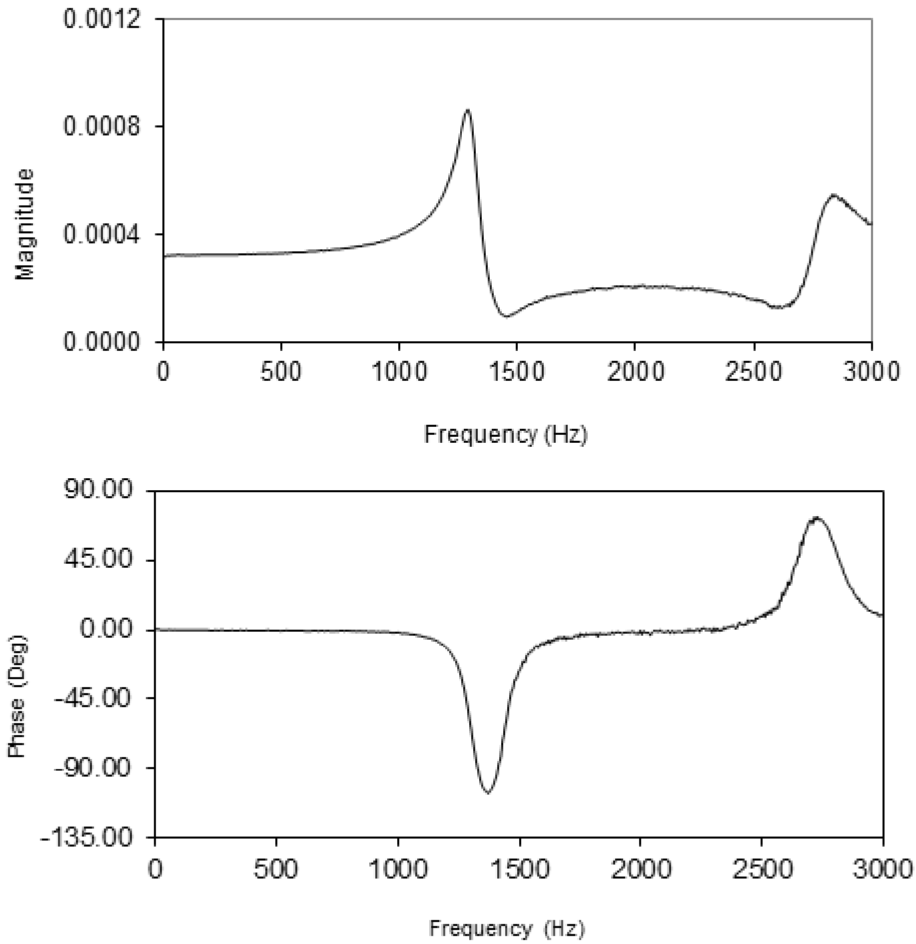


Figure 6.36: 200kV/115V Potential Transformer Frequency Response

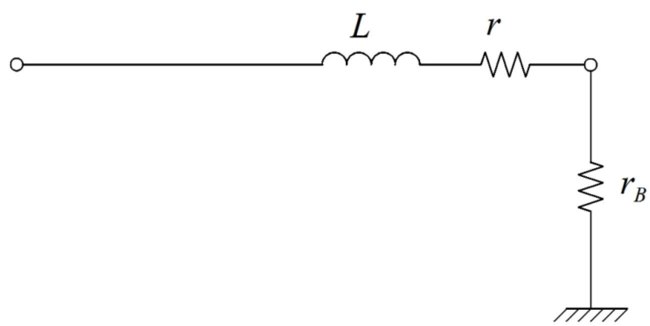
Example E6.x: Consider a 200kV:115V potential transformer. The parameters of the instrument transformer are:

Power Rating: 300 VA
Leakage Reactance: 0.02 pu
Burden Impedance: 100 Ohms
Inter-winding capacitance: 5 pF

(a) Neglect inter-winding capacitance. Compute the transfer function of the transformer in the frequency range 10 Hz to 10 kHz.

(b) Compute the transfer function of the transformer in the frequency range 10 Hz to 10 kHz, taking into account the inter-winding capacitance

Solution: (a) The transformer equivalent circuit, neglecting the inter-winding capacitance, and referring all components to the secondary winding is:



The impedance base on the secondary side is:

$$Z_{BASE} = 115V^2/300VA = 44.1 \text{ Ohms}$$

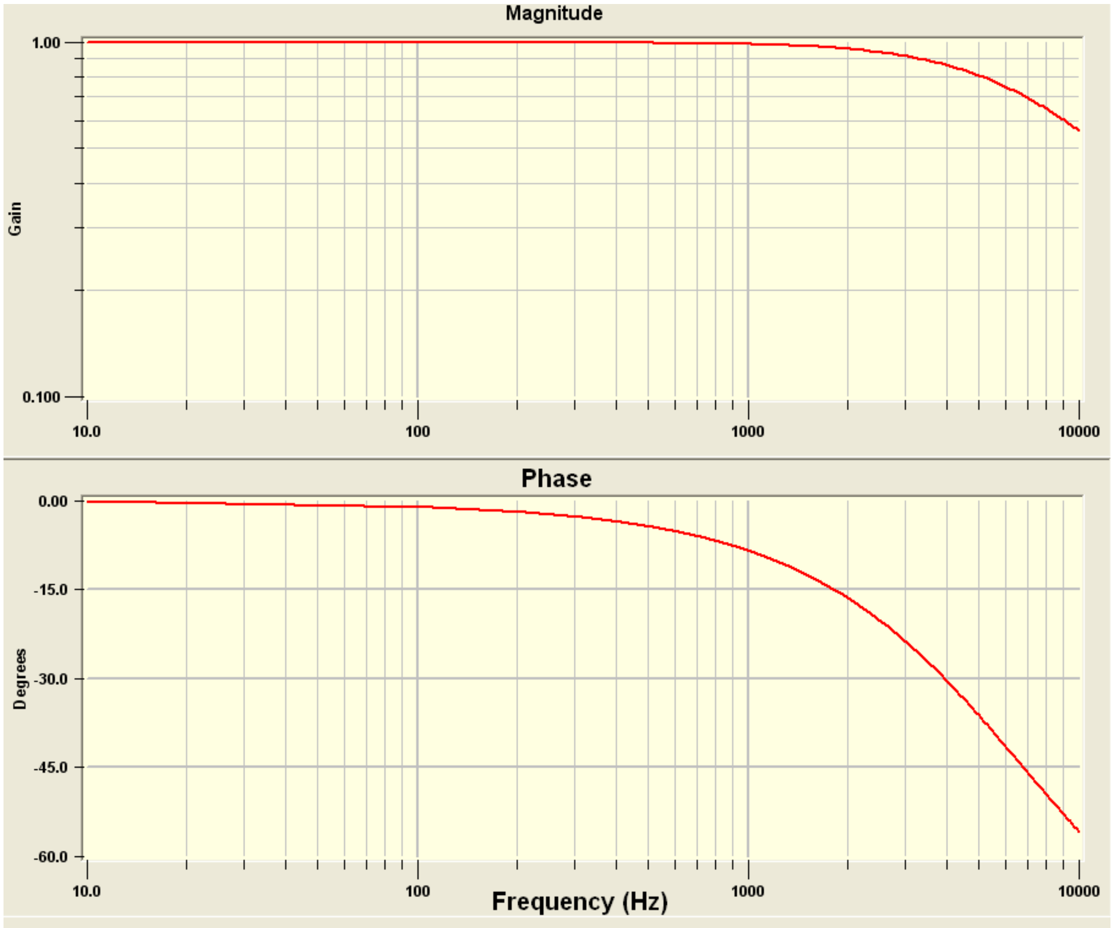
Thus the leakage inductance is:

$$L = 0.02 \text{ x } 44.1 / 377 = 2.34 \text{ mH}$$

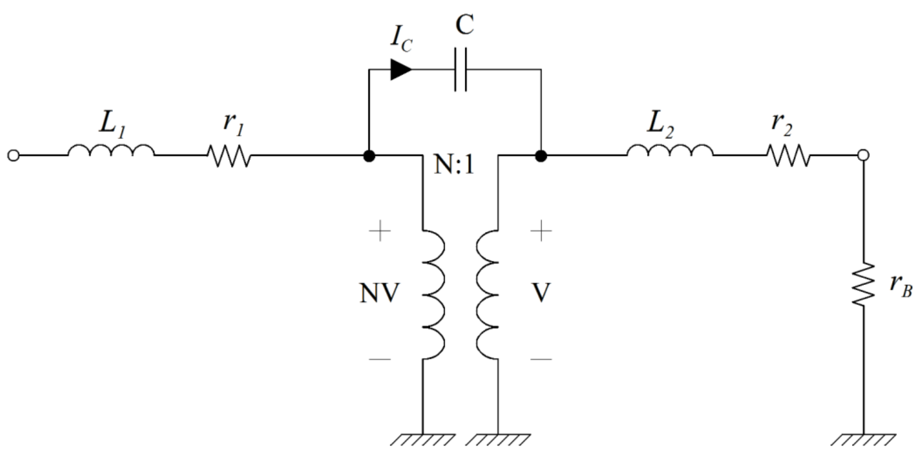
The above circuit is a first order low pass response with cutoff frequency at:

$$f_0 = \frac{R}{2\pi L} = \frac{100}{2\pi 0.00234} \text{Hz} = 6.80 \text{kHz}$$

The frequency response of the above circuit is given below:



(b) The transformer equivalent circuit, including the inter-winding capacitance is:

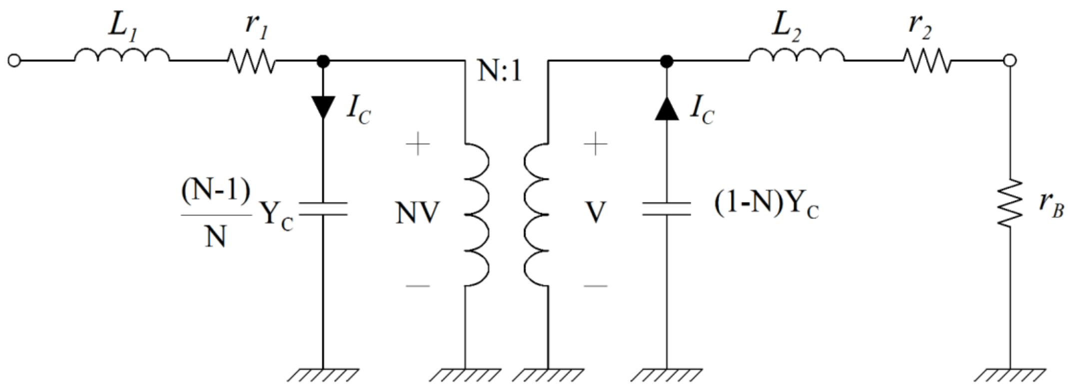


The current through the parasitic capacitance can be expressed as follows:

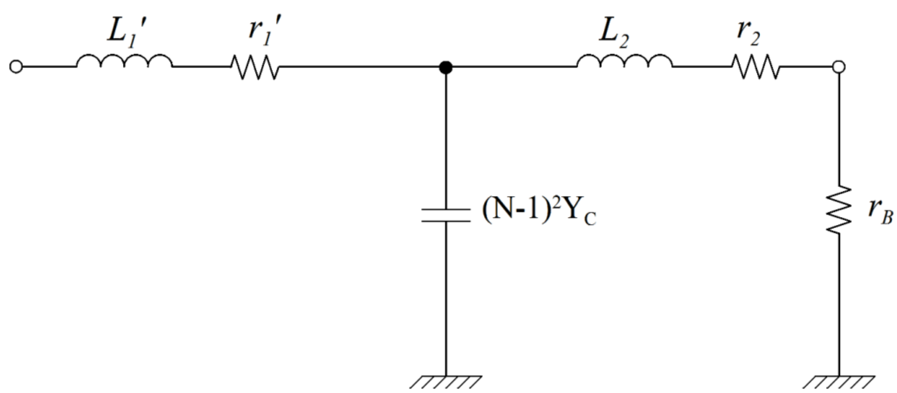
$$I_C = (NV - V)Y_C$$

The capacitor across the transformer can be replaced by two equivalent shunt capacitors. Setting the capacitor currents equal yields the following equivalent shunt capacitance values:

$$C_1 = \frac{(N-1)}{N} Y_C$$
$$C_2 = (1-N) Y_C$$



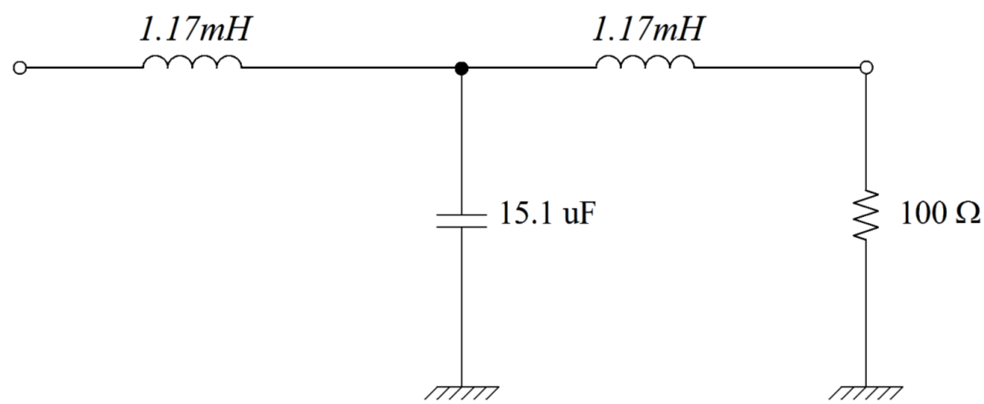
Referring all components to the secondary (low side), the two capacitors are placed in parallel, and the following equivalent circuit is obtained:



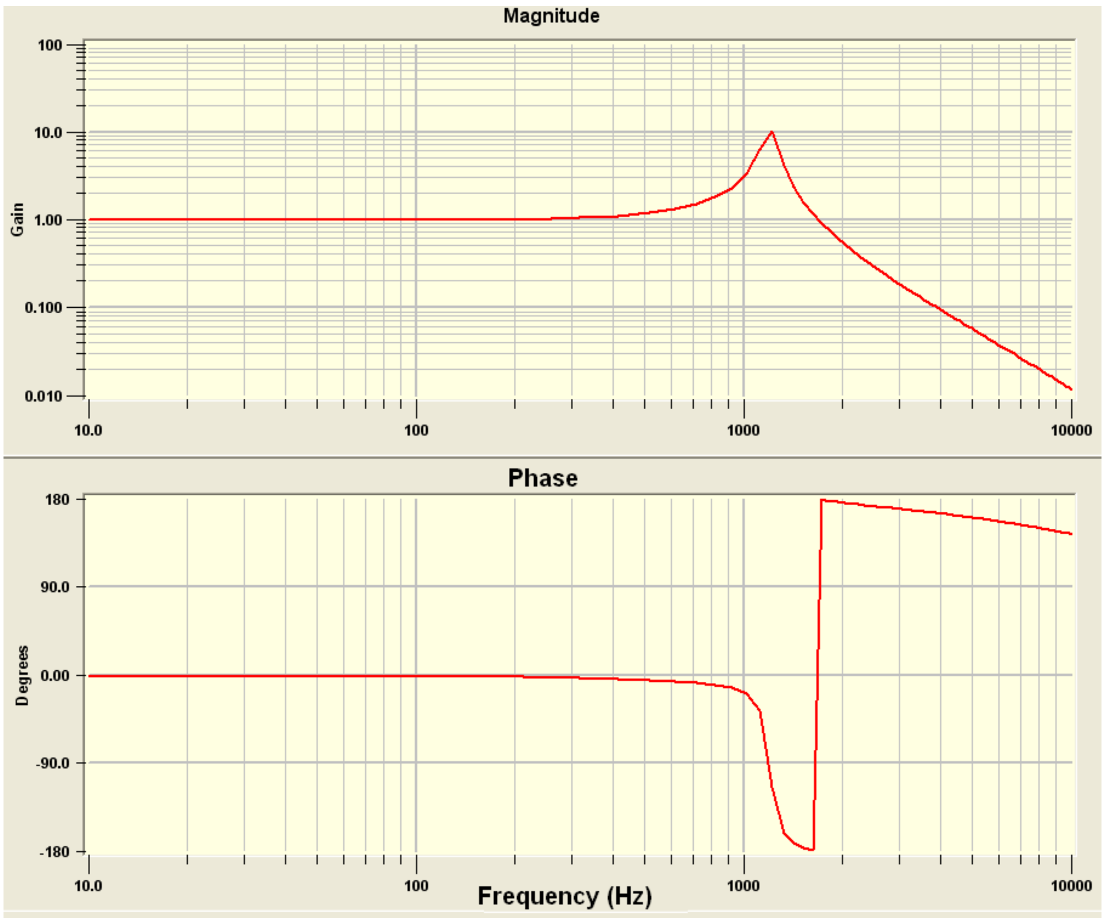
The capacitor value is:

$$C_s = (N-1)^2 C = 15.1 \mu\text{F}$$

Assuming equal leakage inductance split in the per unit sense, and neglecting winding resistance, yields the equivalent circuit:

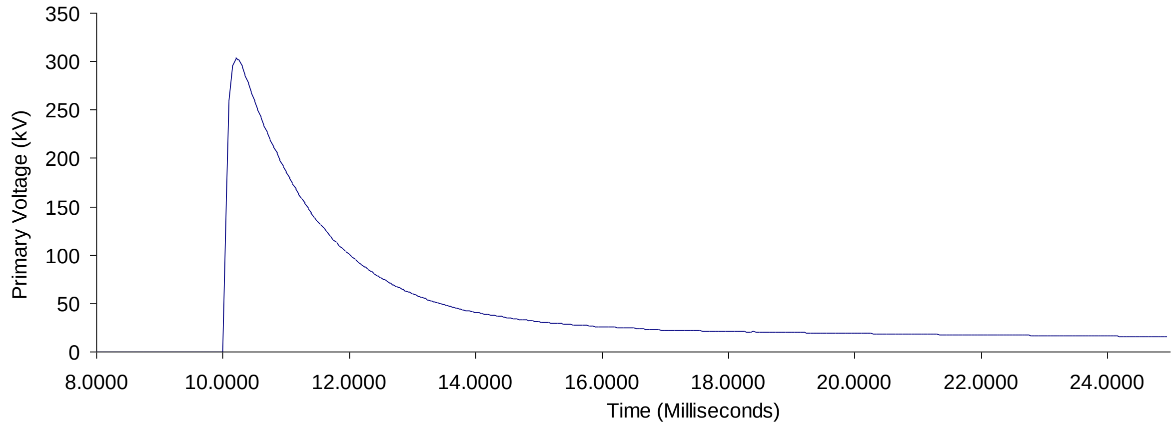


The frequency response of the above circuit is given below:

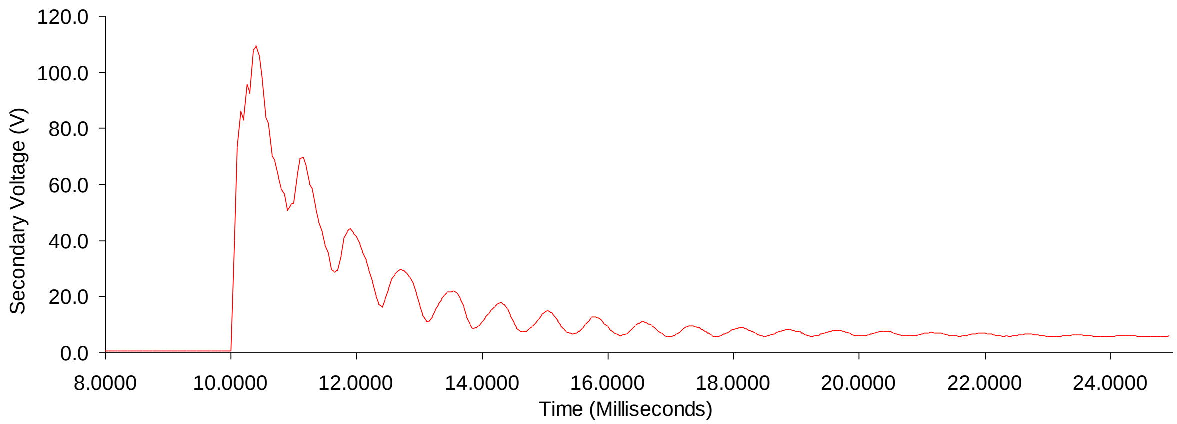


6.3.6 PT Transient Response

The transient response of potential transformers can be determined from frequency response tests, using Fourier transformation techniques, or by direct time domain test. Figure 6.x illustrates the measurement data from a direct time domain test, performed on the same PT for which the frequency response was given in Figure 6.x. A double exponential waveform was applied across the primary winding, while both primary and secondary voltage waveforms are recorded. Note the 1,250 Hz oscillation on the secondary voltage due to resonance.



(a)



(b)

Figure 6.37: Transient Response of a 200kV / 65V PT
Open Circuit Secondary
(a) Primary Voltage Waveform
(b) Secondary Voltage Waveform

The transient response of PTs can be also determined by computation. The approach will be demonstrated with an example.

Example E6.x: Consider a 200kV:115V potential transformer. The parameters of the instrument transformer are:

Power Rating: 300 VA
Leakage Reactance: 0.02 pu
Burden Impedance: 100 Ohms
Inter-winding capacitance: 5 pF

Assume that the transformer operates under normal operating conditions with the primary voltage at 198 kV. Due to a fault the primary voltage is reduced to 56 kV for five cycles. Compute the transient voltage at the secondary of the transformer (assume that the fault occurs at the voltage peak value).

Solution: The transformer equivalent circuit derived in Example E6.x was implemented within a transient analysis program (WInIGS-T) along with a 113.8 V (RMS) voltage source with 0.1 ohm Thevenin resistance. A switch is set to close at 0.1 seconds adding a 0.0394 ohm resistor across the source causing the source output voltage to drop to 32.18 Volts RMS for 5 cycles.

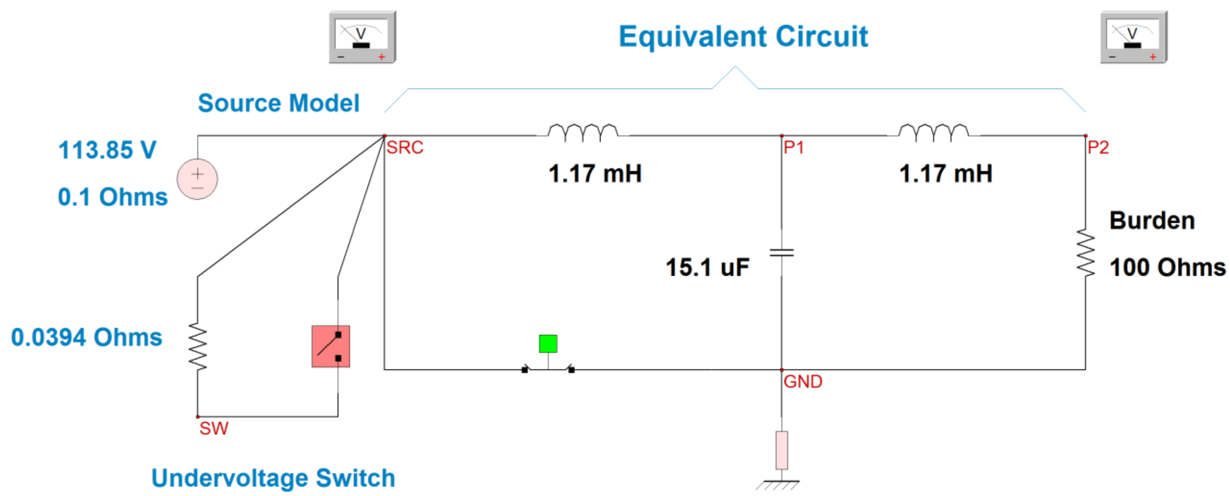
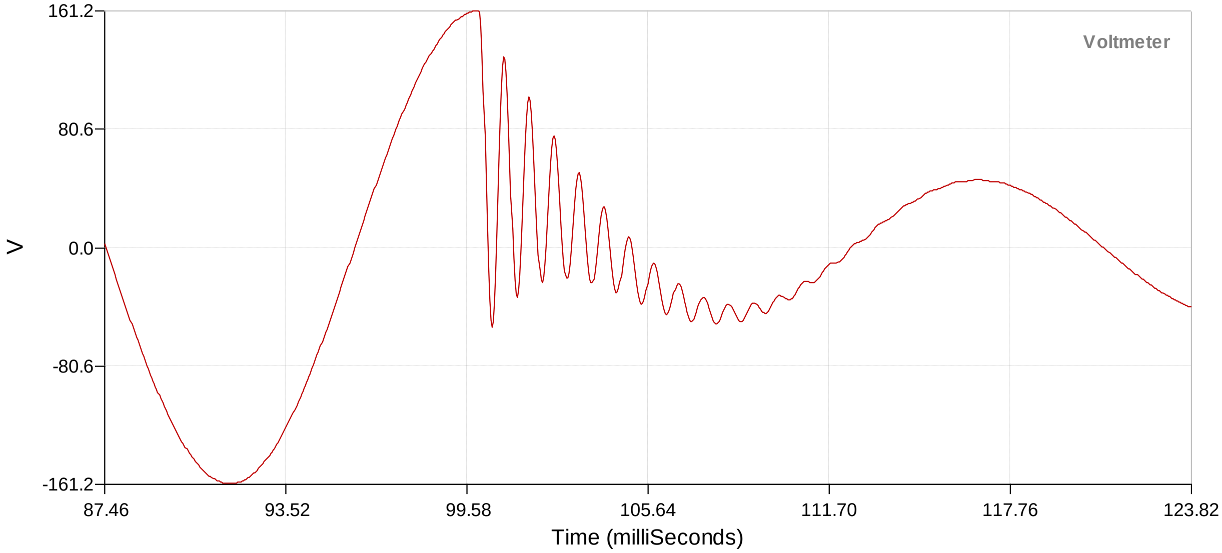
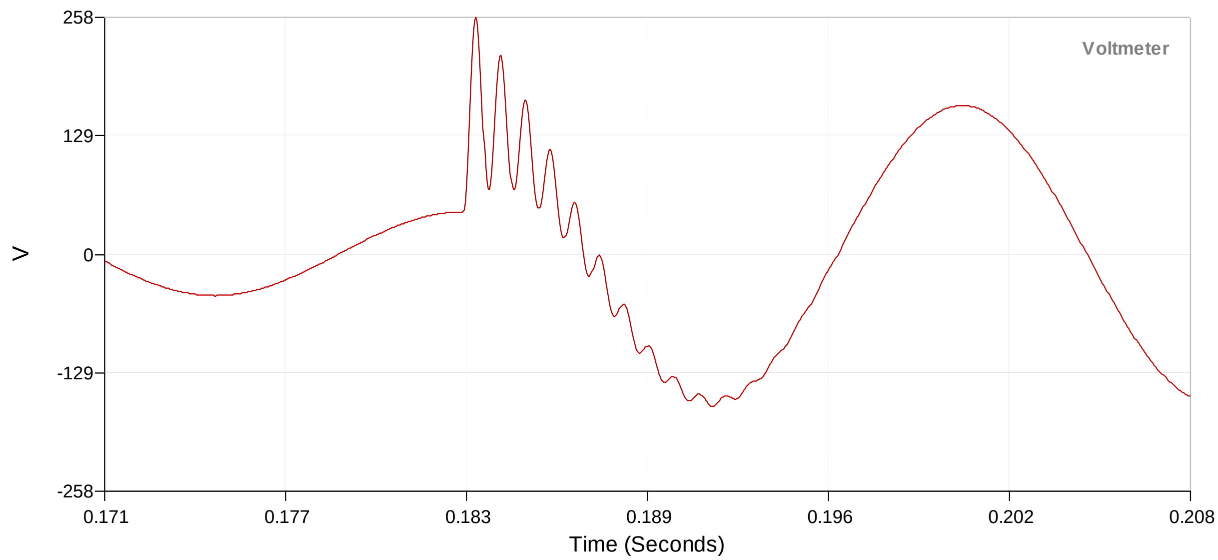


Figure E6.x: Single Line Diagram of Transformer Equivalent Circuit

The computed voltage waveform across the burden resistor is plotted in Figure E6.x. Note that at the fault initiation and clearing the transformer secondary voltage contains a decaying oscillation of about 1.2 kHz. This oscillation is due to the resonance between the transformer leakage inductance and its parasitic capacitance.



(a)



(b)

Figure E6.x: Transformer secondary voltage
(a) Voltage at fault initiation (0.1 msec)
(b) Voltage at fault clearing (0.1833 msec)

6.3.7 Voltage Transformer Connections

Potential transformers can be connected in wye, delta, open delta, or broken delta configurations as illustrated in figure 6.x. The open delta configuration has the advantage of requiring only two single phase transformers. However, due to the circuit asymmetry, in general it introduces larger measurement errors than equivalent wye or delta configurations. The broken delta configuration is used to measure the zero-sequence voltage component.

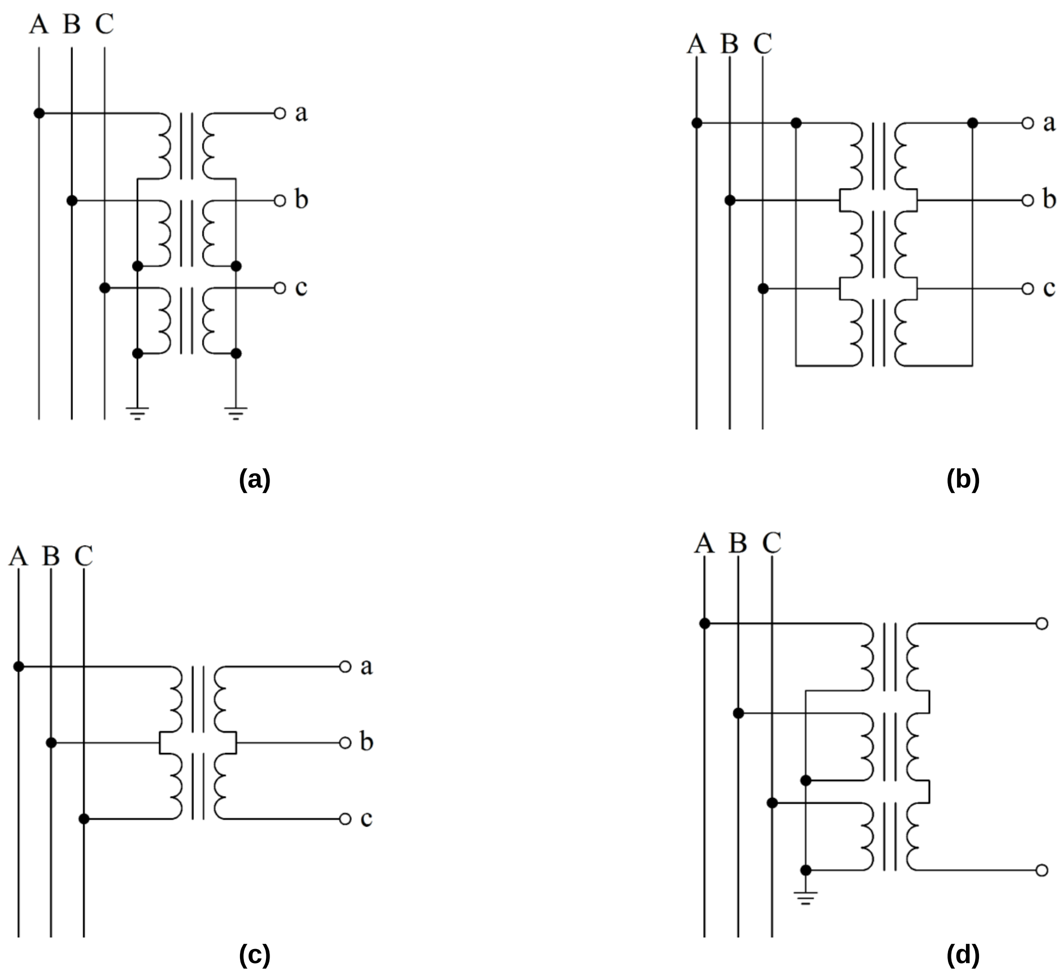


Figure 6.38: Potential Transformer Connections
(a) Wye-Wye, (b) Delta-Delta, (c) Open Delta, (d) Broken Delta

6.3.8 Capacitively Coupled Voltage Transformers (CCVTs)

Due to economic factors, in very high voltage applications (345 kV and above) CCVT's are commonly used instead of magnetic voltage transformers. These devices consist of a capacitive divider and a magnetic core transformer at the output. CCVT's are by design optimized for steady state power frequency measurements (60 or 50 Hz). For relaying applications the steady state as well as the transient response is of major importance. In this section we present the typical circuit configurations of CCVT. We also present parametric error analyses for steady state and transient response.

The basic CCVT configuration is illustrated in Figure 6.x. The capacitive divider is formed by capacitors C_1 and C_2. In typical high voltage CCVT's (345-500kV) the capacitor values are selected so that the voltage at the tap point A is about 4 to 10 kV. The transformer scales this voltage to standard instrumentation voltage level, i.e. 69V or 120V. The output of the transformer is connected to a burden via instrumentation cable (usually a coaxial cable). The burden represents the relay input impedance, which is typically resistive. Since the interaction of the capacitive divider and the resistive burden introduces considerable phase shift, a series inductor L is added to compensate the divider output capacitance.

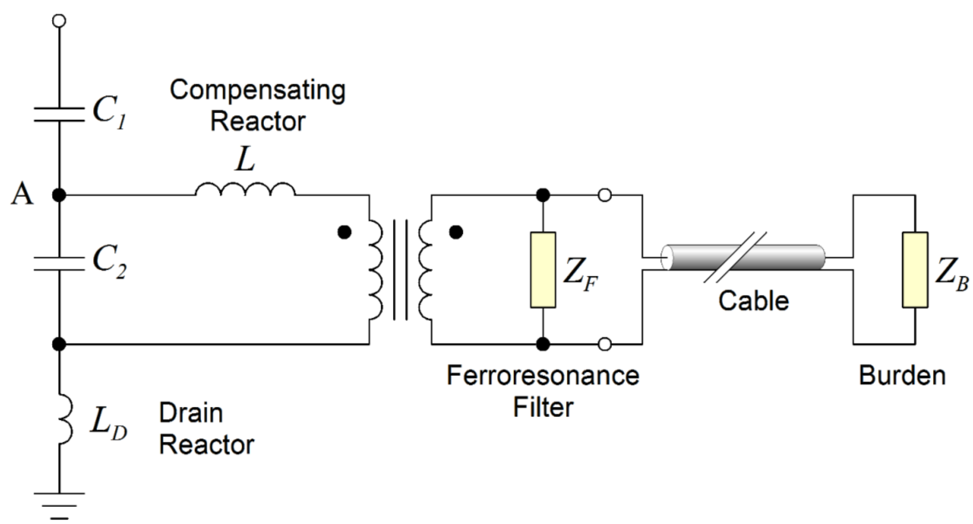


Figure 6.39: CCVT Physical Circuit

The inductor L is typically selected to provide full compensation at the power frequency. Specifically, the inductor L is selected by setting the sum of the equivalent capacitive reactance and the inductive reactance to zero at the power frequency. Note that the Thevenin equivalent capacitive reactance at point A is the sum of the upper and lower leg capacitances, $C_1 + C_2$, thus:

$$j\omega L + \frac{1}{j\omega(C_1 + C_2)} = 0$$

or equivalently:

$$L = \frac{1}{\omega^2 (C_1 + C_2)}$$

Another application of the CCVT is its use for power line carrier applications. For this reason a drain reactor, L_D, is added to the circuit. The drain reactor serves the purpose of power line carrier filtering and may be optionally shorted by a manual switch. For the purpose of low frequency analysis (0-1kHz) this reactor has negligible effect and is ignored.

In standard CCVT designs typical values of capacitor dividers are selected so that the sum of the capacitances $C_1 + C_2$ are in the order of 100 nF. The corresponding compensating reactor inductance for 60 Hz power frequency is in the order of 70 Henries. A reactor of such large inductance must have a magnetic core, and will also have a substantial resistance. As a result, the compensating reactor is also subject to saturation. Furthermore, its resistance makes the CCVT transformation ratio dependent on the burden resistance. Transient analysis has also shown that the stored energy in this large reactance causes poor transient response. Recently, CCVT's with larger capacitance values and smaller compensating reactors have become available. These devices exhibit improved transient response, as well as less sensitivity to burden resistance. Some manufacturers classify CCVT's as (a) Normal Capacitance, (b) High Capacitance, (c) Extra High Capacitance. It is important to note that the threshold values separating these classes are vague. CCVT's of total capacitance value ($C_1 + C_2$) as high as 400 nF are presently commercially available.

The interaction of the transformer saturation characteristics with the divider capacitance makes this circuit subject to ***ferroresonance***. Specifically, during transients a resonance may occur at the frequency determined by the transformer magnetizing reactance and the circuit equivalent capacitance. This results in overvoltages developing across the transformer which drive the core into saturation. This nonlinear high amplitude oscillation causes severe measurement errors and can damage the circuit components. For this reason, CCVT's include a ferroresonance suppression circuit, usually located across the transformer secondary winding (the impedance Z_F in Figure 6.x). Several ferroresonance suppression circuit topologies are presently in use. These circuits are considered proprietary by some manufacturers, and thus the circuit details are not readily available. However, two generic circuit models capture the basic behavior of these filters: The "active" suppression circuit illustrated in Figure 6.x, and the "passive" suppression circuit illustrated in Figure 6.x.

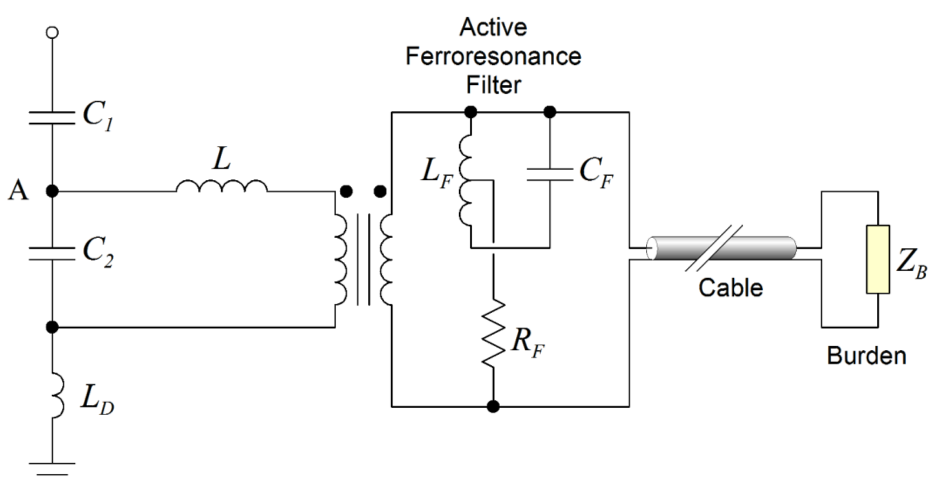


Figure 6.40: Equivalent Circuit of a CCVT with Active Ferroresonance Suppression Circuit

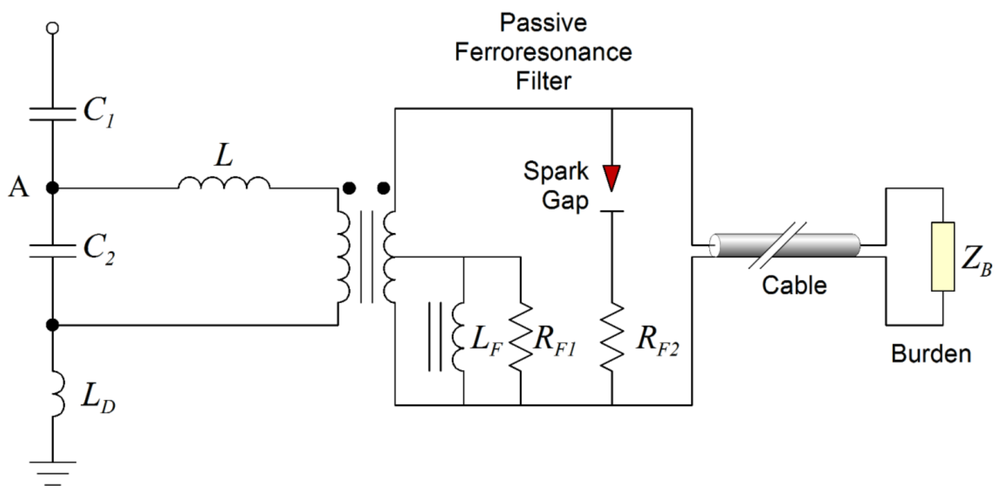


Figure 6.41: Equivalent Circuit of a CCVT with Passive Ferroresonance Suppression Circuit

The active suppression circuit inductor L_F and capacitor C_F are tuned to the power frequency. Thus during normal steady state operation, the impedance of the $L_F//C_F$ branch is very large and the suppression circuit draws negligible current. During transients the impedance of $L_F//C_F$ branch is lower and thus the resonating energy is dissipated through the filter resistor RF.

The passive suppression circuit (see Figure 6.x) consists of a saturable core reactor L_F and a damping resistor R_{F1} connected at the center tap of the transformer secondary, plus a spark gap in series with a second damping resistor R_{F2}.

During steady state 60 Hz operation both of these filter circuits have negligible effect on the CCVT response. However during transients they generally prolong the CCVT transient response. It has been shown that CCVT's with passive ferroresonance suppression circuits have better transient response characteristics (error decays to negligible levels faster). They are also more expensive than active circuits.

CCVT Steady State Response: By appropriate selection of the circuit components a CCVT can be designed to generate an output voltage with any desirable transformation ratio and most importantly with zero phase shift between input and output voltage waveforms. In this section we examine the possible deviations from this ideal behavior due to various causes by means of a parametric analysis, namely:

- Power Frequency Drift
- Circuit Component Parameter Drift
- Burden Impedance

The parametric analysis was performed using the CCVT equivalent circuit model illustrated in Figure 6.x. The model parameters are given in Table 6.x:

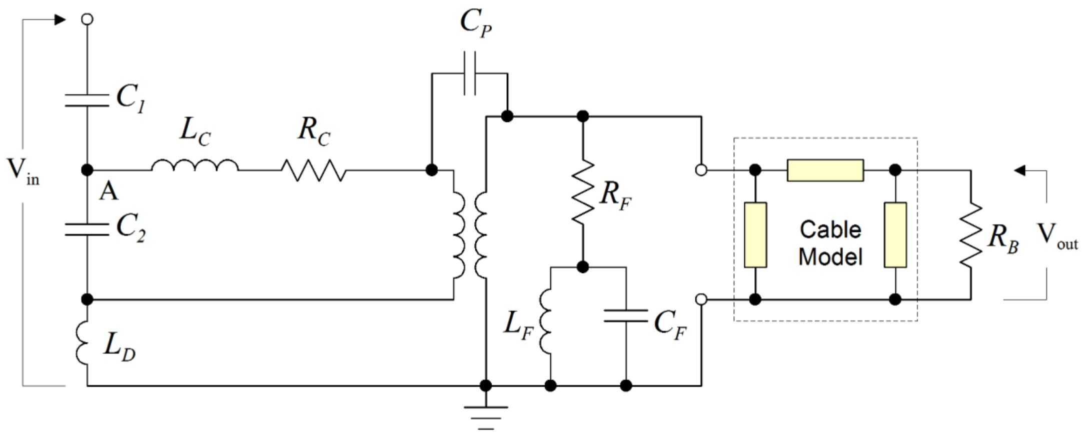


Figure 6.42: CCVT Equivalent Circuit

Table 6.x: CCVT Equivalent Circuit Parameters

Parameter Description	Schematic Reference	Value
CCVT Capacitance Class		Normal
Input Voltage		288 kV
Output Voltage		120 V
Upper Capacitor Size	C1	1.407 nF
Lower Capacitor Size	C2	99.9 nF
Drain Inductor	L_D	2.65 mH
Compensating Reactor Inductance	L_C	68.74 H
Compensating Reactor Resistance	R_C	3000 Ohms
Burden Resistance	R_B	200 Ohms
Ferroresonance Suppression Damping Resistor	R_F	70 Ohms
Ferroresonance Suppression Circuit Inductor	L_F	0.398 H
Ferroresonance Suppression Circuit Capacitor	C_F	17.7 uF
Cable Type		RG-8
Cable Length		100 Feet
Transformer Power Rating		300 VA
Transformer Voltage Rating		4kV/120V
Leakage Reactance		3%
Parasitic Capacitance	C_P	500 pF

Figure 6.x shows the results of a frequency scan. Note that over the frequency range of 0 to 500 Hz the response varies substantially both in magnitude and phase. Near 60 Hz (55 to 65 Hz) the response magnitude is practically constant but the phase varies at the rate of 0.25 degrees per Hz.

Table 6.x shows the results of a parametric analysis with respect to Burden resistance and instrumentation cable length. Note that the system is tuned for zero phase error for a short instrumentation cable and with a 200 Ohm Burden.

Table 6.x shows the results of a parametric analysis with respect to CCVT component parameter inaccuracies. Specifically the varied parameters were the compensating reactor inductance and the capacitive divider capacitance.

Table 6.x: Phase Error (in Degrees) vs Burden Resistance and Cable Length

	Cable Length (feet)		
Burden Resistance	**10'**	**1000'**	**2000'**
50 Ohms	0.077	-0.155	-0.365
100 Ohms	0.026	-0.096	-0.213
200 Ohms	0.000	-0.063	-0.127
400 Ohms	-0.013	-0.047	-0.080
1000 Ohms	-0.022	-0.036	-0.052

Table 6.x: Phase Error (in Degrees) Versus Capacitance and Inductance

	Inductance Error (%)		
Capacitance Error (%)	**0%**	**1%**	**5%**
0%	0.000	-0.066	-0.331
-1%	-0.066	-0.132	-0.397
-5%	-0.330	-0.396	-0.661

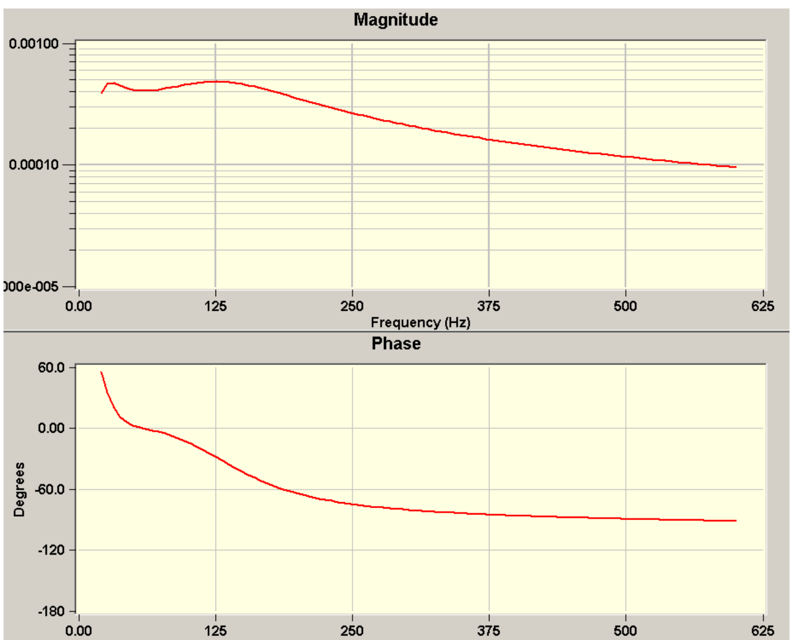


Figure 6.43: CCVT Computed Frequency Response over 10-600 Hz

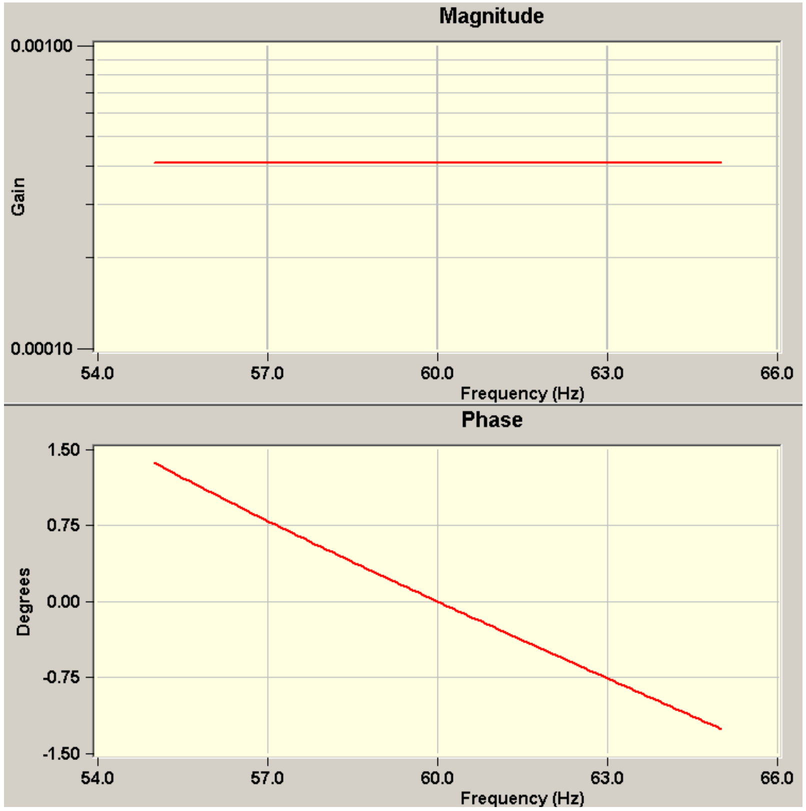


Figure 6.44: CCVT Computed Frequency Response over 55-65 Hz

CCVT Transient Response: The transient response of CCVTs is an important consideration for relaying applications. A prolonged oscillatory transient response can delay correct identification of the power system voltages and can delay relay protective action or worst can cause incorrect relaying operation. Early CCVT's suffered from poor transient response, typically lasting up to two power frequency cycles. The main reason for this is the resonance between the divider capacitance and the compensating reactor inductance. The main factors affecting the CCVT transient response duration and magnitude of the resulting measurement errors are:

- Divider Capacitance Value
- Intermediate Voltage Level
- Burden Resistance
- Ferroresonance Circuit Type (Active versus Passive)
- Input Waveform Characteristics

In general, the higher capacitance and higher intermediate voltage CCVT's have better transient response. The capacitance effect is illustrated in Figure 6.x, which shows typical transient response for two CCVT classes (A) High capacitance and (B) Extra High Capacitance. The input waveform is sinusoidal up to the zero crossing at 0.012 seconds, and remains at zero beyond this time. Both transient response outputs last for about 0.04 seconds, however the transient amplitude of the extra high capacitance CCVT is substantially lower [1], [2].

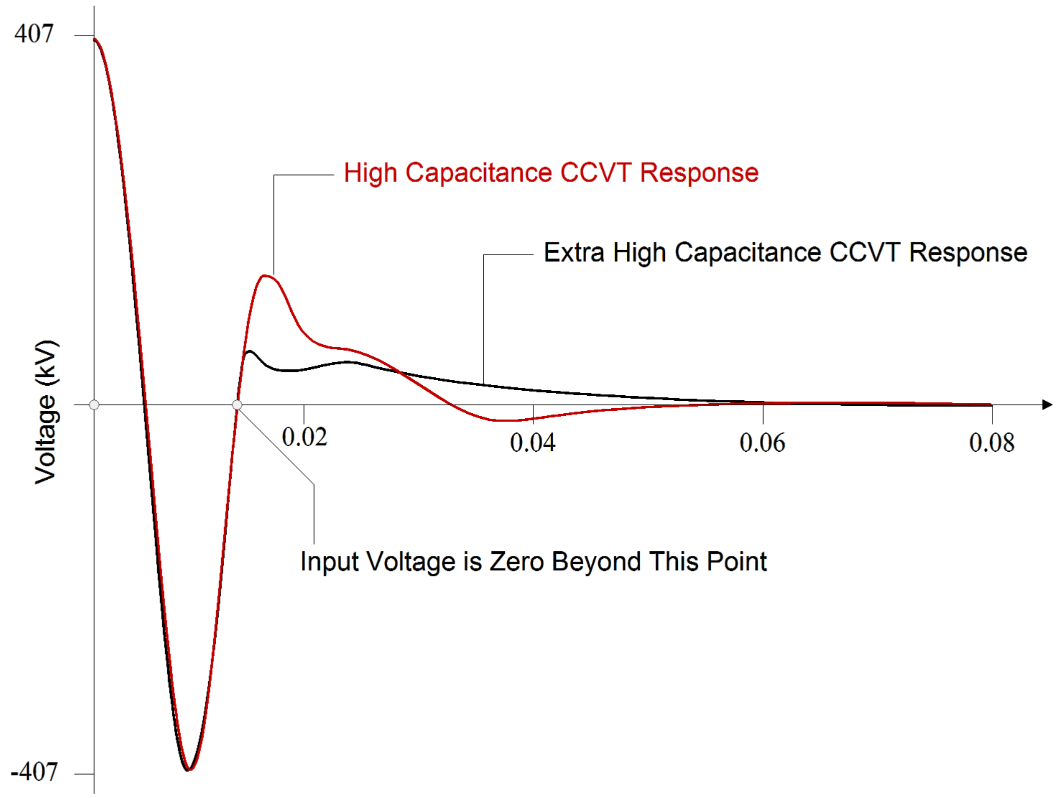


Figure 6.45: CCVT Transient Response (Primary Voltage Drops to Zero at t=0)

The burden resistance affects the transient waveform damping. Lightly loaded CCVT's exhibit slower transient response than if loaded at the rated load (Typically 100-400 VA).

The ferroresonance suppression circuit greatly affects the waveform distortion. Passive type circuits generate much lower waveform distortion and lower fundamental component errors during transients than active type circuits. Figure 6.x illustrates the response of two CCVT's, one with active ferroresonance suppression circuit (FSC) and one with passive FSC. All other parameters are equal. Note that the passive FSC CCVT exhibits non oscillatory (overdamped transient response) [3].

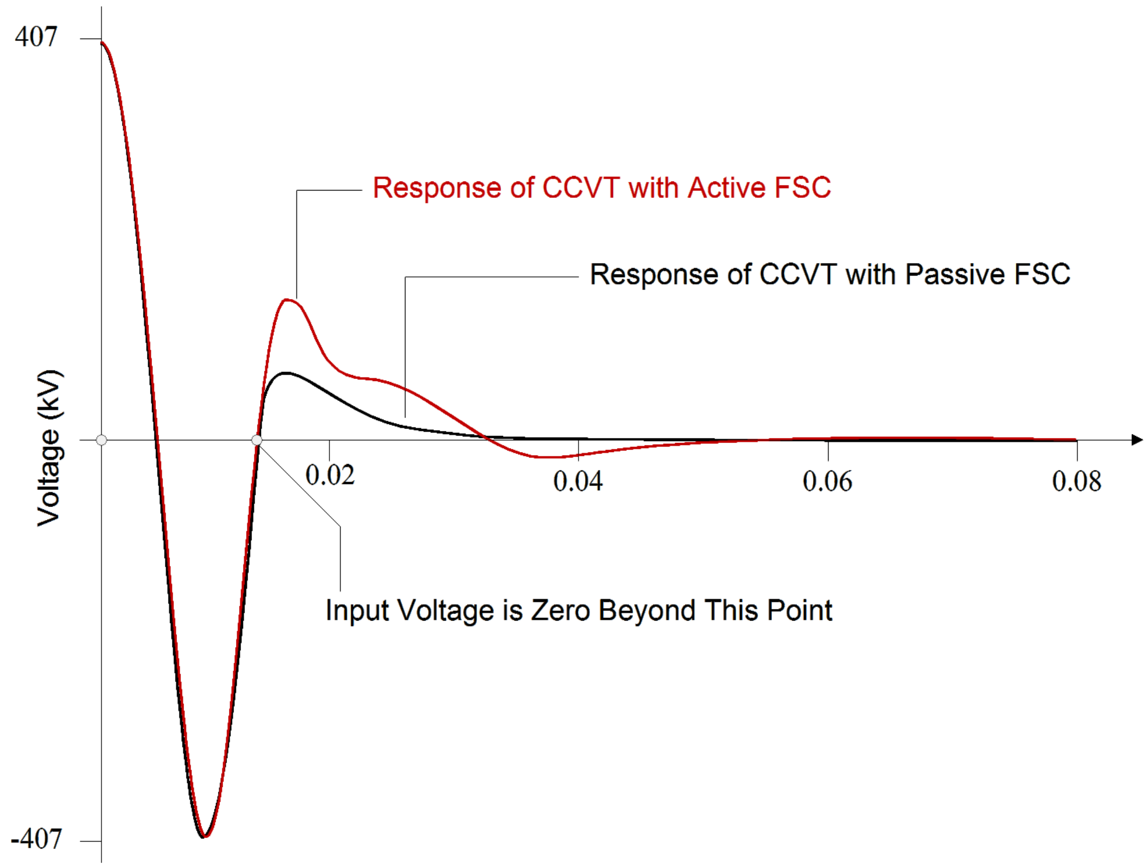


Figure 6.46: CCVT Transient Response

Finally, the transient response error depends on the input waveform shape. For example, for an interrupted sinusoidal input such as the waveform of Figure 6.x, the worst case occurs when the input voltage waveform is interrupted at the zero crossing. The reason is that for this case the energy stored in the compensation reactor at the time of the transient initiation is at a maximum [3].

The CCVT output transients may have a considerable impact on relaying applications. The effect is most critical in microprocessor relays which are designed for high speed tripping. For example, the CCVT voltage transient may result in significant transient overreach on a numerical distance relay set for Zone 1 protection. The errors due to CCVT transients are somewhat less critical in electromechanical relay applications. Figure xxx illustrates the compares the operation of a Mho relay with voltage input from a CCVT and an ideal voltage transformer.

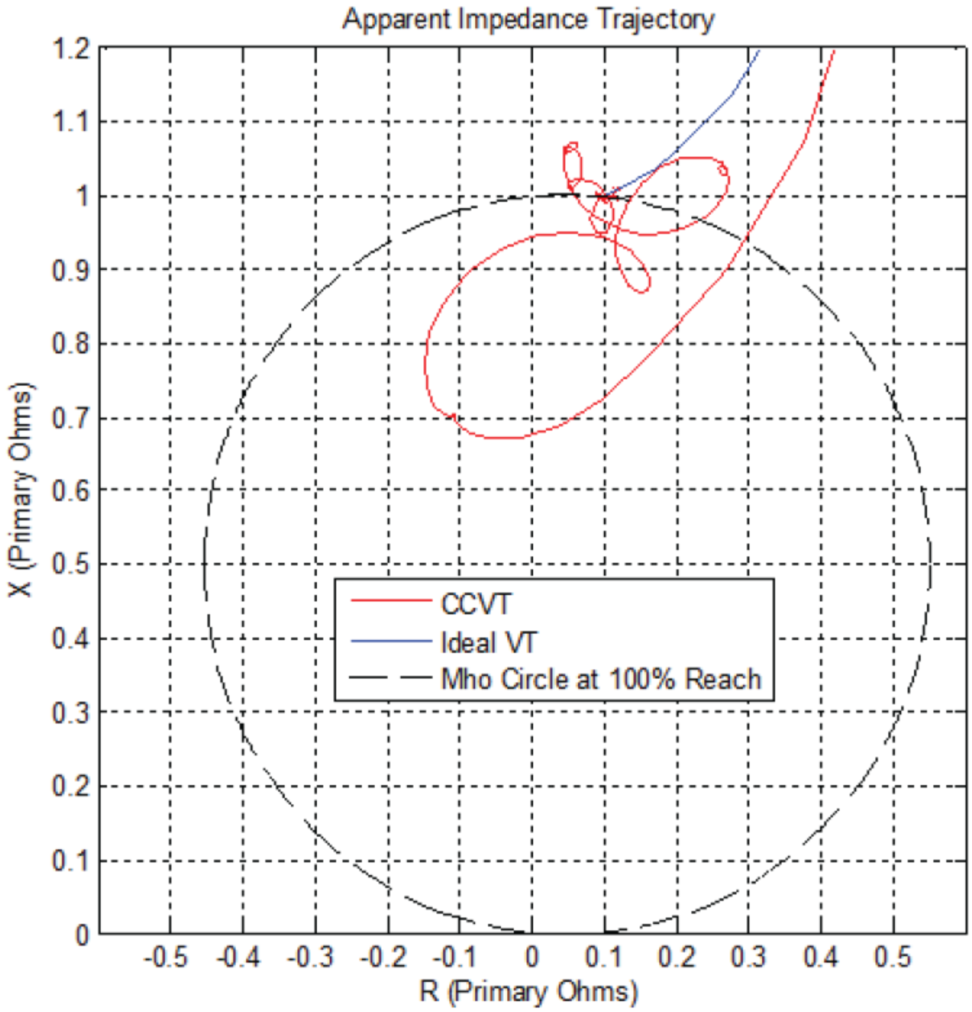


Figure 47: Comparison of Impedance Trajectories During a Fault of a Mho Relay with Voltage Input from an Ideal VT (Blue Trace) and a CCVT with Normal Capacitance and Active Ferroresonance Suppression Circuit (Red Trace)
From: G. A. Franklin and R. Horton "Determining Distance Relay Zone-1 Reach Settings to *Prevent CCVT Transient Overreach*", Proceedings of the 2011 IEEE Southeastcon.

6.3.9 Non-Conventional Voltage Transformers

Recent advances in electronics have resulted in several practical voltage instrument transformer implementations based on technologies not involving magnetics. These technologies generally provide wider and flatter frequency response, better transient performance (no resonance effects), and avoid saturation effects common in magnetics. One drawback of these technologies is that they require a specialized electronic interfaces which convert the output signal to voltage and current levels compatible with standard relay inputs (120V, 5A). However, the introduction of ***Merging Units*** circumvents this requirement. Specifically, specialized merging units can be made which directly convert the measured voltages and currents to standardized Sample-Value packet streams. (See section 6.10 for more information on Merging Units). In the following sections we briefly describe three such technologies, namely the Electro-Optical Voltage Transformer (EOVT), the Capacitive Divider Transformer (CVD), and a method to extend the useful frequency range of CCVTs.

Electro-Optical Voltage Transformer

The operation of electro-optical voltage transformers is based on the effect of the electric field in certain materials such as glass, crystals, and plastics on the polarization angle of a light beam passing through them. The principle of operation of an electro-optical voltage transformer (EOVT) is illustrated in Figure 6.x. A laser diode with a polarizer filter generates a beam with a certain polarization angle. The beam passes through a glass block and is collected by a polarization detector. This detector generates an output voltage which is dependent on the light polarization angle. When a voltage is applied across the grass block, the generated electric field rotates the light beam polarization angle, thus changing the polarization detector output voltage.

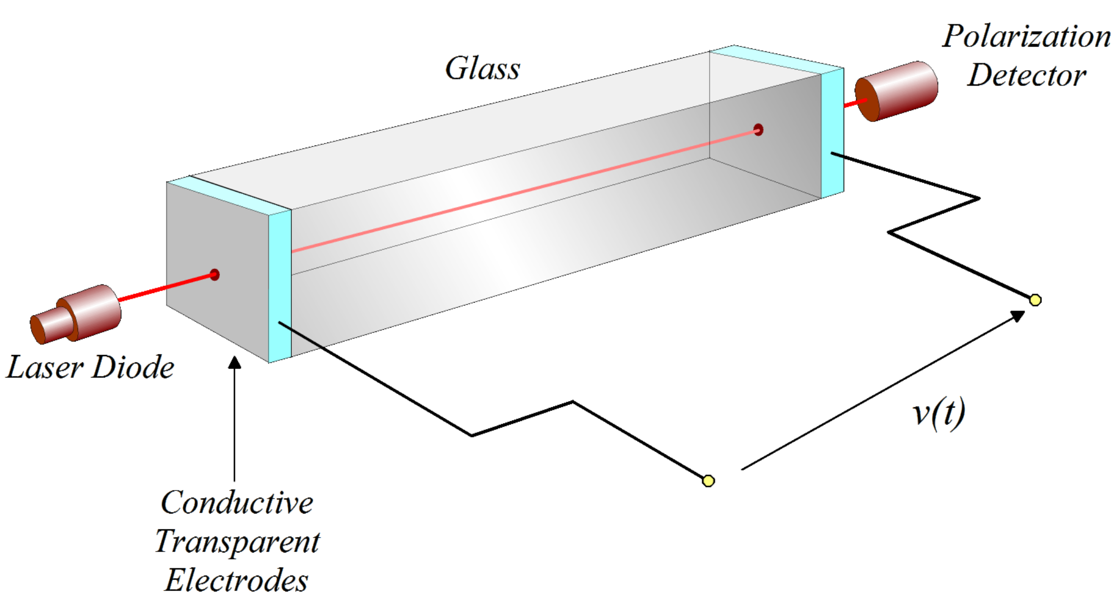


Figure 6.48: Electro-Optical VT Configuration

Obviously, Electro-optic voltage transformers require an electronic interface device to generate the laser beam, and convert the polarization detector output to standard relaying instrumentation voltage levels (nominally 69 or 120 Volts). Implementations can be fully-optical, using optical fibers to carry the sensing light beam to the substation control house where the electronic interface device is located. The advantage of this implementation is the avoidance of measurement errors caused by induced voltages and currents on instrumentation cables.

The advantage of electro-optical voltage transformers as compared with conventional magnetic and capacitive units is that they provide high measurement accuracy, wide frequency range, and wide dynamic range. Furthermore, the optical fiber implementation naturally provides galvanic isolation to very high voltages. The obvious disadvantage is that they require complex support circuitry and power supply. However, integration with dedicated merging units can make this technology practical for power system applications.

Capacitive Divider Voltage Transformer

The capacitive divider voltage transducer is an alternative voltage transducer technology that provides high measurement accuracy, wide frequency range, and wide dynamic range. As with EOVTs, capacitive divider voltage transducers can be made practical by combining them with specialized merging units. An example CDV / Merging Unit implementation is illustrated in Figure 6.x.

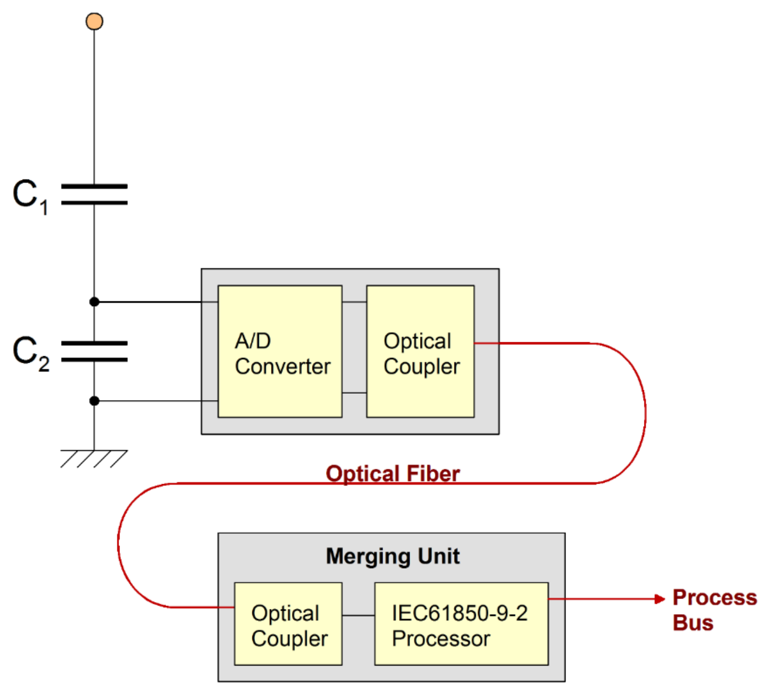


Figure 6.49: CVT with Dedicated Merging Unit Configuration

CCVT Enhancement

Capacitively-coupled voltage transformers are designed to perform accurately over a narrow frequency range about the power frequency (typically 96% to 102% of the nominal power frequency for relaying applications). Outside this frequency range, the CCVT output error may be very large. Measurements have shown that typical CCVT errors at harmonic frequencies are as large as 300%). Therefore, these devices are not useful for applications such as harmonic and transient analysis. For such applications, an added module can be installed to a standard CCVT as illustrated in Figure 6.x (ABB's PQSensor™ Module). This module consists of two current transformers monitoring the currents I_2 and I_3 and appropriate signal processing electronics. The voltage-drop across the capacitors C_1 and C_2 is computed from the currents flowing through the capacitors I_1 and I_2, as follows:

$$V(\omega) = \frac{I_1(\omega)}{j\omega C_1} + \frac{I_2(\omega)}{j\omega C_2} = \frac{I_2(\omega) + I_3(\omega)}{j\omega C_1} + \frac{I_2(\omega)}{j\omega C_2}$$

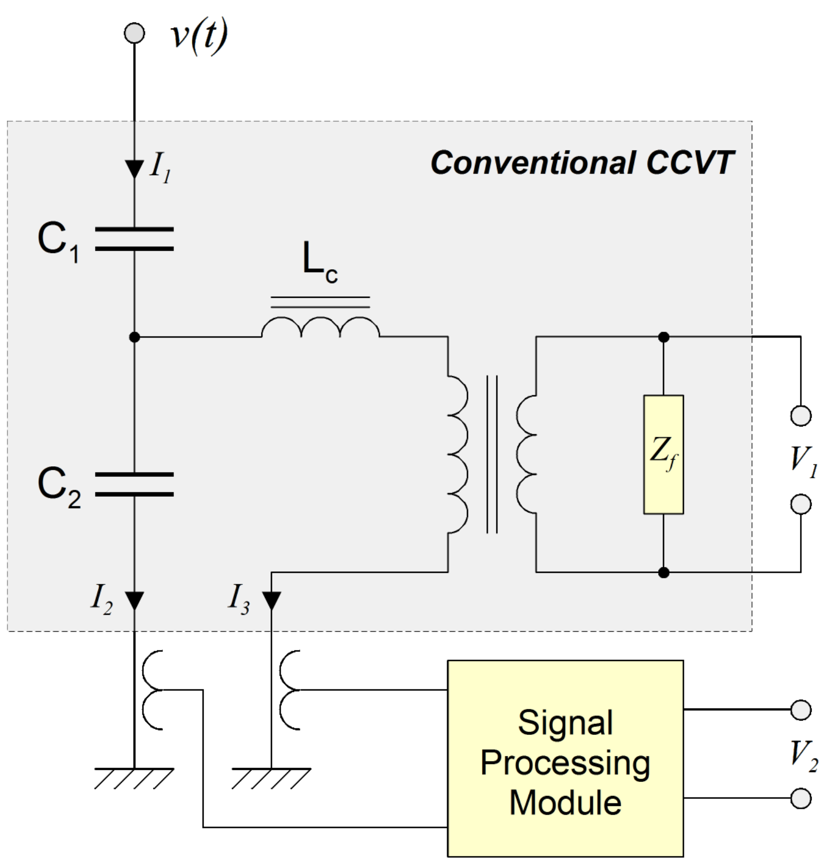


Figure 6.50: CCVT with Added Module Providing Accurate Voltage Measurements Over Wide Frequency Range (Source: ABB PQSensor™ Module).

Note that the added current transformers are both located on grounded conductors. Therefore, these CT's do not require high voltage insulation, thus reducing the implementation cost.

For harmonic computations the signal processing module evaluated the above formula at each frequency of interest, after Fourier transformation of the measured current waveforms $i_2(t)$ and $i_3(t)$. Furthermore, the time domain input voltage waveform can be evaluated by the integration formula:

$$v(t) = \int_{t=0}^{T} \frac{(C_1 + C_2) i_2(t) + C_2 i_3(t)}{C_1 C_2} dt$$

The above operation can be performed numerically by the signal processing module after sampling the current waveforms i_2 and i_3.

(Note that the ABB PQSensor specification states that the output of this device meets standard accuracy requirements to at least 5 kHz).

6.4 Instrumentation Channel Grounding

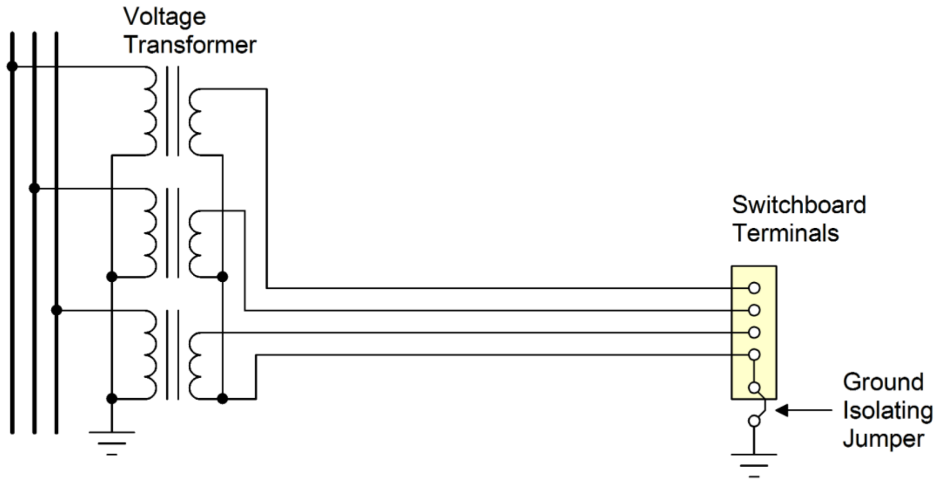
Instrumentation channels must be grounded to prevent excessive voltage buildup due to magnetic and capacitive coupling to power circuits. The goal is the protection of personnel who may contact instrument transformers or any equipment connected to the instrument transformers, and also to prevent damage to equipment connected to the instrument transformer secondary circuits, such as relays and in general IEDs. In general, each instrumentation circuit must be connected to the facility grounding system at a single point, thus avoiding *ground loops*. The reason for this is to prevent ground potential differences causing currents to flow through the instrumentation cables. Such currents can introduce significant errors in the measured quantities causing relays to malfunction. Furthermore, during faults or switching operations these currents may become so high as to damage instrumentation cables as well as equipment connected to instrumentation secondary circuits. In most cases the grounding point is selected to be at the switchboard or relay panel side of the instrumentation secondary circuit in order to minimize ground potential differences at the locations where personnel is most likely to be in contact with equipment, and also to reduce overvoltages appearing at the inputs of relays and other electronic equipment connected to the instrumentation secondary circuit.

Figure 6.51 illustrates the most common grounding scheme for voltage and current transformers. Note that the secondary circuits are grounded at a single point near the switchboard (typically located in the control house). A grounding jumper is provided for facilitating circuit insulation testing. Specifically, the jumper can be temporarily removed for the purpose of measuring the secondary insulation impedance.

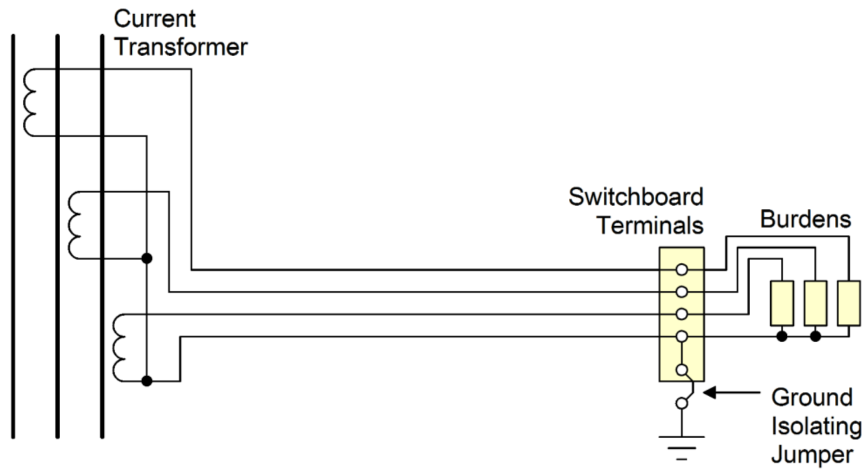
Figure 6.52 illustrates a case where a distribution transformer is used instead of a dedicated voltage transformer. Since the distribution transformer secondary is normally grounded at the transformer location, an isolation transformer is required so that the grounding switchboard does not create a ground loop.

In addition to secondary circuit grounding, the metallic case of all relays, recorders, and other devices installed at the control house switchboard must be bonded to the substation grounding system.

IEEE Standard C57.13.3 provides detailed guidance in proper instrument transformer secondary circuit grounding for many other configurations such as delta connected current transformers for differential relays, unused voltage and current transformer secondaries, optical voltage and current transformers, etc.



(a)



(b)

Figure 6.51: Instrument Transformer Grounding
(a) Voltage Transformer
(b) Current Transformer

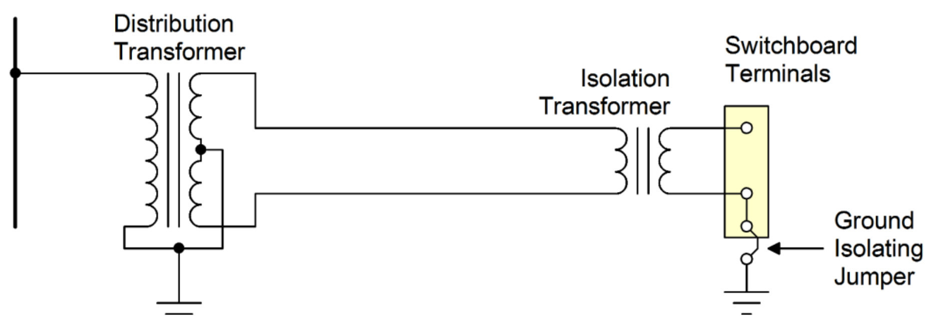


Figure 6.52: Distribution Transformer Secondary Grounding
(Isolation Transformer Required to Prevent Ground Loop)

6.5 Instrumentation Protection and Isolation

Electronic instrumentation installed in substations are vulnerable to transients caused power system faults, switching operations, lightning induced voltages etc. Such phenomena may damage electronic components of instrumentation input circuits. Specifically, relay input circuits are typically connected to transducers via long instrumentation cables which are subject to induced voltages from transients occurring on nearby power circuits. Substations are thus considered harsh EMI environments, and electronic instrumentation must be appropriately designed for such environments.

Techniques for protection of electronic instrumentation input circuits can be classified into two categories: (a) transient suppression devices, and (b) isolation devices.

Transient suppression devices are shunt devices installed across instrumentation inputs that conduct when the voltage exceeds a certain threshold. Commonly used transient suppression devices are MOV's (Metal Oxide Varistors). Shunt capacitors are also used (typically in parallel to MOV's) providing additional protection against very high rise time voltage spikes. Note that the typical MOV parasitic inductance may limit their effectiveness to high rise time spikes. It is important to note that these devices may introduced errors, both in magnitude (gain) as well as in phase. For most relaying applications these errors are not important. However, for specific applications, such as metering, high accuracy GPS-synchronized measurements, etc., these errors may be important and should be compensated for.

Isolation techniques provide additional protection to electronic instrumentation inputs against transients resulting in ground potential differences along paths of instrumentation cables. A commonly used isolation device in digital relay implementations is the isolation transformer. A transformer isolated voltage input is illustrated in Figure 6.x. Isolation transformers are simple and robust but may introduce measurement errors. The leakage inductance and the parasitic capacitance between windings of an isolation transformer cause both magnitude and phase deviations which are functions of frequency. Such errors can be somewhat mitigated by appropriate calibration techniques.

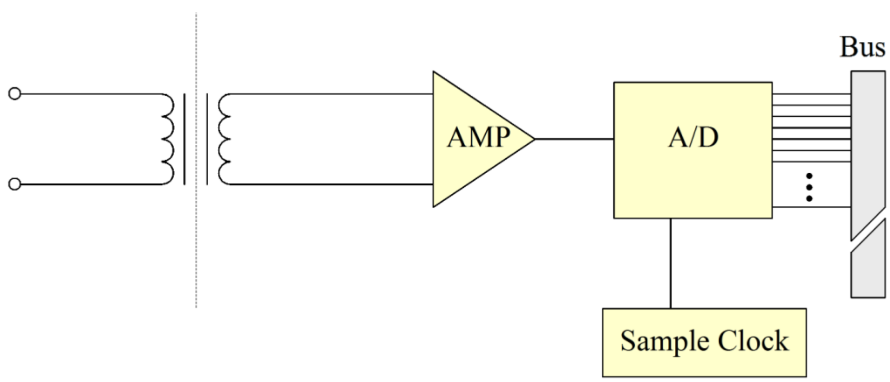


Figure 6.53: Analog Magnetic Isolation
(Show IED in a box)

Alternative isolation techniques are available using optical, capacitive, and magnetic circuits that reduce measurement errors. Figure 6.x illustrates an analog optical isolator applied on the analog front end of an instrumentation channel.

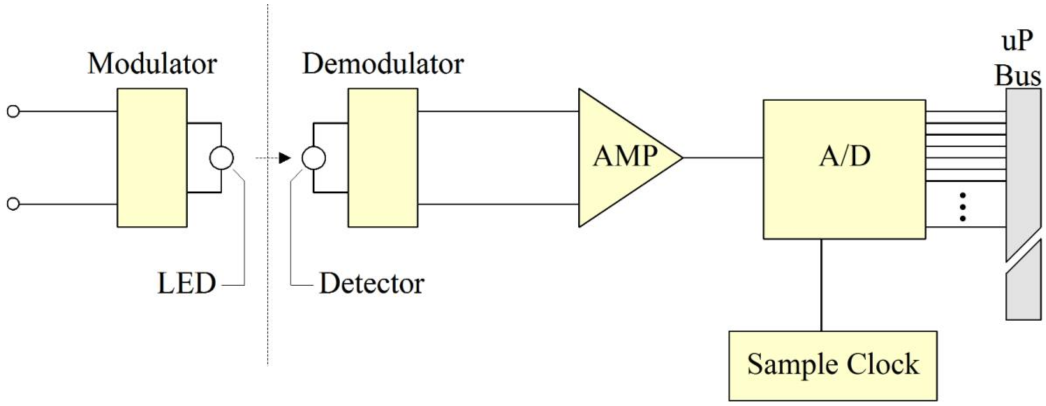


Figure 6.54: Analog Optical Isolation

Most notably, isolation techniques can be totally free of measurement error if applied on digital signals, rather than at the analog front end of an instrumentation channel. Figure 6.x illustrates a "digital optical isolation" scheme. Note that the part of the circuit on the input side of the channel requires a dedicated isolated power supply providing power to the analog electronics and the A/D converter. The A/D converter digital output is converted to serial form, and thus reducing the number of signals that must be optically isolated.

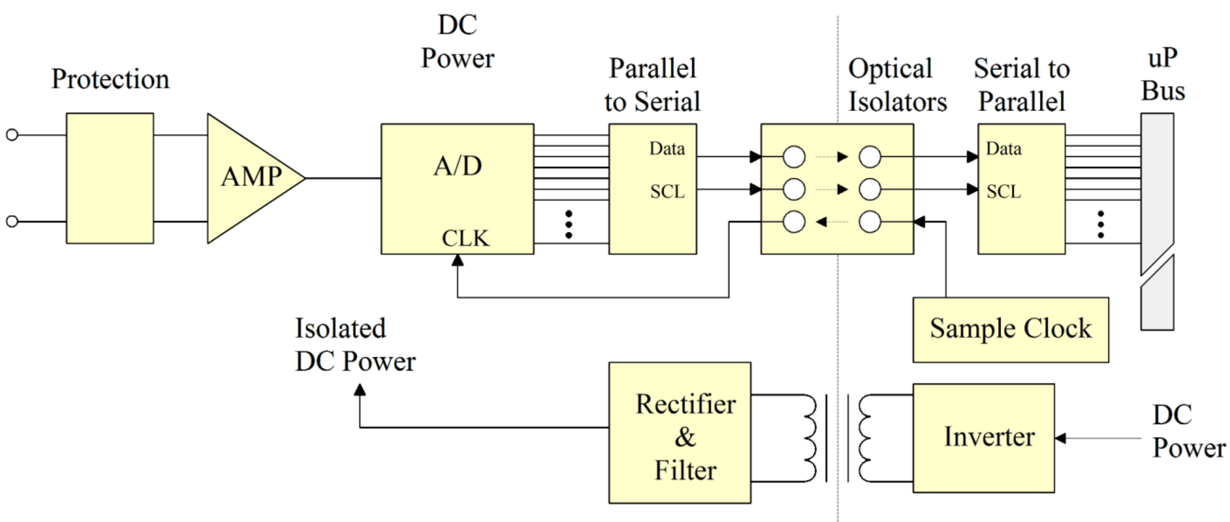


Figure 6.55: Digital Optical Isolation

An example realization of an input protection circuit that combines nonlinear voltage limiting components (MOV's), linear filters consisting (capacitors), and common mode rejection inductors is illustrated in Figure 6.56.

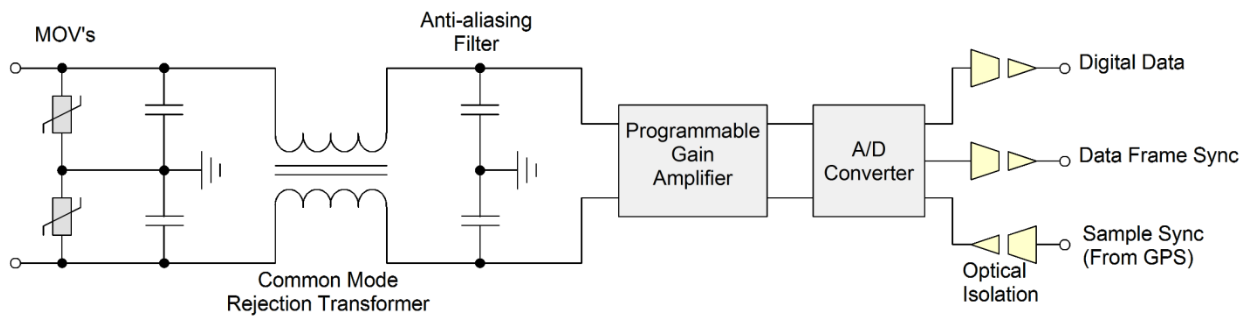


Figure 6.56: Example of a Digital Data Acquisition Protection Scheme

Recently various alternative digital isolation methods have been implemented, most not notably based on magnetic and capacitive isolation techniques. (Examples are Analog Devices iCoupler magnetic isolation devices and Texas Instruments ISO72x capacitive isolation devices). These devices generally provide higher reliability and better immunity to electromagnetic interference than the optical isolation technology.

6.6 A/D Conversion

This section presents the fundamental characteristics of the present Analog to Digital Conversion technology, as well as the additional components comprising a typical digitizing channel. Modern digital relays convert the output of instrument transformers to digital signals by means of Analog to Digital Converters (ADC's). However, the harsh substation environment where relays are usually located presents a special challenge to digitizer electronic design. Substation environments may exhibit high electromagnetic fields, which induce voltages along instrumentation cables connecting instrument transformers to the A/D converters. In addition, during transients (caused by switching operations, lightning storms, faults, etc), substantial ground potential difference may develop across the substation grounding system. This ground potential differences appear as a transfer voltages along the instrumentation cables. These voltages can damage sensitive electronic circuits of A/D converters. Note that A/D converters are implemented with integrated circuit technology operating at very low voltage (typically 5 Volts, and recently as low as 1.25 Volts), and can be easily damaged if not properly protected.

Another issue arising from the use of A/D converters is signal aliasing. In relaying applications, the frequency range of interest is near 60 Hz. However, actual measured signals contain higher frequencies due to nonlinear loads (causing harmonic distortion), as well as higher frequencies due to switching, lightning and fault transients. The aliasing problem is addressed by appropriately filtering the signal before A/D conversion.

The technology of A/D Converters and associated protection and isolation circuitry has improved dramatically over the recent years, making possible reliable implementation of digital relays. The architecture of an Analog to Digital converter channel, which addresses the above-mentioned issues, is illustrated in Figure 6.x. The protection against the harsh substation electromagnetic environment is achieved by the combination of an input overvoltage protection stage and optical isolation stage. The input overvoltage protection limits both common and differential mode voltages induced along the instrumentation cable, while the optical isolation stage removes the sensitive circuits of the A/D converter and other support electronics from ground loops formed by the instrumentation cables and the substation grounding system.

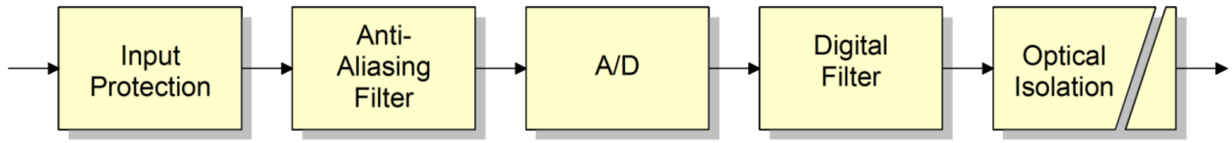


Figure 6.57: Block Diagram Illustrating Digital Data Acquisition Channel Structure

We next examine the basic structure and the fundamental characteristics of the various components comprising a modern A/D conversion channel for relaying applications.

6.6.1 Aliasing and Low Pass Filtering

Aliasing occurs whenever a signal is sampled at a rate slower than twice its highest frequency component. The frequency components that are above half of the sampling rate appear as signals of lower frequency in the sampled signal representation. Figure 6.x illustrates this condition. The solid sinusoidal line represents the analog input signal. The dots are the signal samples. Observe that the sampling rate is slightly lower than the signal frequency.

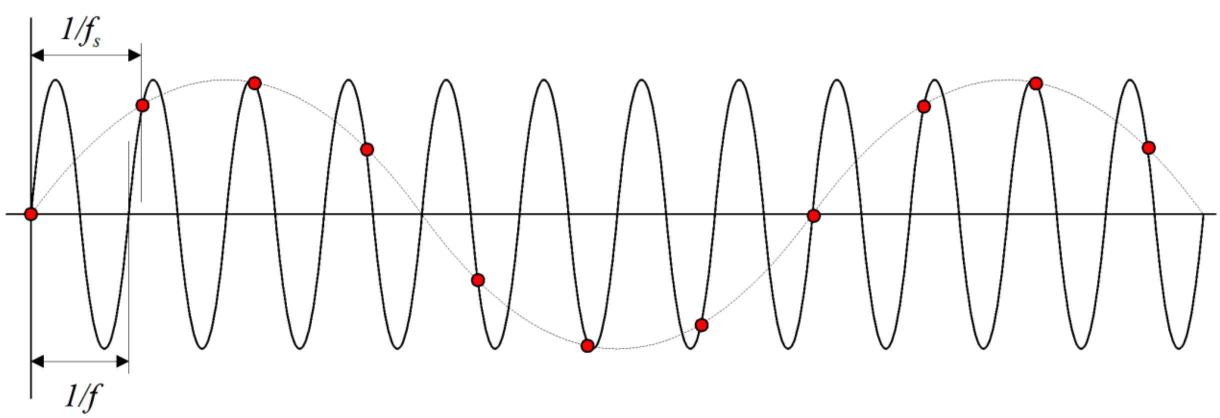


Figure 6.58: Implementation example of a Digital Data Acquisition Channel

Note that the signal samples appear to trace a sinusoidal signal of a much lower frequency than the actual analog signal frequency. In fact it can be shown that sampling always generates aliased components whose frequency is related to the actual analog signal frequency and the sampling rate as follows:

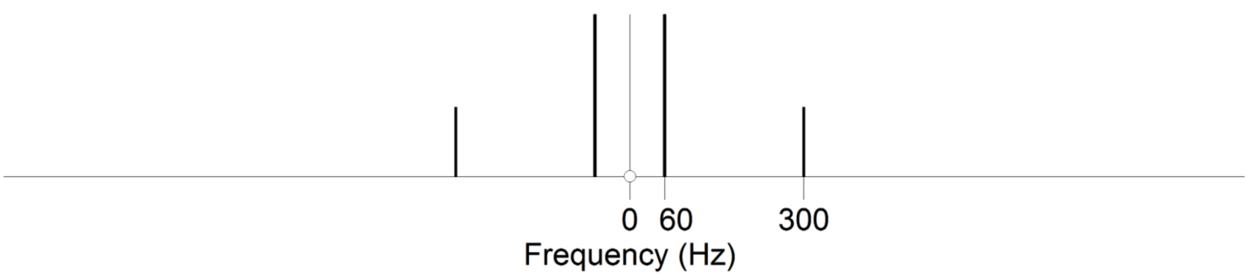
$$f_a = | f_s \pm f |$$

Where: f_a is the aliased frequency, f_s is the sampling frequency and f is the signal frequency. Consider for example a signal that contains the 17^{th} harmonic of 60 Hz, i.e. 1020 Hz. If this signal is sampled at 1024 samples per second, a 4 Hz component will appear in the sampled representation as a result of aliasing. The magnitude of this aliased component will be equal to the 17^{th} harmonic magnitude.

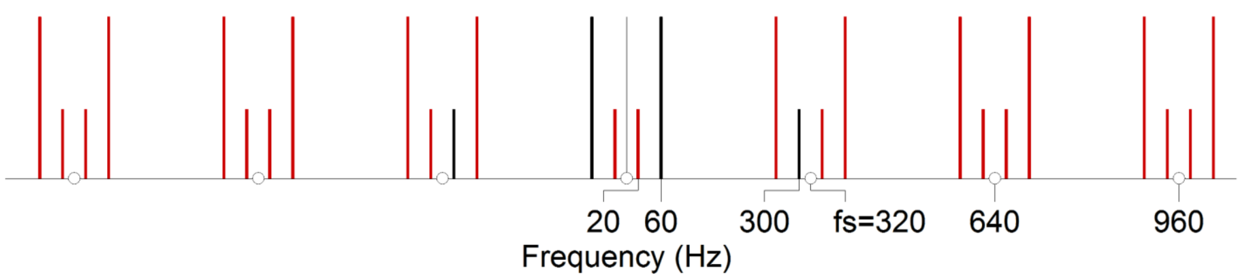
In general sampled signals are represented by a “train” of impulse functions with amplitude equal to the corresponding original continuous signal values. The spectrum of such a signal contains an infinite number of aliased frequencies given by the formula:

$$f \pm kf_s, \; k = 1, 2, 3, \ldots\infty$$

where f is the frequency of the original signal and f_s is the sampling frequency. A graphical illustration of this property is given in Figure 6.x. Figure 6.x(a) shows the spectrum of a continuous signal containing two frequency components: 60 and 300 Hz. After sampling at 320 samples per second, the spectrum of the sampled signal contains components at 20 Hz, 60 Hz, 260 Hz, 300 Hz, 340 Hz, 380 Hz, 580 Hz, 620 Hz etc, as illustrated in Figure 6.x(b)



(a)



(b)

Figure 6.59: Illustration of Aliasing in the Frequency Domain
(a) Original Signal Spectrum
(b) Sampled Signal Spectrum

Aliasing is particularly undesirable if aliased frequency components fall within the bandwidth of interest. For example, in this case the 20 Hz aliased frequency component is within the frequency of interest (0 to 100 Hz). This undesirable aliasing can be avoided by sampling the signal at a sampling rate that is equal or higher than twice the frequency of the highest signal frequency component. This fact is known as the **Nyquist** criterion, and the frequency equal to twice the highest signal frequency component is known as the ***Nyquist Rate***. In fact, the ***Nyquist theorem*** states that if any signal is sampled at a rate equal or higher that the Nyquist Rate, no information is lost in the sampling process, and the original signal can be precisely reconstructed from the samples. In this example, the Nyquist rate is 600 Hz.

Although in relaying applications the frequencies of interest are near the power frequency fundamental, the actual signals may contain higher frequencies arising from nonlinear loads (harmonics), and transients due to switching operations, faults etc. Thus the aliasing problem must be addressed.

A solution to the aliasing problem is to introduce an analog filter before the A/D converter which attenuates all frequencies above half of the A/D converter sampling rate. The ideal filter for this application is a low pass filter with a flat pass-band and a very steep cutoff slope. Unfortunately such filters are necessarily of high order, thus requiring complicated circuitry, and consequently are plagued by stability and drift problems. The filter characteristic drift is exacerbated by substation control house widely varying temperature, in case no heating and air conditioning is provided.

A more practical approach is to use a low order low pass filter (thus of gradual cutoff slope) and sample the input signal at a rate much higher than the Nyquist rate. This approach is known as *Oversampling*. Figure 6.x illustrates the characteristics of an example oversampling system. Note that the minimum sampling frequency that avoids aliasing within the frequency of interest band is at:

$$f_s = 2\ f_{max} + f_t$$

Where: f_s is the sampling frequency, f_{max} is the maximum frequency of interest and f_t is the filter transition bandwidth, i.e. the frequency range starting from f_{max} and ending where the filter response decays to a negligible value.

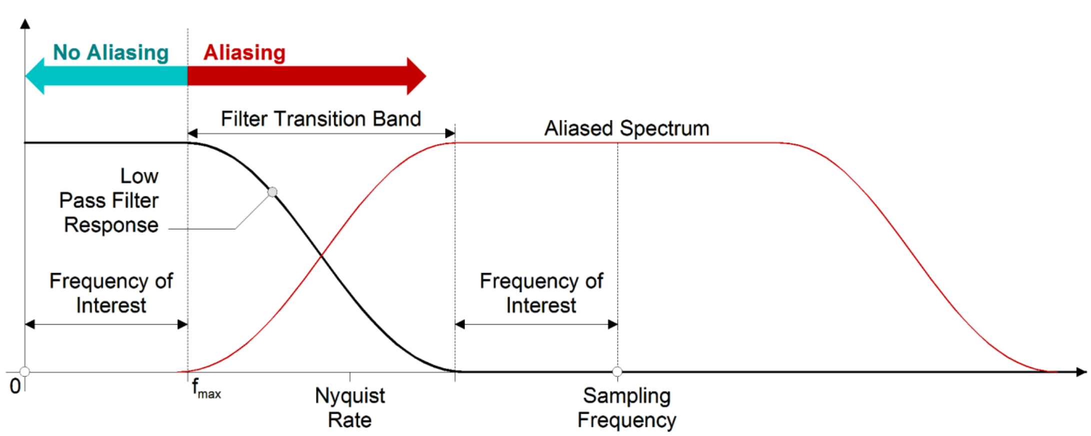


Figure 6.60: Oversampling Anti-Aliasing System Characteristics

6.6.2 Analog Filter Implementation

Analog filters can be implemented using either passive component circuits (consisting of resistors capacitors and inductors), or active circuits, which are implemented using operational amplifiers.

Passive circuit implementations are generally limited to relatively simple (low order) filters. They generally consist of inductors and capacitors and resistors. A passive circuit disadvantage is that they tend to have input impedance and output impedance in the same order of magnitude. This complicates cascading filters with other systems, since the overall system response is affected by the source and load impedances. Active circuits overcome this problem using operational amplifiers as buffers. (Operational amplifiers typically have very high input impedance and very low output impedance). Another advantage of active circuits is that any rational transfer function can be implemented without the use of inductors (i.e. only using operational amplifiers, resistors and capacitors). This allows for higher performance filter implementations, by avoiding the common inductor problems, such as magnetic core saturation, unintentional mutual coupling, noise coupling by magnetic induction, etc.
Examples of passive 1st and 2nd order low pass filters are illustrated in Figure 6.x. An example of an active filter using operational amplifiers is illustrated in Figure 6.x.

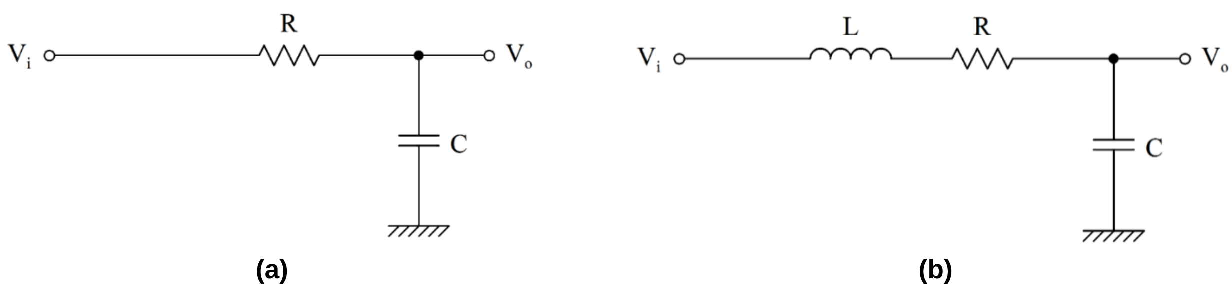


Figure 6.61: First Order Passive Low Pass Filter Examples (a) First Order, (b) Second Order

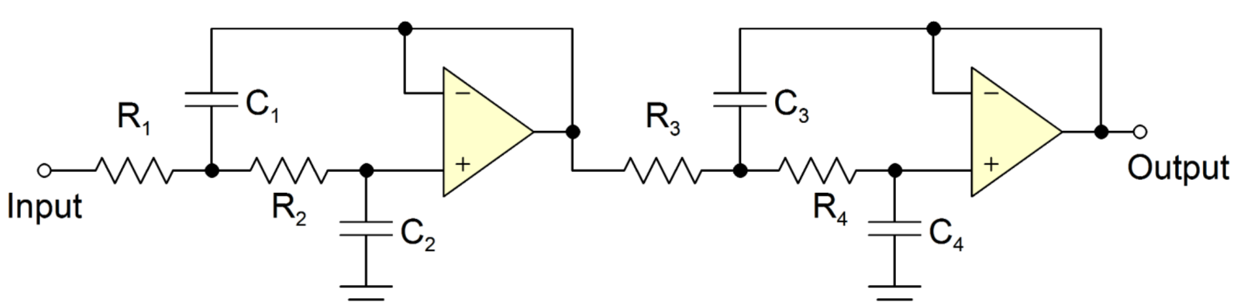


Figure 6.62: Example of 4th Order Active Low Pass Filter (Sallen-Key Circuit)

Both active and passive analog filter circuits are subject to frequency response drift, due to component aging, temperature dependence. In general, the sensitivity to temperature and aging increases with the filter complexity (filter order).

A low pass filter is typically used to eliminate high frequencies from a certain signal. A typical application is to prevent aliasing during A/D conversion. The ideal low pass filter impulse and frequency response is illustrated in Figure 6.x (a). It is characterized by a flat response from DC up to a certain frequency (known as the filter cutoff frequency) and zero response for any frequency above the cutoff frequency. Unfortunately, such a filter is not practical, since its impulse response has infinite duration. However, the ideal response can be approximated, by a smooth transfer functions illustrated in Figure 6.x (b). The filter order (and thus its complexity) increases as the response approaches the ideal filter response.

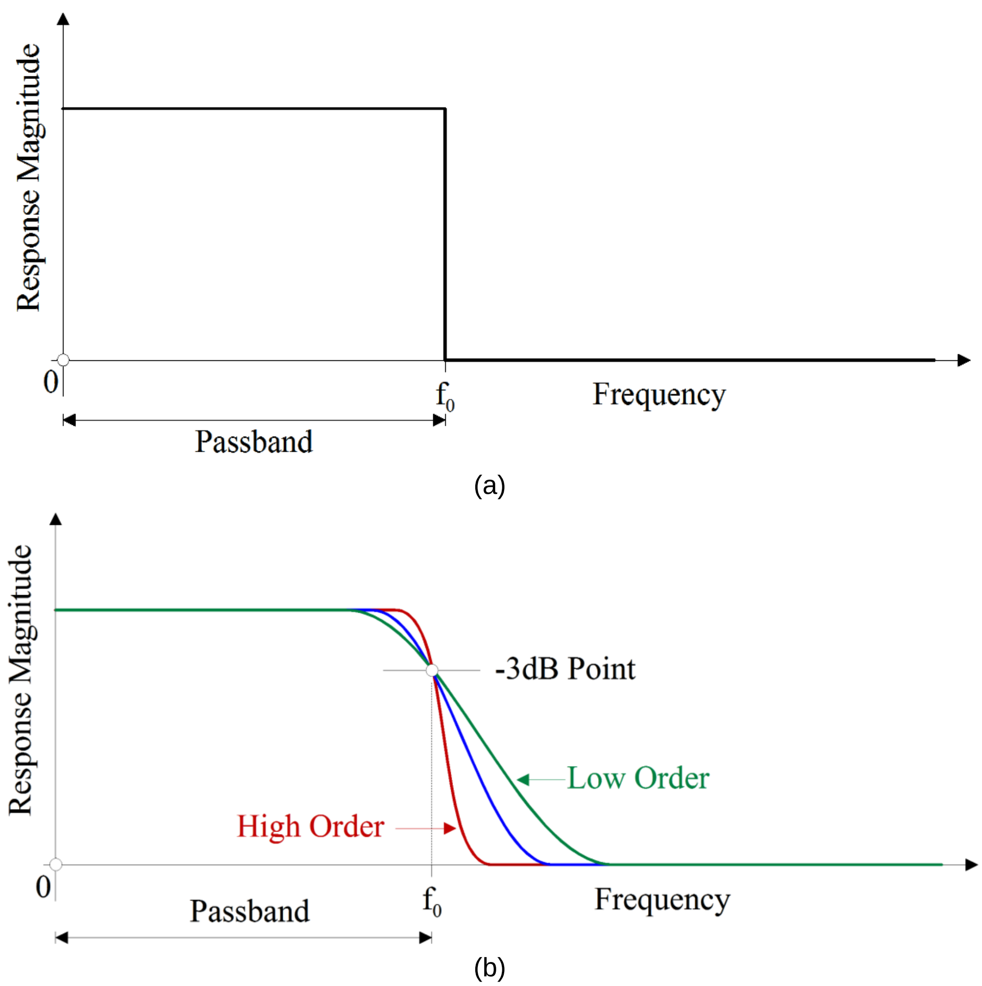


Figure 6.63: Low Pass Filter Frequency Response
(a) Ideal Response, (b) Practical Response

An important issue in using low pass filters for relaying applications is the filter delay. In fact, the filter delay generally increases with filter order. Figure 6x shows the impulse response of a low pass filter with cutoff frequency of 2 kHz. This filter could be used to reduce aliasing of a data acquisition system sampling with at least 4000 samples per second (for example 4800 s/s, a common sampling rate for power system applications). Note that tis filter introduces a 3.5 millisecond delay, which corresponds to 75.6 degrees of phase error at 60 Hz.

Since relaying applications require fast real-time operation, many data acquisition systems used in relaying applications do not use any filtering. This approach is usually justified since typical transmission level power system voltage and current waveforms mostly consist of 60 Hz waveforms with very low harmonic content.

Recent advances in A/D converter technology simplified the anti-aliasing filter requirements. Specifically, converters based on the Sigma-Delta modulation technique sample the input signal at a very high rate, and subsequently decimate the signal to the output rate using digital filters. For these analog to digital converters a simple first order low pass anti-aliasing filter is usually sufficient.

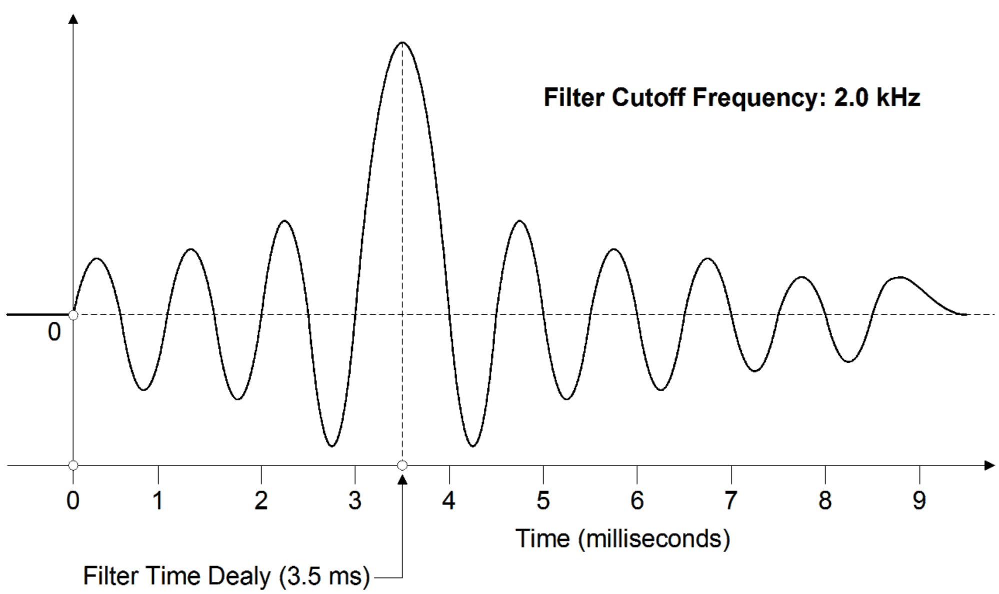


Figure 6.64: 2 kHz Low Pass Filter Impulse Response

6.6.3 A/D Converter Technology Overview

The **A/D converter** generates a series of binary numbers, which represent the input analog signal. This process is known as sampling, since these numbers are samples of the input signal taken at uniformly spaced time instants. Presently there is a plethora of available sampling technologies. In this book we emphasize the technologies most applicable to power system applications. The major A/D Converter technologies can be classified as follows:

- Sigma Delta Modulation
- Successive Approximation
- Subranging-Pipelined
- Flash

Figure 6.65 illustrates the typical resolution and bandwidth of these technology classes.

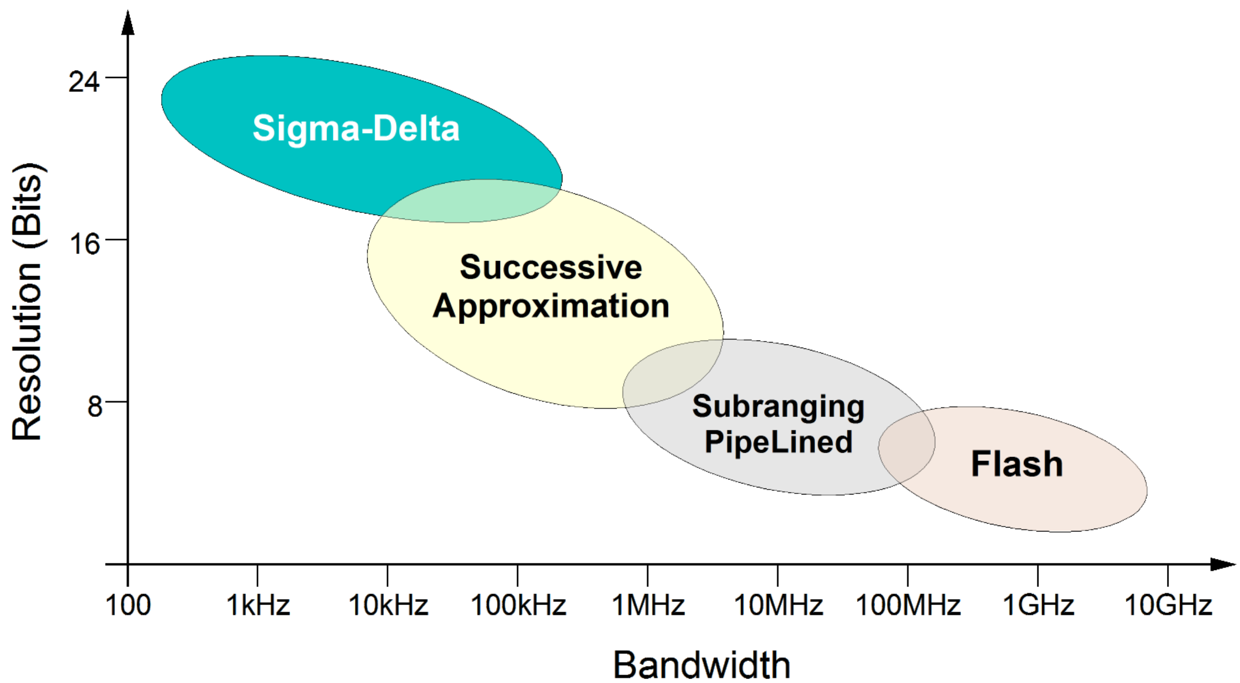


Figure 6.65: Resolution & Bandwidth of Major A/D Converter Technologies

Table 6.x: Converter Technology Comparison

Technology	Resolution	Linearity	Latency	Bandwidth
Sigma-Delta	Very High (Up to 24 bits)	Very High	High	Low
Successive Approximation	High (Up to 16 bits)	High	Low	Medium
Subranging-Pipelined	Medium (12 bits)	Medium	Very Low	High
Flash	Low (8 bits)	Medium	Extremely Low	Very High

Table 6.x: Resolution and Dynamic Range

Number of Bits	Levels of Discretization	Dynamic Range (dB)

8	256	48 dB
12	4,096	72 dB
16	65,536	96 dB
24	16,777,216	144 dB

Of particular interest is a class of A/D converter known as Sigma-Delta modulation converters. In recent years the improvement in integrated circuit technology made possible Sigma-Delta modulation converters realizations with sampling rates adequate for most power system applications (up to 400 kHz). These converters have the following desirable characteristics:

Advantages of Sigma-Delta Technology:

- Wide Dynamic Range (16-24 bits SNR > 86 dB)
- Simple Analog Anti-Aliasing Requirements (1-pole Low Pass Filter sufficient)
- No Sample-Hold Requirement
- Inherently Linear & Monotonic (1-Bit Input Stage)
- Low Drift Due to Aging & Temperature Variations (Mostly Digital)
- Relatively inexpensive

Disadvantages of Sigma-Delta Technology:

- Large latency between sampling and digital output due to digital filter stage. The digital filter is typically an FIR filter. The latency increases with the FIR filter order (number of samples comprising the filter impulse response. It is important to note that for a specific design the latency is fixed and can be taken into consideration in the application of the data.

Figure 6.x illustrates the architecture of a Sigma-Delta Modulation A/D Converter.

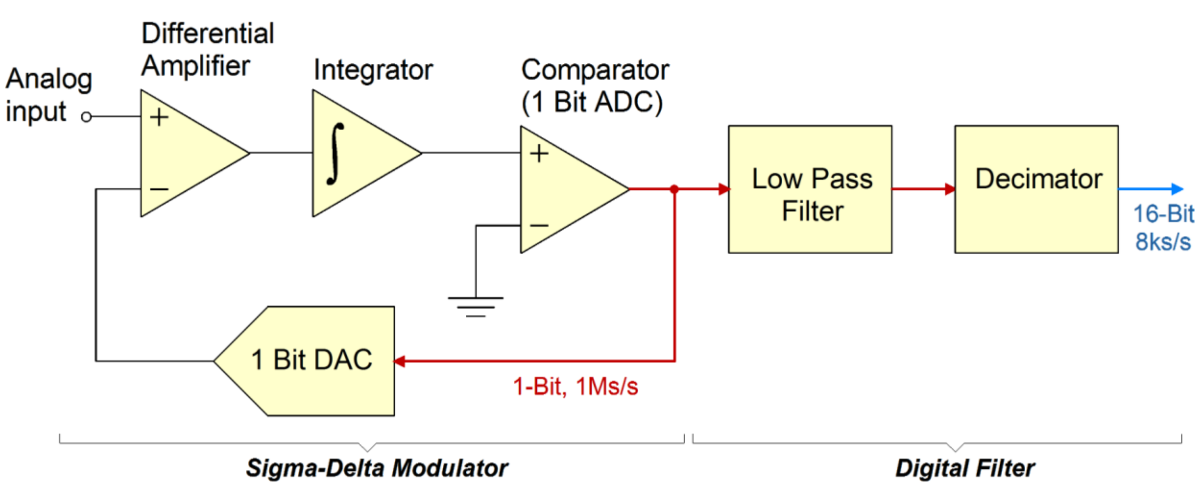


Figure 6.66: Basic Architecture of 1st Order Sigma Delta Modulation A/D Converter

The sigma-delta modulation A/D converter technology has been steadily improving. Typical improvements include:

- Increasing resolution (24 bit output is presently typical)
- Serial Data Output enables simple galvanic isolation schemes.
- Automatic Calibration ensures high accuracy
- Higher order digital filters provide immunity from aliasing

Note that as previously mentioned in relaying applications where fast real-time response is often an essential requirement, sigma-delta modulation A/D converters with high order digital filters may be unsuitable due to the long time delays generated by digital filtering.

6.6.3 Gain Control Stage

The **gain control** stage allows the dynamic range of the A/D converter to be optimally utilized. Note that in many power system applications a wide channel dynamic range is one of the main design challenges. The dynamic range of an A/D converter is mainly limited by the number of bits used in the representation of the signal samples. This parameter is commonly referred to as the converter word length. Common word lengths are 8, 10, 12, 14, 16, and 24 bits. Presently, 16 bit converters at sampling rates up to 1 MHz are widely available at relatively inexpensive prices. The maximum theoretical dynamic range of a 16 bit converter is $1:2^{16}$ or 1: 65536. Many times the dynamic range is expressed in dB. For example, the 16 bit converter dynamic range is $20 \times \log_{10}(2^{16})$ dB or 96 dB, approximately. Recently 24 bit converters are readily available at relatively low prices. However, taking advantage of the full theoretical dynamic range of 24 bit converters is difficult because of the limits imposed by the analog front-end circuit noise. Thus, 16-bit converters are presently preferable for most power system applications.

For relaying applications it is important to allow for substantial signal “headroom” so that the A/D converter does not saturate during transients. For other monitoring applications, for example harmonic monitoring, the signals of interest (harmonics) may be of much lower amplitude that the fundamental component signals. Figure 6.x illustrates this condition. In both of these situations it is evident that a wide dynamic range converter is desirable. Furthermore appropriate gain selection is necessary so as to maximize signal resolution and at the same time avoid saturation. A programmable gain control stage provides this capability.

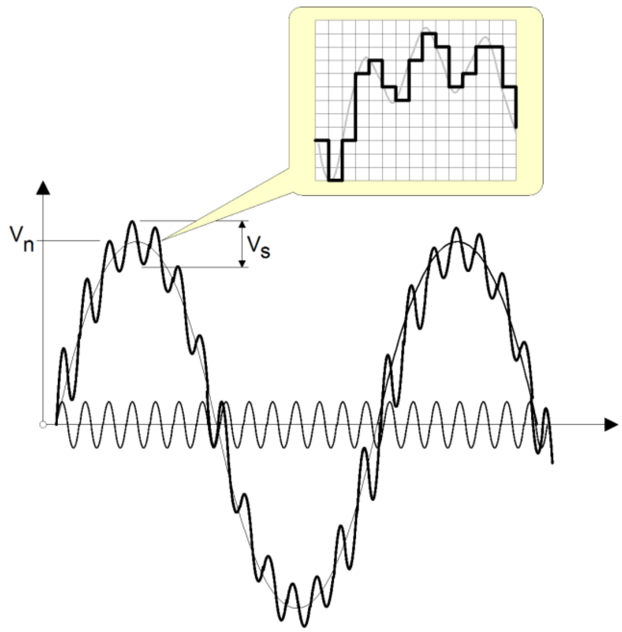


Figure 6.67: The Importance of Dynamic Range

6.6.4 Sampling Synchronization

In relaying applications such as transmission line fault detection and fault distance computations it is desirable to process measured voltages and currents from both ends of the transmission line. (See also Chapter 10 on Fault Monitoring and Recording). Furthermore, additional accuracy can be obtained if the data from the two ends of the line are sampled synchronously. Synchronous sampling with remotely located digitizers can be achieved by synchronizing the A/D converter clocks to a common timing signal. Note that the accuracy of relative phasor angles computed from synchronously sampled signals depends of the consistency of the common timing signal. Today, the most practical source of a highly consistent timing signal is the Global Positioning Satellite system (GPS). GPS based timing signals are typically consistent within 0.5 microseconds.

Historical Note: An earlier implementation of a satellite based timing source was the Geostationary Operational Environmental Satellites (GOES) created in 1975 by the National Oceanic and Atmospheric Administration (NOAA). It provided timing accuracy of about 100 microseconds by broadcasting a timing code generated by the National Institute of Standards and Technology (NIST) through a geostationary satellite network. This service was discontinued in 2005. NIST also transmits timing signals via short and long wave radio transmitters located in Fort Collins, Colorado (WWV and WWVB) and in Kauai, Hawaii (WWVH). This service was initiated in 1962 and is still in operation. However the timing accuracy it provides is in the order of 10 milliseconds, which is inadequate for the requirements of power system data acquisition synchronization.

The digitizer sampling clock synchronization to GPS signals can be achieved via a hybrid (analog/digital) feedback loop, which is illustrated in Figure 6.x. The system is based on a voltage controlled crystal oscillator (VCO or VCXO). This oscillator generates a signal whose frequency can be varied over a small range by changing its input voltage (control signal). The oscillator output frequency is monitored by a zero crossing counter. This counter is read and reset precisely once every second using the 1PPS (one pulse per second) signal derived from the GPS receiver. An adjustment of the oscillator control voltage made every second based on the deviation from the correct zero crossing count.

This scheme, known as GPS disciplined oscillator, can be made stable enough to provide timing accuracy of less than 0.5 microseconds. Typical GPS disciplined oscillator implementations can maintain accuracy below 1 microsecond even if the GPS clock looses satellite communications for up to several minutes. This is an important feature since in many GPS installations obstructions to the GPS antenna view of the satellites (buildings, trees, mountains etc) may cause momentary loss of satellite reception several times each day.

Figure 6.x illustrates a block diagram of a digitizer channel, with protection and isolation features, as well as synchronization to external timing signal.

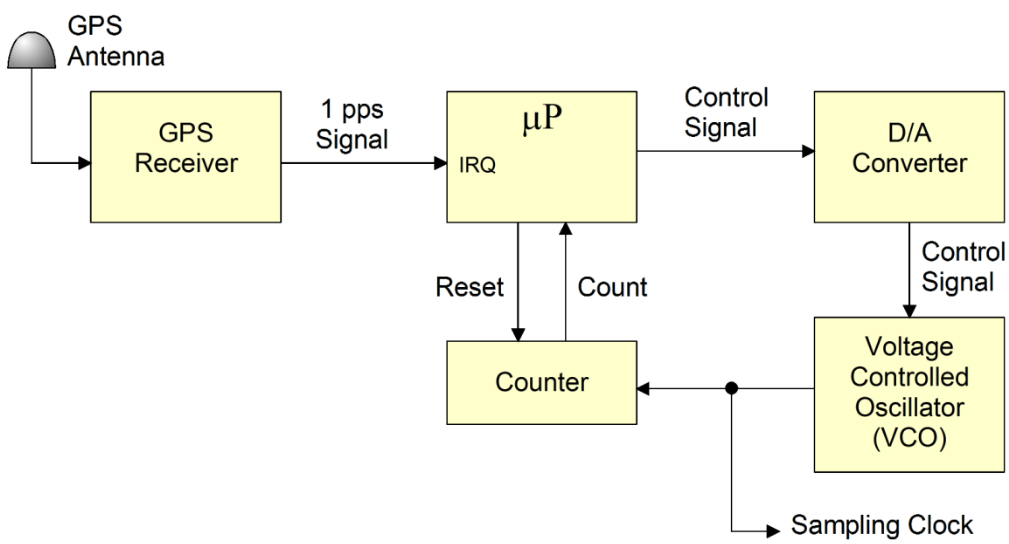


Figure 6.68: Sampling Clock Synchronization Implementation

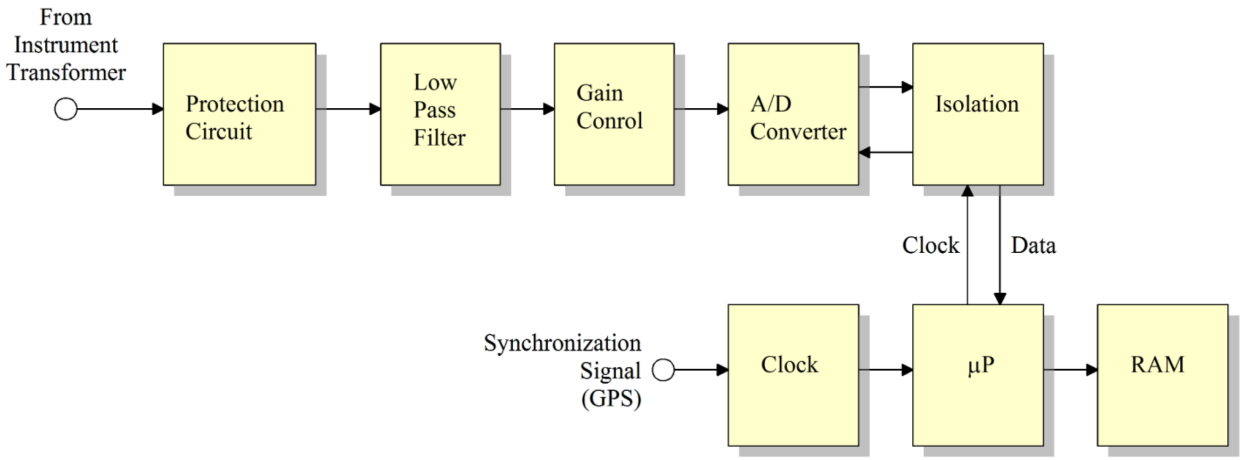


Figure 6.69: Block Diagram Illustrating Digital Data Acquisition Channel Structure With Isolation and Synchronization Capability

The GPS system of satellites started in 1989 with the US placing six to nine satellites in orbit per year, creating a system that provides the GPS signal anywhere on earth. GPS synchronized data acquisition was first introduced in Phasor Measurement Units (PMU's), i.e. devices dedicated to direct measurement of voltage and current phasors with UTC time reference. The first PMU was introduced by Macrodyne in 1992. In the Macrodyne PMU, the GPS receiver was integrated within the PMU. However, GPS synchronized data acquisition is now becoming common to many digital relays.

Figure 6.70: Typical GPS Clocks for Power System Instrumentation Applications
Top: Schweitzer Engineering Laboratories SEL-2407
Bottom: Arbiter Systems 1064C

In order to reduce equipment cost, many relays and other data acquisition systems capable of synchronous sampling do not include a built in a clock source such as a GPS receiver, but accept a timing signal from an external clock. Thus, a single GPS receiver can be installed in a substation and provide timing signals to any number of data acquisition equipment. Two commercially available GPS receivers designed for substation applications are illustrated in Figure 6.70.

The timing signal standard most commonly used for transmitting timing information from GPS receivers to relays and fault recorders is the IRIG time code format. The IRIG time code standard was developed by the Telecommunications Working Group of Inter-Range Instrumentation Group (IRIG), which is the standards part of Range Commanders Council (RCC). Work on this standard started in 1956, while the latest version is IRIG Standard 200-04, published in September 2004. The standard contains several different time-codes identified by alphabetic designations A, B, D, E, G, and H. The most commonly used code is IRIG-B, which is briefly described next

6.6.5: The IRIG-B Time Code

The IRIG-B code is a serially transmitted binary signal consisting of a sequence of "frames". Each frame contains time and date information plus some additional "Control Function" bits that are reserved for special applications. A frame is transmitted every second and consists of binary signal containing 74 bits, and each bit is represented by a variable width pulse occurring within a 10 millisecond interval. The information in the 74 bits of a frame is listed in Table 6.4. Note that the first 6 numbers forming the time stamp are encoded in Binary Coded Decimal (BCD). The Control bits are not defined by the standard as they are for internal use by the Range Commanders Council. The seconds of day are in *straight* binary form, and represent the number of seconds since midnight.

Table 6.4: IRIG-B Frame Information Encoding

Number of Bits	Encoding	Information
7	BCD	Seconds of Minute (0-59)
7	BCD	Minutes of Hour (0-59)
6	BCD	Hours of Day (0-24)
10	BCD	Days of Year (0-366)
9	BCD	Year (last two digits)
18	Binary	Control Bits
17	Binary	Seconds of Day (0-86399)

IRIG-B signal may be transmitted in three modes:

- Unmodulated
- Amplitude Modulated
- Modified Manchester Modulated

A snapshot of an unmodulated and an amplitude modulated IRIG signal are illustrated in Figure 6.71. The modulated carrier signal frequency is specified at 1 kHz. Therefore each 10ms period during which a bit is transmitted contains 10 carrier waveform cycles. An advantage of the modulated IRIG-B transmission is that it contains no DC component and thus it can be easily transmitted through galvanic isolation circuits such as transformers or capacitive coupled stages. A major disadvantage is that the time resolution is considerably inferior to the un-modulated transmission. Specifically, the un-modulated signal time resolution is typically in the order of 100 nanoseconds. This is facilitated by the short rise time of the pulses comprising the un-modulated signal. On the other hand, the modulated signal rise time is determined by the 1 kHz carrier frequency, resulting in time resolution in the order of 100 microseconds. A minor disadvantage of the un-modulated IRIG signal is that it contains a significant DC component which is a function of the transmitted data. Thus passing this signal through DC blocking galvanic isolation circuits may cause signal deterioration and data errors. The Manchester Modulated transmission mode combines the advantages of the other two modes. Detailed description of Manchester Modulated schemes are described in the IRIG standard. Note also that transmission of un-modulated IRIG signal over long distances is prone to degradation and corruption, and thus the connecting coaxial cable length must be limited to about 100 feet.

Figure 6.72 shows an example of a complete un-modulated IRIG-B frame. Note that the frame starts with two 8 millisecond pulses which indicate the beginning of the frame (P0). Binary ones are represented by 5 millisecond pulses while binary zeroes are represented by 2 millisecond pulses. Additional single 8 millisecond pulses separate the sequential data fields (seconds, minutes, hours, etc.).

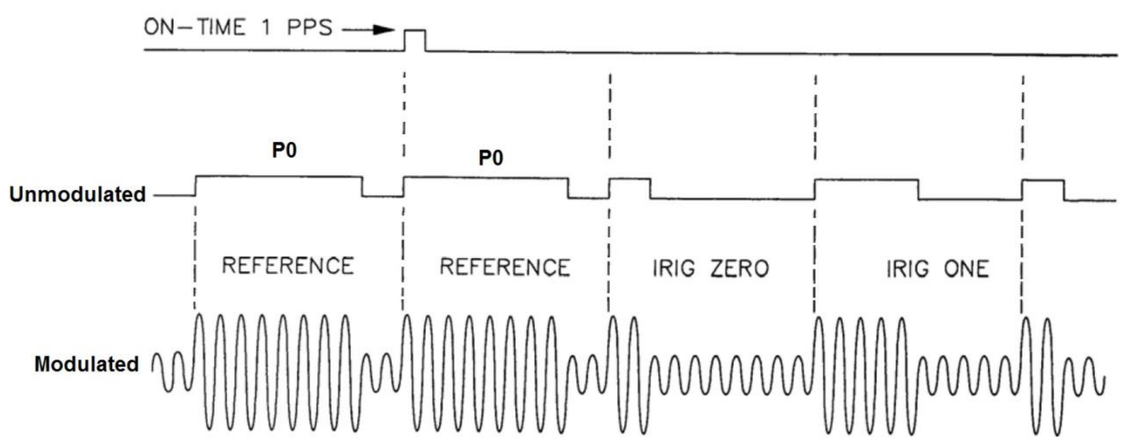


Figure 6.71: A Snapshot of an Unmodulated and a Modulated IRIG-B Signal

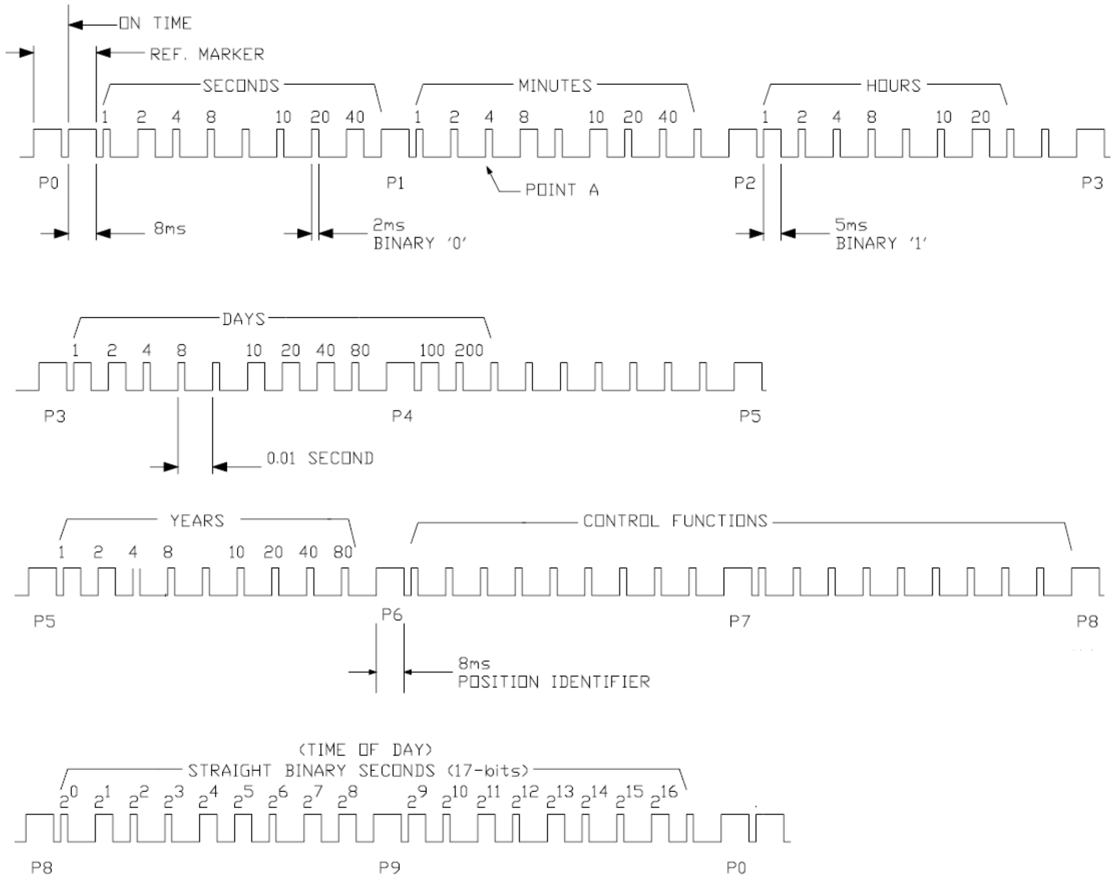


Figure 6.72: Example of IRIG-B Un-modulated Frame

6.6.6 Time Tagging Versus True Synchronous Sampling

PMUs use a UTC synchronized reference clock to provide a sequence of phasor measurements with phase angle referenced to UTC time using two approaches: (a) Time tagging and (b) synchronized sampling. In the **time tagging** approach the sampling clock of the A/D converters is free running (i.e. not synchronized to UTC in any way) and each sample is assigned a time tag by reading the reference clock at the time each sample is taken. From the time tagged data samples at the desired time instants are estimated by interpolation.

In the **synchronized sampling** approach the A/D converter clock is synchronized to UTC so that a sample is always taken within a microsecond or less from the at the UTC second rollover. This approach ensures that phasors can be directly calculated at the desired time intervals without requiring interpolation. The obvious advantage of the synchronized sampling approach is that it avoids interpolation errors, which may be significant during transients. Of course, the hardware required for the implementation of synchronized sampling is more complex as it requires a dedicated A/D converter for every channel, driven by a common sampling clock "disciplined" to the UTC second rollover. (Phase-locked-loop technology is a common approach in achieving the required clock synchronization). Figure 6.73 illustrates the hardware organization required for synchronized sampling.

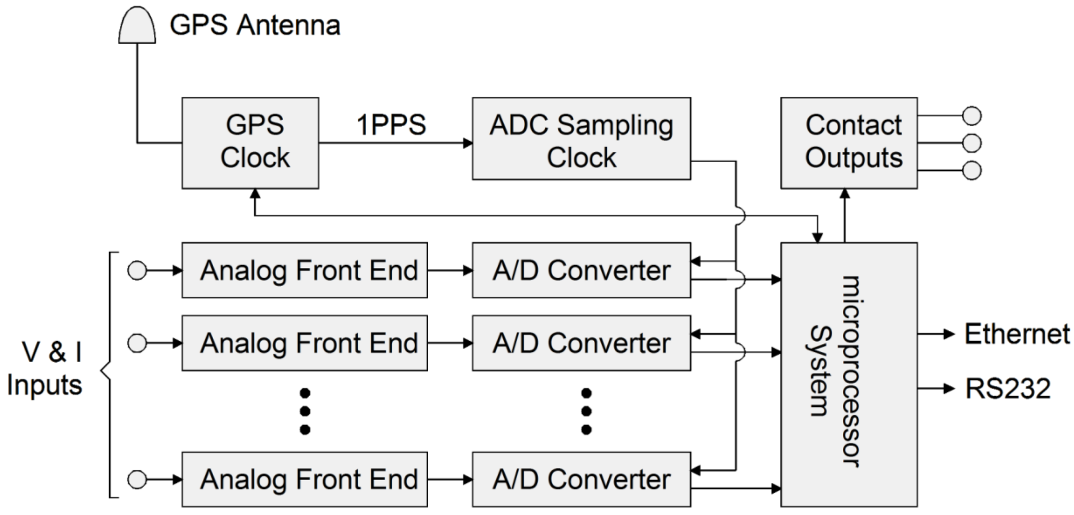


Figure 6.73: Block Diagram of Data Acquisition System with Synchronized Sampling Capability

6.6.7 Interoperability

Interoperability is a property referring to the ability of diverse systems to work together.

Additional Topics to be Added

- Synchronized sampling versus time stamping.
- Effects of A/D conversion
- Cost Reduction
- Relaying Versus Metering Requirements
- Isolation Technologies (Optical/Magnetic/Capacitive)

6.7 Data Fidelity and Measurement Error Correction

Relaying, metering and disturbance recording uses a system of instrument transformers to scale the power system voltages and currents into instrumentation level voltages and currents. Standard instrumentation level voltages and currents are 67V or 115V and 5A respectively. These standards were established many years ago to accommodate the electromechanical relays. Today, the instrument transformers are still in use but because modern relays, metering and disturbance recording operates at much lower voltages, it is necessary to apply another transformation from the previously defined standard voltages and currents to another set of standard voltages of 10V or 2V. This means that the modern instrumentation channel consists of typically two transformations and additional wiring and possibly burdens. Figure 6.74 illustrates typical instrumentation channels, a voltage channel and a current channel.

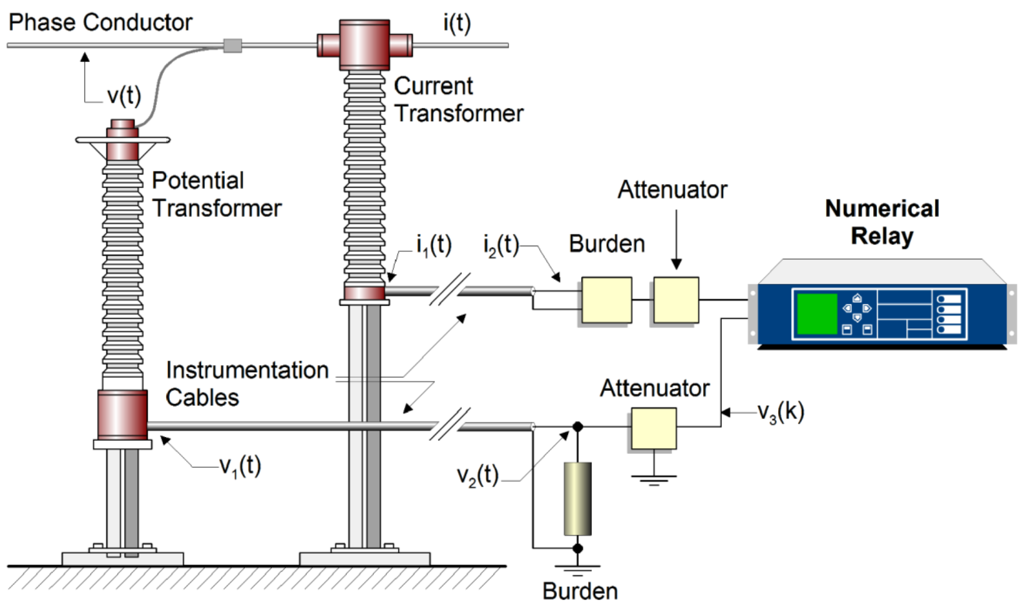


Figure 6.74: Typical Voltage and Current Instrumentation Channel

Each channel consists of the following components, each of which is a potential source of error:

- Instrument transformer
- Control Cable
- Burden
- Attenuator (optional)
- Digitizer (A/D Conversion)

Of importance is the net error introduced by all the components of the instrumentation channel. The overall error can be defined as follows. Let the voltage or current at the power system be:

$$v_a(t), \quad i_a(t)$$

An ideal instrumentation channel will generate a waveform at the output of the channel that will be an exact replica of the waveform at the power system. If the nominal transformation ratio is k_v and k_i for the voltage and current instrumentation channels respectively, then the output of the ideal channels will be:

$$v_{ideal}(t) = k_v v_a(t), \quad i_{ideal}(t) = k_i i_a(t)$$

The error is defined as follows:

$$v_{error}(t) = v_{out}(t) - v_{ideal}(t), \quad i_{error}(t) = i_{out}(t) - i_{ideal}(t)$$

where the subscript "out" refers to the actual output of the instrumentation channel. The error waveform can be analyzed to provide the rms value of the error, the phase error, etc. The instrumentation error can be computed by appropriate models of the entire instrumentation channel. It is important to note that some components may be subject to saturation (CTs and PTs) while other components may include resonant circuits with difficult to model behavior (CCVTs).

In subsequent paragraphs we present examples of errors introduced by three classes of instrumentation channel based on conventional CTs, PTs and CCVTs.

Conventional CTs Based Instrumentation Channels: The conventional CT steady state response is very accurate. The steady state response can be extracted from the frequency response of the device. Figure 6.75 provides a typical frequency response of a CT. Note that the response is flat in the frequency range of interest. It is important to note that errors may be present due to inaccurate determination of the transformation ratio. These errors are typically small.

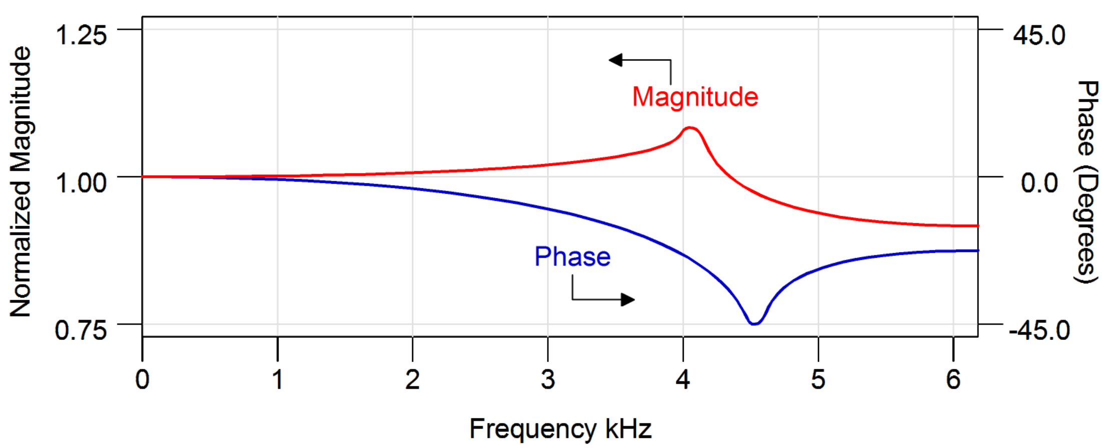
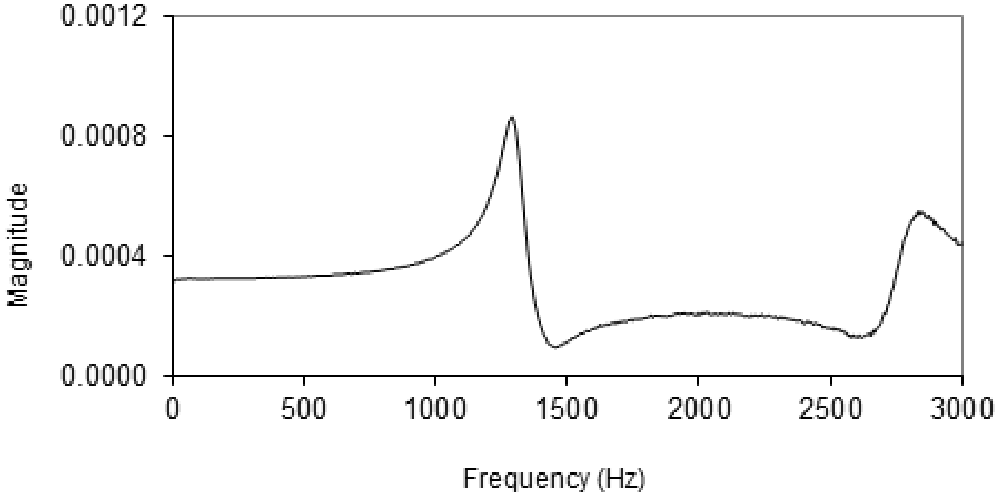


Figure 6.75: Typical 600 V Metering Class CT Frequency Response

Wound PTs Based Instrumentation Channels: Wound type PTs are in general less accurate than CTs. Again the steady state response can be obtained from the frequency response of the device. Figure 6.76 provides a typical frequency response of a wound type PT. Note that the response is flat in a small frequency range around the nominal frequency. Our work has shown that the higher the transformation ratio of the PT the higher the errors will be.



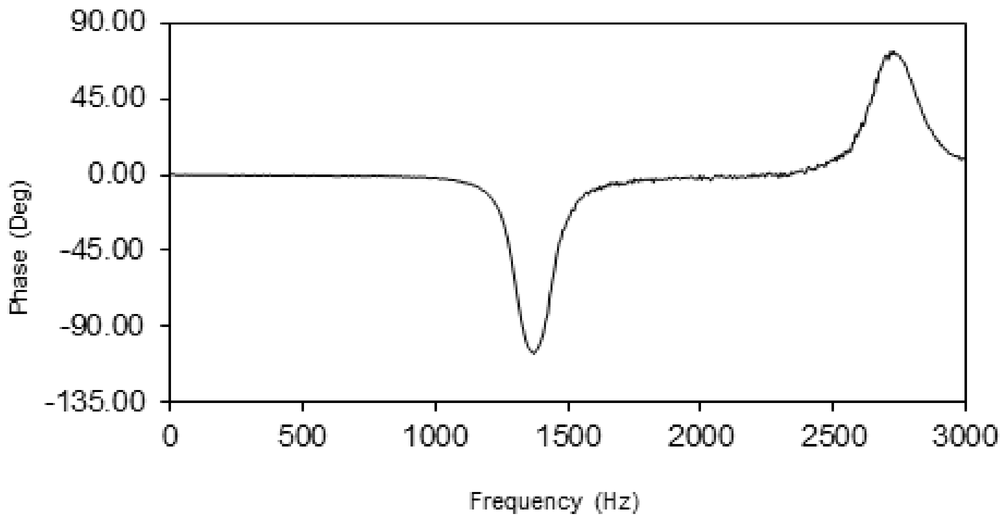


Figure 6.76: 200kV/115V Potential Transformer Frequency Response

CCVT Based Instrumentation Channels: By appropriate selection of the circuit components a CCVT can be designed to generate an output voltage with any desirable transformation ratio and most importantly with zero phase shift between input and output voltage waveforms. In this section we examine the possible deviations from this ideal behavior due to various causes by means of a parametric analysis, namely:

- Power Frequency Drift
- Circuit component parameter Drift
- Burden Impedance

The parametric analysis was performed using the CCVT equivalent circuit model illustrated in Figure B.3. The model parameters are given in Table 6.x:

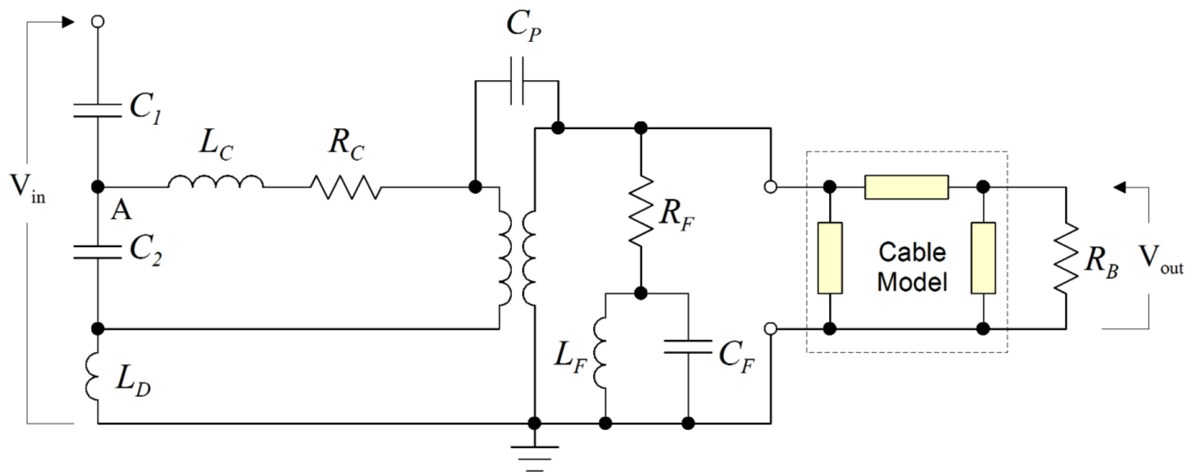


Figure 6.77: CCVT Equivalent Circuit

Table 6.x: CCVT Equivalent Circuit Parameters

Parameter Description	Schematic Reference	Value
CCVT Capacitance Class		Normal
Input Voltage		288 kV
Output Voltage		120 V
Upper Capacitor Size	C1	1.407 nF
Lower Capacitor Size	C2	99.9 nF
Drain Inductor	L_D	2.65 mH
Compensating Reactor Inductance	L_C	68.74 H
Compensating Reactor Resistance	R_C	3000 Ohms
Burden Resistance	R_B	200 Ohms
Ferroresonance Suppression Damping Resistor	R_F	70 Ohms
Ferroresonance Suppression Circuit Inductor	L_F	0.398 H
Ferroresonance Suppression Circuit Capacitor	C_F	17.7 uF
Cable Type		RG-8
Cable Length		100 Feet
Transformer Power Rating		300 VA
Transformer Voltage Rating		4kV/120V
Leakage Reactance		3%
Parasitic Capacitance	C_P	500 pF

Figure B.4 shows the results of a frequency scan. Note that over the frequency range of 0 to 500 Hz the response varies substantially both in magnitude and phase. Near 60 Hz (55 to 65 Hz) the response magnitude is practically constant but the phase varies at the rate of 0.25 degrees per Hz.

Table 6.x shows the results of a parametric analysis with respect to Burden resistance and instrumentation cable length. Note that the system is tuned for zero phase error for a short instrumentation cable and with a 200 Ohm Burden.

Table 6.x shows the results of a parametric analysis with respect to CCVT component parameter inaccuracies. Specifically the varied parameters were the compensating reactor inductance and the capacitive divider capacitance.

Table 6.x: Phase Error (in Degrees) Versus Burden Resistance and Cable Length

	Cable Length (feet)		
Burden Resistance	**10'**	**1000'**	**2000'**
50 Ohms	0.077	-0.155	-0.365
100 Ohms	0.026	-0.096	-0.213
200 Ohms	0.000	-0.063	-0.127
400 Ohms	-0.013	-0.047	-0.080
1000 Ohms	-0.022	-0.036	-0.052

Table 6.x: Phase Error (in Degrees) Versus Capacitance and Inductance

	Inductance Error (%)		
Capacitance Error (%)	**0%**	**1%**	**5%**
0%	0.000	-0.066	-0.331
-1%	-0.066	-0.132	-0.397
-5%	-0.330	-0.396	-0.661

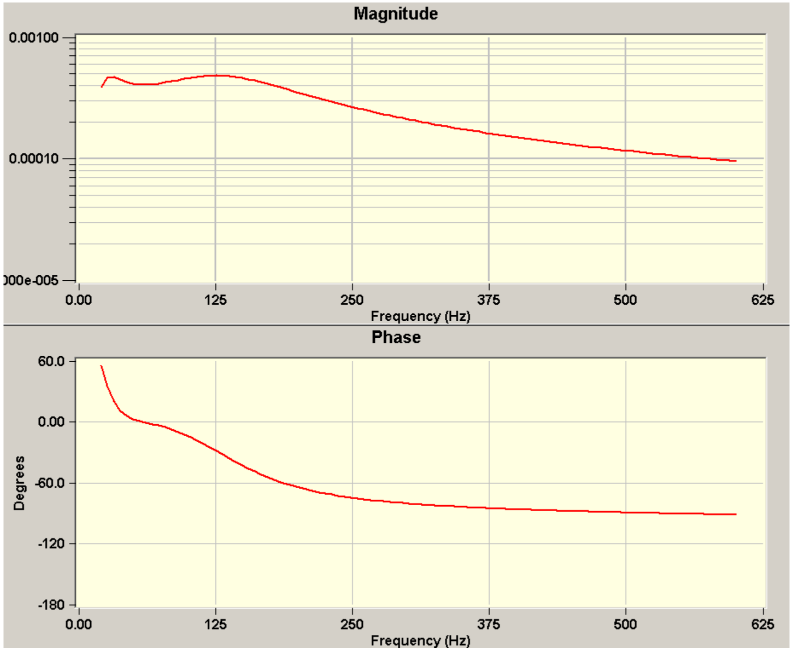


Figure 6.78: CCVT Computed Frequency Response over 10-600 Hz

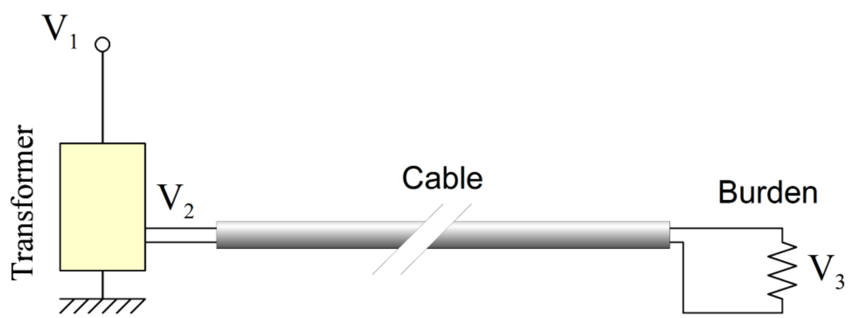
6.7.1 Instrument Transformer Error Correction

The error introduced by the instrumentation channel is usually computable. In this case it can be corrected. This concept is introduced by an example, next.

Example E6.X: Consider the voltage instrumentation channel of Figure E6.x. The parameters of the various components of the circuit are:

Compute the error introduced by the instrumentation channel, both magnitude and phase.

TO BE COMPLETED – State system parameters, ask for error at fundamental power frequency.



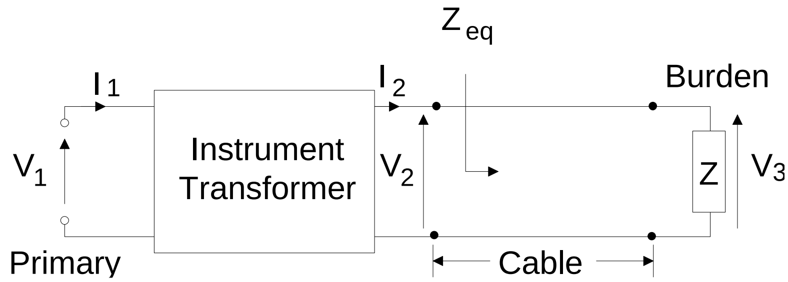


Figure E6.x

Solution: **TO BE COMPLETED**

$$V_1 = A(\omega)V_2 + B(\omega)I_2$$

$$\frac{V_1}{V_2} = A(\omega) + B(\omega)/\ Z_{eq}$$

$\frac{V_2}{V_3}$ Computed from Cable / Burden Parameters

$$H(\omega) = (\frac{V_1}{V_2}) * (\frac{V_2}{V_3})$$

6.7.2 Control Cable Error Correction

Control cables can introduce frequency response errors to a data acquisition system. The frequency response error can be corrected by time or frequency domain techniques, once the frequency response of the cable is known. The cable frequency response is computed by constructing a physically based mathematical model of the cable.

TO BE COMPLETED

6.7.3 Digitizer Error Correction

Digitizers introduce frequency response errors. These errors are mainly due to the A/D converter technology that is used. This error can be corrected in software. Figure 6.79 illustrates the frequency response of a Crystal Semiconductor A/D converter. Note that the phase response indicates a considerable delay mostly due to the digital filter computations.

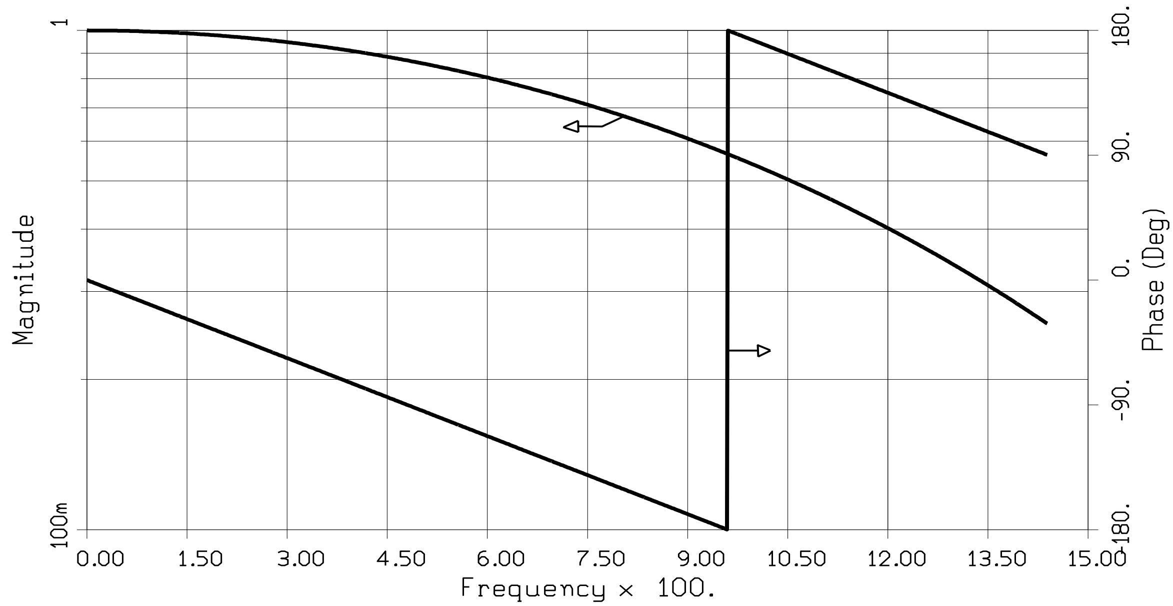


Figure 6.79: Crystal Semiconductor CS1707 A/D Converter Frequency Response.

The frequency response error correction can be performed either in the frequency domain or in the time domain. The frequency domain error correction consists of simply multiplying the Fourier transform of the converter output by the inverse of the converter response. However, in relaying applications it is often necessary to work with time domain signals. Time domain error correction can be accomplished by convolution of the converter signal output with the inverse Fourier transform of the converter inverse of the converter response.

TO BE COMPLETED

Additional Topics:

- Amplitude Linearity and Quantization Error
- Time Jitter Error

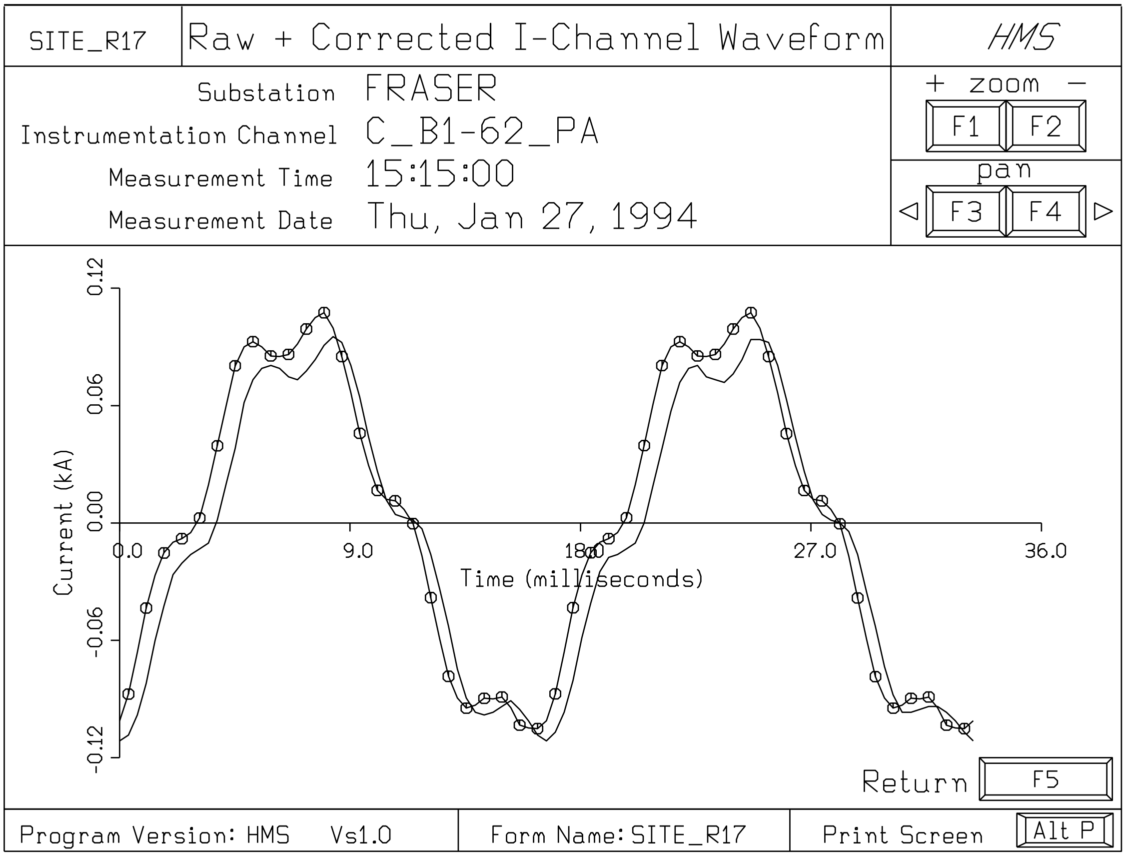


Figure 6.80: Raw and Corrected Digitally Acquired Waveform Data.

Next, we present a method for the simulation of the instrumentation channel. The method predicts with precision the instrumentation channel error. The method is also suitable for visualization of the operation of the instrumentation channel. Continuous visualization of the instrumentation channel errors provides a useful animation of the evolution of the errors.

6.8 Instrument Channel Computer Simulation

The instrumentation channel model provides the voltages and currents at any point of the instrumentation channel at any time. It is possible to generate a snapshot of the voltages and currents at the various location of then instrumentation channel and to compute metrics of the error between the actual voltages and currents and the ideal values. These metrics can be displayed on the same frame. As the simulation progresses, the visualization display is refreshed providing the sense of animation.

In the next section, we present two applications: (a) one involving a current instrumentation channel and (b) another that involves a voltage instrumentation channel. For each one of these instrumentation channels, we present visualization results of the overall channel error for specific operating conditions. Several applications of the proposed instrumentation channel are presented here. The first example illustrates the visualization of CT saturation and its effects on recorded data accuracy. It is demonstrated that CT saturation is affected by control cable length as well as total burden on the CT. The second example illustrates the effect of instrumentation channel error on the operation of relays.

TO BE COMPLETED

6.9 The Emergence of Merging Units

Recent technological advances resulted in the development of merging units. This technology is intended to address the issues associated with the fidelity of the data acquisition systems for relaying, metering and other applications. Traditional substation instrumentation typically includes long runs of cables connecting the secondary outputs of instrument transformers to multiple relays (burdens). Long cables and multiple burdens introduce measurement errors. Furthermore, the resulting system complexity increases the installation and maintenance effort and cost.

Merging units are devices which digitally sample the output of instrument transformers and transmit the sampled waveforms in digital form to a substation local-area-network known as the *process bus.* Figure 6.81 shows the hardware setup of a typical merging unit. It consists of several voltage and current data acquisition channels, status inputs and contact outputs. Voltage and current data acquisition sampling rates have been standardized to 80 samples per nominal power frequency cycle (i.e.: 4800 samples per second for 60 Hz systems and 4000 samples per second for 50 Hz systems). Optionally the sampling may be synchronized to UTC time, thus allowing devices and applications using data from different locations to accurately compute phasor angles referenced to a common clock. Both time synchronization signals, as well as the merging unit digital output are typically transmitted using optical fibers. Optical fiber transmission provides immunity from EMI which is a common issue in substations. Figure 8.82 illustrates several commercially available Merging Units.

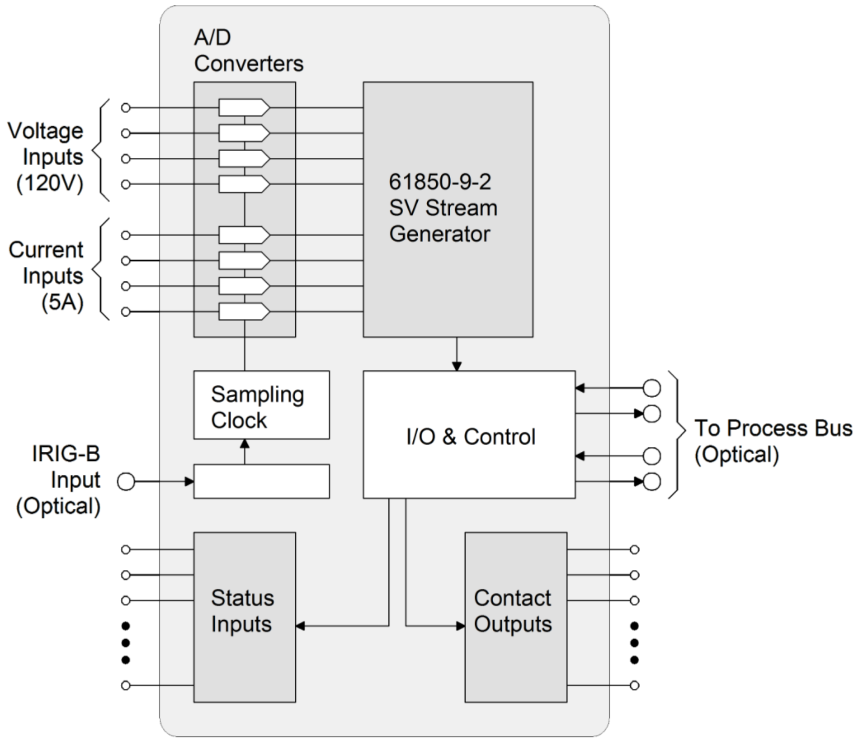
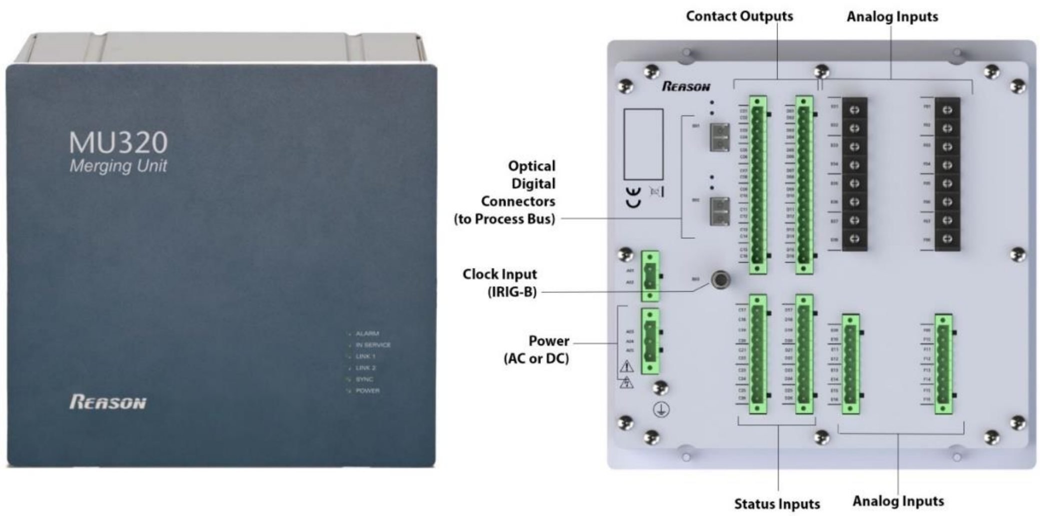


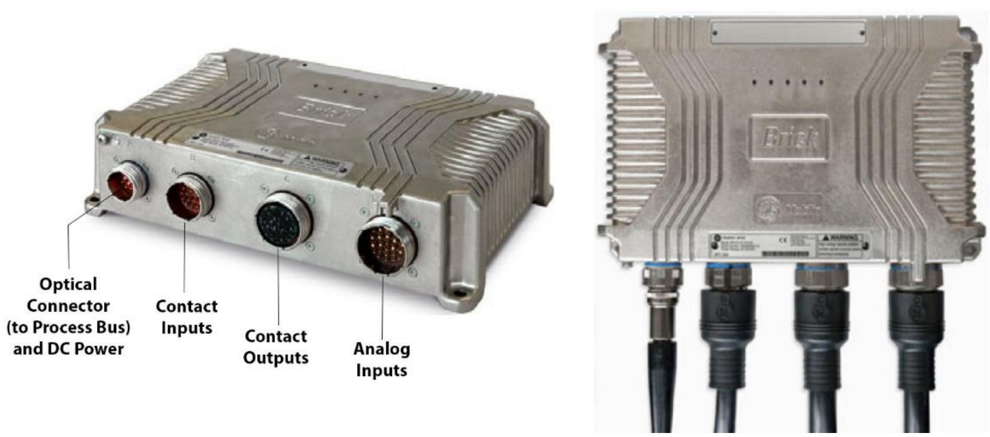
Figure 6.81: Typical Merging Unit Hardware Organization

The merging unit output data format and communication protocol are based on the IEC 61850 standard. It is expected that this approach provides interoperability, i.e. merging units, relays, and other intelligent electronic devices from various manufacturers should be able to operate together and share the measured data. However, in practice, since the IEC61850 standard is very complex, and leaves many aspects of data definitions up to the equipment manufacturers, this may not be so straightforward.

The application of merging units to typical substation arrangements is illustrated in Figures 6.83, 6.84, an 6.85. Protective relays and other devices pick up the desired measurements from the process bus in digital form. This approach reduces the required wiring as each instrument transformer analog output must only be connected to a single data acquisition device. Furthermore, some merging units are designed to be located outdoors, (and in close proximity to the instrument transformers) thus further reducing the secondary cable lengths.



(a)



(b)

(c)

Figure 6.82: Examples of Merging Units by Reason and GE
(a) GE Reason MU320, (b) GE Hard Fiber, (c) Siemens Siprotec

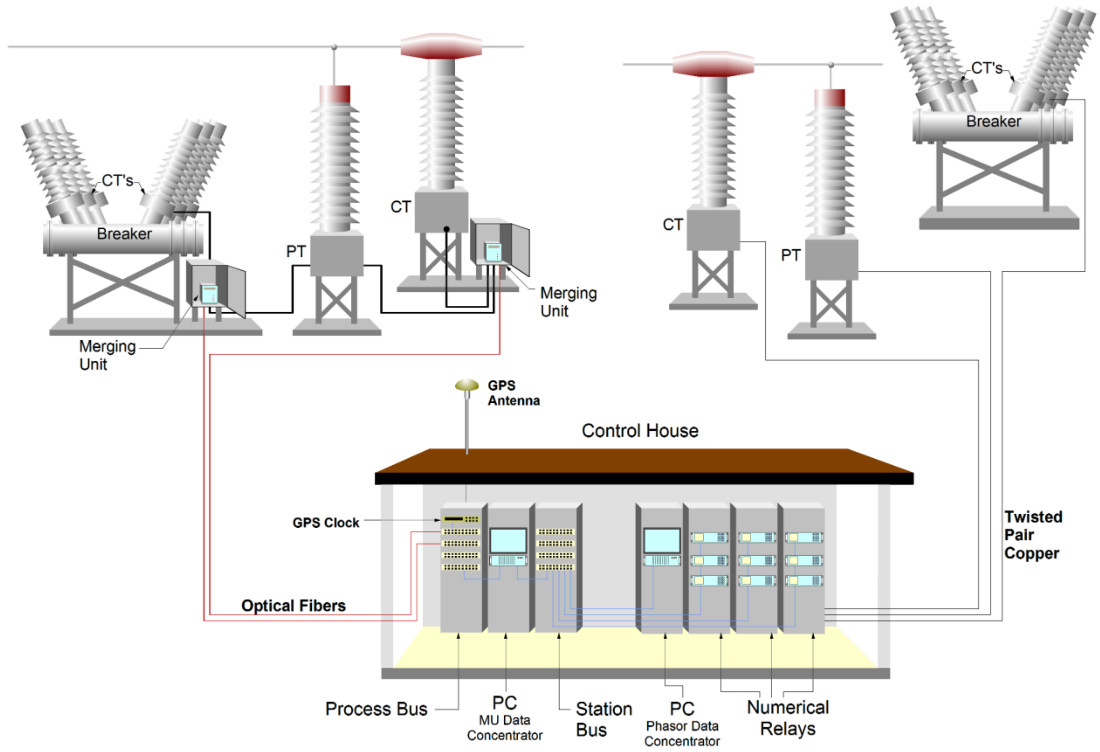


Figure 6.83: Merging Units Physical Arrangement Example

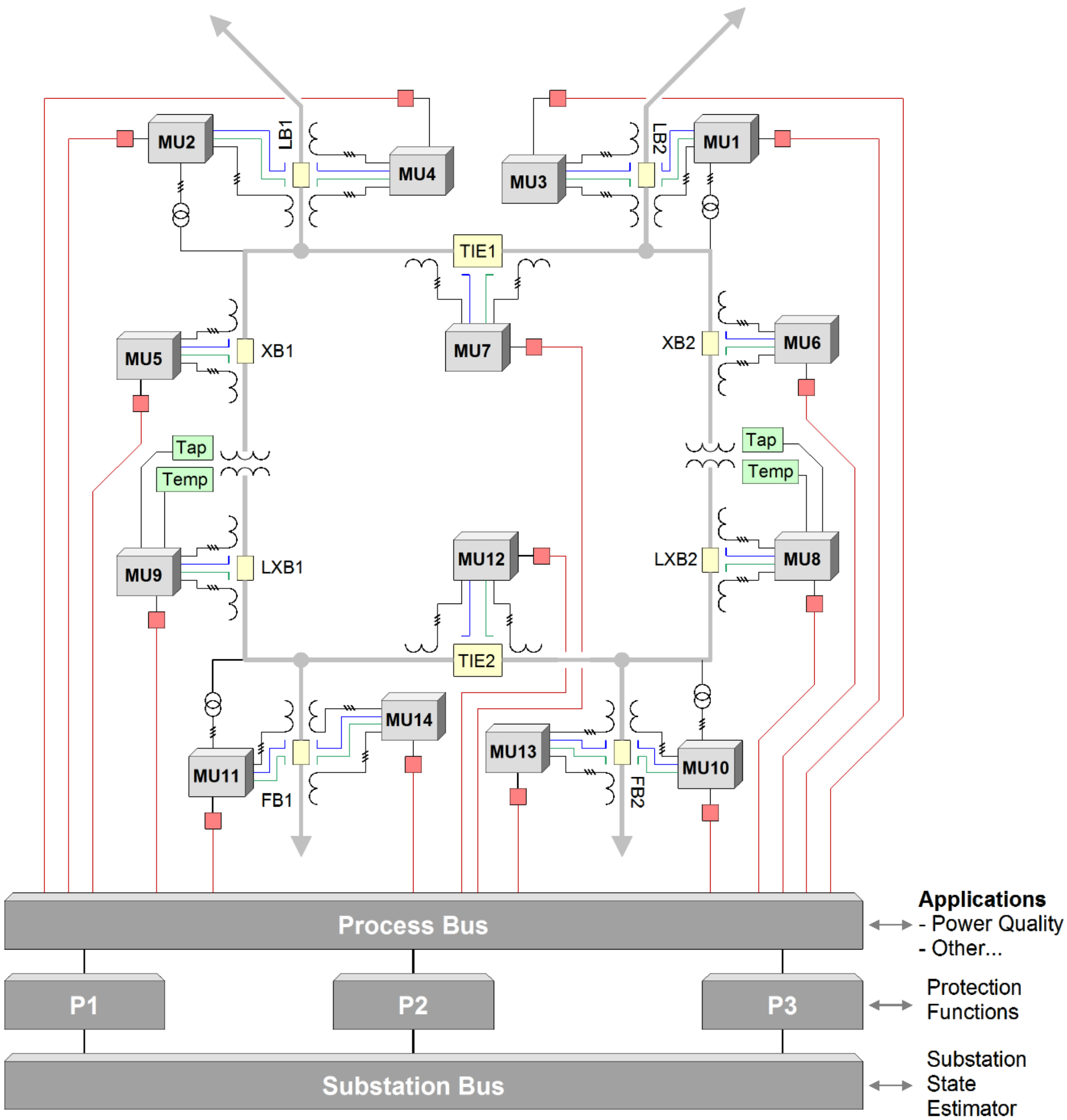


Figure 6.84: Merging Unit Application Organization

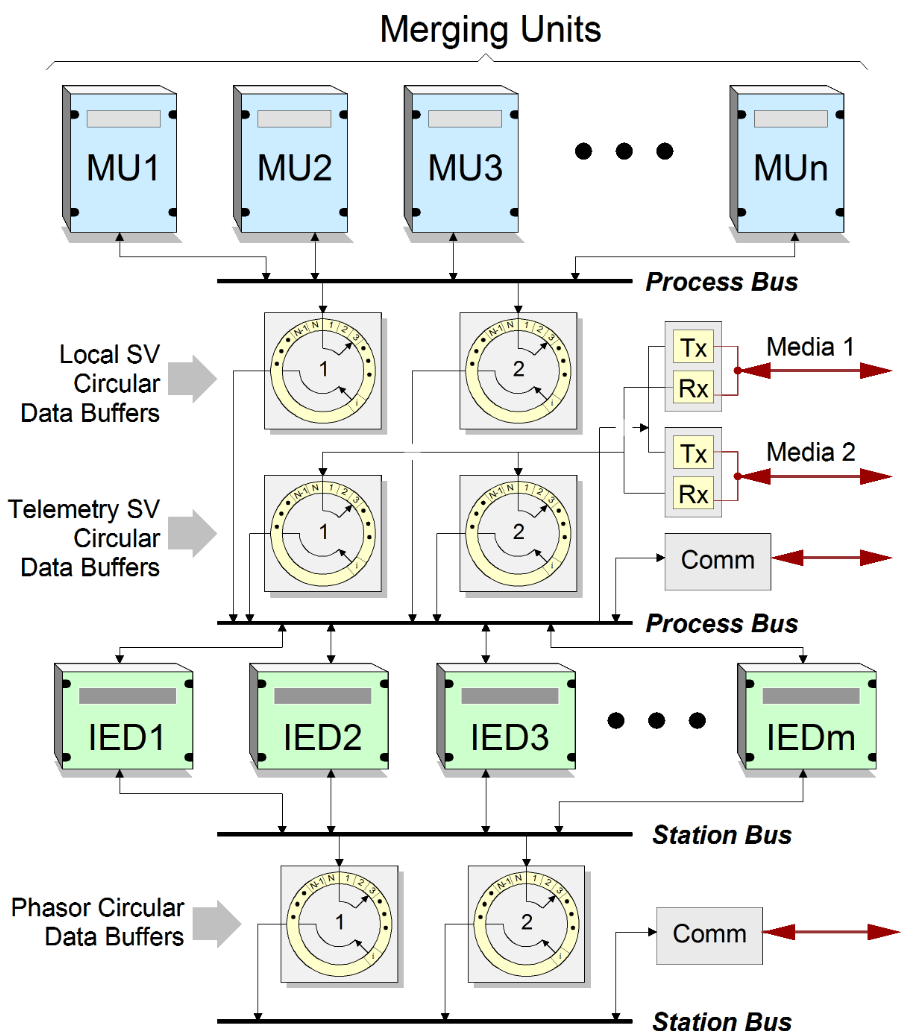


Figure 6.85: Merging Unit Application Organization

6.10 The IEEE Synchrophasor Standard

A standard defining both the measurement procedures and transmission of phasor measurements has been created by IEEE. The original standard was created in 1995, updated in 2005 and in 2011. The 2011 version has been split into two documents, Std-C37.118.1 and Std-C37.118.2. The first document covers measurement techniques while the second document with communications. A brief description of the concepts defined in the standard follows.

The standard introduces the term "Synchrophasor" defined as a phasor with magnitude equal to the RMS value of a monitored voltage or current waveform, and phase angle defined using a cosine function, at the nominal system frequency, referenced to "Universal Coordinated Time" (UTC). Specifically the magnitude X_m and phase φ angle are defined by the equation:

$$x(t) = \sqrt{2}X_m \cos(\omega t + \varphi)$$

where x(t) is the monitored voltage or current waveform and ω is the nominal system base frequency. From this definition follows that a synchrophasor with 0 degrees phase angle corresponds to a sinusoidal function which reaches its maximum value at the UTC second rollover, while a sinusoidal function with a positive slope zero crossing occurring at the UTC second has a phase angle of -90 degrees.

Note that since the synchrophasors are always referenced to the nominal system base frequency, the phase angle of a synchrophasor with frequency lower than the nominal frequency will decrease with time. This phenomenon is illustrated graphically in Figure 6.86.

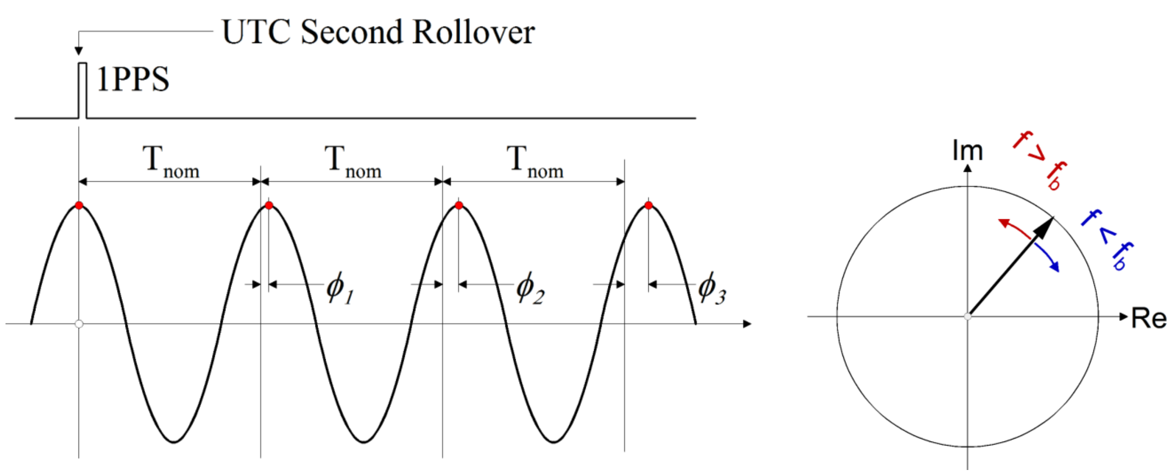


Figure 6.86: Synchrophasor Phase Angle Variation at Off-Nominal Frequencies

The first cycle in this Figure reaches its maximum value on the UTC second rollover time instant, while successive cycles reach their maximum values at time instants which are increasingly delayed from the start of the corresponding nominal period window. Plotting this synchrophasor on a complex plane it will appear to rotate clockwise at a rate equal to the difference between the nominal and actual frequencies. Conversely, if the actual frequency is higher than the nominal frequency the synchrophasor will rotate counterclockwise.

Note that the UTC second rollover time instant corresponds to the rising edge of a timing signal referred to as a "One Pulse per Second" signal (1PPS) (See Figure 2.21). This signal a common output of GPS based clocks. It is a signal that provides synchronization accuracy with typical resolution of less than a microsecond. This signal is typically transmitted along with an AM modulated IRIG signal in order to achieve high resolution synchronization. It is unnecessary if the IRIG signal is unmodulated, or "Modified Manchester" modulated.

The IEEE synchrophasor standard requires that a device that measures phasors, referred to as Phasor Measurement Unit (PMU) generates a data stream containing synchrophasors of the monitored voltage and current waveforms, as well as the system frequency and the rate of change of frequency (ROCOF) derived from the monitored waveforms. These data are to be written into "Data Frames" and transmitted typically via a wide area network to other devices. The standard defines a minimum set of data frame transmission rates that a PMU must be capable of generating. For 60 Hz systems PMUs must be able to generate synchrophasor data frames at the rates of 10, 12, 15, 20, 30, and 60 synchrophasors per second. At all rates, there should always be a phasor corresponding on the UTC hour rollover, and the remaining samples taken at uniformly spaced time intervals.

The IEEE synchrophasor standard specifies limits on the synchrophasor measurement error, as well as frequency and rate of change of frequency measurement errors. The synchrophasor error is expressed in terms of the "Total Vector Error". The total vector error (TVE) is illustrated in Figure 6.87 and is defined as the magnitude of the difference between the measured phasor and the theoretical exact phasor. A disadvantage of the TVE is that it does not differentiate between magnitude and phase errors. For certain applications for which the phase error is more critical than the magnitude error (for example real power flow computations), a separate magnitude and phase error specification is preferable.

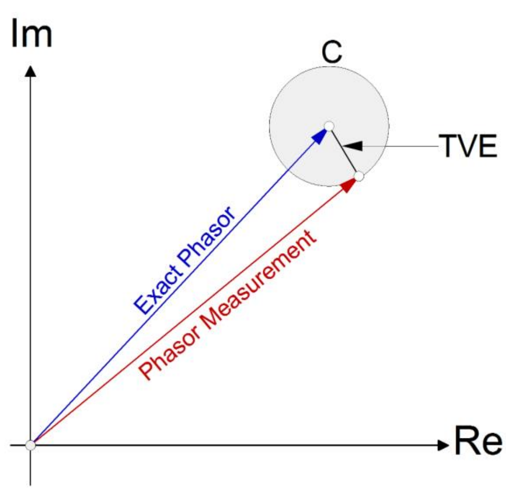


Figure 6.87: Total Vector Error

Note that the synchrophasor measurement error may vary considerably depending on the conditions of the measured signals. Depending on the algorithm used to extract phasors from sampled waveform data, the error may be considerable higher during transients (such as magnitude or frequency variation), and waveform harmonic distortion. For example in several PMU implementations averaging filters are used in order to reduce errors during steady state conditions. However, such filters may actually increase the computed phasor errors during transient conditions. Thus it is important to test phasor computation algorithms against a variety of signal conditions.

The IEEE synchrophasor standard includes a detailed specification of the format of the transmitted data. Specifically, two types of synchrophasor frames are defined: (a) configuration frames and (b) data frames. Configuration frames contain PMU setup information such as number of input channels, channel names, station name, numeric encoding (floating point or integer) etc. The information in configuration frames changes very infrequently, and thus configuration frames are transmitted only after a request from a device receiving the synchrophasor stream. Data frames contain the frame time stamp, clock quality indicators, the measured phasor values, the measured system frequency, and the ROCOF. The standard also allows the transmission of a number of analog and discrete values. These values are typically used to transmit station status data such as transformer tap settings and breaker status. All values in data frames are encoded in binary form, resulting in a very compact message format.

6.11 Summary and Discussion

This chapter presented typical instrumentation for relay applications.

TO BE COMPLETED.

6.12 Problems

Problem P6.1: A 3000:5 current transformer consists of a toroidal magnetic core. The cross section of the core is circular with a diameter of 4.0 cm. The core material saturates at B=0.5 Tesla. The total impedance on the secondary of the CT, including coil/wiring and burdens, is 2.8 ohms.

(a) Compute the maximum 60 Hz primary current that will not saturate the CT.

(b) An engineer desires that this CT should be able to develop a maximum voltage of 350 volts (rms) on the secondary without saturation. What should be the radius of the core in this case?

Solution: The number of turns on the secondary are: 3000/5=600. The maximum flux in the core is:

$$\varphi = (0.5T)\pi r^2 = 0.000628318 \text{ Wb}$$

And the maximum voltage that can be developed:

$$V = \omega\varphi(600) = 142.1\, V$$

The secondary current that causes this voltage will be: 35.89 Arms.

Problem P6.2: A protection engineer requires a 3000:5 current transformer for a specific application in which the net impedance of the secondary circuit will be 1.6 ohms and the maximum current in the secondary circuit may be 100 Amperes. The CT is designed with a toroidal magnetic core of circular cross section. The core material saturates at B=0.6 Tesla. Compute the radius of the core so that the CT will not saturate for any of the anticipated conditions.

Problem P6.3: The 60 Hz saturation characteristics of a 1200:5 current transformer are given in Figure P6.4. The total resistance on the secondary of the CT is 0.2 ohms (burden plus wire and cable resistance). (a) Compute the CT magnetizing inductance referred to the secondary assuming no saturation is occurring. (b) Compute the maximum primary direct current that the CT can sustain indefinitely without saturating. (c) Compute the maximum steady state 10 Hz sinusoidal primary current that the CT can sustain without saturation. (d) At t=0 a direct current of 240 Amperes starts flowing through the primary (no other current is present). Compute the time instant at which the CT will saturate. (Assume that prior to the DC current flow the CT core was not magnetized).

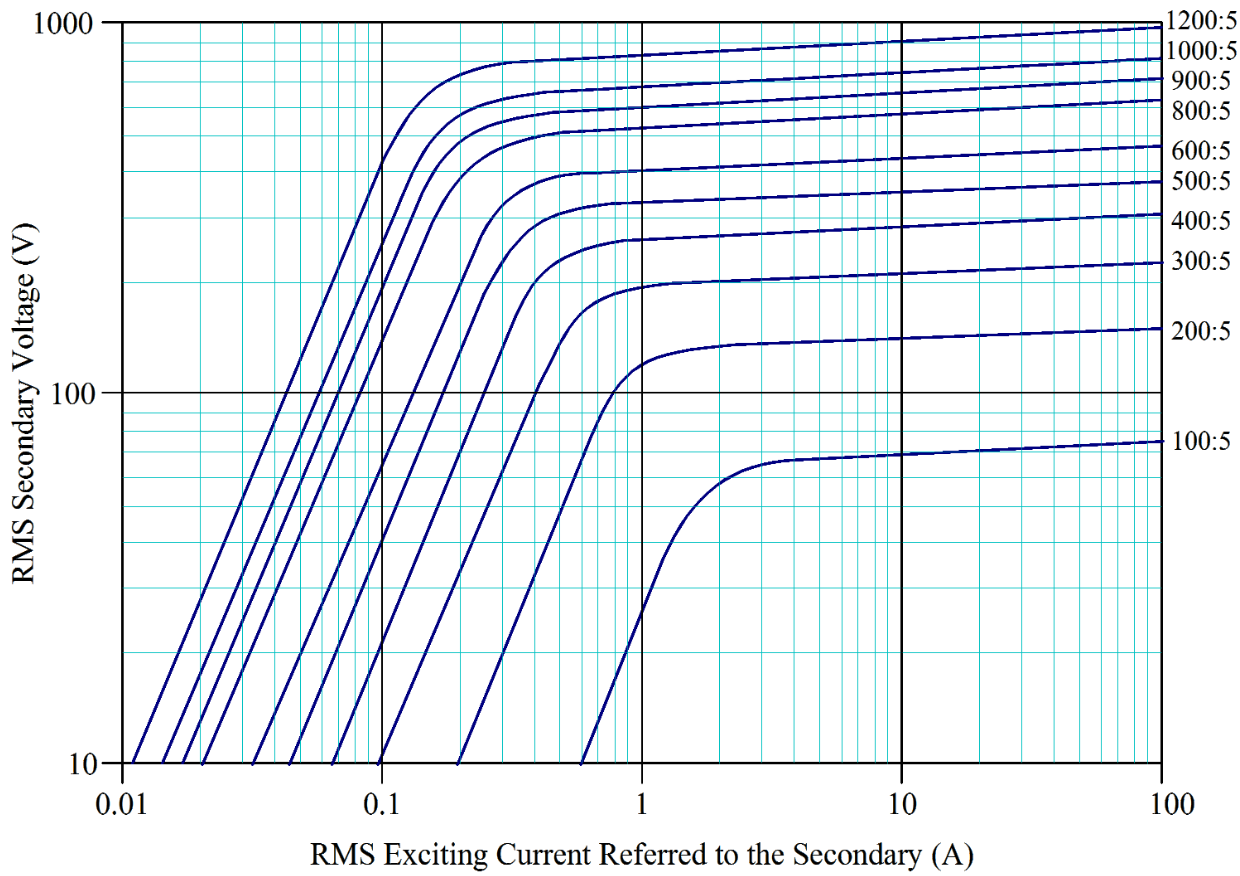


Figure P6.4

Solution: (a) From Figure P6.4 we get:

at 0.1 A the voltage is 425 volts. Thus

$$L_m = \frac{x_m}{\omega} = \frac{425V}{(0.1A)(377 rad/\sec)} = 11.27\,H$$

Problem P6.4: The equivalent circuit of a 200kV:67V potential transformer is illustrated in Figure P6.5. Compute the transfer function of this PT in the frequency range 10 Hz to 1200 Hz. The indicated parameters in the figure are: L_m=20,000 H, r_1=24,000 ohms, L_1=640 H, r_2=0.003 ohms , L_2=0.00007 H, C=2.5 nF.

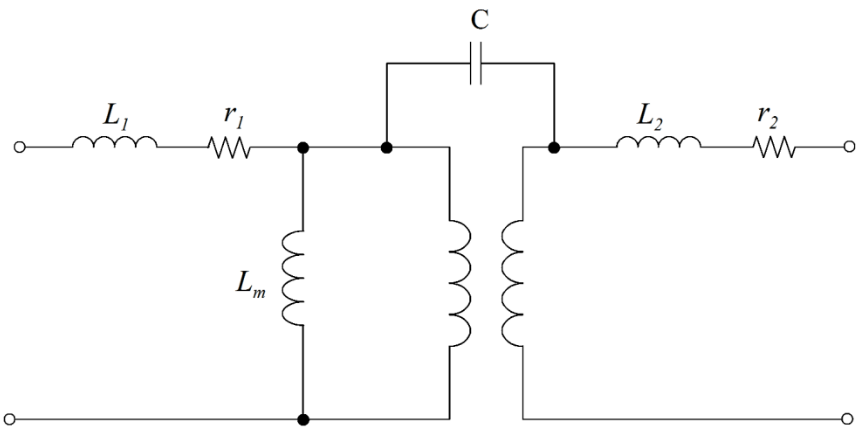


Figure P6.5

Problem P6.5: The equivalent circuit of a 2400:5 current transformer is illustrated in Figure P6.6. The total impedance at the secondary of this CT is Z=2.3+j0.0012f ohms, where f is the frequency in Hz. Compute the transfer function of this CT in the frequency range 10 Hz to 1200 Hz. The transfer function is defined as the ratio of the primary current over the current in the burden of the circuit.

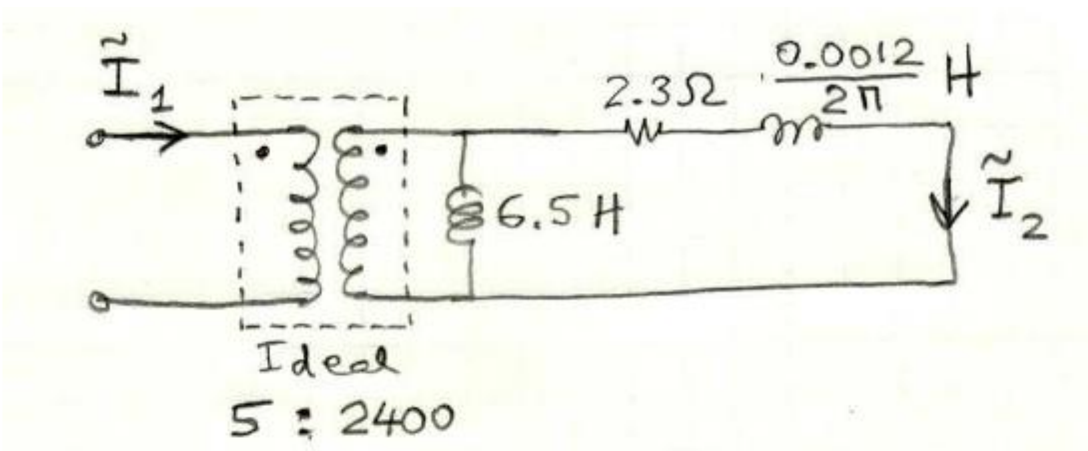


Figure P6.6

Problem P6.6: The electric current in a circuit has the following actual harmonics:

Harmonic Order	Harmonic Current Magnitude (Amperes)
1	120
3	25
5	11
7	5

The electric current is measured with a harmonic meter which has an 8 bit A/D converter and current setting of 1, 10, 100, 200 Amperes. Compute the minimum digitization error for the 7th harmonic.

Solution: The resolution of the A/D converter is:

$$\text{Resolution } = \frac{1}{2^8} = 0.00391$$

The minimum range that we can select is 200 A. At this range, the resolution in absolute amperes is:

(0.00391)(200A)=0.78125 A.

The maximum digitized error for the 7th harmonic is:

$$error = \frac{0.78125A}{5A} 100 = 15.6\%$$

Problem P6.7: It is desirable to monitor the harmonics on the 13.8 kV side of an incoming service to a power plant. For this reason, PTs and CTs are installed on the 13.8 kV side of the power transformer. The output of the PTs and CTs is brought into the control room that is located 600 ft from the transformer via 10 gauge copper conductor pairs. The meters are 12 bit digital recorders. Compute the overall error of the 11th harmonic measurement.

check below...

Problem P6.8: A 2400:5 current transformer consists of a toroidal magnetic core. The cross section of the core is circular with a diameter of 3.5 cm. The core material saturates at B=0.5 Tesla. The total impedance on the secondary of the CT, including winding impedance and burdens, is 2.8 ohms. Compute the maximum 60 Hz primary current that will not saturate the CT (i.e. compute the maximum 60 Hz primary current that will result in a maximum magnetic flux density in the core material of 0.5 Tesla).

An engineer desires that this CT should be able to develop a maximum voltage of 400 volts (rms) on the secondary when the secondary current is 100 Amperes without saturation. What should be the radius of the core in this case?

Solution: First the number of turns in the secondary circuit is computed from:

N1/N2=5/2400, N1=1. Thus N2=2400/5=480.

to be continued...

Problem P6.9: A 2400:5 current transformer consists of a toroidal magnetic core. The cross section of the core is circular with a diameter of 3.5 cm. The core material saturates at B=0.5

Tesla. The total impedance on the secondary of the CT is 2.8 ohms. Compute the maximum 60 Hz primary current that will not saturate the CT (i.e. compute the maximum 60 Hz primary current that will result in a maximum magnetic flux density in the core material of 0.5 Tesla).

An engineer desires that this CT should be able to develop a maximum voltage of 400 volts (rms) on the secondary when the secondary current is 100 Amperes without saturation. What should be the radius of the core in this case?

Solution:

(a) $A = \pi a^2 = \pi \left(\frac{0.035}{2}\right)^2 m^2 = 0.000962112 \ m^2$

At onset of Saturation $\varphi = A(0.5T) = 0.000481056 \ Wb$

$\lambda = \frac{2400}{5}\varphi = 0.230906 \ Wb$

$v(t) = \omega\lambda \cos\omega t = 87 \cos\omega t$

$\Rightarrow \quad V = 61.55 \ V_{rms}$

$I_{sec} \, 2.8 = V \quad \Rightarrow \quad I_{sec} = 21.987 \ A$

$\Rightarrow \quad I_{pri} = \frac{2400}{5} 21.987 \ A = 10{,}552 \ A$

(b)

$\frac{\omega\lambda}{\sqrt{2}} = 400 \qquad \lambda = \frac{2400}{5} A(0.5T)$

$\frac{\omega}{\sqrt{2}} \frac{2400}{5} A(0.5) = 400$

$\Rightarrow \quad A = 0.006252 = \pi a_1^2$

$\Rightarrow \quad a_1 = 0.0446 \ m = 4.46 \ cm$

DRAFT and INCOMPLETE

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from

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Chapter 7
Transformer Protection

7.1 Introduction

In this chapter, we focus on the protection of power transformers. There are many different designs of power transformers and the design and size of the transformer affects the protection functions and philosophy. The majority of power transformers are oil-filled (oil insulated), iron core transformers. Many of these transformers have regulating capability of (a) the voltage or (b) the phase shift between the two sides of the transformer can be changed under load. Voltage regulating transformers are equipped with a tap changing mechanism. Phase shifting regulating transformers are more complicated as they have additional coils that are tap controlled and can be inserted in series to the circuit. This chapter focuses on the power transformers with or without regulating capability. There are other types of power transformers, for example air insulated transformers. For the protection of other types of transformers, the reader is encouraged to consult a number of excellent references addressing the protection of these transformers.

The protection of the transformer requires that we protect against electrical abnormal conditions as well as mechanical abnormal conditions. The mechanical abnormalities that we must guard against are:

- low oil level in the main tank and load tap changing (LTC) compartment, if one exists
- high oil temperature in the main tank and LTC compartment
- high winding temperature (hot spot)
- sudden oil pressure increase due to a high through fault current or other events
- pressure relief device operation in the main tank and LTC compartment
- gas accumulation in oil
- loss of AC power for fans and pumps

For large power transformers continuous monitoring of the transformer mechanical quantities is justifiable. The most usual monitoring system may include sensors for oil temperature in the main tank and LTC compartment, hot spot temperature, oil level, winding temperature, status of fans and pumps. For any abnormal conditions in the oil, fans, pumps, etc. the monitoring system will issue alarms that must be acknowledged by an operator. The reason is that mechanical problems can be tolerated for a relatively long time during which the problem can be corrected without interrupting the operation of the transformer.

In addition, a number of electrical abnormal conditions may occur in the transformer. The types of phenomena that must be considered are:

- Internal faults, phase to ground, phase to phase, etc. (winding failures)
- Tap-hanger failures

- Turn to turn faults
- Bushing failures
- Core failures
- Terminal board failures
- Overexcitation
- Underfrequency
- Transformer core saturation
- Excessive harmonics and transformer overheating

For these conditions, relays should recognize the condition and disconnect the transformer before any additional damage can be inflicted on the transformer. Some of these faults require immediate disconnection of the transformer (for example internal faults) and some others develop over time to the point where damage will occur in the transformer (for example transformer core saturation). The protective schemes for protection of the transformer against mechanical and electrical abnormalities will be discussed next.

7.2 Transformer Protection Devices

Monitoring of transformer mechanical parameters is accomplished by means of several types of sensors and specialized relays. Figure 7.1 illustrates typical sensor arrangement on an oil cooled power transformer, including temperature sensors, and various types of oil pressure and oil flow rate sensors and relays. In the next paragraphs we provide brief descriptions of these devices.

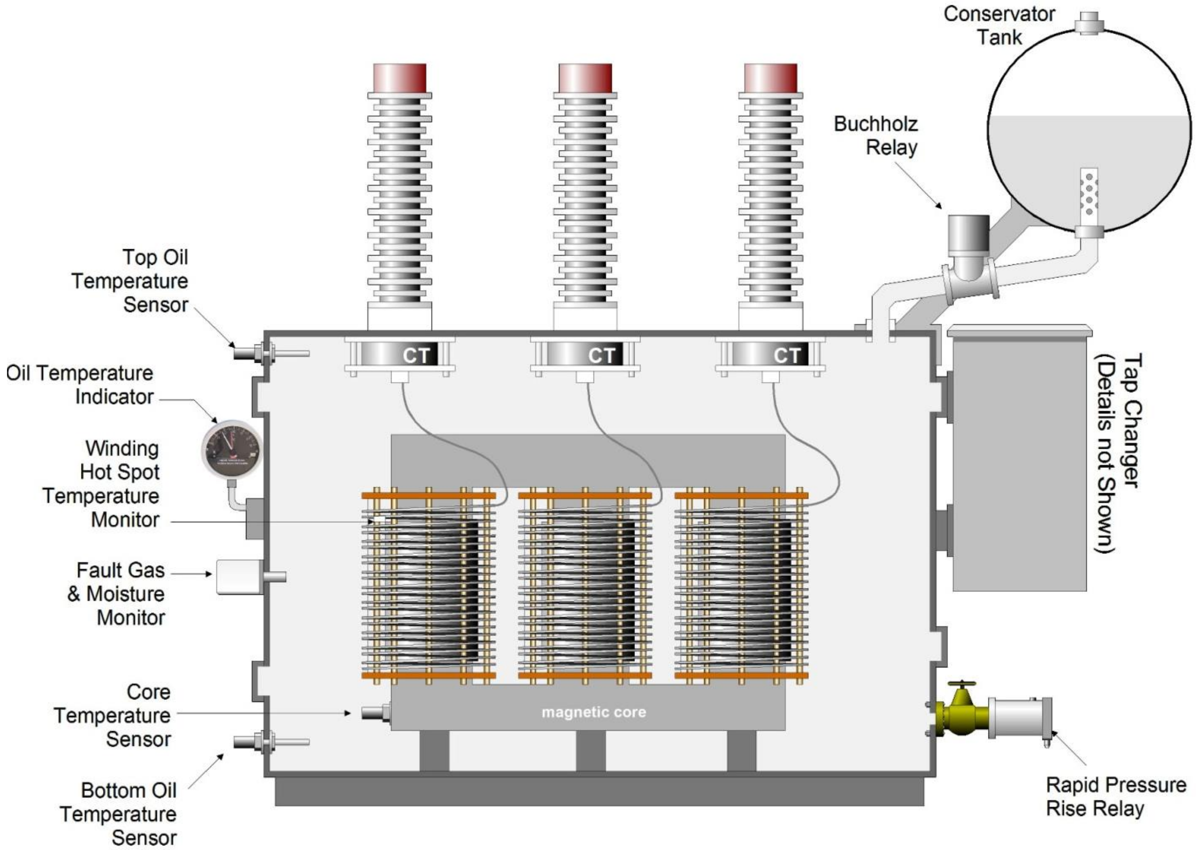


Figure 7.1: Typical Transformer Mechanical Monitoring and Protection Devices

Temperature Sensors monitor the temperature of transformer components, namely, the windings, magnetic core, cooling oil, tank, as well as the ambient temperature. Tracking the temperature of transformer components is important since temperature is a determining factor of insulation deterioration and failure. Furthermore, extreme temperature rise may lead to pressure buildup and catastrophic failure. Oil temperature sensors are typically placed near the top and bottom of the transformer enclosure tank. Additional temperature sensors measure the magnetic core temperature, the enclosure temperature, as well as the ambient temperature (see Figure 7.1). From these temperature readings the temperature of the hottest point in the transformer (hot spot temperature) is typically continuously estimated using computer models. The accuracy of these models is limited and thus a conservative margin of error (about 5 degrees C^0) is added to the estimate. Recently developed fiber optic based sensor technology allows direct temperature sensing of winding conductors. Note that the hot spot location will be typically at a coil/oil interface. The estimated or measured hot spot temperatures generate alarms when specific temperature limits are exceeded, and may be also used in schemes that limit the transformer load in order to keep the hot spot temperature below the desirable threshold (Dynamic Loading).

Several types of oil flow and pressure sensors are commonly used in oil filled power transformers. The **"*Rapid Pressure Rise Relay*"** (lower right side of Figure 7.1) monitors the oil pressure and sends a signal if the oil pressure increases faster than a certain threshold. Rapid increase of oil pressure may be caused by an internal fault or an overload, and may result in severe damage or rupture of the transformer enclosure. This signal can be used as an alarm indicator or as a breaker trip signal to disconnect the transformer depending on the severity of pressure buildup.

Another commonly used transformer oil monitoring device is the **"*Buchholz Relay*".** This device is usually installed along the pipe connecting the transformer enclosure tank to the conservator tank (See Figure 7.1). The conservator tank is an oil reservoir usually suspended above the main transformer enclosure and ensures that all electrical components of the transformer are immersed in oil. Oil from the transformer enclosure can flow into the conservator via a pipe allowing oil expansion with temperature rise. A Buchholz relay can also be placed between the main transformer enclosure and the tap changer enclosure. In both cases, this device senses (a) the rate of oil flow between the main enclosure and conservator or tap changer enclosure (b) the oil level and (c) the presence of gas. Sensing is achieved by means of a mechanical system consisting of vanes and floats. Figure 7.2 illustrates the Buchholz relay mechanism. The upper float drops if gas accumulates in the upper part of the relay cavity. The lower float drops if the entire relay cavity is empty of oil. A vane attached to the bottom float forces the float to drop if the oil flow-rate exceeds a certain threshold. Each float activates a mercury switch that can be used to set off alarms or provide trip signals to breakers that isolate the transformer. In typical applications the top float switch only activates an alarm.

High oil flow rate or oil level severe drop are associated with more serious conditions such as internal or external faults, thus the lower float/vane switch typically provides a breaker trip signal which automatically takes the transformer out of service.

Gas bubbles in the transformer oil are typically due to arcing caused by insulation deterioration. Prolonged arcing can eventually lead to internal faults which can result in catastrophic transformer failure. High concentration of gases in the oil indicates deteriorating conditions of the insulating capability of the oil and over time will lead to failure. Today there are on-line systems that sample the oil and perform gas analysis. The Buchholz relay enclosure usually includes valves that allow extraction of oil and gas samples. These devices are very useful for assessing the status of the transformer to determine when the transformer oil should be replaced.

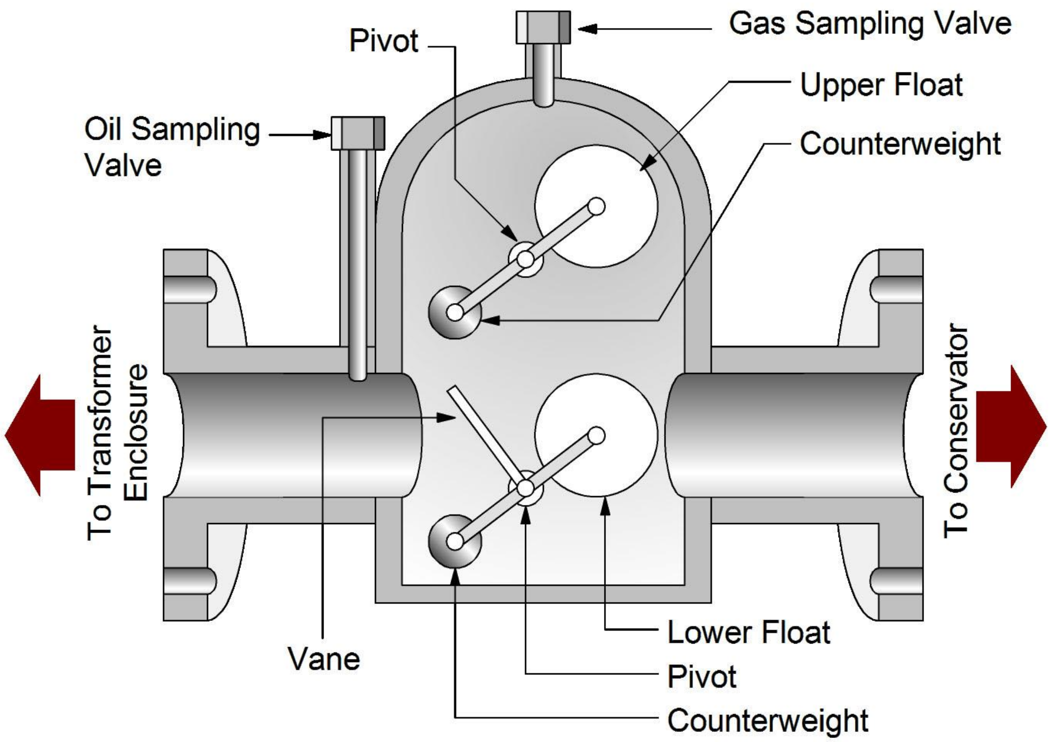


Figure 7.2: Buchholz Relay Mechanism

7.3 Typical Transformer Protection Schemes

Transformers may be typically protected with fuses, overcurrent relays, differential relays, overexcitation (volts over Hz) relays, temperature relays, etc. The specific scheme depends on the importance of the transformer, the number and type of interrupting devices and the type of the transformer. Typical transformer protection schemes for a two-winding transformer with various type of interrupting devices are illustrated in Figures 7.3, 7.4 and 7.5.

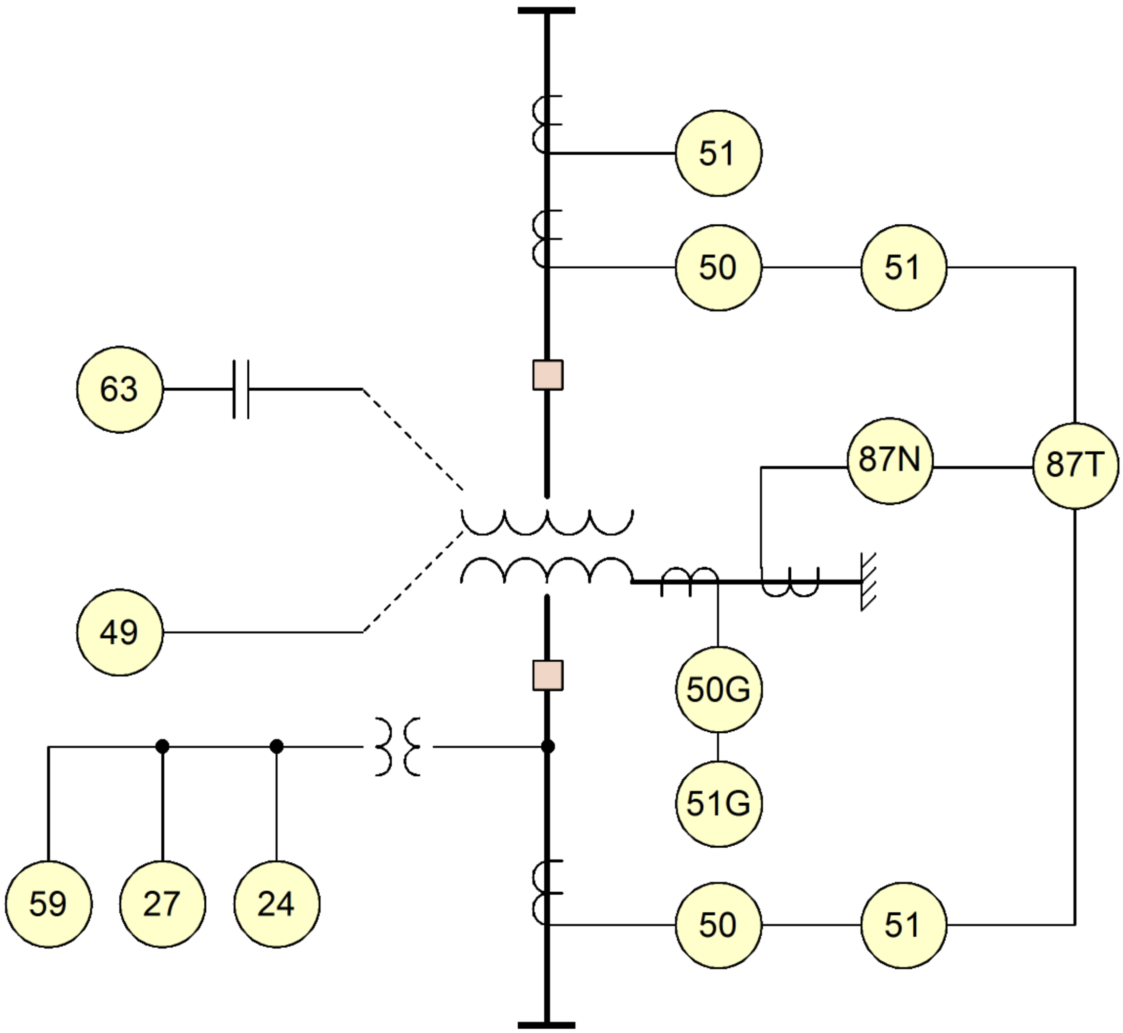


Figure 7.3: Typical Transformer Protection Scheme – Transformer with Single Breakers at the High and Low Side

87: Differential
50: Current instantaneous
51: Inverse Time Overcurrent
63: Pressure Switch
49: Thermal
59: Overvoltage
27: Undervoltage
24: Volts per Hertz

ADD:
26
64

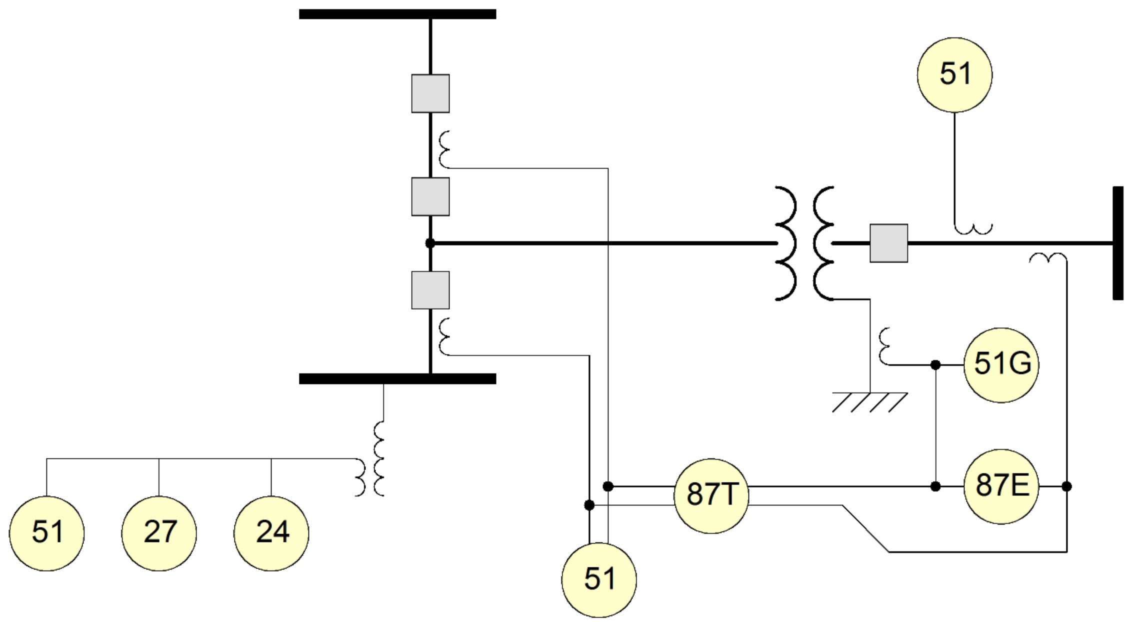


Figure 7.4: Typical Transformer Protection Schemes – Transformer connected to breaker and a half scheme at the high and single breaker at the low side
(87: Differential, 51: Inverse Time Overcurrent, 63: Pressure Switch, 49: Thermal, 59: Overvoltage, 27: Undervoltage, 24: Volts per Hertz)

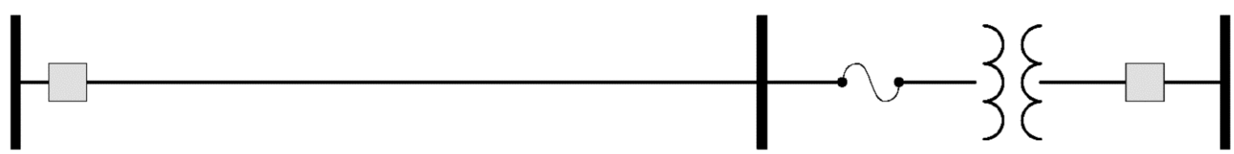


Figure 7.5: Typical Transformer Protection Schemes – Transformer is protected with fuse on the high side and breaker at the low side

The protection scheme of a transformer typically includes the differential function. Differential relays provide effective protection of transformers for practically all internal faults. Because of specific characteristics of transformers, as it will be discussed later, differential relays are somewhat desensitized to prevent false trips. These characteristics and phenomena will be discussed in subsequent paragraphs. It is important to note that numerical relays provide methods for mitigating many of the reasons for desensitizing differential relays and therefore they offer more sensitive protection for transformers. These issues will become evident in the discussion of the next paragraphs.

7.4 Transformer Over-current Protection

Overcurrent protection of transformers is achieved with fuses or overcurrent or directional overcurrent relays. The selected scheme depends on the selected interrupting devices. Typical configurations are: (a) fuse on the high side of the transformer, breaker on the low side, (b) single breakers on the high and low side of the transformer, (c) breaker and a half scheme on the high side of the transformer and single breaker on the low side of the transformer, and (d) ring bus breaker scheme on the high side of the transformer and single breaker on the low side of the transformer.

It should be understood that overcurrent protection is only part of the overall protection scheme.

Transformer with high side fuse/low side breaker: This protection scheme may be applied in cases that a transformer is connected via a single transmission line to the rest of the system. The secondary of the transformer typically serves a distribution system. The power flow in this case is typically from the high side to the low side. The fuse is typically selected to blow for faults anywhere in the transformer including the low voltage side of the transformer. The breaker on the low side protects against faults on the distribution circuits connected to the transformer. Specifically, the overcurrent protection is set in such a way that any fault downstream from the breaker will be cleared by the breaker before the fuse blows. In other words the overcurrent protection settings for the breaker must be coordinated with the fuse.

Transformer with single breaker at both sides: In this case directional instantaneous and time overcurrent is provided on each side. The settings are selected as to provide protection for faults anywhere in the transformer reaching to the other side of the transformer and to coordinate with other functions to avoid tripping for external faults.

Transformer with side 1 breaker and a half/side 2 single breaker: This case is similar to the previous case, except the current on side 1 is obtained via two CTs. Overcurrent protection is similar as in previous case.

Transformer with side 1 ring bus breaker scheme/side 2 single breaker: This case is also similar to the previous case. The current on side 1 is also obtained via two CTs. Overcurrent protection is similar as in previous case.

An example will illustrate the settings for overcurrent protection in transformers.

Example E7.1: Consider the transformer of Figure 7.3. Assume the transformer being a 36/42/54 MVA, 115 kV / 13.8 kV transformer. An extensive fault analysis provided the following results: (a) fault current range on the high side of the transformer is: direct network contribution: 3Phase: 4.7 kA, SinglePhase: 3.8 kA, contribution via the transformer: 3Phase: 1.7 kA, SinglePhase: 1.4 kA, and (b) fault current range on the low voltage side of the transformer is: direct network contribution: 3Phase: 14.1 kA, SinglePhase: 11.4 kA, contribution via the transformer: 3Phase: 14.6 kA, SinglePhase: 11.9 kA.

Solution: The nominal current at the two sides of the transformer are:

$$I_{n,115kV} = \frac{54MVA}{115kV\sqrt{3}} = 0.2711kA, \quad I_{n,13.8kV} = \frac{54MVA}{13.8kV\sqrt{3}} = 2.259kA$$

Considering these currents, the CT selection for the high voltage side and low voltage side will be 400:5 and 4000:5 respectively.

To be continued…

7.5 Transformer Differential Protection

Differential protection is popular, effective and secure protection function for transformers. Transformers present several unique challenges in the application of differential relaying protection due to variable transformation ratios, saturation characteristics and inrush current phenomena.

Faults affecting transformers can be classified as external and internal. External faults that may damage transformers include external system short circuits, overloads, overvoltages, and under-frequency. Internal faults are subdivided into incipient faults and active faults. Incipient faults include overheating, over-pressure and over-fluxing. Incipient faults cannot be detected by differential relays since they do not significantly affect terminal currents. Active faults include short circuits between windings and the core or the transformer tank, and short circuits between turns of same or different windings. The main objective of differential relaying is the detection of active internal faults and disconnect the transformer as fast as possible.

For simplicity, the application of differential relaying to transformers is introduced by a single-phase two-winding transformer. However this protection scheme can be easily generalized to three-phase multi-winding transformers. Consider a single phase transformer with N_1/N_2 turns ratio. Let I_1 and I_2 be the primary and secondary currents. The differential transformer protection is based on the observation that under normal operating conditions, the ratio of the primary and secondary currents is constant, and approximately equal to the inverse of the transformer turns ratio. Thus the quantity $I_O = N_1 I_1 - N_2 I_2$ will remain nearly equal to zero, unless an internal fault occurs. However, variable tap transformers to a large extent and instrumentation errors to a lesser extent make this simple criterion inappropriate for practical applications. Specifically, ratio errors of the current transformers result in I_O being proportional to the transformer load current. To overcome this shortcoming the concept of a *restraining quantity* is introduced, defined as $I_R = N_1 I_1 + N_2 I_2$. An internal fault is detected by monitoring if the ratio I_O / I_R exceeds a certain threshold.

Figure 7.6 shows a differential relay implementation based on the above criterion. Two current transformers (CT's) monitor the transformer primary and secondary currents. The CT secondary windings are connected to the coils of the balanced beam electromechanical relay, which is illustrated in Figure 7.7. The relay will trip (i.e. the contacts will close) if the net torque on the moving beam is positive. Neglecting the torque due to the spring, it can be shown that the relay will trip if:

$$\frac{I_O}{I_R} > K$$

where:

$$I_O = I_{S1} - I_{S2}, \text{ and } I_R = (I_{S1} + I_{S2})/2$$

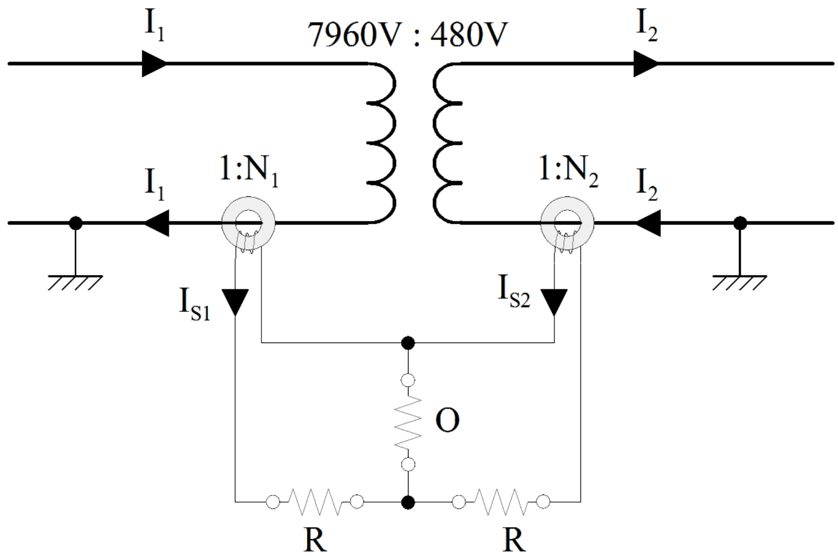


Figure 7.6: Single Phase Transformer Percentage Differential Protection

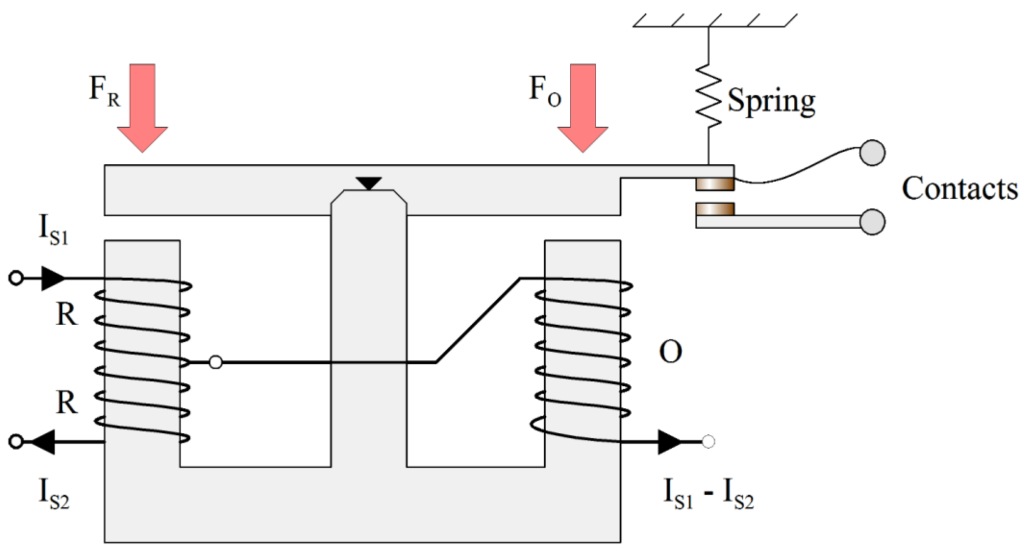


Figure 7.7: Percentage Differential Relay Implementation via a Balancing Beam

The constant K depends on the relay construction (number of winding turns, beam arm lengths etc.). Percentage differential relays can be also implemented with induction disk relays where one induction disk energized with the operating current and another with the restraining current. Both implementations are electromechanical relays. The constant K in electromechanical relays can be selected among several values (such as 10%, 25% and 50%). Percentage differential relays are also implemented in numerical relays in which case the constant K can be continuously defined.

Figure 7.8 illustrates the trip and block regions of the above system. The slope of the line separating the Trip and Block regions is equal to the factor K. Note that for low current values, the trip/block line curves away from the graph origin due to the spring action. This is a desirable property since it prevents false tripping due to the magnetizing current, when the transformer is unloaded.

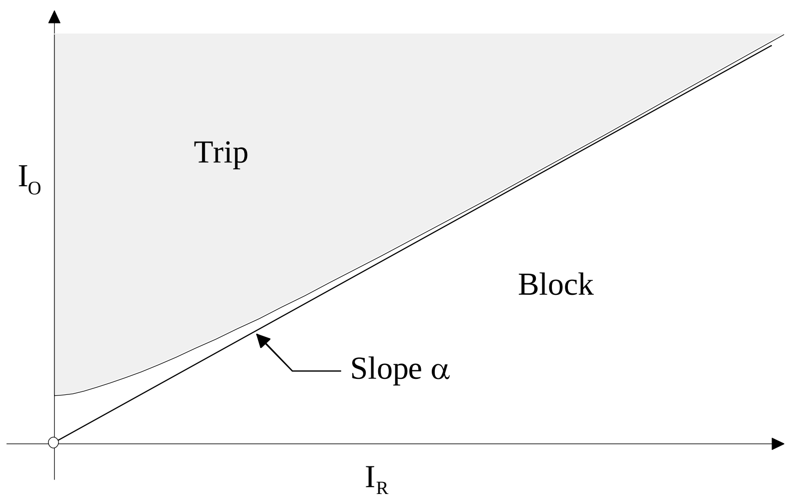


Figure 7.8: Electromechanical Differential Relay Characteristic

Note that the CT ratios must be appropriately selected so that the operating current quantity $I_O = I_{S1} - I_{S2}$ is nearly zero under normal operating conditions. For this example, the following CT ratio relationship will satisfy this requirement:

$$\frac{N_2}{N_1} = \frac{7960}{480} = 16.58$$

In practice, CT's must be selected from commercially available standard ratios. The following selection from standard CTR ratios approximates the above requirement:

Primary CT - 100:5
Secondary CT - 1600:5

Note that the above selection represents a ratio mismatch of 100 x 0.58 / 16.58 = 3.5%. Most electromechanical differential relays provide additional adjustments by means of operating and restraining coil taps, usually in 1% increments. In numerical relays, CT matching is not required as the operating current is numerically computed by first computing the primary currents and then forming the operating current using the transformer transformation ratio.

In addition to the ratio mismatch, the CT ratio error should be considered for the maximum fault currents. This error is determined from CT manufacturer data. Note that the CT error is a function of the measured current and burden impedance. If CT core saturation occurs, the CT error increases by a large factor.

Finally, if the monitored transformer is equipped with a load tap changer, the maximum deviation from the nominal turns ratio should also be considered. For example, assume that the CT ratio errors are 5% and that the tap changer maximum setting is 10%. Then the maximum total ratio mismatch will be 3.5% + 5% + 10% = 17.5%. In this case setting the percentage restraint factor K at 25% will provide adequate restraint under all operating conditions and external faults to prevent false tripping.

7.6 Factors Affecting Transformer Differential Protection

Under normal operating conditions the weighted sum (with the transformation ratio) of the electric currents on the high voltage side and the low voltage side of a transformer is equal to zero. This property is utilized in the application of transformer differential relaying. However, there are some phenomena and design options of transformers that deviate from this observation. These are:

(a) non-matching current transformer ratios,
(b) tap changers under load,
(c) differences in CT accuracy on the two sides of the transformer,
(d) phase shifting (phase shifting transformers),
(e) transformer overexcitation, and
(f) inrush currents.

These issues will be discussed in detail next.

7.6.1 Non-Matching Current Transformer Ratios

CTs have standard transformation ratios. As a result, for a given transformer using standard CTs may result in some operating current through the operating coil under normal operating conditions.

The issue of matching CT ratios is important and pertinent only for electromechanical relays. For numerical relays, this is not an issue. Specifically, in numerical relays the operating current is numerical computed and therefore any transformation ratio can be used.

7.6.2 Transformers with Tap Changers Under Load (TCUL)

Many transformers are equipped with tap changer that can operate under load. In this case the transformation ratio of the transformer changes during the period of one day depending on the loading of the system. For these transformers the CT ratio selection is typically based on matching the CTs when the transformer is at the neutral tap. Then allowance should be made to account for the variable transformation ratio during the operation of the transformers. This allowance is in the form of providing higher percentage ratio. The following exercise indicates the process.

Example E7.2: Consider a 36/42/54 MVA, 115 kV / 13.8 kV transformer. The transformer has a Tap Changer Under Load (TCUL) with the following capability: 16 raise steps of 0.00625 each and 16 lower steps of 0.00625 each. Select the CT ratios that best match the transformer at neutral. Then compute the maximum operating current that can occur during normal operating conditions as the transformer may be changing taps.

Solution: xxx to be continued.

Numerical relays have provided the ability to remove this issue. Specifically, numerical relays can monitor the tap of the transformer and adjust the computations of the operating current accordingly.

7.6.3 Transformer Inrush Currents

When transformers are energized a large magnetizing current may occur. We refer to these transients current as inrush current. Typically inrush currents decay fast and become negligible after a fraction of a second. However, under certain system conditions, inrush currents have been observed to last up to several seconds. The magnetizing current appears only on the source side of the transformer and therefore it appears on its entirety on the operating coil of the differential relay. Thus, large magnetizing currents can cause false tripping of the differential protective relay scheme presented above.

The generation of the inrush current can be explained with the aid of Figure 7.9.

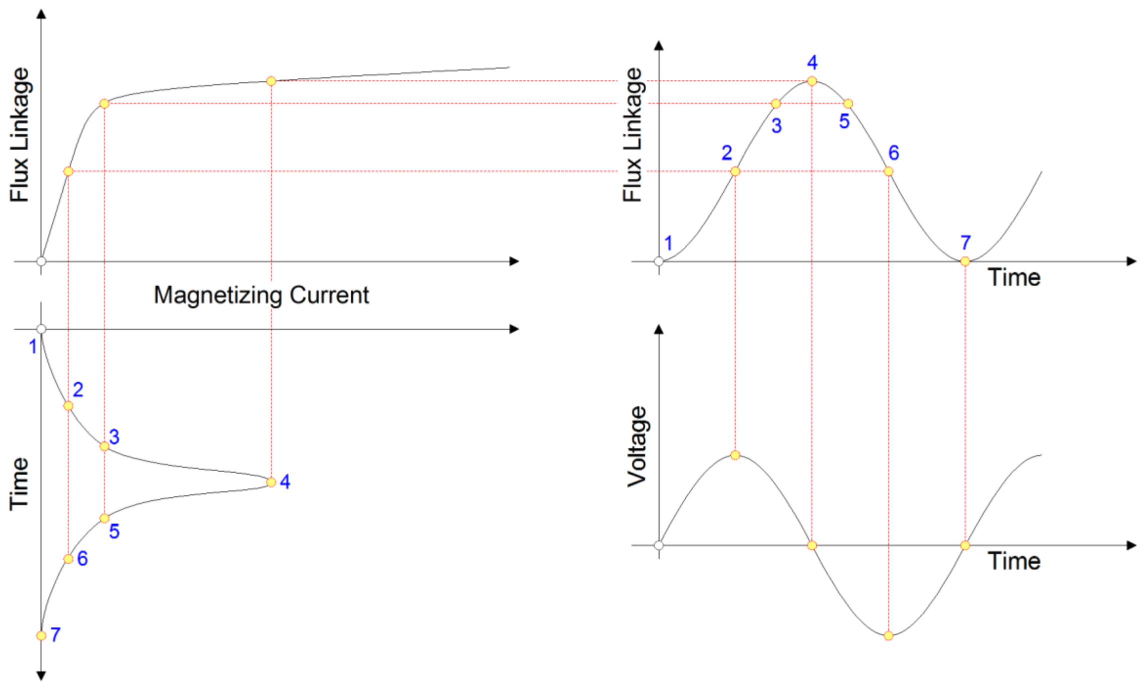


Figure 7.9 Generation of Inrush Currents

A typical inrush current waveform is illustrated in Figure 7.10.

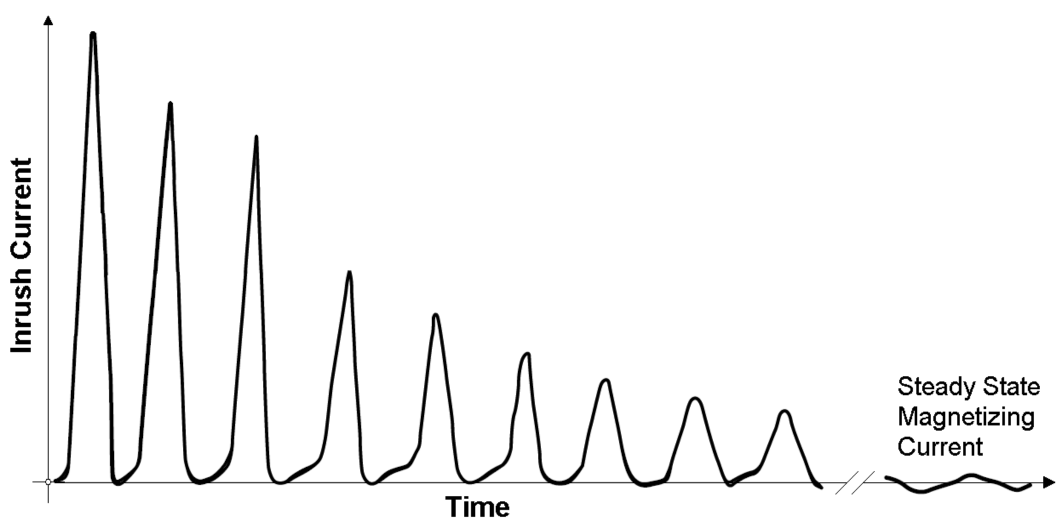


Figure 7.10 Typical In-Rush Current Waveform

The nature of the magnetizing currents in power transformers is illustrated by an example problem, next.

Example E7.3: An iron core 14.4 kV/240V, 30 kVA, 60 Hz transformer is energized from an ideal voltage source. The voltage source produces a voltage equal to:

$$e(t) = \sqrt{2}E\cos(\omega t + 1.00) \quad volts,$$
where: E=14.44 kV, ω=377 sec^{-1}

The transformer has the following magnetic flux linkage versus magnetizing current relationship:

$$i_m(t) = i_0 \left(\frac{|\lambda(t)|}{\lambda_0} \right)^8 sign(\lambda(t))$$

where, i_0 equals 0.01 pu and λ_0 equals 1.0 pu on the transformer ratings. The transformer is energized at time t=0. Compute the maximum value of the magnetizing current in Amperes. What is the second harmonic current value of the inrush current for this transformer and for the specified condition?

Solution: The differential equation governing the relationship of flux and voltage is:

$$\frac{d\lambda(t)}{dt} = e(t)$$

Initial condition of the flux linkage of the transformer is:

$$\lambda(0) = 0$$

$$\lambda(t) = \int_0^t e(\tau)\,d\tau = \frac{\sqrt{2}E}{\omega}\sin(\omega t + 1.0) - \frac{\sqrt{2}E}{\omega}\sin(1.0) = 54.1677\sin(\omega t + 0.1) - 45.5806 \quad Wb$$

The maximum magnetizing current $i_m(t)$ is obtained at maximum flux linkage $\lambda_{\max}(t)$:

$$\lambda_{\max}(t) = (54.1677)(-1) - 45.5806 = -99.748\,Wb$$

$$i_{m,\max}(t) = 0.01\left(\frac{30\times10^3}{14.4\times10^3}\right)\left(\frac{99.748}{54.1677}\right)^8(-1) = -2.7547\ A$$

The second harmonic current value of the inrush current for this transformer is:

$$\frac{1}{\sqrt{2}}\sqrt{a_2^2 + b_2^2}$$

where: $a_2 = \int_0^T i_m(t)\sin(n\omega t)\,dt \qquad b_2 = \int_0^T i_m(t)\cos(n\omega t)\,dt$

The above integrals were computed numerically for n = 0, 1, 2, 3, and 4 yielding the corresponding harmonics. The results are listed in the Table xxx:

Table E7.3 Excitation Current Harmonics

Harmonic Order	RMS Value (A)
DC	-0.525
Fundamental	0.658
2^{nd}	0.475
3^{d}	0.273
4^{th}	0.123
RMS Value	1.009

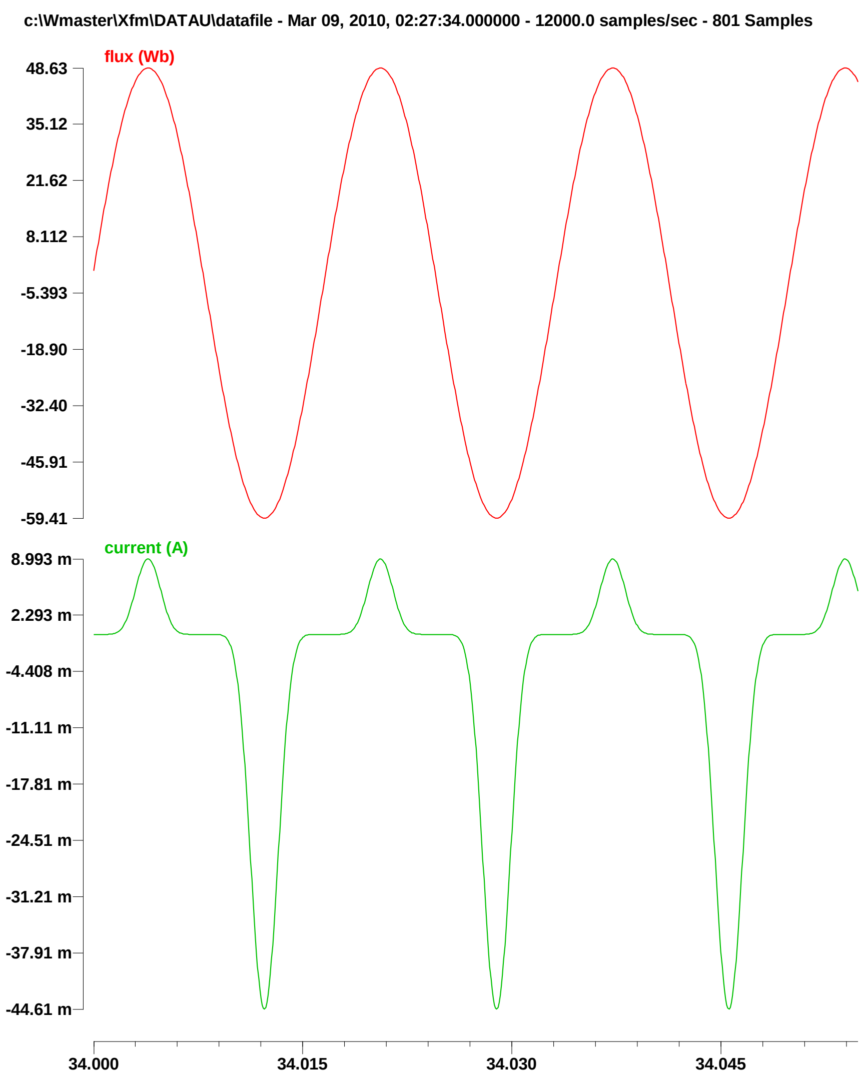


Figure E7.3a: Flux Linkage and Excitation Current for θ = 0.1

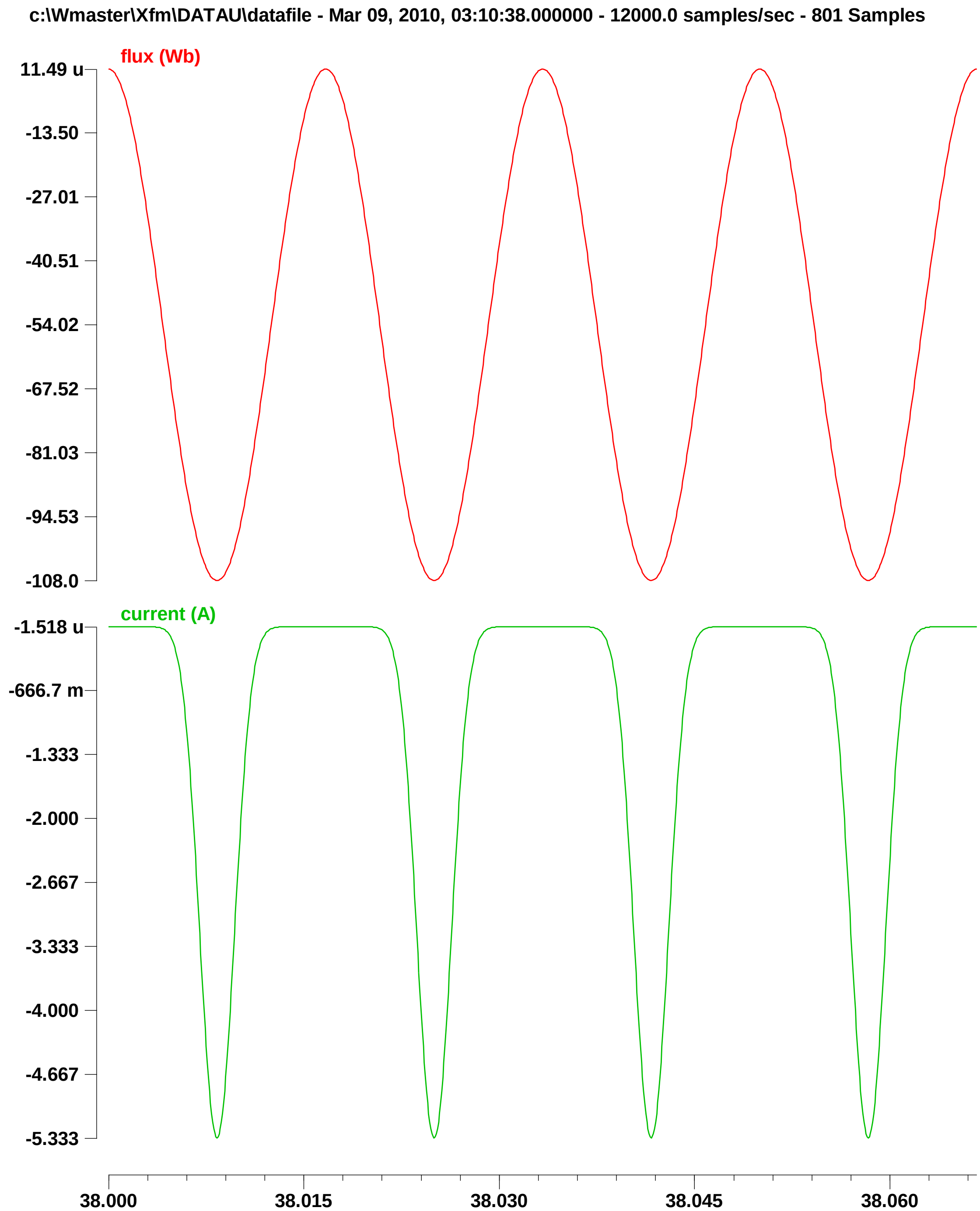


Figure E7.3b: Flux Linkage and Excitation Current for θ = π/2

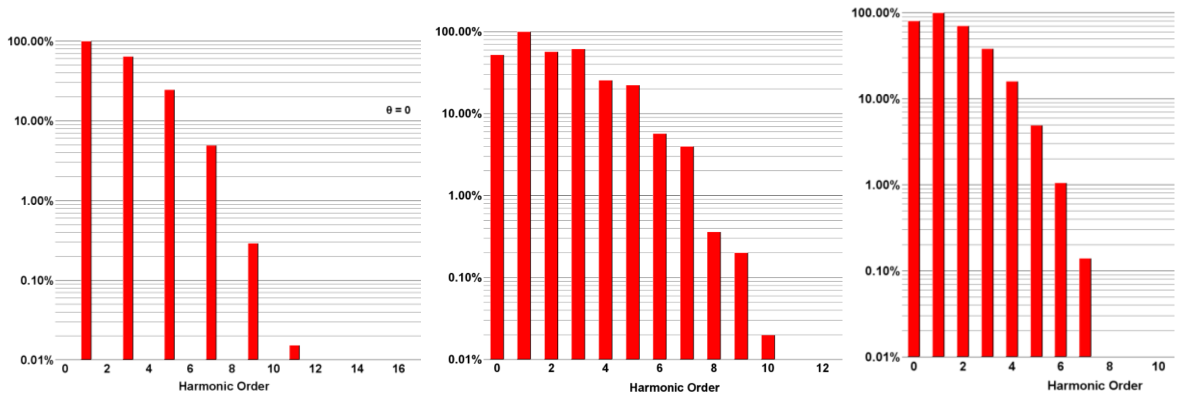


Figure E7.3c: Normalized Excitation Current Harmonics for θ = 0, θ = 0.1 rad, and θ = π/2 rad

From the above example problem it can be seen that magnetizing currents are characterized by large harmonic distortion. Both even and odd harmonics are present due to the asymmetry of the current waveform (positive and negative pulses have different magnitudes). This characteristic can be used to identify magnetizing currents and prevent differential relay false tripping. Figure 7.11 illustrates a circuit providing "harmonic restraint" to a differential relay. The circuit elements L_1 and C_1 form a band pass filter. L_2 and C_2 form a band reject filter. Both filters are tuned to the power frequency fundamental. Thus the operating coil responds mostly to the fundamental while the harmonic restrain coil responds to the harmonic components. An additional restraint coil provides the RMS current restraint function.

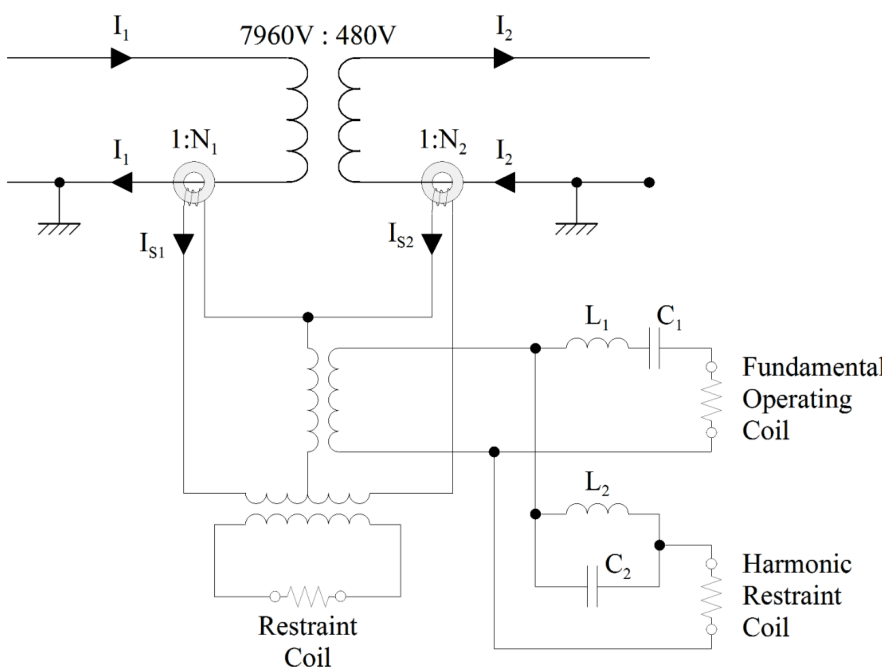


Figure 7.11: Electromechanical Proportional Differential Relay with Harmonic Restraint

The above passive filter based approach has several limitations. The filter selectivity (quality factor) is limited by the passive component losses. The required capacitors and inductors are typically large and are subject to drift with aging and temperature. Active analog filters based on differential amplifiers can provide improved performance, although are still subject to component parameter drift. Digital relay implementations mitigate all such limitations, and in addition provide practically unlimited flexibility in designing advanced detection schemes.

There are several ways to deal with in-rush currents. Some of them are:

1. Desensitizing the differential relay
2. Insert a time delay upon energization of the transformer
3. Filtering inrush current harmonics
4. Use harmonic current restraint
5. Use time domain relaying methods

These methods are discussed next.

7.6.4 CT Saturation

CT saturation during external faults can result in differential protection false tripping. Figure 7.12 illustrates typical saturated CT primary and secondary current waveforms. Note that saturation results in a distorted waveform. Furthermore the RMS value of a saturated CT is lower than the one expected by the CT ratio.

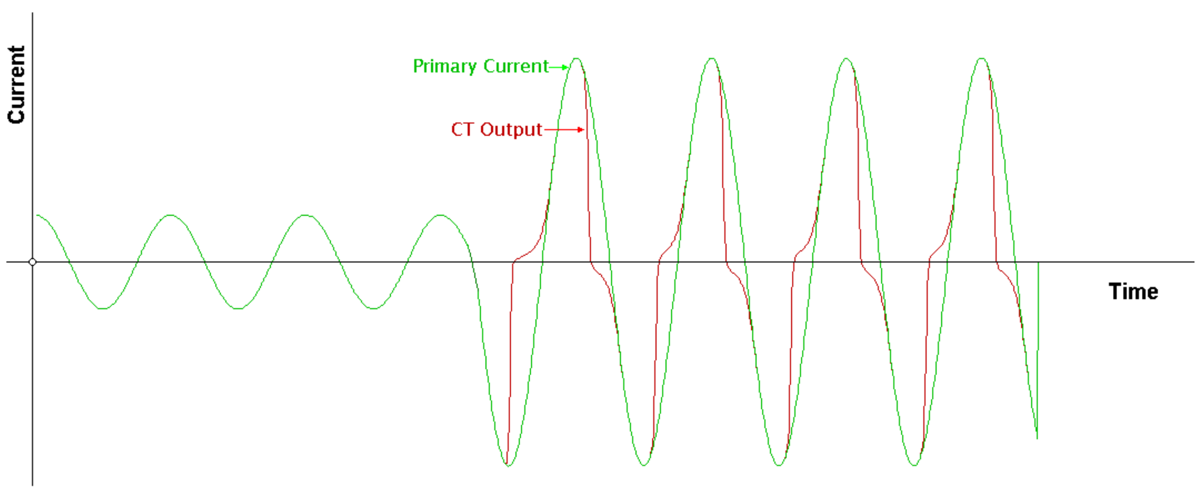


Figure 7.12: Typical Saturated CT Current Waveform

One technique that reduces the possibility of false tripping in the event of CT saturation, while retaining the relay sensitivity for lower current internal faults is the use of Dual Slope Trip/Block curve. Figure 7.13 illustrates a dual slope Trip/Block characteristic. Note that the slope is lower for low current values and higher past a transition point. This allows for a high sensitivity to low current internal faults and lower sensitivity in regions where the CT's may saturate.

In digital relay implementations the trip/block characteristic curve is user defined. Typically the user specifies the two slopes (in %) and the restraint current value at the transition point. These parameters are selected by considering the accuracy and saturation characteristics of the CT's.

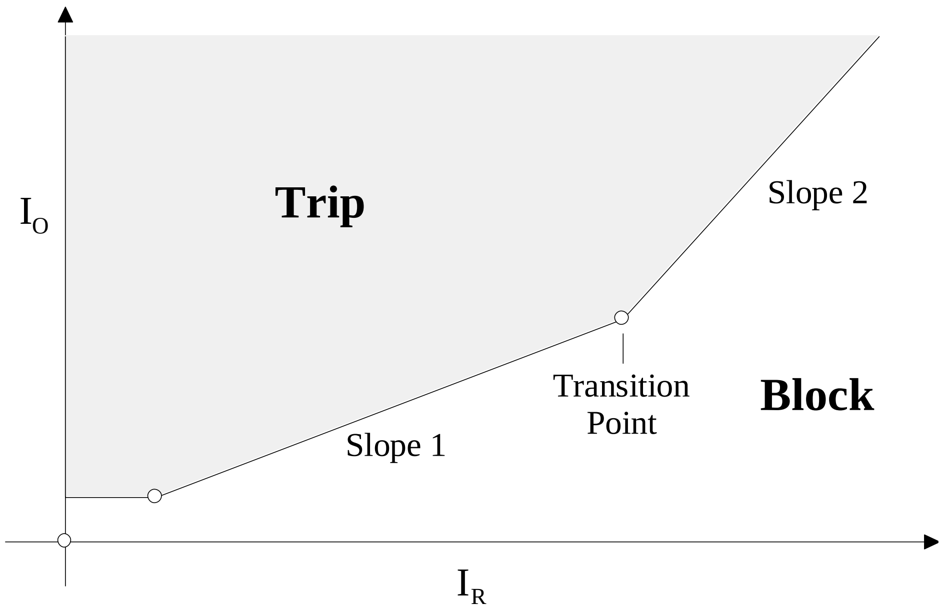


Figure 7.13: Dual Slope Differential Protection

7.7 Three Phase Transformer Protection

The presented differential protection scheme is generalized to three phase transformers by monitoring all transformer terminal currents as illustrated in Figure 7.x. The principle of operating and restraining currents are derived in a similar manner as for the single phase case, however attention must be paid to the phase shifts introduced in case of a Delta-Wye or Wye Delta connected transformer. One simple approach that cancels this phase shift is to connect the delta side CT secondary windings in Wye configuration, and the Wye side CT secondary windings in Delta configuration, as illustrated in Figure 7.14. It is important to note that there are many types of three phase transformers: two winding transformers (delta-wye, wye-wye, etc.), autotransformers with or without tertiary, as well as three winding transformers. In each case the phase relationships must be accounted for. Note that most digital relay implementations can remove the delta-wye phase shift computationally, thus being able to work with any CT arrangement.

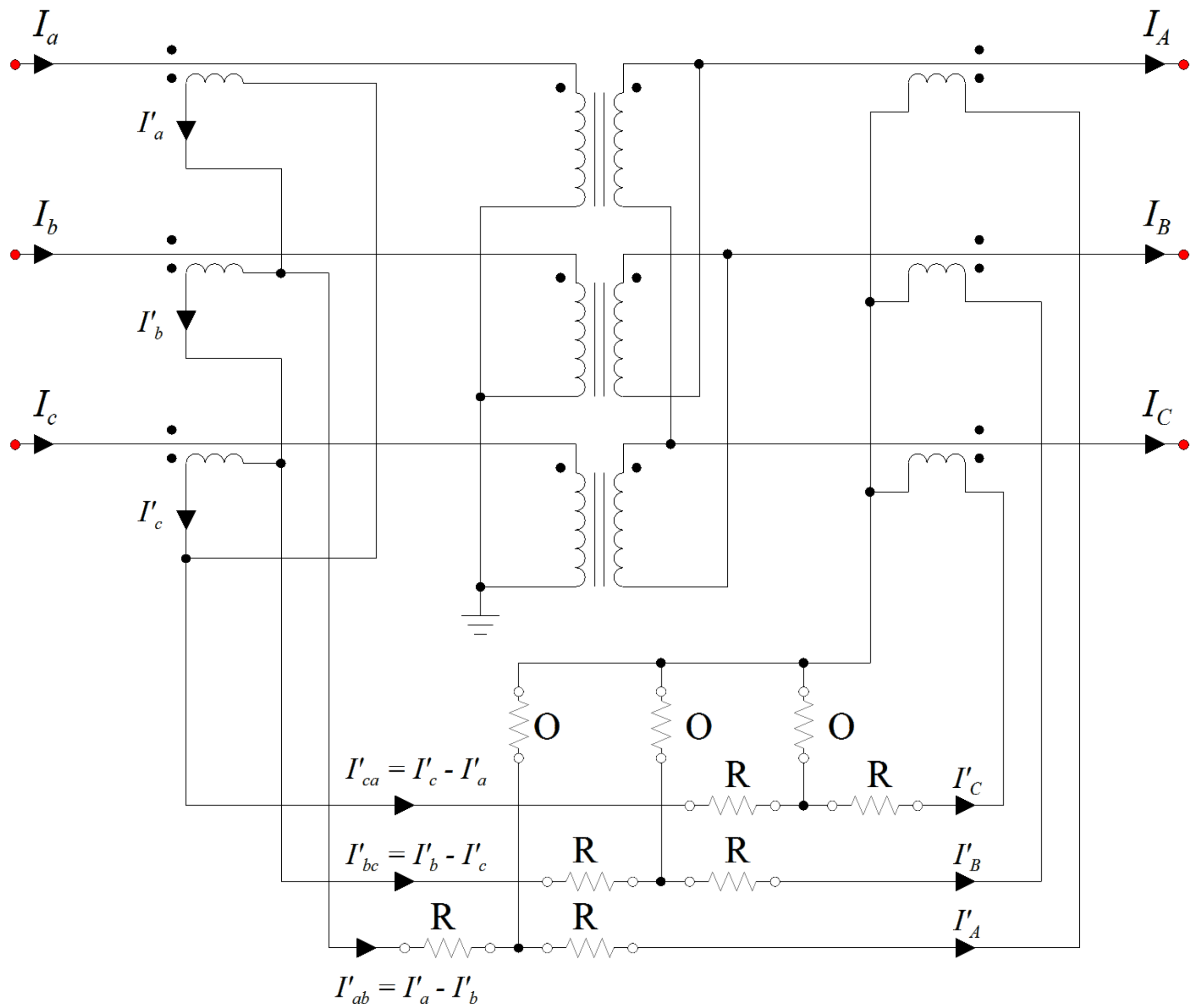


Figure 7.14: Three Phase Transformer Differential Protection
(Wye / Delta Connected Transformer, Delta Side is the High Voltage Side)

Note that in selecting the CT ratios in the above configuration the factors arising from Delta versus Wye connection ($\sqrt{3}$) must be taken into account. The procedure is illustrated next by an example.

Example E7.4: Consider a delta-wye connected power transformer, 36/48/60 MVA, 115 kV/13.8 kV. The transformer has the following taps: 115 kV side: three fixed taps 110 kV to 120 kV, (b) 13.8 kV side: variable taps under load, 16 raise taps 13.8 to 15.18 kV and 16 lower taps 13.8 kV to 12.42 kV. The transformer is connected to a 115 kV system and the fault current at the 115 kV bus is 37.8 kA (three phase fault) and 35.6 kA (single phase to ground fault). The fault currents on the 13.8 kV side are: 22.8 kA (three phase fault) and 24.6 kA (single phase to ground fault). Select the CTs for a percentage differential protection scheme. Provide settings for the percentage differential scheme.

Solution: The nominal primary and secondary currents are:

High Voltage Side: $\dfrac{36.0MW/3.0}{115kV/\sqrt{3}} = 180\ A$

Low Voltage Side: $\dfrac{36.0MW/3.0}{13.8kV/\sqrt{3}} = 1{,}506\ A$

Since this is a 36/48/60 MVA transformer the maximum rated current will be 300A and 2,500A for the high and low voltage sides respectively.

For better matching, select the following CT ratios:
Option One: High Voltage Side: 300:5, Low Voltage Side: 4000:5
Option Two: High Voltage Side: 300:5, Low Voltage Side: 5000:5

Of interest is to compute the maximum operating coil current under various tap settings. The following table summarizes these computations. These results were obtained by observing that the operating coil current as a function of the CT and transformer ratios, under balanced conditions, is:

$I_0 = \left(k_1 k - k_2\sqrt{3}\right)\tilde{I}_a$, where k_1, k_2, k are the transformations ratios of the two CTs and the transformer. For maximum balanced load current, the operating coil current will be:
Option One: High Voltage Side: 300:5, Low Voltage Side: 4000:5:
$I_0 = (0.01666k - 0.0021651)(2500)$
Option Two: High Voltage Side: 300:5, Low Voltage Side: 5000:5
$I_0 = (0.01666k - 0.0017321)(2500)$

Table E7.4 Operating Coil Current Under various Tap Positions – Max Load Current

CT Selection	Tap: 15.18/110	Tap: 12.42/120
300:5 & 4000:5	0.337 A	1.10 A
300:5 & 5000:5	1.41 A	0.018 A

Note that for a three phase fault on the secondary bus, the maximum operating coil current will be as in the table below.

Table E7.4 Operating Coil Current Under various Tap Positions and Restraining Coil Current – Three Phase Fault

CT Selection	Tap: 15.18/110	Tap: 12.42/120
300:5 & 4000:5	3.073 A / 52.44 A	10.03 A / 39.33 A
300:5 & 5000:5	12.85 A / 52.44 A	0.164 A / 39.33 A

A good selection of relay settings will be: pickup current: 1 A, percentage at least 30%.

- End of Example

Under normal operating conditions the electric current through the operating coil (phase A) will be:

$$I_0 = k_1\widetilde{I_A} - k_2 k\widetilde{I_A}$$

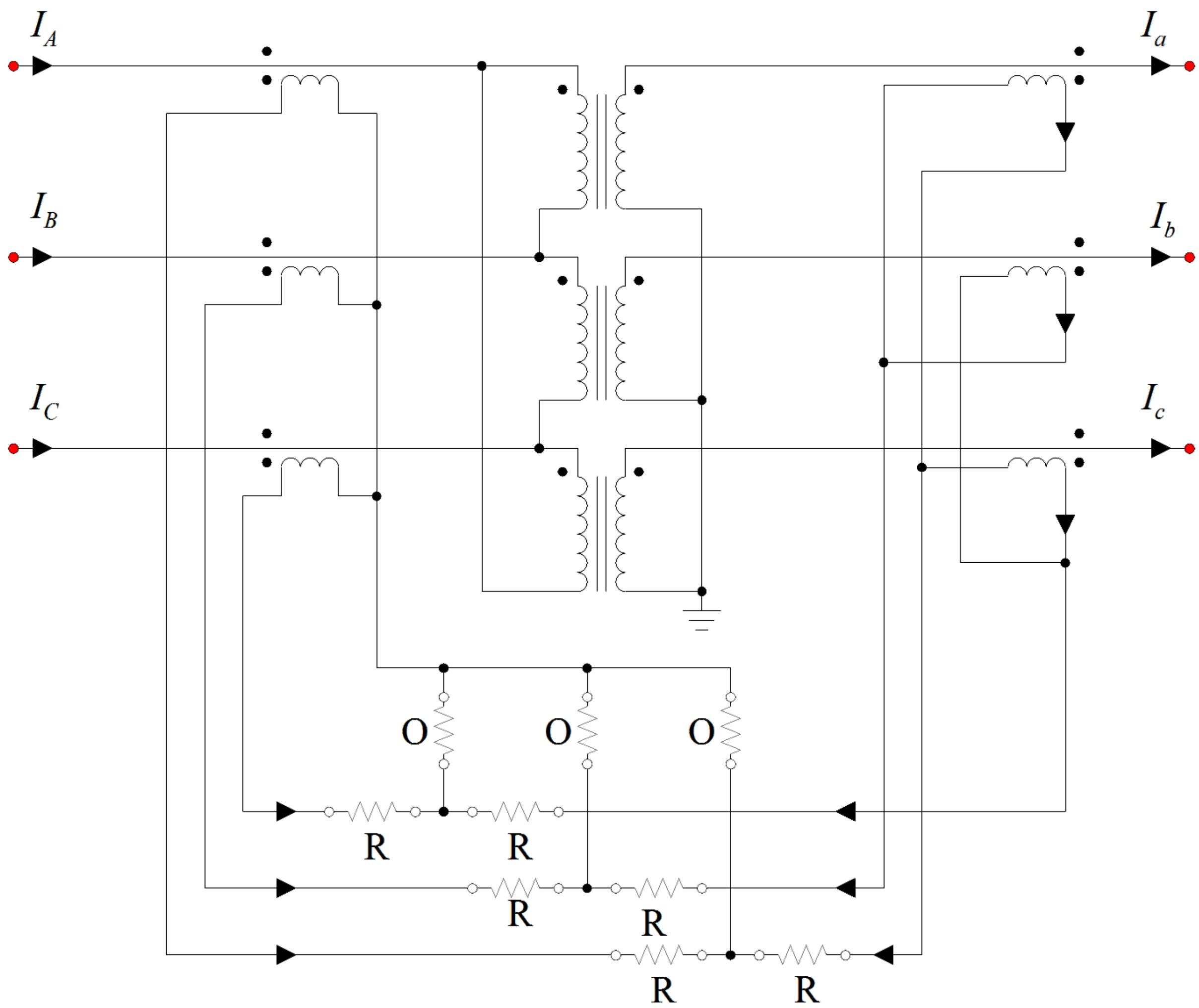


Figure 7.15: Three Phase Transformer Differential Protection
(Delta / Wye Connected Transformer, Delta Side is the High Voltage Side, Standard Connection)

Another advantage of the above CT connection is that the zero sequence current in the event of an external ground fault on the Wye transformer side is blocked by the delta connection of the CT's on that side. This prevents false tripping since the delta side zero sequence currents are also zero. In the case of the computational phase shift correction afforded by digital relay implementations the zero sequence currents should also be removed computationally. Modern digital differential relays perform these tasks by appropriate numerical transformations.

In general percentage differential is used to avoid false tripping due to CT mismatch, tap changers and other causes of mismatch in the differential scheme. The percentage may be set at relatively high value. This in turn generates the case where a ground fault near the neutral of the transformer may not be seen by the differential scheme. For this purpose a more sensitive differential scheme is applied to the wye side of the transformer known as restricted earth fault protection. This type of protection is discussed in section 7.8.

The process by which differential settings are selected will be discussed by an example.

Example E7.5: Consider a delta-wye connected power transformer, 36 MVA, 115 kV/13.8 kV. The transformer has the following taps: 115 kV side: five fixed taps 112 kV to 124 kV, (b) 13.8 kV side: variable taps under load, 16 raise taps 13.8 to 14.2 kV and 16 lower taps 13.8 kV to 12.4 kV..The transformer is connected to a 115 kV system and the fault current at the 115 kV bus is 37.8 kA (three phase fault) and 35.6 kA (single phase to ground fault). Select the differential protection scheme and the settings.

Solution: The transformer, CT, and relay connection circuit is shown in Figure E7.x. We first perform a fault analysis in order to determine the minimum requirement for current transformer ratings. Figure 7.x illustrates the sequence component equivalent circuit for a single phase to ground fault at the 13.8 kV side. Note that all impedances are referred on the 115 kV side. Solving for the zero sequence current yields:

$$I_0 = 66.39 \text{ kV} / (2 \times 1.756 + 3 \times 29.39) = 0.7242 \text{ kA}$$

From the above value the 13.8 kV side fault current is computed as:

$$I_f = 3 \times 0.7242 \times (115 / 13.8) = 18.105 \text{ kA}$$

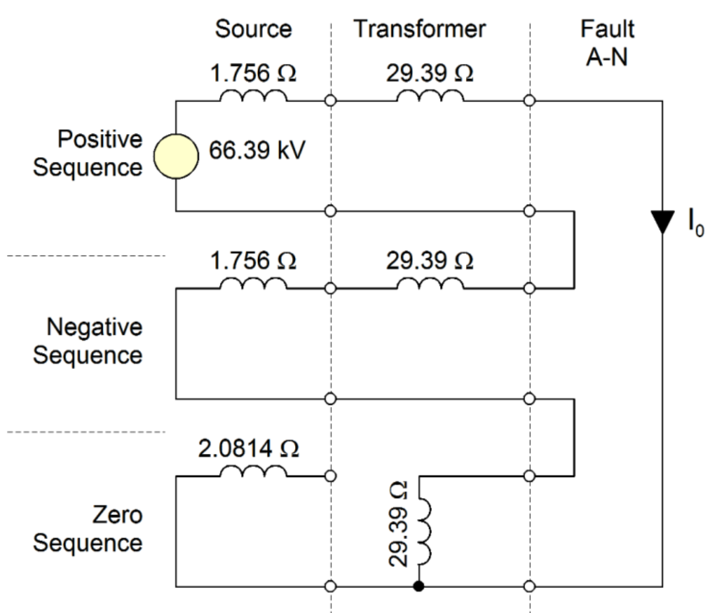


Figure E7.5: Equivalent Sequence

Using the above sequence component circuit to compute 3-Phase and L-L-N fault levels it can be shown that the highest fault current at the 13.8 kV side is in fact 18.105 kA. Thus the minimum secondary side standard CT's rating is 1000:5. The primary side CT rating is next selected for proper operation of an electro-mechanical differential relay, as follows:

$$\frac{1000A}{115/13.8/\sqrt{3}} = \frac{1000A}{14.35} = 69.28A$$

where the $\sqrt{3}$ factor is needed to compensate for the delta connection of the 13.8 kV side CTs. Since this value is far from any standard CT rating, the next higher rating for the secondary CT side is considered, i.e: 1500:5, for which the primary CT rating should be:

$$\frac{1500A}{115/13.8/\sqrt{3}} = \frac{1500A}{14.35} = 104.5 \cong 100A$$

Thus 100:5 CTs are selected for the primary side. *Note that these CTs will go into deep saturation if exposed to primary side fault current levels. However, assuming that these CTs are inside the transformer bushings, they will not see any fault current from any external primary side faults*.

Next the restraining factor is computed by considering the worst case tap positions, i.e. the positions for which the transformer ration deviates the most from the selected CT ratios.

Specifically, taking into account all possible tap settings, the exact ratio of CT secondary to primary ratings should range

$$\text{from:} \quad \sqrt{3}\,\frac{112kV}{14.2kV} = 13.66 \quad \text{to:} \quad \sqrt{3}\,\frac{124kV}{12.4kV} = 17.32$$

Expressed in percent of the selected ratio of CT secondary to primary ratings (15.0) is from -8.9% to +15.5%. Thus a restraining factor of at least 15.5 % is necessary to avoid false tripping with all possible tap settings.

7.8 Restricted Earth Fault (REF) Protection

Differential schemes that are based on phase currents only for transformers with wye grounded sides, present a special challenge. Specifically, in this case the zero sequence currents in the wye connected side must be removed to avoid tripping for outside ground faults. In electromechanical relays the removal of the zero sequence currents is achieved by connecting the CTs in a delta arrangement. In numerical relays the removal of the zero sequence current can be done via software.

Another set of issues are: Many times it is desirable to provide special protection to a transformer against ground faults on the wye connected side. There are a couple of reasons for this protection: (a) for solidly grounded transformers a ground fault near the neutral of the transformer may not produce enough differential fault current to trip the differential scheme. (b) for an impedance grounded transformer the ground fault may be limited to the point that makes the differential scheme unable to detect the fault.

For the above reasons, sensitive protection is needed against internal ground faults and secure protection for ground faults outside the transformer zone. This type of protection can be provided by a differential scheme that includes all the terminals of the wye side of the transformer as it is shown in Figure 7.x. We refer to this scheme as restrictive earth fault protection or sensitive ground fault protection. Note that the restrictive earth fault protection is a differential scheme restricted only to the wye side of the transformer and it is a differential sceme that operates on zero sequence current only.

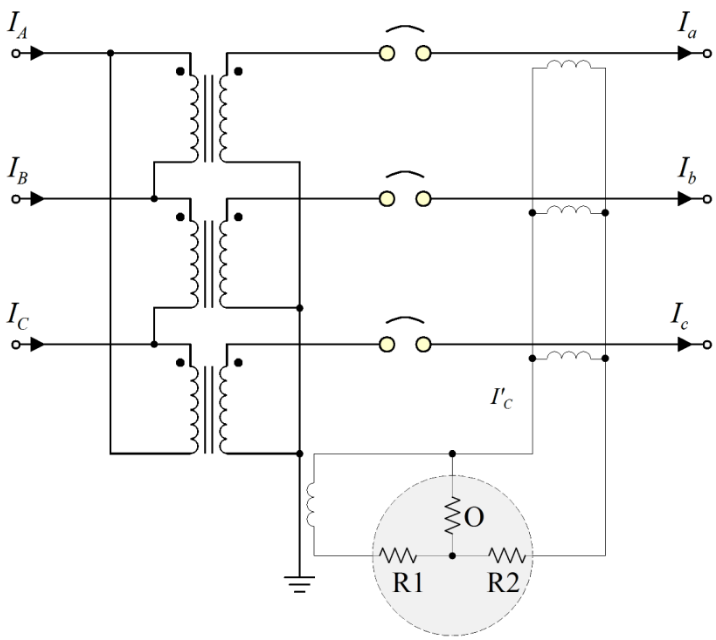


Figure 7.16 Restricted Earth Fault Protection Scheme for a Delta-Wye Connected Transformer

Restrictive earth fault protection can be also provided for three phase autotransformers. Figure 7.y illustrates the application of the restricted earth fault scheme for an autotransformer.

To be added

Figure 7.y: Restricted Earth Fault Protection Scheme for an Autotransformer

7.9 Auto-Transformer Protection

Autotransformers are widely used to interconnect power transmission systems operating at different kV levels. The autotransformers do not incur a phase shift. Most of the times autotransformers have a tertiary winding to provide a path for zero sequence currents.

7.10 Three-Winding Transformer Protection

Three winding power transformers are used for a variety of reasons: (a) to connect two generators to the transmission system (step-up transformer for two units), (b) to provide isolation between two transmission systems with the third winding used as a tertiary, (c) to interconnect a single generation unit to two different kV transmission units, etc.

7.11 Phase Shifting Transformer Protection

Phase shifting transformers are used to control power flow in transmission systems. Typical phase shifting transformer topologies are shown in Figures 7.x and 7.y. In general these regulating transformers can create a phase shift between the two sides of the transformer that is variable depending on the needs of the network. Typical capabilities are plus/minus 18 degrees.

The protection of these transformers with differential schemes will require that the differential scheme be desensitized to allow for the variations of the transformation ratio.

7.12 Volts per Hertz Protection

Transformer saturation occurs whenever the magnetic flux density in the core of the transformer exceeds a certain value. This value is in the order of half Tesla for typical core materials. Under steady state conditions the magnetic flux density is proportional to the voltage and inversely proportional to the frequency. Transformers are designed to operate very close to the saturation knee of the core material. This means that when the applied voltage and the system frequency is close to nominal the magnetic flux density will be sinusoidal and the maximum value will be close to the saturation knee. Any increase in voltage, decrease in frequency or transient increase of the ratio voltage over frequency will drive the transformer into saturation. It is important to recognize the magnetic flux density in the core of the transformer depends on the frequency and voltage as follows:

$$B \approx \frac{1}{2\pi NA}\left(\frac{V}{f}\right)$$

It is apparent that either increases in voltage or decreases in frequency or combination of the two will increase the magnetic flux density of the core and it will drive the transformer core into saturation. When the core is driven into saturation, the transformer becomes a generator of harmonics and is subjected to excessive heating, temperature rise and eventually failure. It is therefore important to monitor this condition and trip the transformer before it is too late. This is achieved with the "Volts over Hertz" relay (relay function 24) that monitors the ratio of the voltage over frequency. When this ratio exceeds the setting it will trip the transformer with a specified time delay. A typical setting of a "Volts over Hertz" relay is shown in Figure 7.x.

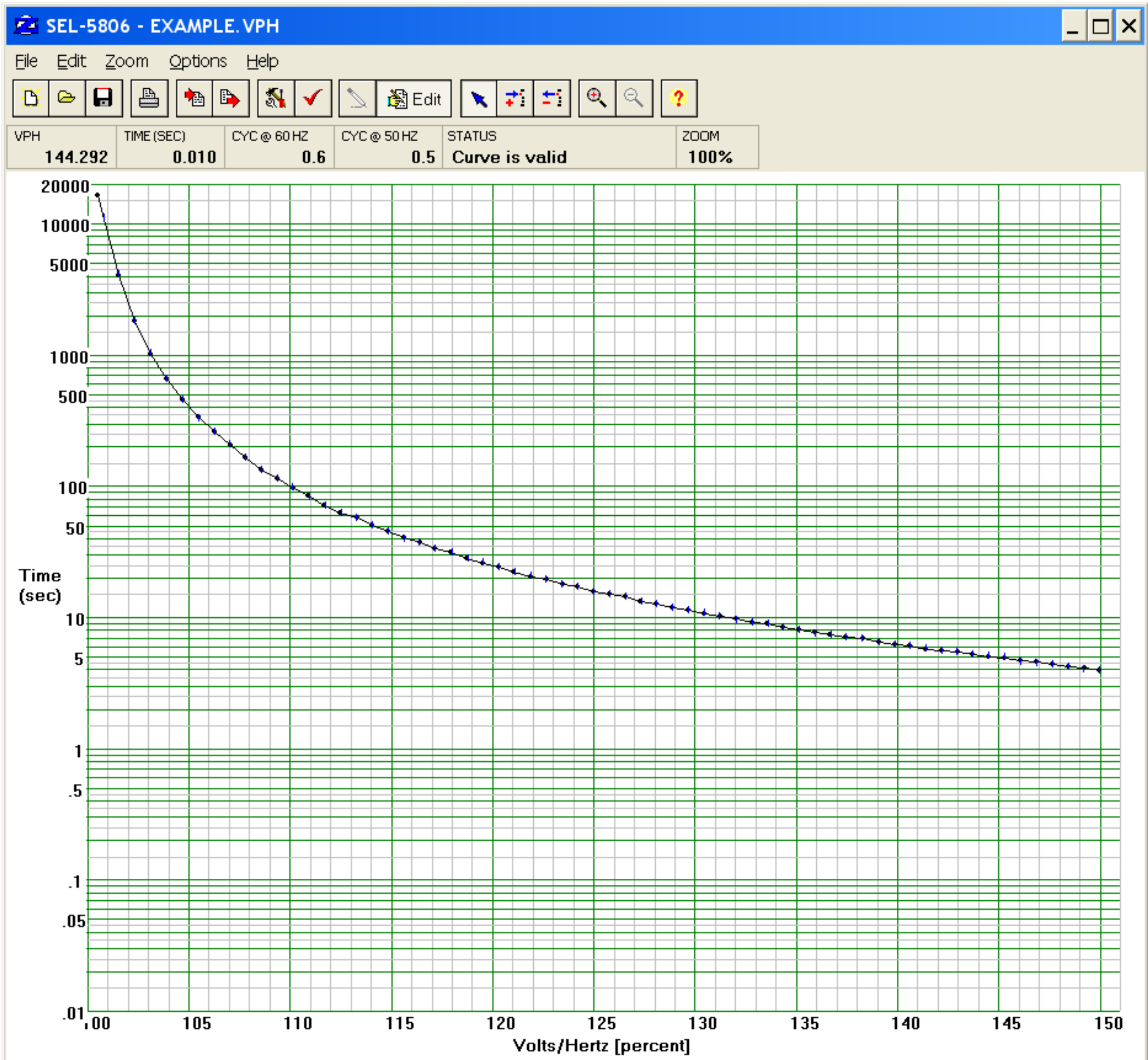


Figure 7.17: Example of PC based Volts-per-Hertz entry user interface for digital relays (Courtesy SEL Inc.)

The volts per Hertz protection should match the transformer limit due to heating from overexcitation. This can be achieved by using an instantaneous definite-time characteristic coupled with an inverse time characteristic. By combining these characteristics the following overall characteristic is obtained as illustrated in Figure 7.x. For convenience the transformer thermal limit curve (due to overexcitation) is superimposed on the relay characteristic.

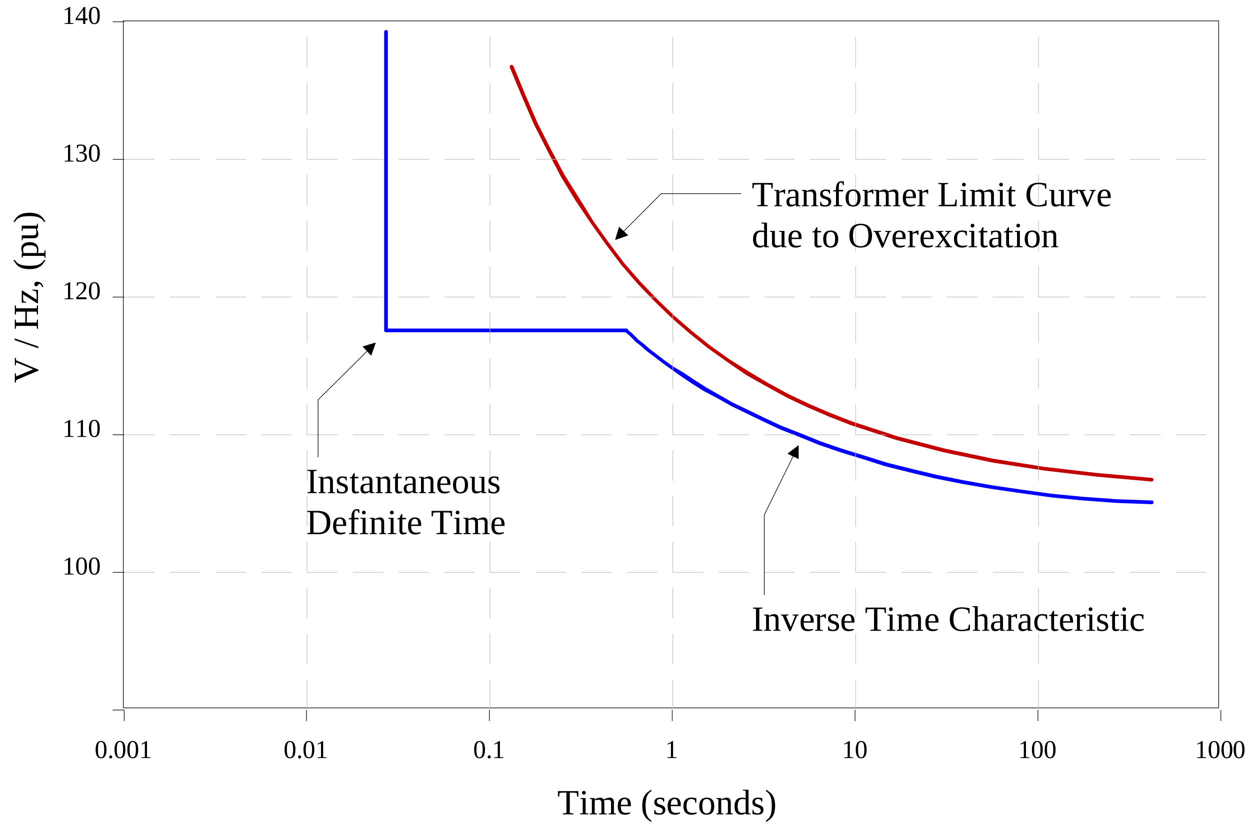


Figure 7.18: Volts per Hertz Relay Characteristic Matching Transformer Limit Curve

7.13 Differential Relay Visualization Example

Computer simulation provides a powerful method for evaluating protective relay performance. Furthermore, modern computer graphics provide visualization of the simulation results. This section presents differential relay visualization example. The simulated system, illustrated in Figure 7.x includes a Delta-Wye Connected 138kV/13.8kV 3-Phase Transformer. The transformer is fed from a 138 kV source through a 10-mile transmission line. The low voltage winding supplies power to a three phase load through a 2.5-mile distribution line. The simulation starts by the transformer energization. A phase to ground fault occurs 0.5 seconds past the energization.

The simulation includes a balanced beam differential relay model. The relay monitors the six transformer terminal currents and computes the net force acting on the relay beam element. This is achieved by computation of the phasors of all measured terminal currents, computation of operating and restraint currents, and finally computation of the operating and restraining forces. All quantities are continuously updated to reflect the corresponding instantaneous values. Note that the simulation includes physically based models of all system components and takes into account the current transformer characteristics and connections, the typical relay settings (percentage factor K, and trip threshold) (See Figure 7.x). During the simulation the user can observe the transformer instantaneous terminal current values, the corresponding current phasors, and the net force on the relay beam element, in an animated graphical display, which is illustrated in Figure 7.x.

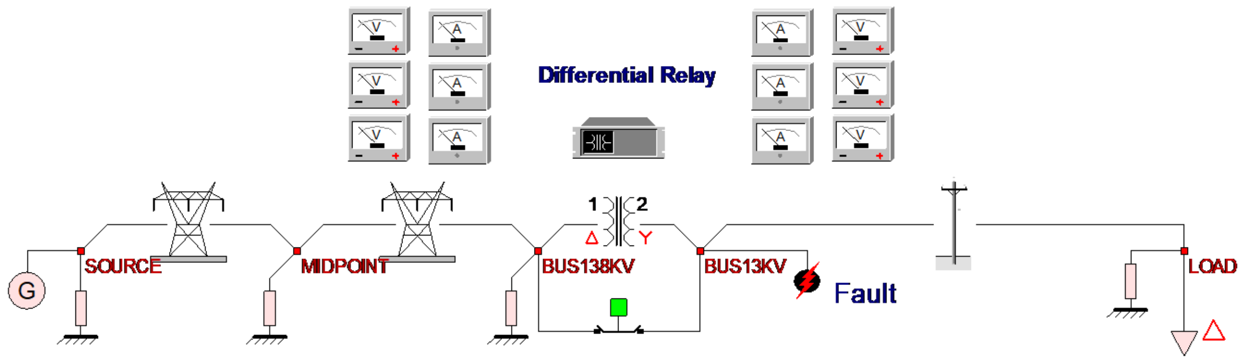


Figure 7.19: Single Line Diagram of Simulated System for Differential Relay Visualization Example

This simulation based approach is useful in evaluating the performance of existing electromechanical relays, as well as modern digital relays. A variety of tests can be quickly performed to test the relay security and sensitivity for internal faults, external faults, energization transients, overvoltages etc.

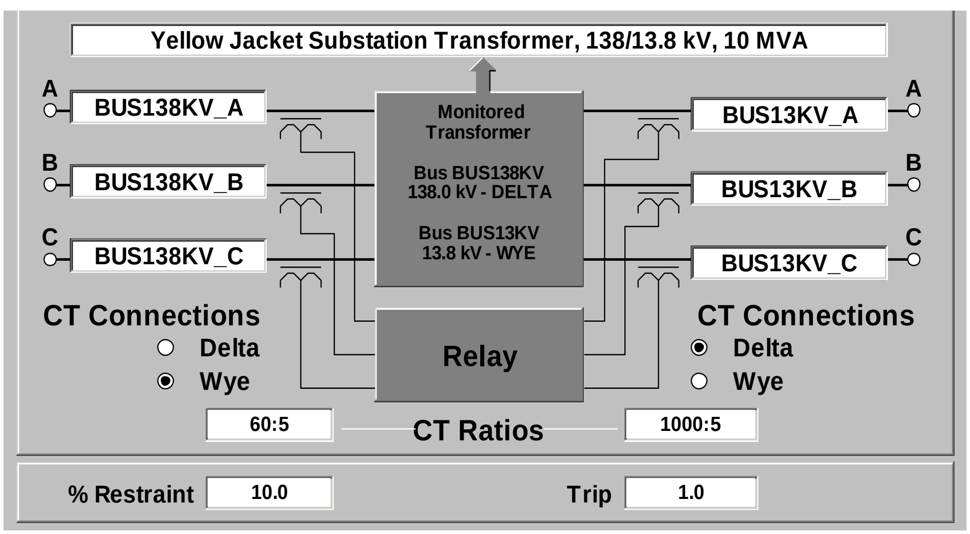


Figure 7.20: Differential Relay Parameter and Settings Entry Form

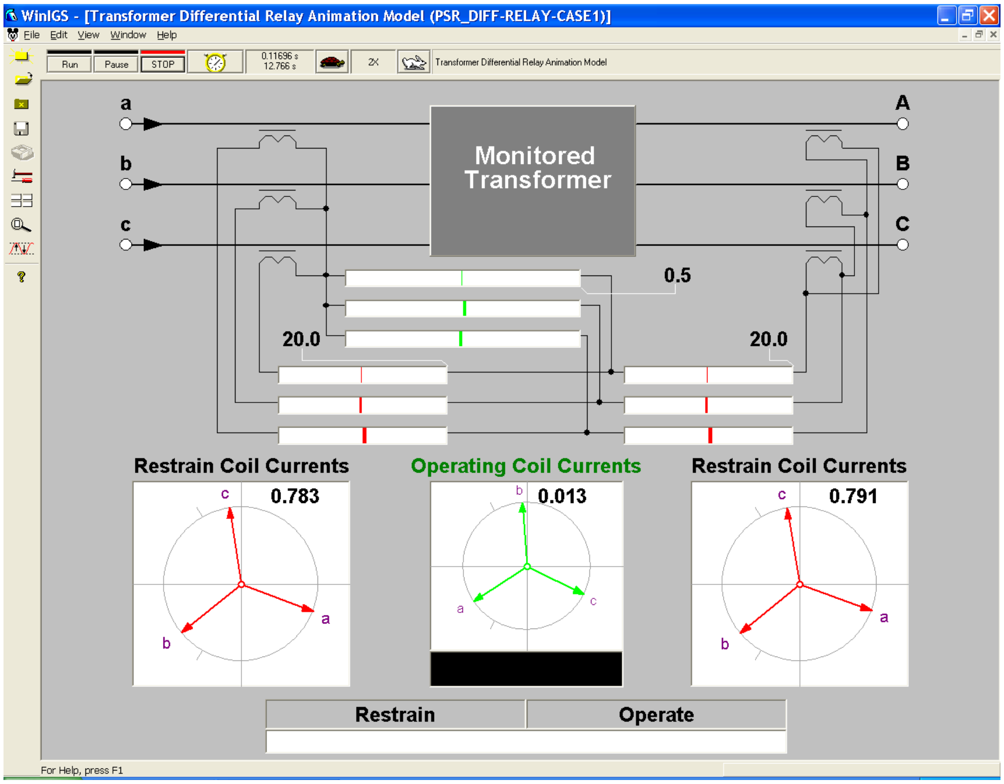


Figure 7.21: Differential Relay Operation Visualization Display

7.14 Dynamic State Estimation Based Transformer Protection

Recently a new protection approach for transformers was introduced.

To be added

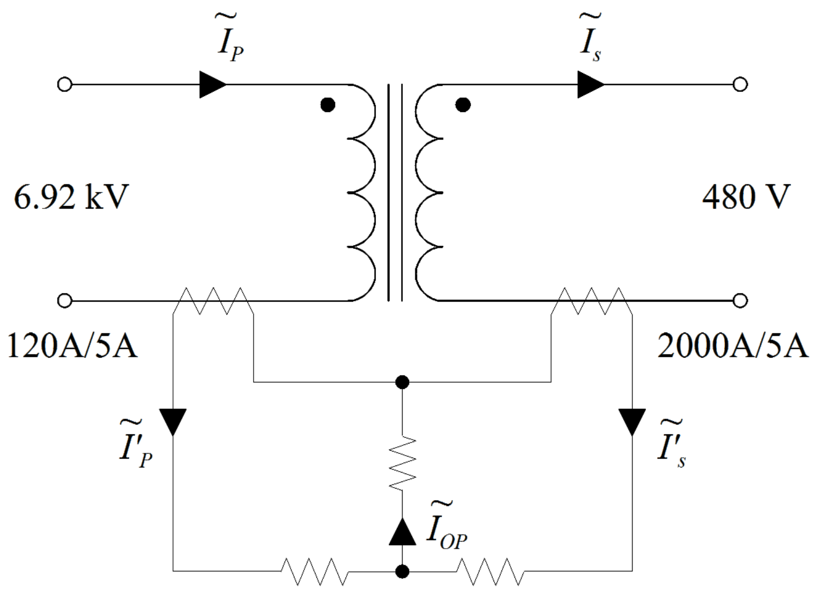
7.15 Summary and Discussion

In this chapter we have discussed protection schemes for transformers and reactors.

7.16 Problems

Problem P7.1: A single phase 6.92 kV/480V, 45 kVA, 60 Hz transformer is protected with a differential relay that uses the following current transformers: Primary: 120:5 and secondary: 2000:5. Compute the current in the operating coil of the differential relay as a function of the transformer current in the secondary.

Solution: Let the transformer current in the secondary be I_S. Then (see figure):



$$I_P = 0.06936 I_S$$

$$I_{op} = \frac{5}{2000} I_S - \frac{5}{120} 0.06936 I_S = -0.00039 I_S$$

Problem P7.2: An iron core 138 kV/13.8 kV, 10 MVA, 60 Hz, single phase transformer is protected via a differential relay. The CT ratio on the high side is 300:5 and the CT ratio on the low side is 3000:5. Assume that the transformer is energized from an ideal voltage source. The voltage source produces a voltage equal to:

$e(t) = \sqrt{2} E \cos(\omega t + 1.05 rads) \quad volts$, where: E=138.0 kV, ω=377 sec^{-1}

The transformer has the following magnetic flux linkage versus magnetizing current relationship: $i_m(t) = i_0 \left(\frac{|\lambda(t)|}{\lambda_0} \right)^8 sign(\lambda(t))$, where, i_0 equals 0.01 pu and λ_0 equals 1.0 pu on the transformer ratings.

The transformer is energized at time t=0.

(a) Assume ideal instrumentation and compute the waveform of the operating coil current of the differential relay in Amperes.
(b) Compute the maximum value of the current in the operating coil of the differential relay in Amperes.
(c) Compute the rms value of the operating current in (a).

Solution: (a) The magnetic flux is:

$$\lambda(t) = 517.7\sin(\omega t + 1.05) - 449 \quad Wb$$

The magnetizing current is:

$$i_m(t) = 0.01\frac{10,000}{138}\left(\frac{|517.7\sin(\omega t+1.05)-449|}{517.7}\right)^8 sign(517.7\sin(\omega t+1.05)-449)$$

Since the transformer is not loaded the current on the secondary will be zero and the current on the proimary will be the magnetizing current. The operating coil current will be:

$$i_0 = \frac{5}{300} i_m(t) \quad A$$

Figure P7.2a provides a sketch of the operating coil current.

(b) The maximum value of the current occurs when the magnetic flux linkage is negative maximum

$$\lambda_{\max} = -966.70 \quad Wb$$

The maximum value of the operating coil current is

$$i_{o,\max} = \frac{5}{300} 0.01\frac{10,000}{138}\left(\frac{|-966.7|}{517.7}\right)^8 (-1) = -1.7857 \; A$$

(c) The rms value is computed by using the rms definition.

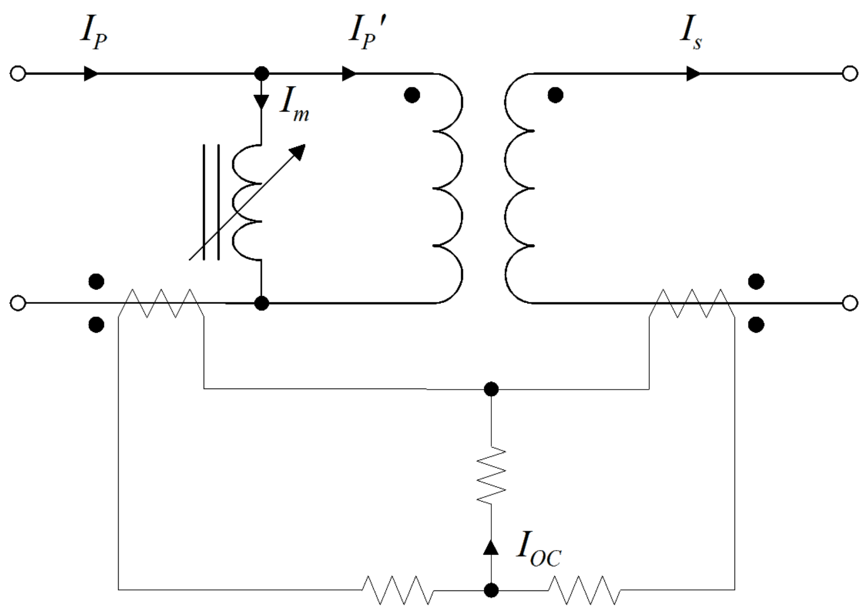
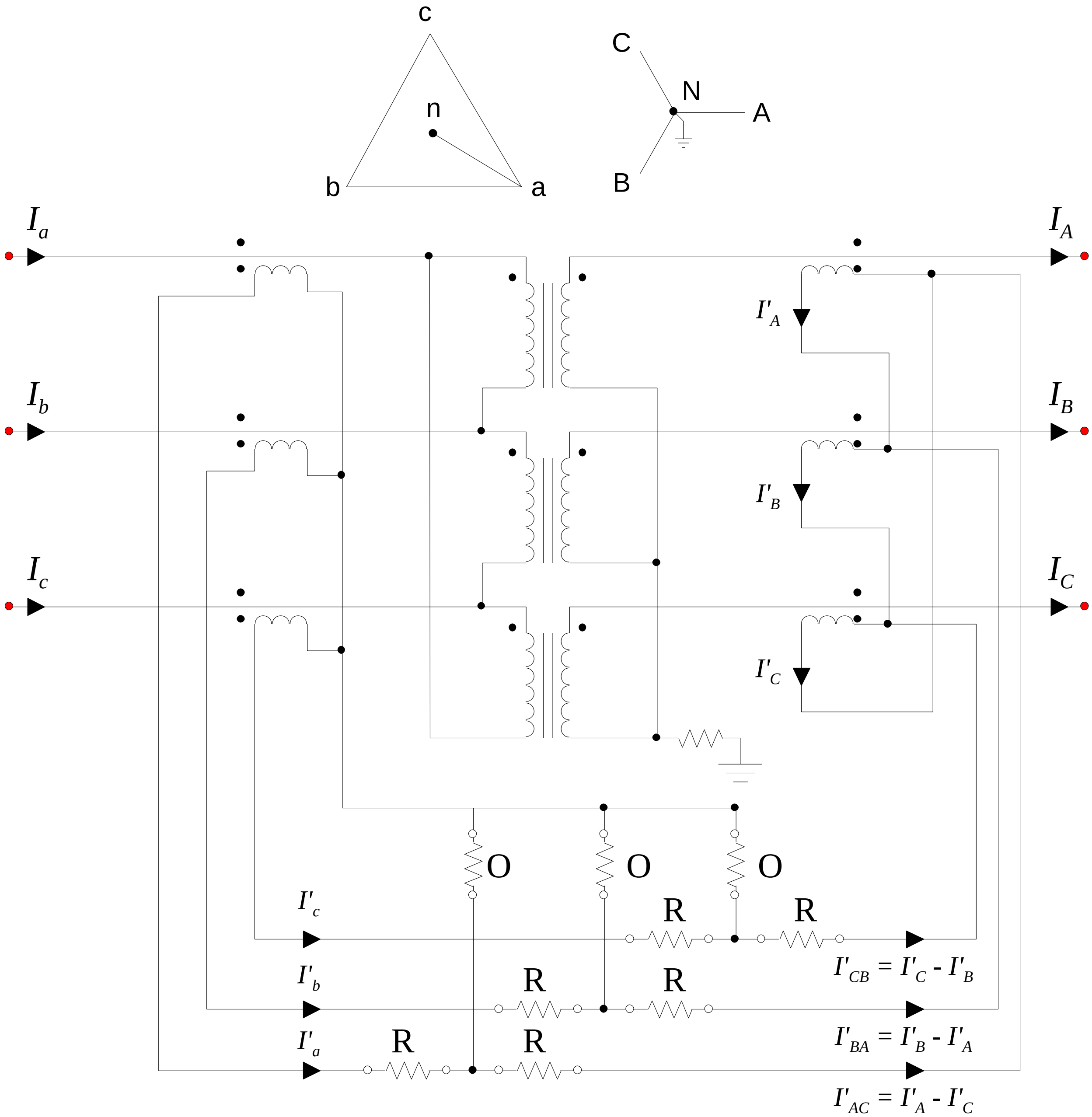


Figure P7.2

Problem P7.3: Consider a three-phase, 30 MVA, 115 kV/13.8 kV, delta-wye connected, 60 Hz transformer with variable tap under load on the 13.8 kV side. The tap may vary between the values 0.9 to 1.1 pu. The transformer is protected with a differential relay. The CT ratios on the high side are 300/5A while the CT ratio on the low side is 2400/5A. The CTs are connected wye on the delta side of the transformer and they are connected delta on the wye side oif the transformer. Determine the maximum current in the operating coil of the differential relay under all normal operating conditions of the transformer. The normal operating conditions are defined as follows: (a) secondary current (0 - 1.0 pu), (b) tap setting (0.9 to 1.1 pu), and (c) power factor (0.95 capacitive to 0.95 inductive). For simplicity, ignore transformer impedances.

Solution: The extreme cases should be considered. Since we can neglect transformer impedances for simplicity, the relationship of the currents between high side and low side will be agffected only by the transformation ratio. The current relationship appears in Figure P7.3. The extreme cases will be two:



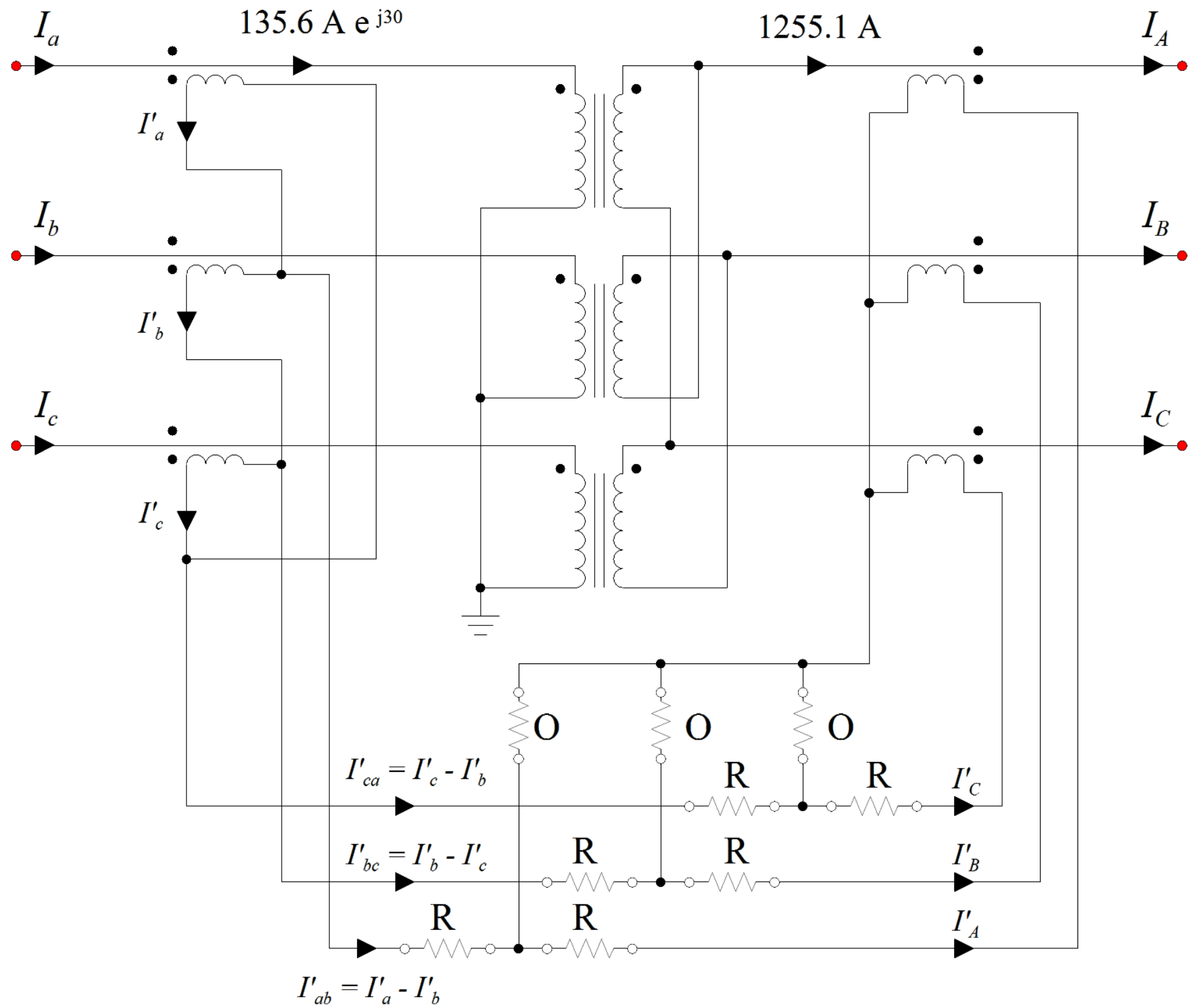
Case 1: current = 1.0 pu, tap=0.9

Note in this case the currents are (without loss of generality, assume the phase angle of one current to be zero):

$$\tilde{I}_A = -1{,}255 \quad A$$

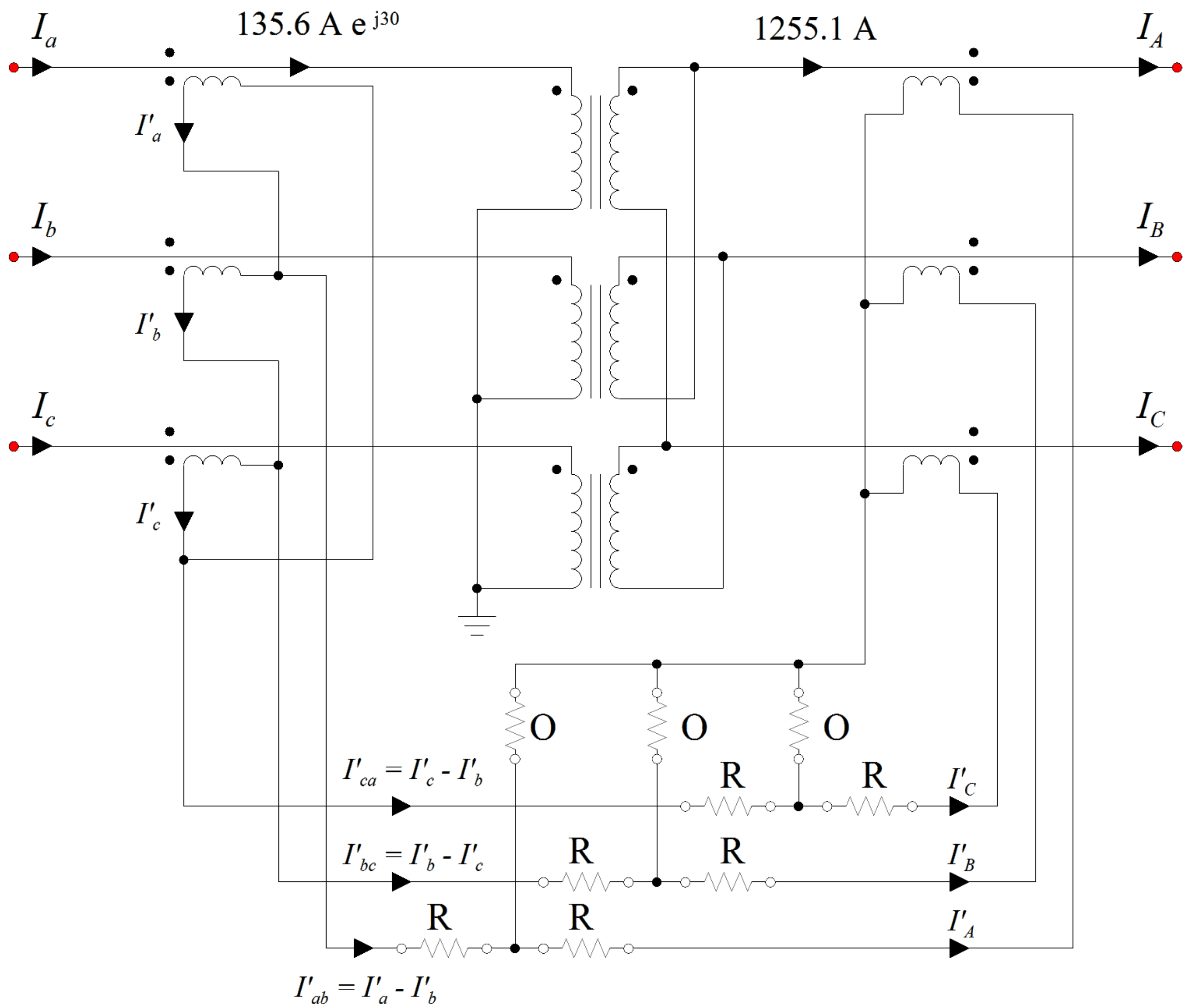
$$\tilde{I}_a = 135.55 e^{-j30^0} \; A$$

In this case the current in the operating coil is 0.3 A (see figure for case 1 below - CHECK)



Case 2: current = 1.0 pu, tap=1.1, pf=0.95 inductive

In this case the current in the operating coil is 0.2 A (see figure for case 2 below - CHECK).



Problem P7.4: An iron core, single phase, 14.4 kV/240V, 75 kVA, 60 Hz transformer is energized from an ideal voltage source. The voltage source produces a voltage equal to:

$e(t) = \sqrt{2}E\cos(\omega t + 8^0)\quad volts,$ where: E=14.44 kV, ω=377 sec^{-1}

The transformer has the following magnetic flux linkage versus magnetizing current relationship:

$i_m(t) = i_0\left(\frac{|\lambda(t)|}{\lambda_0}\right)^8 sign(\lambda(t))$, where, i_0 equals 0.01 pu and λ_0 equals 1.0 pu on the transformer ratings.

The transformer is energized at time t=0. Compute the maximum value of the magnetizing current in Amperes.

What is the second harmonic current value of the inrush current for this transformer and for the specified condition?

Solution:

$$\lambda(t) = \int_0^t (\sqrt{2}E\cos(\omega\tau + 8^0))d\tau = \frac{\sqrt{2}E}{\omega}\sin(\omega\tau + 8^0) + c$$

The constant c is computed from the initial condition: $\lambda(t=0)=0$, yielding:

$$\lambda(t) = 54.017\sin(\omega\tau + 8^0) - 7.530\,Wb$$

The magnetizing current is:

$$i_m(t) = 0.01(\sin(\omega\tau + 8^0) - 0.1394)^8\,sign(\lambda(t))$$

and

$$i_{m,\max}(t) = 0.0284\,pu$$

The second harmonic is:

$$a_2 = 0.005563\,pu$$
$$b_2 = -0.001595\,pu$$

and $I_2 = \frac{1}{\sqrt{2}}\sqrt{a_2^2 + b_2^2} = 0.004092\,pu$

Problem P7.5: Consider the electric power system illustrated in Figure P7.5. The indicated three phase, 30 MVA, 138kV:12.47kV, $z = j0.085\,pu$, delta-wye connected transformer is resistance grounded and it is protected with a differential relay. The delta side is the 138 kV side. The grounding resistance is $R_g = 20\,ohms$. The settings of the relay are: 5% restrain, minimum pickup 0.5 Amperes. The CTs are 240:5 and 2400:5 for the high voltage side and low voltage side respectively. The high voltage side CTs are connected wye and the low voltage side CTs are connected delta. The parameters of the equivalent source and line are:

Source: $Z_1 = Z_2 = j5.1\,ohms, \quad Z_0 = j4.8\,ohms$
Transmission Line: $Z_1 = Z_2 = j13.2\,ohms, \quad Z_0 = j32.8\,ohms$

(a) Assume a three-phase fault at the indicated location (this is a fault outside the differential protection zone, or a “through” fault). Determine whether the relay will trip or not for this fault.

(b) Assume a single phase to ground fault at the indicated fault location (this is also a fault outside the differential protection zone, or a "through" fault). Determine whether the relay will trip.

The use of program WinIGS is encouraged.

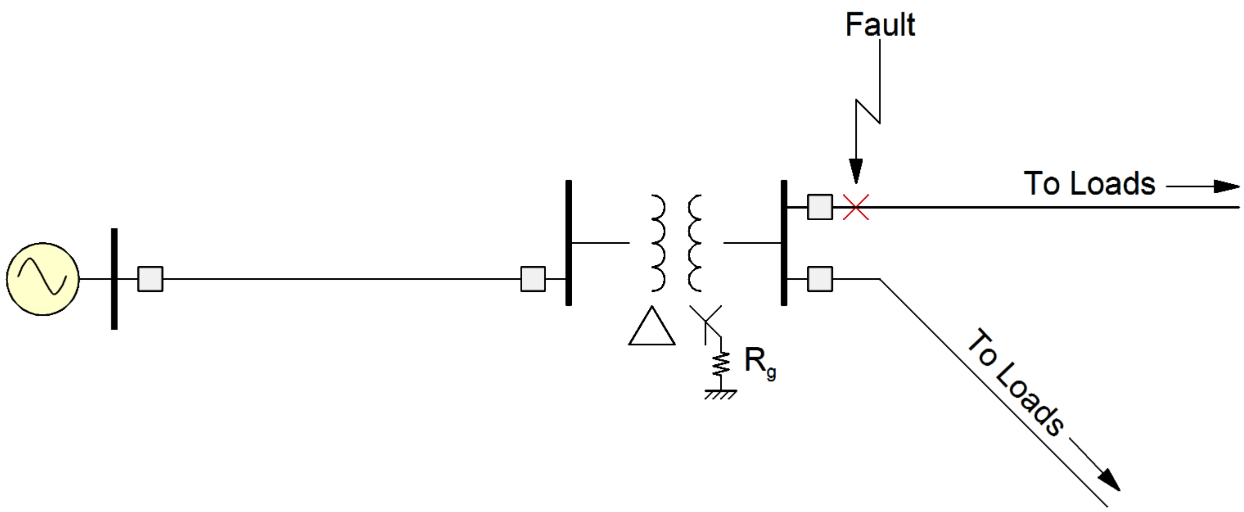


Figure P7.5

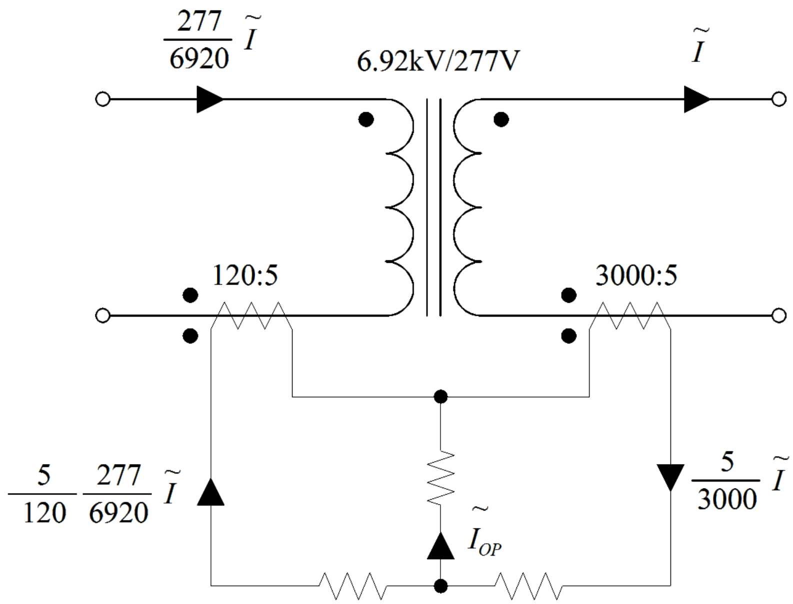
Solution: The short circuit analysis has been performed with the program WinIGS.

(a) Three-phase fault. The results are:

Problem P7.6: A single phase 6.92 kV/277V, 750 kVA, 60 Hz transformer is protected with a differential relay that uses the following current transformers: Primary: 120:5 and secondary: 3000:5. Compute the current in the operating coil of the differential relay as a function of the transformer current in the secondary, assuming ideal CTs.

Solution: Assume the current in secondary is I. Then:

$$I_{relay} = \left(\frac{5}{3000}\right)I - \left(\frac{277}{6920}\right)\left(\frac{5}{120}\right)I = (0.0000012)I$$



Problem P7.7: Consider the electric power system of Figure P7.7a. The parameters of the various system components are given in Figure P7.7a. Note that the indicated per-unit parameters are always on the device ratings. The transformer is protected with a differential relay fed via current transformers (20,000:5A on the generator side and 2,000:5A on the 230 kV side). The connections are shown in Figure P7.7b. Consider a three-phase fault at location A indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. Compute the current in the operating coil of the differential relay.

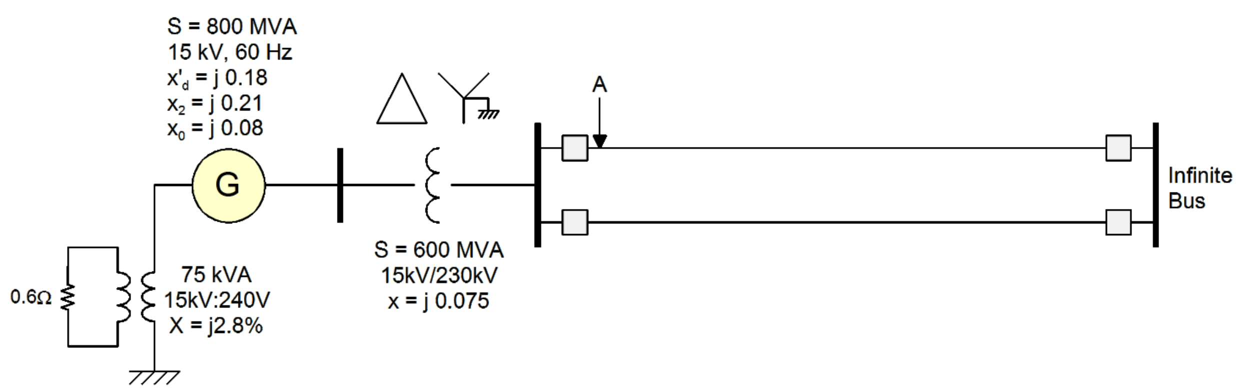


Figure P7.7a

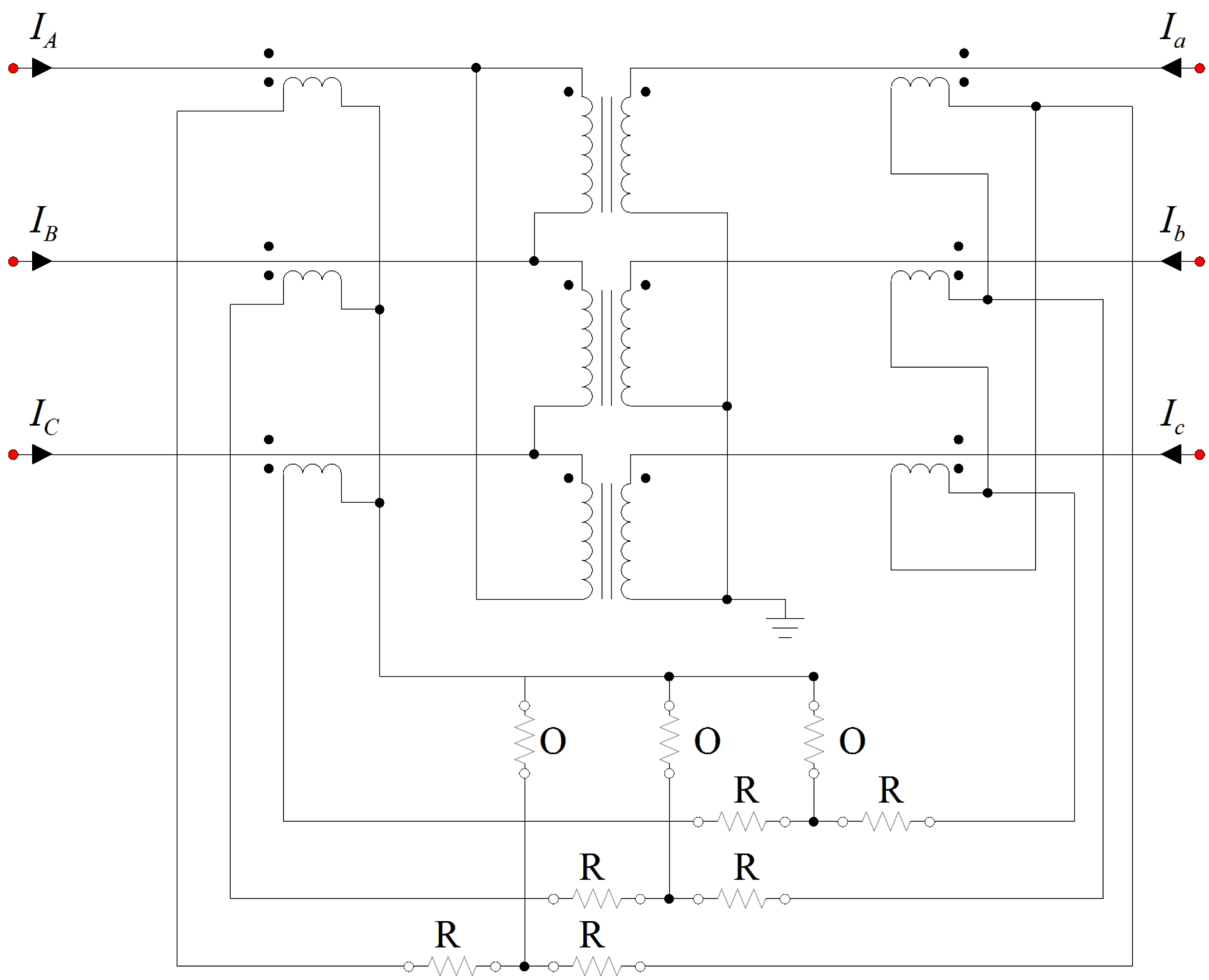


Figure P7.7b

Solution: Performing a fault analysis, the fault current on the high side is:

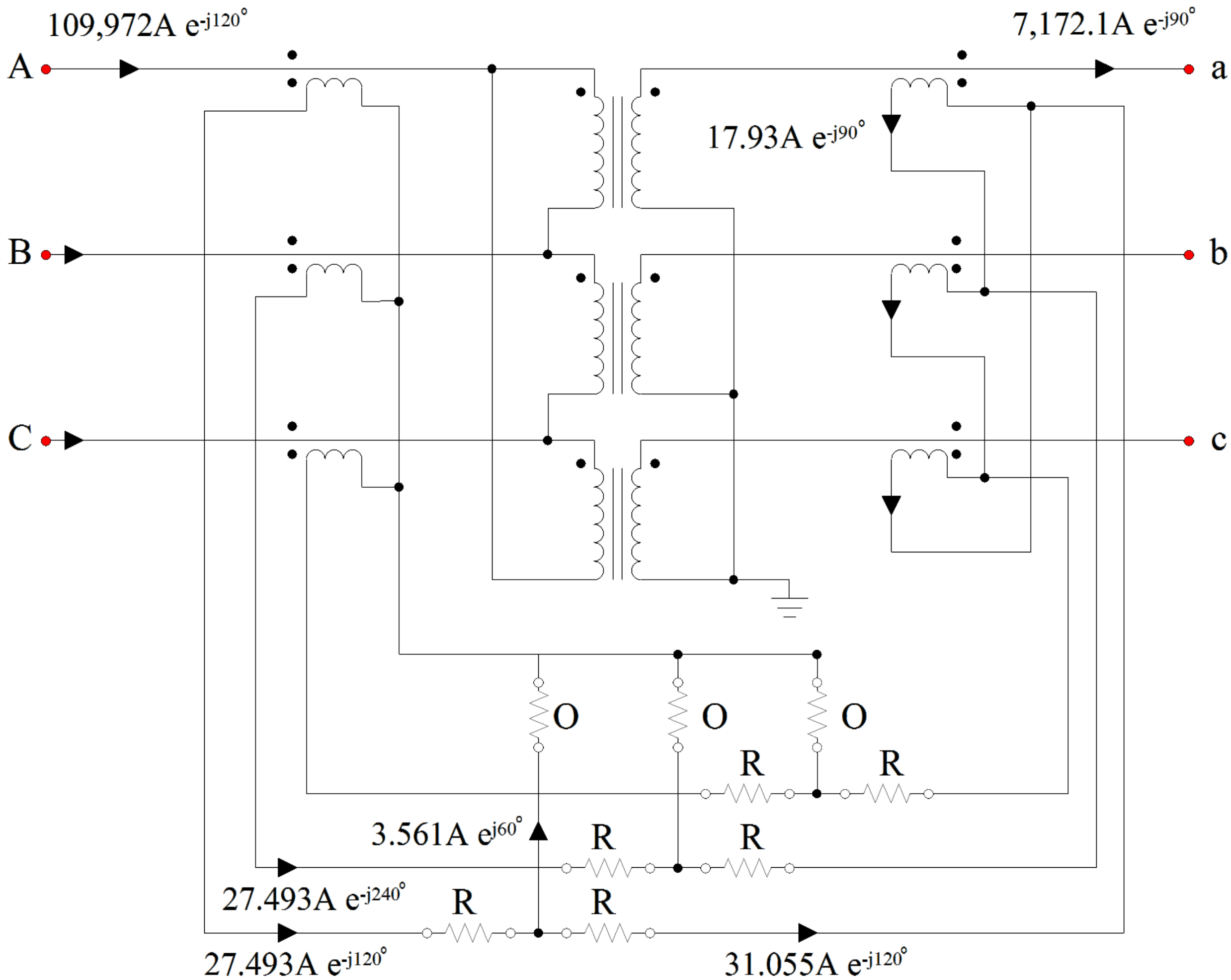
$$\tilde{I}_1 = -j3.5714\ pu = 7{,}172.1\ A$$

The phase currents on the high voltage and low voltage sides will be:

$$\tilde{I}_A = 7{,}172.1 e^{-j90^0}\ Amperes$$
$$\tilde{I}_B = 7{,}172.1 e^{-j210^0}\ Amperes$$
$$\tilde{I}_C = 7{,}172.1 e^{j30^0}\ Amperes$$

$$\tilde{I}_a = 109{,}972.2 e^{-j60^0}\ Amperes$$
$$\tilde{I}_b = 109{,}972.2 e^{-j180^0}\ Amperes$$
$$\tilde{I}_c = 109{,}972.2 e^{j90^0}\ Amperes$$

The figure below shows the fault currents in the transformer and the secondary of the CTs. The current in the operating coil is 3.561 Amperes.



Problem P7.8: Consider the electric power system of Figure P7.8a. The parameters of the various system components are given in the Figure. Note that the indicated per-unit parameters are always on the device ratings. The transformer is grounded via the grounding resistor R. The transformer is protected with a differential relay fed via current transformers (30,000:5A on the generator side and 3,200:5A on the 230 kV side). The connections of the differential relay are shown in Figure P7.8b. The settings of the differential relay are: minimum pickup current 0.5 Amperes, 20% percentage differential.

(a) Assume the grounding resistor to be zero, R=0, (solidly grounded). Consider a single-phase to ground fault at location A indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. Compute the current in the operating coil of the differential relay. Will the relay trip?
(b) Assume the grounding resistor to be zero, R=0, (solidly grounded). Consider a single-phase to ground fault at location B indicated in the Figure. Location B is on the bushings of the

transformer (internal fault). Compute the current in the operating coil of the differential relay. Will the relay trip?
(c) Assume the grounding resistor to be 10 ohms, R=10 ohms, (impedance grounded). Consider a single-phase to ground fault at location A indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. Compute the current in the operating coil of the differential relay. Will the relay trip?
(d) Assume the grounding resistor to be 10 ohms, R=10 ohms, (impedance grounded). Consider a single-phase to ground fault at location B indicated in the Figure. Location B is on the bushings of the transformer (internal fault). Compute the current in the operating coil of the differential relay. Will the relay trip?

The use of the program WinIGS is encouraged for the fault analysis.

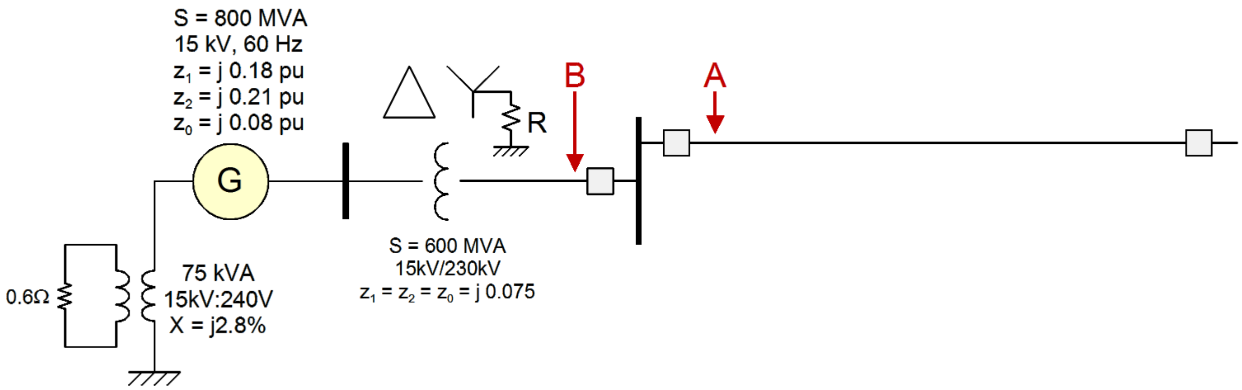


Figure P7.8a

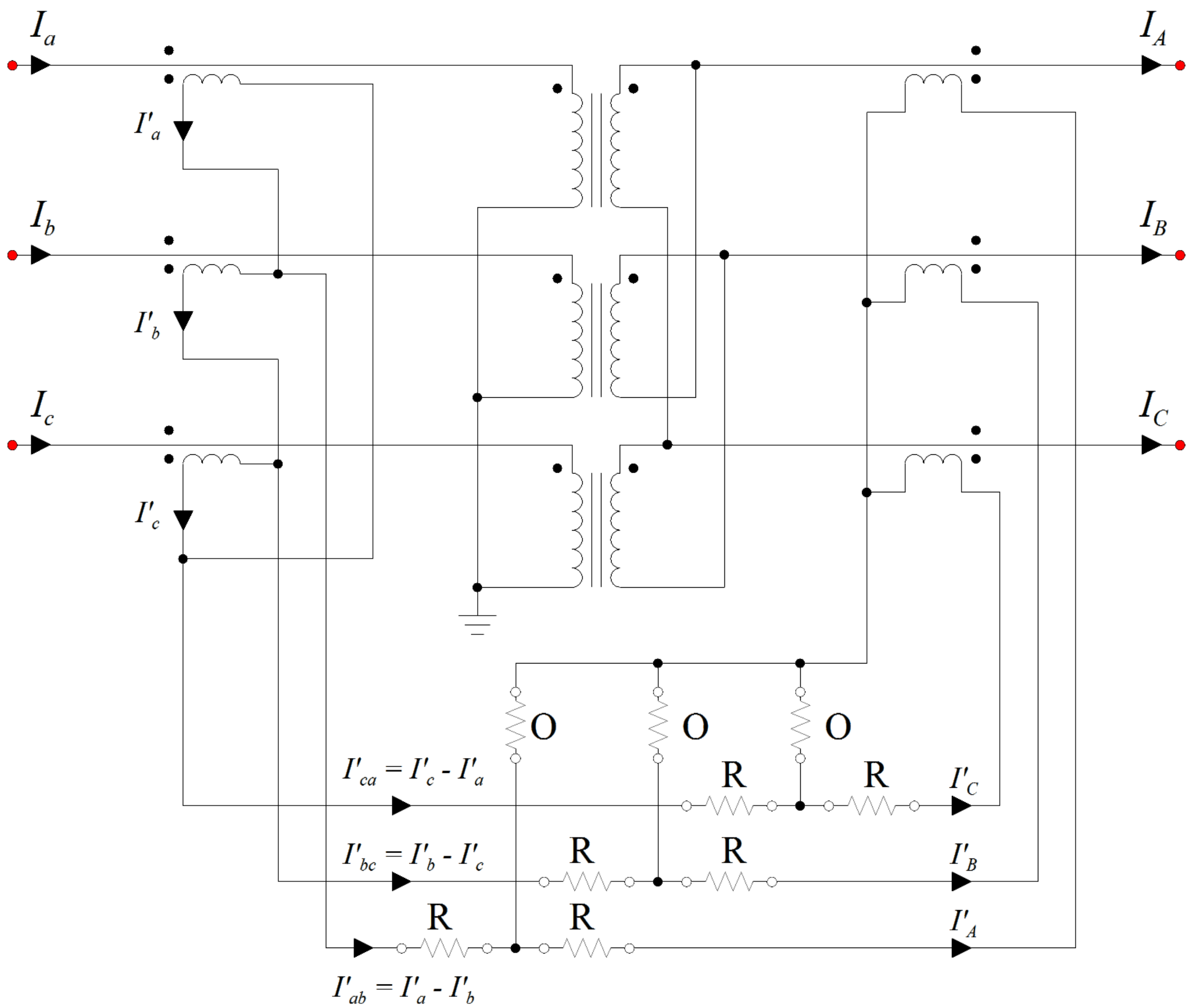
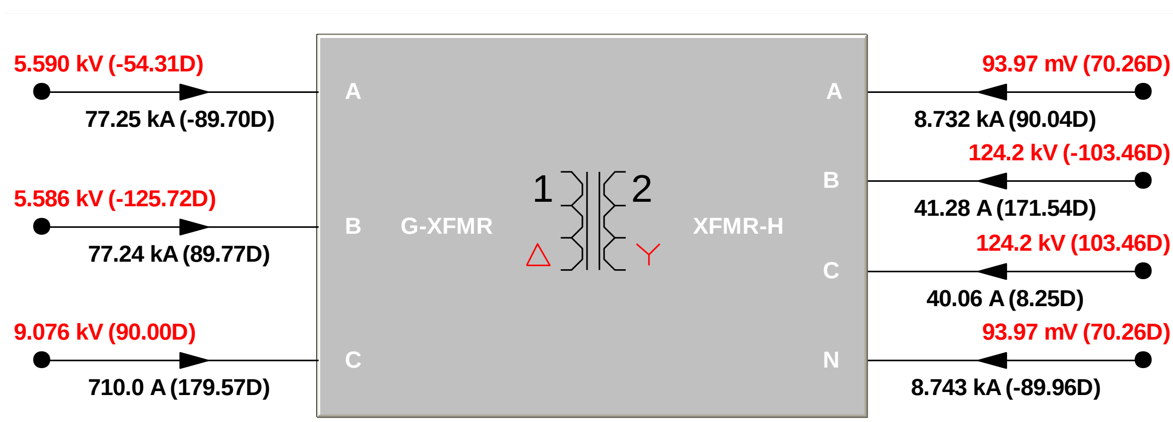
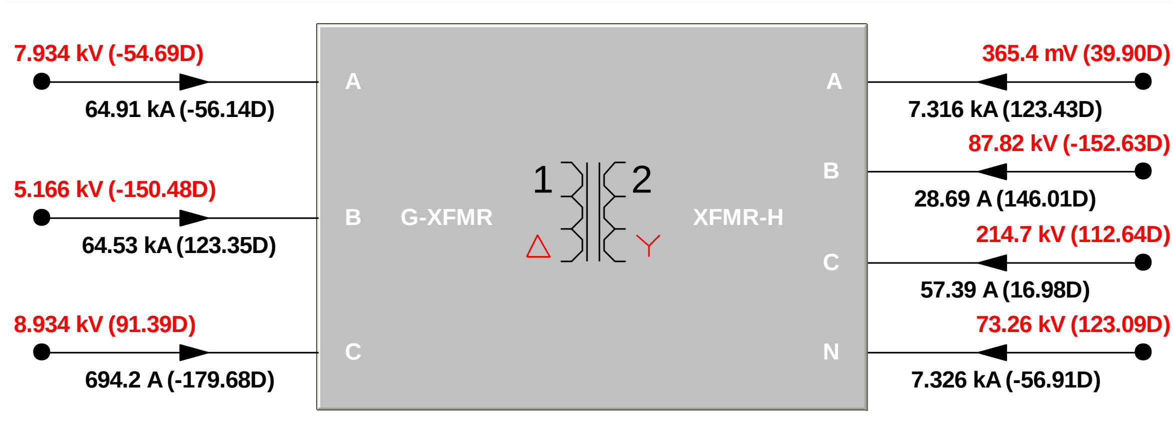


Figure P7.8b

Solution: The system is first modeled in the program WinIGS. The single line to ground fault results for the case of solidly grounded is given below.

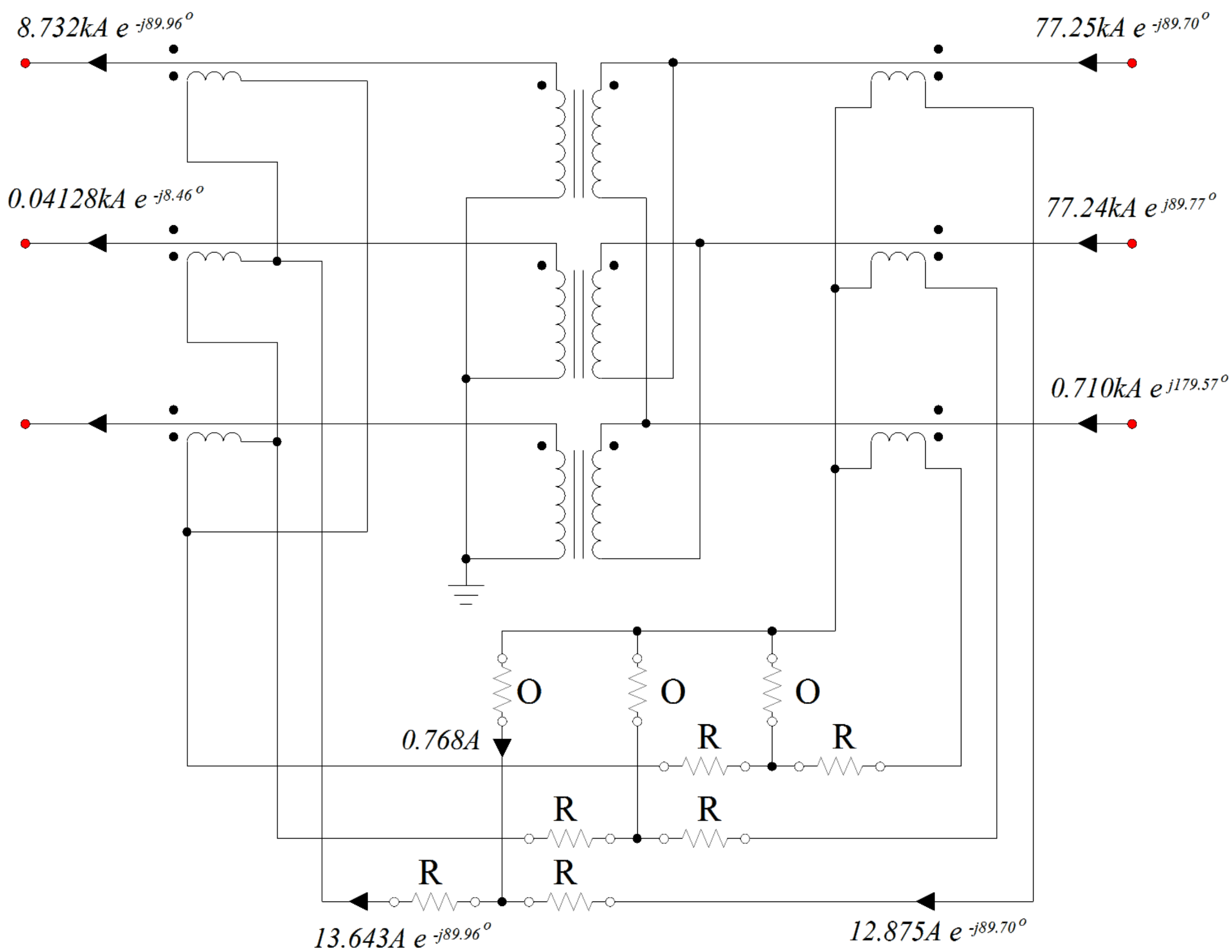


For the case of single line to ground fault with a 10 ohm ground resistor, the results are given below.



Case (a): The analysis of current flow is shown in the figure. The operating coil current is 0.768 Amperes (above the pickup current) but the percentage is

$\% = \frac{0.768}{0.5(13.643+12.875)}(100) = 5.79\%$ which is below 20% and therefore the relay will not operate.



Case (b): In this case the currents are same except that the fault is before the CT on the high voltage side of the transformer and therefore the current through that CT will be practically zero. Then, the operating coil current is 12.875 Amperes (above the pickup current) and the percentage is

$$\% = \frac{12.875}{(0.5)(0+12.875)}(100) = 200\%$$ which is above 20% and therefore the relay will operate.

Case (c): The analysis of current flow is shown in the figure. The operating coil current is 0.62 Amperes (above the pickup current) but the percentage is

$$\% = \frac{0.62}{(0.5)(11.43+10.81)}(100) = 5.57\%$$ which is below 20% and therefore the relay will not operate.

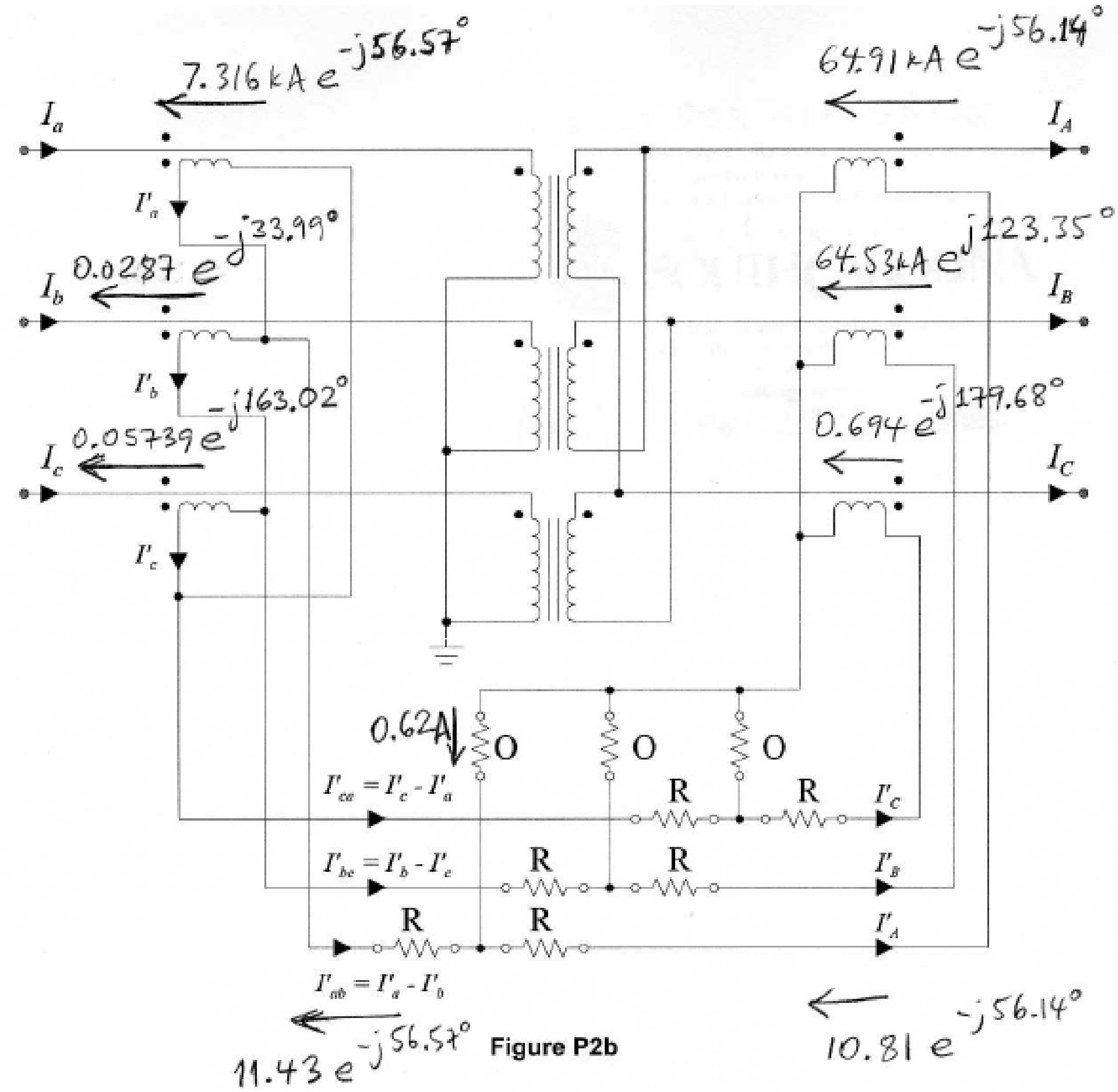


Figure P2b

Case (d): In this case the currents are same as in case (c) except that the fault is before the CT on the high voltage side of the transformer and therefore the current through that CT will be practically zero. Then, the operating coil current is 10.81 Amperes (above the pickup current) and the percentage is

$$\% = \frac{10.81}{0.5(0+10.81)}(100) = 200\%$$ which is above 20% and therefore the relay will operate.

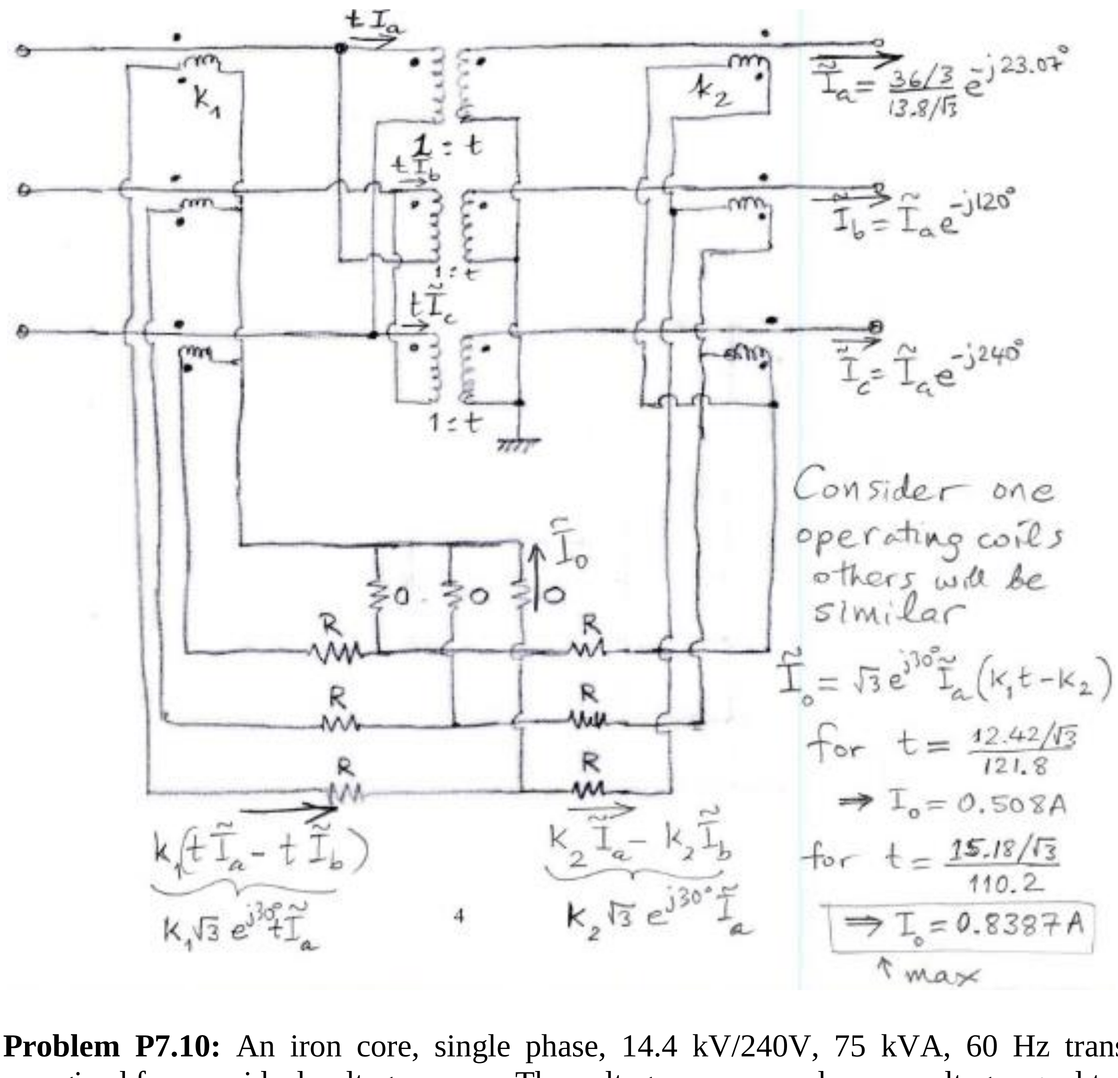
Problem P7.9: Consider a three-phase transformer with the following nominal specifications: 116 kV/13.8 kV, 36 MVA, 60 Hz. The transformer has the following taps:

High Voltage Side (Delta Connected): 110.2 kV to 121.8 kV (five taps)
Low Voltage Side (Wye Connected): 12.42 kV to 15.18 kV (8 raise taps and 8 lower taps)

It is desired to generate a circuit that will have an output as close as possible to the net current flowing into the transformer. For this purpose, an engineer selects the following CTs: High Voltage Side CTs (transformer delta connected side): 200:5, Low Voltage Side CTs (transformer wye connected side): 3000:5. The wire diagram of the system is shown in the figure below.

Compute the maximum operating current (current in the operating coil of the relay) when the transformer load at the secondary is nominal, balanced, at nominal voltage and power factor 0.92 current lagging. (Hint: consider the extreme cases of tap selection yielding the extreme transformation ratios: 110.2kV /15.18kV and 121.8 kV/12.42kV).

Solution:



Problem P7.10: An iron core, single phase, 14.4 kV/240V, 75 kVA, 60 Hz transformer is energized from an ideal voltage source. The voltage source produces a voltage equal to:

$e(t) = \sqrt{2}E\cos(\omega t + 38^0) \quad volts,$ where: E=14.44 kV, ω=377 sec^{-1}

The transformer has the following magnetic flux linkage versus magnetizing current relationship:

$i_m(t) = i_0 \left(\frac{|\lambda(t)|}{\lambda_0} \right)^9 sign(\lambda(t))$, where, i_0 equals 0.01 pu and λ_0 equals 1.0 pu on the transformer ratings.

The transformer is energized at time t=0. Compute the maximum value of the magnetizing current in Amperes.

Problem P7.11: Consider the electric power system of Figure P7.xa. The parameters of the various system components are given in the Figure. Note that the indicated per-unit parameters are always on the device ratings. The generating facility has a ground mat with a total ground impedance of 0.25 ohms. The transformer is grounded via the grounding resistor R. The transformer is protected with a differential relay fed via current transformers (25,000:5A on the generator side and 2,400:5A on the 345 kV side). The connections of the differential relay are shown in Figure P7.xb. The settings of the differential relay are: minimum pickup current 0.5 Amperes, 10% percentage differential.

Assume the grounding resistor to be 10 ohms, R=10 ohms, (impedance grounded). Consider a single-phase to ground fault at location A indicated in the Figure. Location A is very close to the 345 kV bus of the transformer and practically the impedance between the bus and location A is zero. For simplicity neglect the system beyond the fault location A. Compute the current in the operating coil of the differential relay. Will the relay trip?

The use of the program WinIGS is encouraged for the fault analysis.

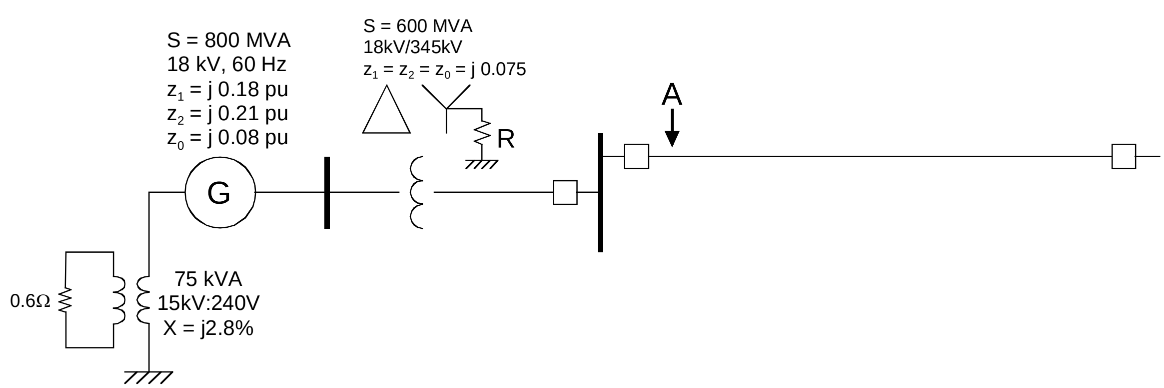


Figure P7.xa

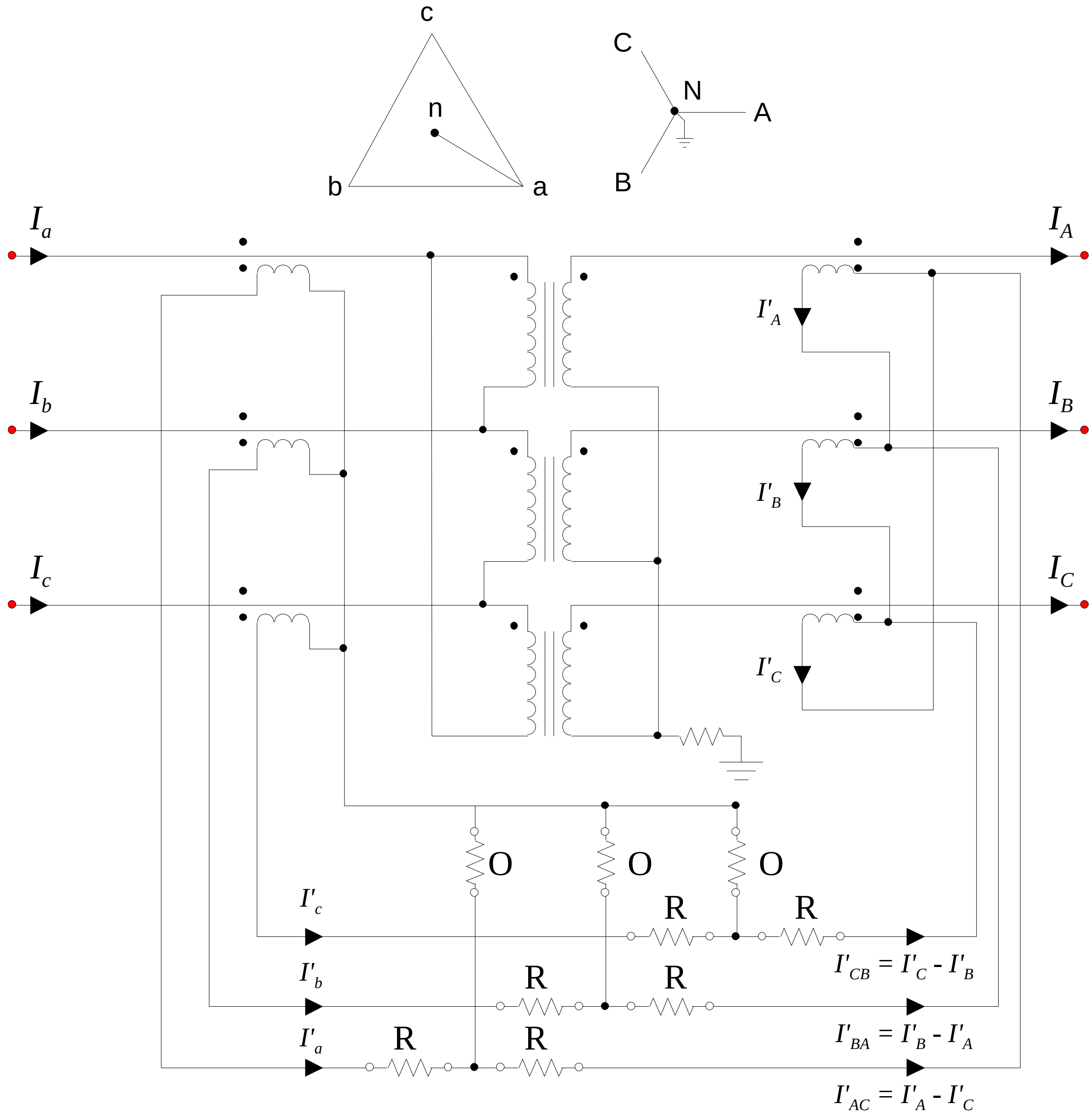
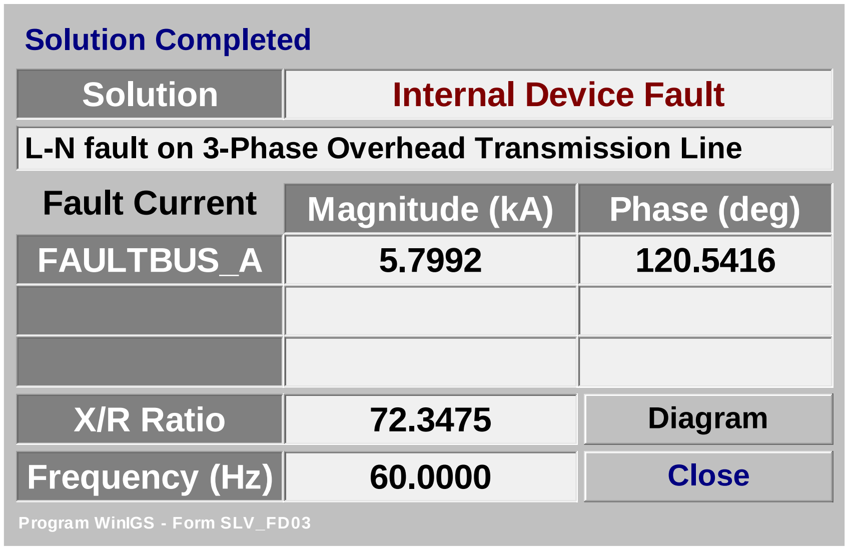
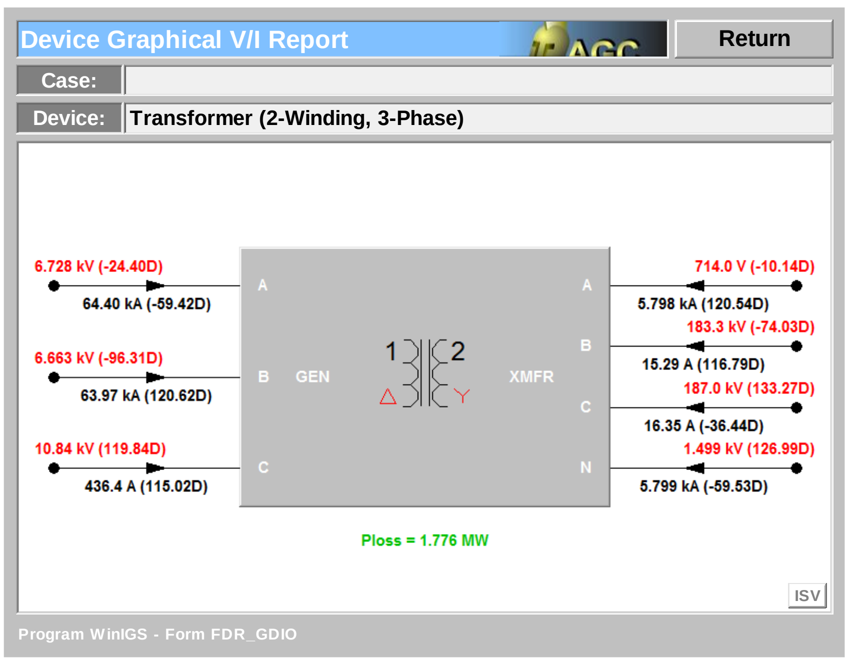


Figure P7.xb

Solution:



The current information of transformer is



The current at relay side:

$$\tilde{I}'_a = \frac{5}{25000} \times \tilde{I}_a = \frac{5}{25000} \times 64.4 e^{-j59.42^\circ} kA = 12.88 e^{-j59.42^\circ} A$$

$$\tilde{I}'_{AC} = \tilde{I}'_A - \tilde{I}'_C = \frac{5}{2400} \times (\tilde{I}_A - \tilde{I}_C)$$

$$= \frac{5}{2400} \times (5798 e^{j(120.54^\circ - 180^\circ)} - 16.35 e^{j(-36.44^\circ + 180^\circ)}) A = 12.1105 e^{-j59.397^\circ} A$$

$$\tilde{I}'_{OA} = \tilde{I}'_a - \tilde{I}'_{AC} = 12.88 e^{-j59.42^\circ} - 12.1105 e^{-j59.397^\circ} = 0.7695 e^{-j59.782^\circ} A$$

$$\frac{\tilde{I}'_{OA}}{\frac{\tilde{I}'_a+\tilde{I}'_{AC}}{2}} = \left| \frac{0.7695e^{-j59.782^\circ}}{\frac{12.88e^{-j59.42^\circ}+12.1105e^{-j59.397^\circ}}{2}} \right| \times 100\% = 6.16\% < 10\%$$

Phase A will not trip.

$$\tilde{I}'_b = \frac{5}{25000} \times \tilde{I}_b = \frac{5}{25000} \times 63.97e^{j120.62^\circ} kA = 12.794e^{j120.62^\circ} A$$

$$\tilde{I}'_{BA} = \tilde{I}'_B - \tilde{I}'_A = \frac{5}{2400} \times (\tilde{I}_B - \tilde{I}_A)$$

$$= \frac{5}{2400} \times (15.29e^{j(116.79-180^\circ)} - 5798e^{j(120.54^\circ-180^\circ)})A = 12.0474e^{j120.5499^\circ} A$$

$$\tilde{I}'_{OB} = \tilde{I}'_b - \tilde{I}'_{BA} = 12.794e^{j120.62^\circ} - 12.0474e^{j120.5499^\circ} = 0.7468e^{j121.7508^\circ} A$$

$$\frac{\tilde{I}'_{OB}}{\frac{\tilde{I}'_b+\tilde{I}'_{BA}}{2}} = \left| \frac{0.7468e^{j121.7508^\circ}}{\frac{12.794e^{j120.62^\circ}+12.0474e^{j120.5499^\circ}}{2}} \right| \times 100\% = 6.01\% < 10\%$$

Phase B will not trip.

$$\tilde{I}'_c = \frac{5}{25000} \times \tilde{I}_c = \frac{5}{25000} \times 436.4e^{j115.02^\circ} A = 0.0873e^{j115.02^\circ} A$$

$$\tilde{I}'_{CB} = \tilde{I}'_C - \tilde{I}'_B = \frac{5}{2400} \times (\tilde{I}_C - \tilde{I}_B)$$

$$= \frac{5}{2400} \times (16.35e^{j(-36.44^\circ+180^\circ)} - 15.29e^{j(116.79-180^\circ)})A = 0.0641e^{j130.6381^\circ} A$$

$$\tilde{I}'_{OC} = \tilde{I}'_c - \tilde{I}'_{CB} = 0.0873e^{j115.02^\circ} - 0.0641e^{j130.6381^\circ} = 0.0308e^{j84.9489^\circ} A < 0.5A$$

$$\frac{\tilde{I}'_{OA}}{\frac{\tilde{I}'_a+\tilde{I}'_{AC}}{2}} = \left| \frac{0.7695e^{-j59.782^\circ}}{\frac{12.88e^{-j59.42^\circ}+12.1105e^{-j59.397^\circ}}{2}} \right| \times 100\% = 41.06\%$$

Problem P7.12: An iron core, single phase, 14.4 kV/240V, 75 kVA, 60 Hz transformer is energized from an ideal voltage source. The voltage source produces a voltage equal to:

$e(t) = \sqrt{2}E\cos(\omega t + 38^0) \quad volts,$ where: E=14.44 kV, ω=377 sec^{-1}

The transformer has the following magnetic flux linkage versus magnetizing current relationship: $i_m(t) = i_0\left(\frac{|\lambda(t)|}{\lambda_0}\right)^9 sign(\lambda(t))$, where, i_0 equals 0.01 pu and λ_0 equals 1.0 pu on the transformer ratings.

The transformer is energized at time t=0. Compute the maximum value of the magnetizing current in Amperes.

Solution: Upon transformer energization the following equation provides the relationship of transformer magnetic flux linkage and transformer voltage:

$\frac{d\lambda(t)}{dt} = e(t)$, with initial condition: $\lambda(0) = 0$. Solution is:

$$\lambda(t) = \int e(t)dt = \frac{\sqrt{2}E}{\omega}\sin(\omega t + 0.6632) - \frac{\sqrt{2}E}{\omega}\sin(0.6632)$$
$$= \frac{\sqrt{2}E}{\omega}\sin(\omega t + 0.6632) - \frac{\sqrt{2}E}{\omega}\sin(0.6632)$$
$$= 54.019\sin(\omega t + 0.6632) - 33.2563 Wb$$

The maximum magnetizing current $i_m(t)$ is obtained at maximum flux linkage

$$\lambda_{\max}(t) = 54.019 \times (-1) - 33.2563 = -87.2753 Wb$$

$$i_{m,\max}(t) = 0.01 \times \left(\frac{75k}{14.4k}\right)\left(\frac{87.2753}{54.019}\right)^9 \times (-1) = -3.9066 \ Amperes$$

Problem P7.13: Consider a three-phase transformer, delta-wye connected, variable tap (on the low side), with nominal ratings $36\,MVA, \quad 138kV/13.8kV, \quad z = j0.08\,pu$ indicated in Figure P7.xa. This transformer is protected with a differential relay. The delta side is the 138 kV side. The settings of the relay are: 5% restrain, minimum pickup 0.5 Amperes. The CTs are 100:5 and 1800:5 for the high voltage side and low voltage side respectively. The high voltage side CTs are connected wye and the low voltage side CTs are connected delta, as indicated in the figure.

At a certain instance of time, the transformer operates at the 138kV/12.45kV tap while a fault occurs on the low voltage system, outside the transformer. The currents on the low side of the transformer are shown in the figure. Note that for this tap the transformation ratio of each of the three single phase transformers is 138kV/7.188kV (138kV:12.45/sqrt(3.0)kV).

(a) Compute the electric current in the three operating coils of the relay.

(b) Determine whether the differential relay will trip the transformer.

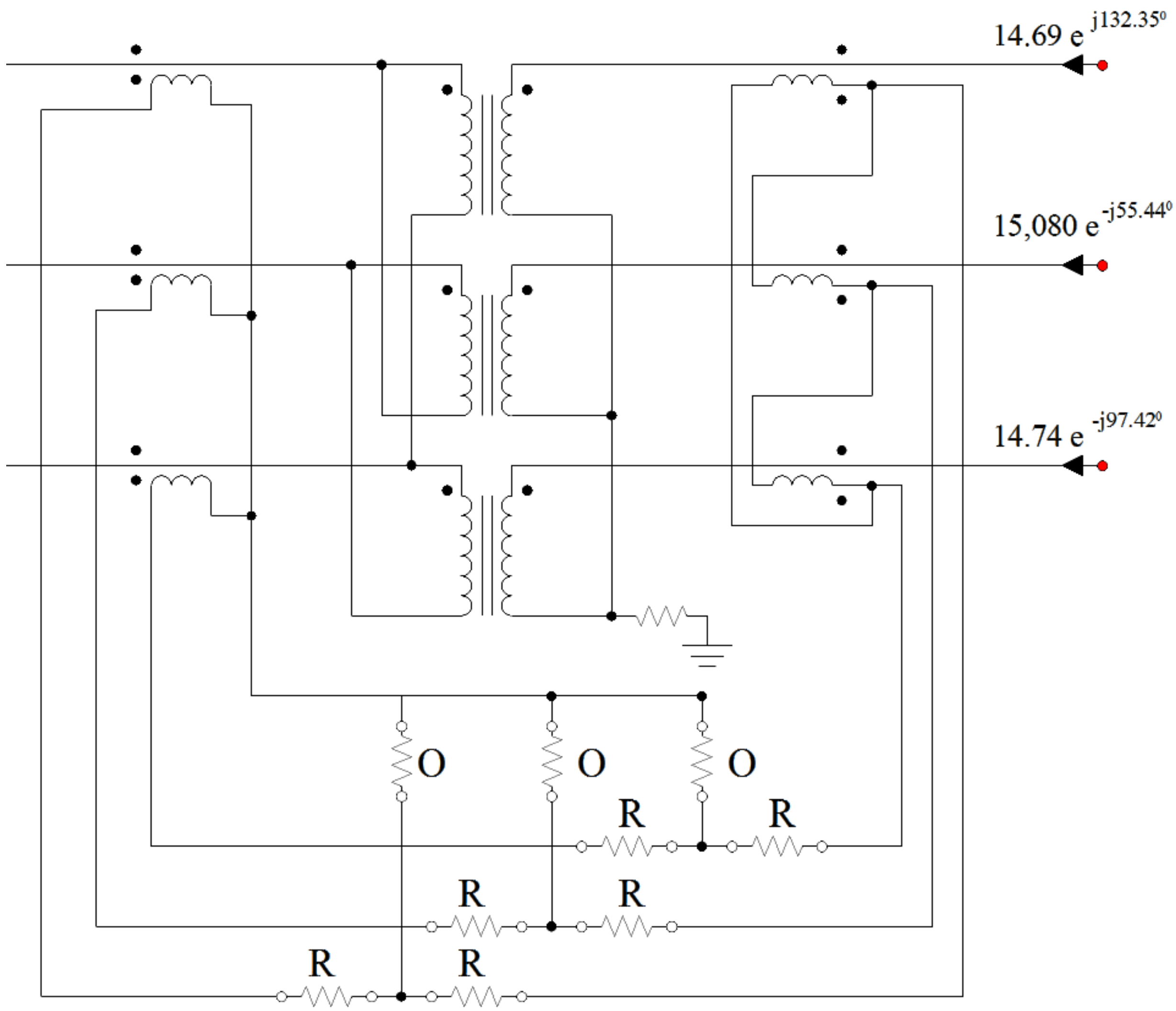


Figure P7.13a: Three-Phase, Delta-Wye Connected Transformer and Differential Scheme Configuration

DRAFT and INCOMPLETE

Table of Contents

from

A. P. Sakis Meliopoulos and George J. Cokkinides
Power System Relaying, Theory and Applications

Chapter 8
Generator Protection

8.1 Introduction

In this chapter, we focus on protection schemes for generators. The majority of generating units are large synchronous machines. The protection of the synchronous generators is a critical issue for the electric power system for many reasons including: (a) synchronous generators are very expensive equipment and (b) they impact the stability of the system and consequently the reliability of the overall power system. Because of the importance of generators, protection schemes of large synchronous generators are quite comprehensive and complex. The philosophy of generator protection is that the generator must be protected against all conditions that may damage the generator or they may affect system stability and security. At the same time the generator should not be tripped for abnormal conditions that do not threaten the health of the generator, the safety of personnel or the security of the system. Such phenomena may be a transient but stable swing of a generator following a fault on the transmission system and successful clearing of the transmission system fault, low voltage due to nearby faults and low frequency due to overall system operating conditions.

Note that we have discussed two distinct classes of abnormal conditions that the generator should be protected against: (a) abnormal conditions that may damage the generator and (b) abnormal conditions that may compromise the security of the system. The former abnormal conditions refer to faults and disturbances within the generating unit. The latter refer to system disturbances (external or internal) that affect the operation of the generator, such as those that may create an unstable swing of the system. Initially, we focus on the former types of disturbances and the associated protection schemes, i.e. component protection. Later in the chapter, we address the issue of generator protection against disturbances that threaten the stability and security of the system. A common protection scheme for these type of disturbances is the out of step protection, discussed in this chapter. This scheme is part of a more general approach of system protection.

The protection philosophy of generators against internal faults and disturbances has evolved over the years. Initially, in the era of electromechanical relays, a generator will be protected with several relays, i.e. overcurrent, differential protection of the stator coils, over and under-frequency, over- and under-voltage, etc. Typically, a large generator is connected to the grid via a step-up transformer. The protection of the step-up transformer was also provided with a set of individual relays, for example overcurrent, differential, volts over Hertz, and other as discussed in Chapter 7. Many generators have dedicated step-up transformers. In this case the generator/transformer can be protected as a unit, if one part of the unit must be tripped, then both components can be tripped as a unit. In other words, the generator and transformer is treated as one protection zone. This approach is attractive from the economic point of view as one can avoid the use of an expensive generator breaker.

The development of numerical relays in the 1980's and their continuous enhancements resulted in relays with multiple functions and multiple elements for each function. As a result, numerical relays were developed that support all the recommended protection functions for a generator. We refer to these relays as generator relays.

Any generator protection scheme must be designed for the particular configuration of the overall system and the associated support components for the operation of the generator, such as exciter, auxiliary transformers, startup transformers, etc. Figures 8.1 and 8.2 illustrate two example configurations of generating units. Figure 8.1 shows a generating unit with a dedicated step-up transformer. Note that at the generator bus there may be a station service transformer and possibly another transformer for the excitation system of the generator. For some large generators the station service transformer may be directly connected to the high voltage bus of the generating plant. Figure 8.2 shows two generators sharing a three winding step-up transformer. Figure 8.3 shows a 3-D rendered view of a generating plant (only one generator is illustrated).

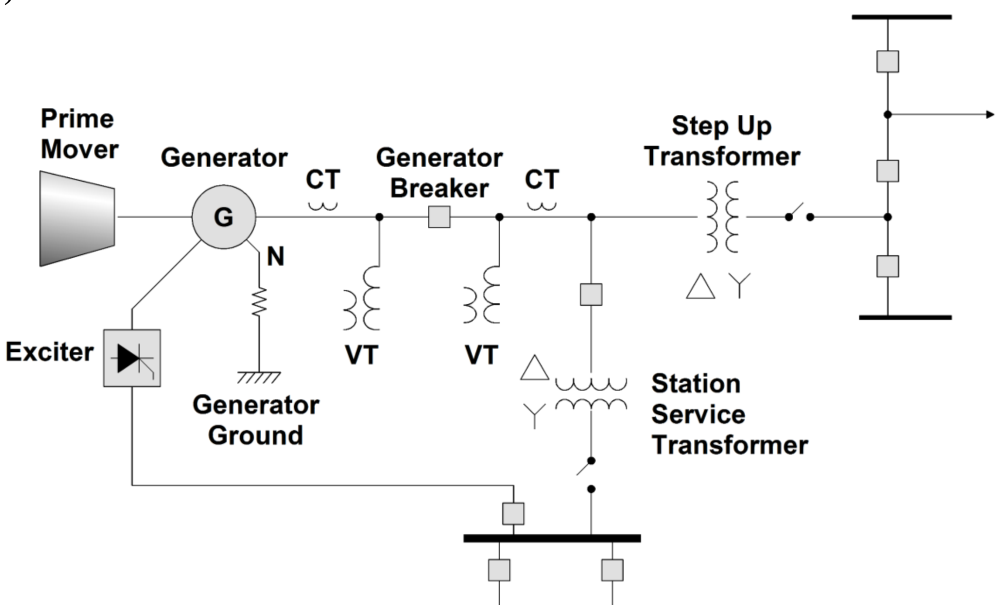


Figure 8.1 Single Generator with Dedicated Step-Up Transformer and Station Service Transformer

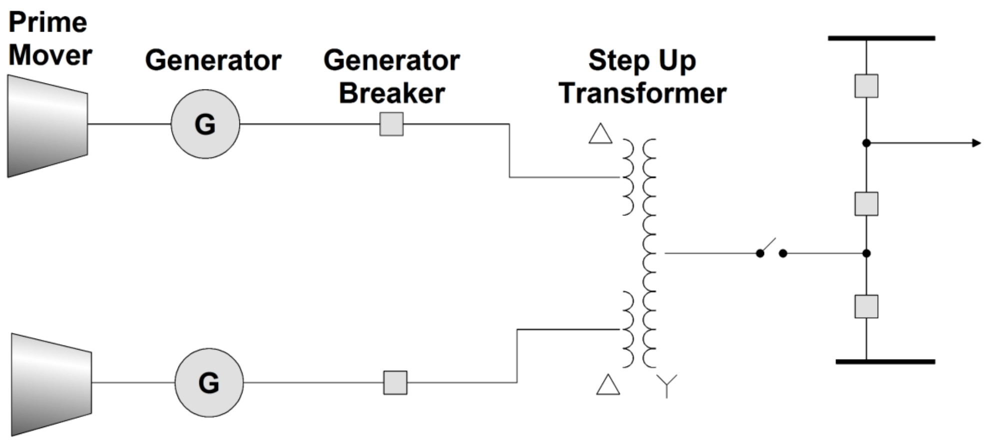


Figure 8.2 Two Generators with Three-Winding Step-Up Transformer

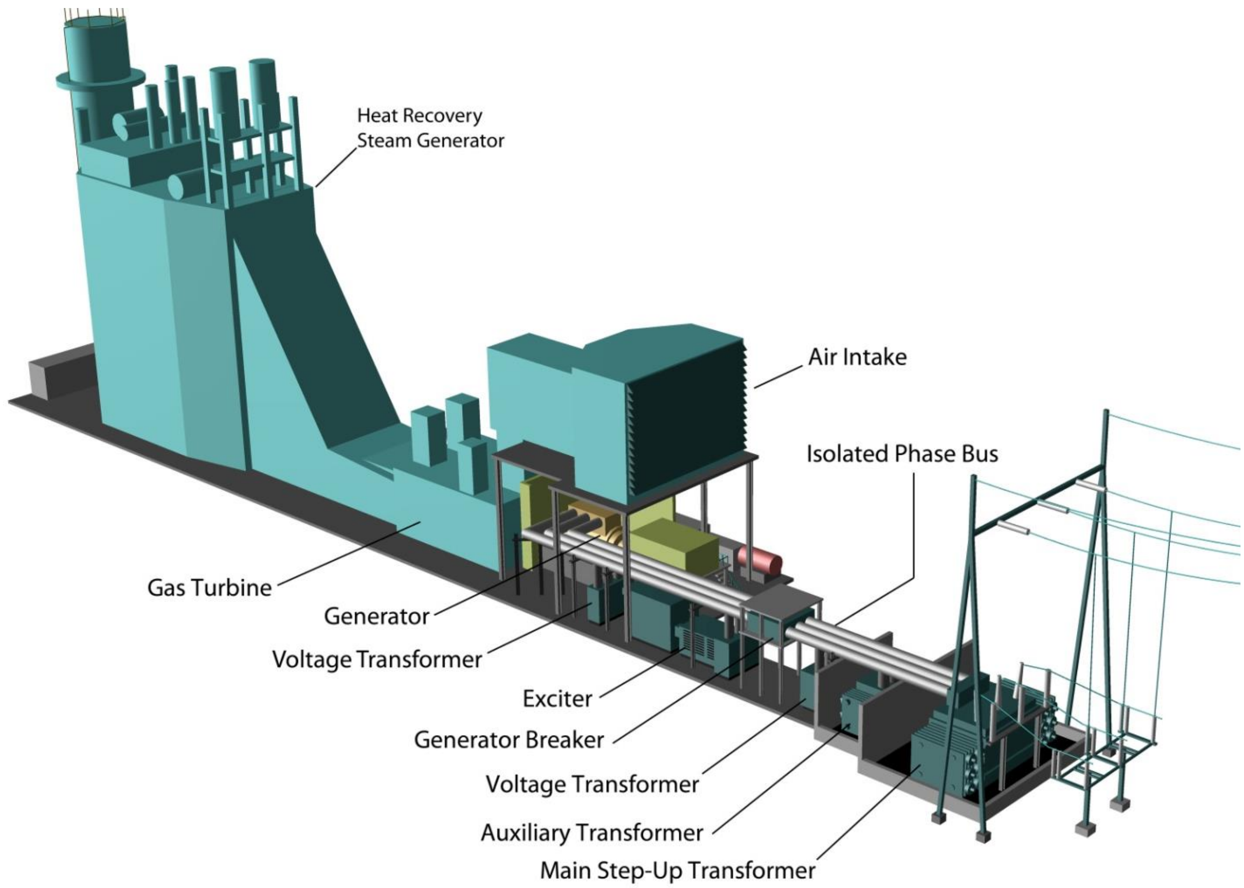


Figure 8.3 3-D Rendered View of a Generating Unit

It should be understood that for certain disturbances, both the generator and the transformer should be disconnected from the system as a unit. This is the case for the systems that shown in Figures 8.1 and 8.3. If the disturbance/fault requires that the generator be tripped, there is no reason to keep the step-up transformer energized. Similarly, if the disturbance (fault) affects the step-up transformer and the transformer must be tripped then the generator will be also tripped because it cannot deliver power to the system anymore. In addition, the generator/step-up transformer system may be connected to additional support systems such as a unit start-up transformer or an auxiliary transformer. For best practice, the protection of the generator should be coordinated with the step-up transformer and possibly with the auxiliary and start-up transformer. Many modern systems treat the generator and the step-up transformer as a "unit" and the protection system is designed to protect the unit as a single entity and to trip the entire unit when warranted. At the same time the auxiliaries of the generation unit should have their separate protection systems and should be able to be energized when the generation unit must restart. We will examine the functions of generator unit protection and then discuss protection philosophies that provide coordination for the overall generation unit. Table 8.1 provides a list of the types of protection functions that are typical in a generation protection system.

Table 8.1 Generator Protection Issues

Phase Fault Protection
Phase to Ground Fault Protection (100% Protection)
Rotor Fault Protection
Unbalance (voltage)
Undervoltage
Overvoltage
Underfrequency
Overfrequency
Overload
Loss of Excitation
Reverse Power
Unbalanced Currents
Over/Underexcitation
Motoring and Start-up
Synchronization
Accidental Energization

Figure 8.4 illustrates the practice for generator protection of a specific utility. One can identify the various protection functions from the function numbers. The objective of the array of the protection functions is to protect the generator against all possible fault conditions that may damage the generator. The philosophy of generator protection is as follows: first one needs to recognize that any fault on the generator armature will damage the generator if the fault current is high enough and before any protection system can respond, no matter how fast. For this reason the generator is design to practically eliminate high fault current faults. This is achieved by isolating the three phases of the generator so that the possibility of phase to phase and three phase faults is practically eliminated. The possible faults that can occur is a phase to the grounded enclosure of the isolated phase, i.e. a ground fault. Also faults from the winding of the transformer to the steel structure of the generator is also a ground fault. The level of currents that can develop during ground faults are minimized by means of high impedance grounding. Figure 8.3 illustrates the isolated phase bus. Figure 8.4 shows the typical high impedance grounding of large generators via a single phase transformer. The secondary of the transformer is loaded with

a resistor R. This system typically provides an equivalent grounding impedance of a few thousand ohms. With this ground impedance a ground fault may draw only a few Amperes. A typical objective is to keep this current below 5 Amperes. The protection of the generator against fault in the generator is specialized for this system.

The settings of the various protection functions will be discussed in subsequent paragraphs.

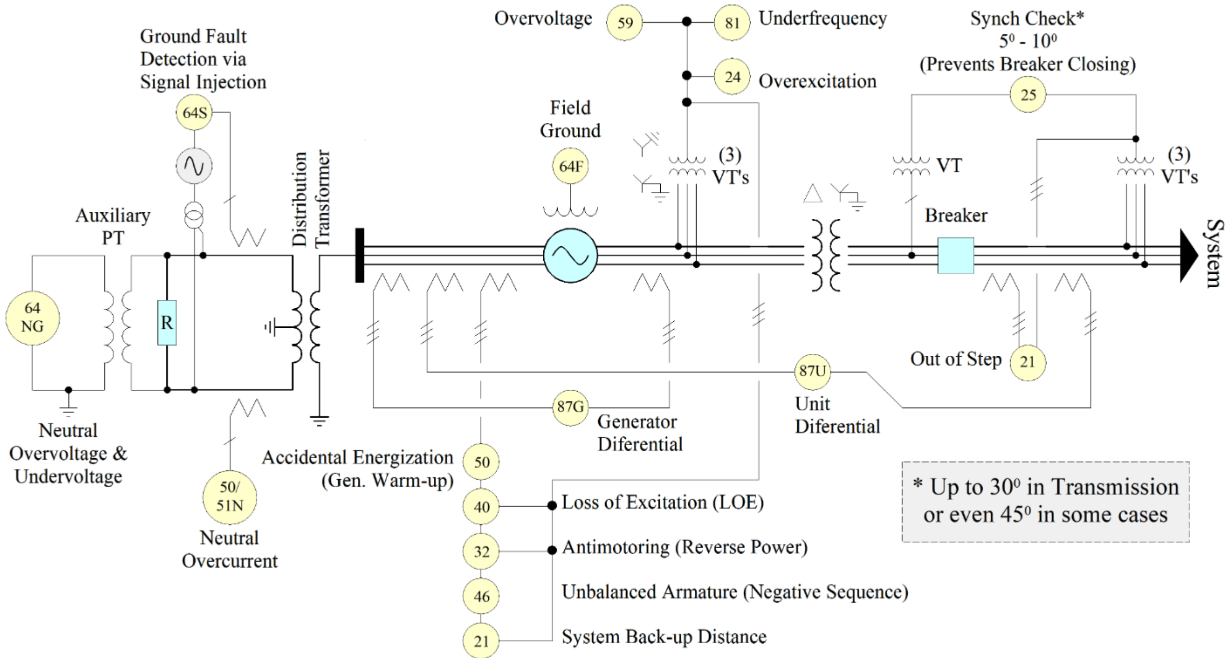


Figure 8.4 Typical Protection Functions for a Synchronous Generator
(64NG uses a 59 tuned to 60Hz and a 27 tuned to 180Hz)

8.2 Generator Protection Philosophy

Figure 8.5 illustrates a typical configuration of a generating unit. It consists of a generator connected to the power system via a delta-wye step-up transformer, a station service transformer, and grounding impedance, consisting of a resistor and distribution transformer. The philosophy of generator protection is as follows: first one needs to recognize that any fault on the generator armature will damage the generator if the fault current is high enough and before any protection system can respond, no matter how fast. For this reason the generator is designed to practically eliminate high fault current faults. This is achieved by isolating the three phases of the generator (i.e. enclosing each phase into a metallic grounded tube) so that the possibility of phase to phase and three phase faults is practically eliminated. Practically any fault will be from the phase to the grounded enclosure resulting in a phase to ground fault. The level of currents that can develop during ground faults are minimized by means of high impedance grounding. Figure 8.3 illustrates the isolated phase bus. Figure 8.5 shows the high impedance grounding of the generator via a single phase transformer. The secondary of the transformer is loaded with a resistor R. This system typically provides an equivalent grounding impedance of a few thousand ohms. With this ground impedance a ground fault may draw only a few Amperes. It is important mentioning that generators have a relatively low impedance to ground faults (low zero sequence impedance) and this results in very high ground fault currents for solidly grounded generators. By impedance grounding the neutral of the generator, ground faults result in low fault current and it allows to coordinate the protection of the generator in a more orderly fashion. The fault current of this type of fault will be limited by the grounding impedance. The level of fault current depends on the grounding impedance.

Another important objective in protecting a generator is the ability to detect and protect against faults in the winding of the generator, anywhere they may occur. It turns out that detecting winding faults near the neutral of the generator are difficult to detect. Methods have been developed to reliably detect faults near the neutral of the generator. These methods are referred to as 100% stator protection because they provide the capability to protect the generator for armature faults anywhere on the stator winding. In this chapter, we will examine these methods.

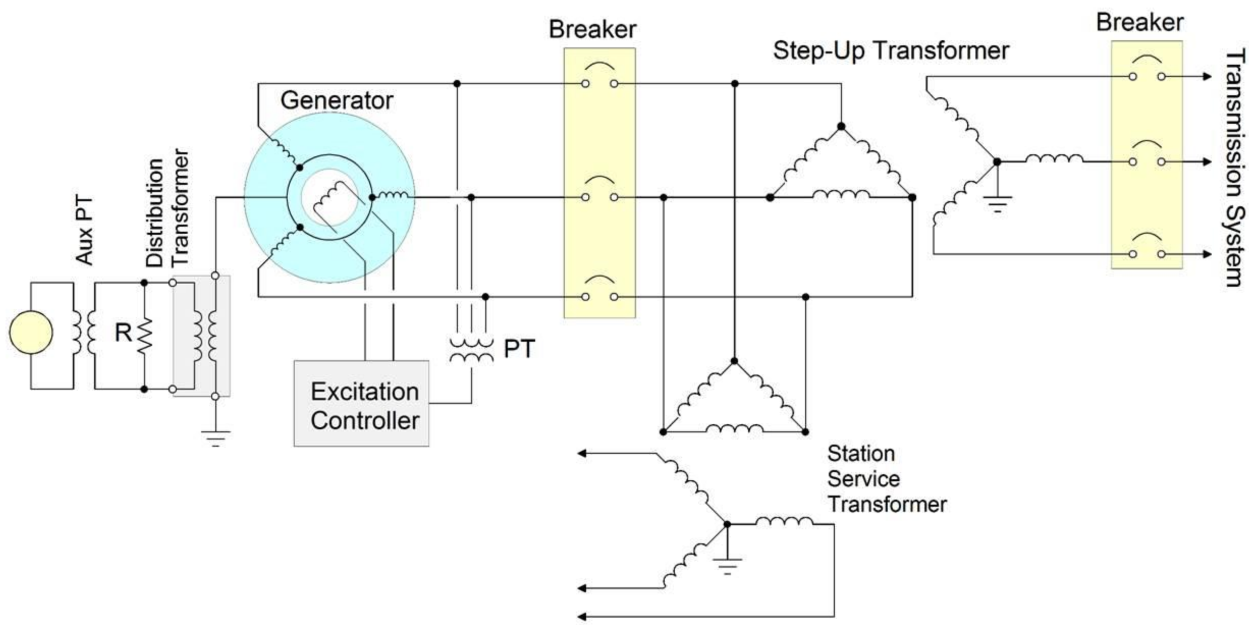


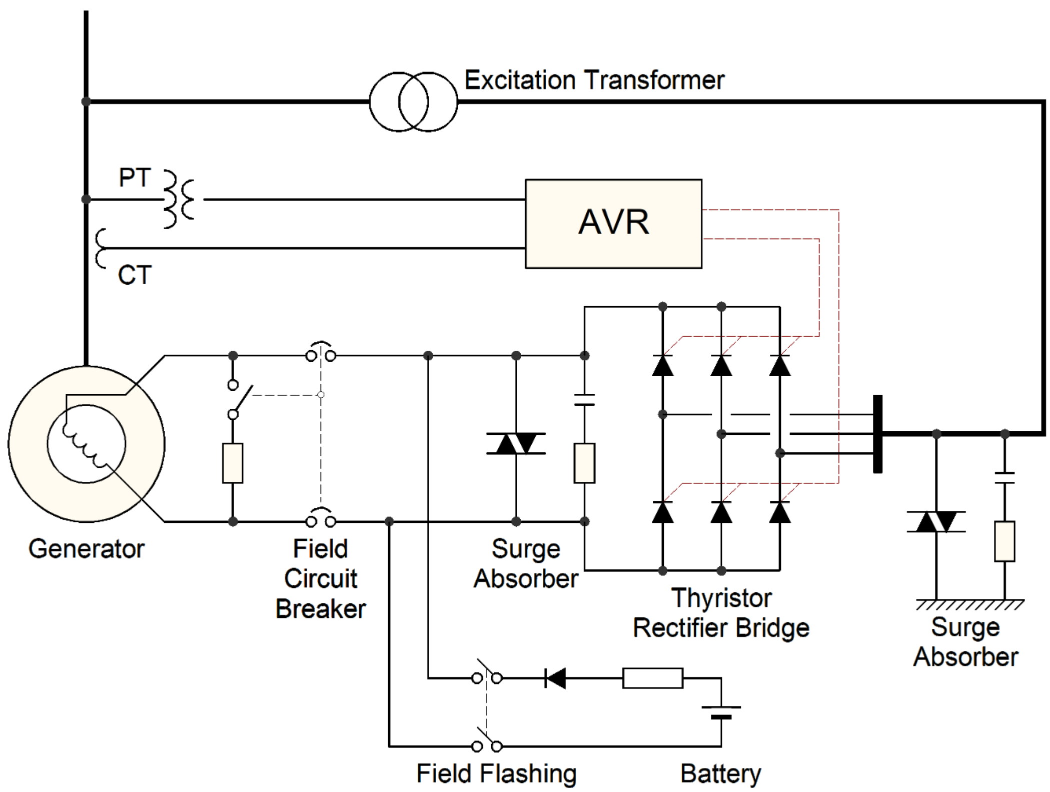
Figure 8.5: Typical Generator Configuration

The generating protection zone may contain only the generator or both generator and step-up transformer. The latter practice is more common. Figure 8.4 illustrates the general protection scheme for generating units. Since the cost of large generating equipment outages is high, the basic idea is to provide protection for any possible fault, in order to prevent major damage to the equipment. In addition, generator faults typically result in acceleration of the generating unit during the fault and the possibility that if the fault persists for a long time the generator will be driven out of synchronism from the network. For this reason, the protection practice for generators is biased towards dependability. This can be achieved by coordinating generator protection and protection of lines and transformers in the vicinity of the generator. Specifically, faults in nearby lines and transformers should be cleared as fast as possible so that the transients induced in the generator be as minimal as possible. In addition, if the transients following the successful clearance of nearby faults are severe to the point that will jeopardize the stability of the generator, the generator should be also tripped, even if there is no fault in the generator.

8.3 Generator Ground Fault Protection

Generators are normally impedance grounded to minimize electric fault currents during a ground fault and minimize generator damage. The ground impedance may be just a resistor or a reactor connected to the generator neutral at one end and to the system ground at the other end or a single phase transformer with the primary connected between the generator neutral and the plant ground and the secondary loaded with a resistor or an inductor of appropriate size and rating. The three grounding methods are illustrated in Figure 8.6 designated as high, medium and low impedance grounding.

The reason that generators are impedance grounded is to minimize the ground fault current. The maximum ground fault current will depend on the value of the grounding impedance. In this respect we classify the grounding of a generator as low impedance (usually less than 1 ohm), medium impedance (typically limiting the current to less than 400 Amperes) and high impedance (typically a few thousand ohms) and limiting the ground fault currents to less than 5 Amperes. The generator ground fault protection scheme will depend on the grounding impedance. In general, large and important generators are protected with high impedance grounding. Only smaller generators are typically grounded with medium or low impedance.



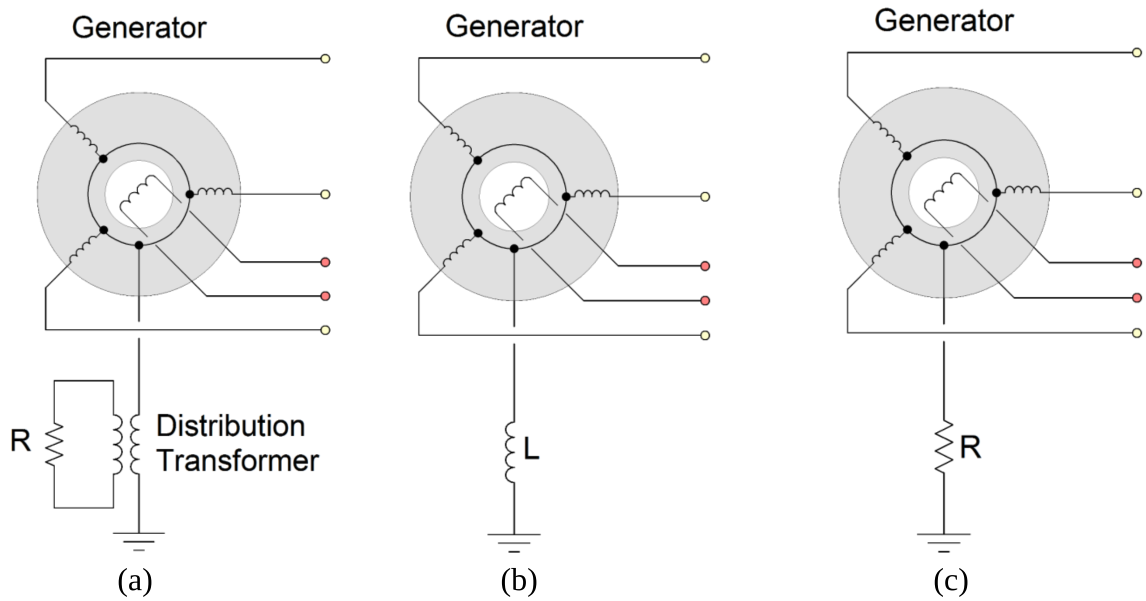


Figure 8.6: Illustration of Generator Grounding Methods
Grounding with (a) High, (b) Medium and (c) Low Ground Impedance

The selection of the grounding method depends on the importance of the generator and the level of protection that will be provided and cost considerations. Low/medium resistance or inductance grounding are the least expensive. These cases also have simplified protection schemes for the generator. We will focus on the most complete and sophisticated protection of generator units which are typically grounded via a distribution transformer as shown in Figure 8.6a.

In the case of grounding via a transformer with a secondary resistance (case a), the size of the grounding impedance of the generator should be so selected as to avoid oscillations in case of arcing ground faults. For this purpose, one must consider the total capacitance of the system that will include the generator parasitic capacitance the step-up transformer parasitic capacitance and other transformers connected to the generator terminals such as the station service transformer parasitic capacitance. The resistance of the grounding resistor should be less than the total magnitude of the impedance of the parasitic capacitances.

The grounding of the generator creates the ability to better coordinate the protection of ground faults in the generator. The selections for this part of the generator protection are described in the example below.

Example E8.1: Consider an 800 MVA, 60 Hz, 18 kV synchronous generator with the parameter values indicated in Figure E8.1. The generator is connected to a large substation via two 17.6-mile-long, 230 kV transmission lines with bundled phase wires. The parasitic capacitance of the generator, step-up transformer and station service transformer is 0.45 microFarads. The generator is to be grounded with a 14.4kV:240V center-tapped distribution transformer with a resistor at the secondary of the transformer. Select the grounding transformer and the size of the grounding resistor.

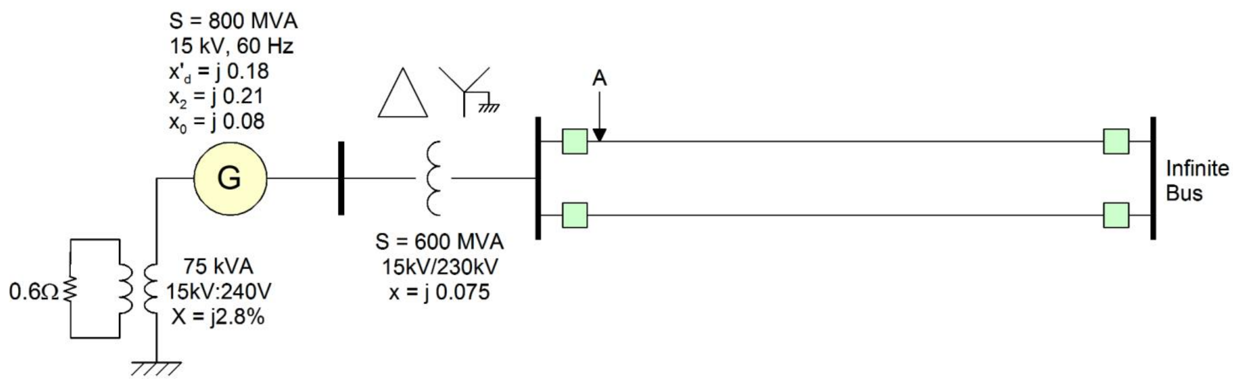


Figure E8.1: An Example Generator Grounding Arrangement

Solution: The size of the grounding transformer should be so selected as to avoid oscillations between the parasitic capacitance of the generator and the reactance of the transformer. For this purpose, first we select the resistor at the secondary of the transformer as follows:

$3R\left(\frac{15,000}{240}\right)^2 = \frac{1}{\omega C}$, which yields R=0.5 ohms

The power rating of the transformer is selected to be at least the reactive power consumed in the parasitic capacitance.

$S = 3\omega CV^2 = 55,000\, VA$

We select a transformer with rating 75 kVA.

Exercise: Discuss the reason for above selections.

8.3.1. Detection of Ground Faults

For grounded generators, the generator grounding circuit can be used to detect ground faults near the neutral of the generator. These faults can be detected by the harmonics in the grounding circuit. A ground fault near the neutral will result in abrupt reduction of harmonics in the grounding circuit. Harmonic currents may be 1% to 10% depending on design and loading. The grounding circuit should be designed as to withstand the harmonics, especially third harmonic. One of the protection functions is based on monitoring the harmonics and tripping the generator when the harmonics are suddenly reduced. Instantaneous tripping is normally set to 50%.

There is a small number of applications, typically in lower size generator or portable generators, that the generator are not grounded. An ungrounded generator is one without a grounding connection at the generator location. This does not mean that the circuit is ungrounded. At least one ground is required somewhere in the circuit. This grounding point may be at a distance from the generator. An example is shown in Figure 8.x.

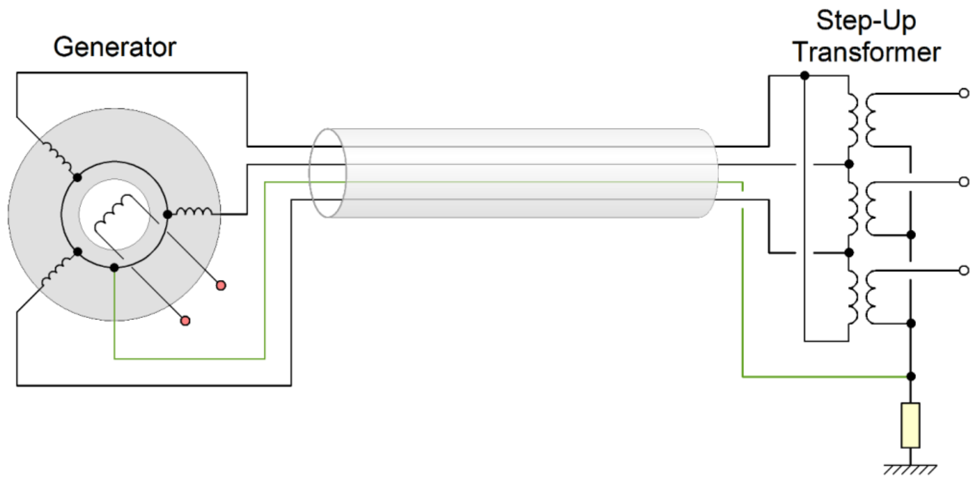


Figure 8.x: Example of Ungrounded Generator

Ungrounded generators require detection of ground faults. A single ground fault is harmless but a second one is detrimental. A single ground fault is detected with a zero sequence voltage relay. Such a relay is illustrated in Figure 8.x.

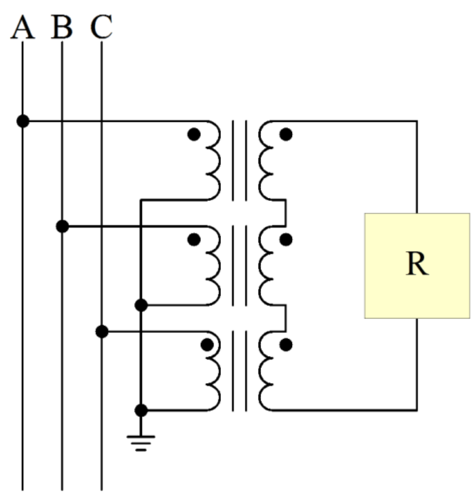


Figure 8.x: Zero Sequence Voltage Relay

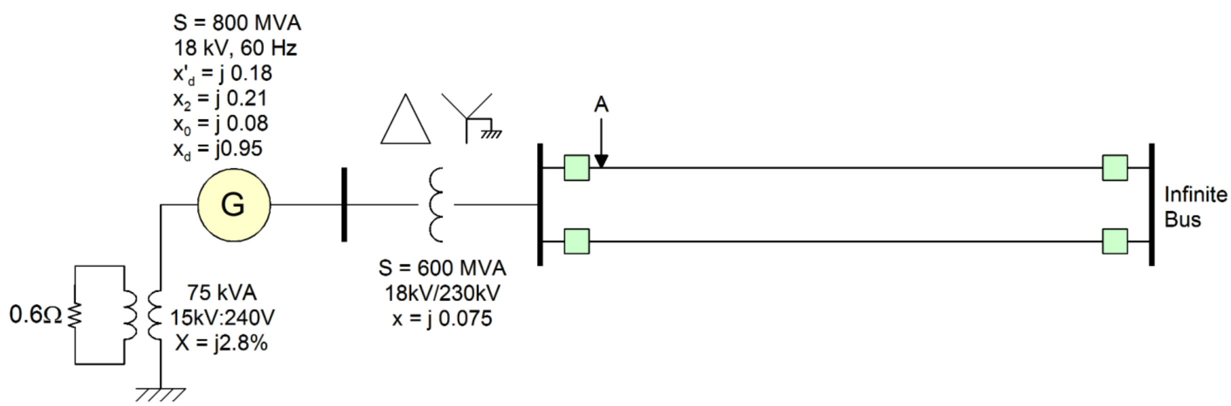
Example E8.2: Consider the circuit of Figure 8.x. Assume that the generator is ungrounded as shown in the figure (note it is grounded at the transformer ground). The length of the cable from generator to the transformer is 150 feet. Determine the signal seen by the zero sequence voltage relay, located at the terminals of the generator in case of a single line to ground fault at the transformer 4.16 kV side. The parameters of the system are as follows: generator rated voltage: 4.16 kV, rated power: 2.5 MVA, Generator impedances: z1=j0.18 pu, z2=j0.20 pu, z0=j0.09 pu. The cable impedances are: series impedance: z1=z2=xx, z0=xx, neglect the shunt impedance. The transformer is rated 2.5 MVA, 4.16kV/25kV, z=j0.08 pu. The equivalent of the system beyond the transformer has the following parameters: z1=z2=j0.01 pu, z0=j0.02 pu.

Solution: to be added.

8.3.2. Selection of Relaying Instrumentation

The selection of Voltage Transformers and Current Transformers is dictated by the loading of these devices. Because currents can be quite high under disturbance conditions and transient voltages can be also high, the selection of the VT and CTs should be such that the instrumentation is designed to operate reliably in these conditions. This means that the ratings of the VTs and CTs should be above the expected highest voltages and currents respectively. The selection process is illustrated with an example.

Example E8.3: Consider an 800 MVA, 60 Hz, 18 kV synchronous generator with the parameter values indicated in Figure E8.1. The generator is ungrounded. The generator is connected to a large substation via two 17.6-mile-long, 230 kV transmission lines with bundled phase wires. The SCC at the large substation is 3,200 MVA (3phase) and 3,100 MVA (1phase). Select the grounding and protection instrumentation for this generator.



Solution: to be added.

8.4 Generator Phase Fault Protection

Faults in the generator phase windings are serious because they involve high levels of energy that can damage the generator. Therefore, they should be cleared as soon as possible. It is important to note that the amount of energy stored in a generator during normal operating conditions is large and when a fault occurs this energy is dumped into the fault. Disconnecting the generating unit from the system does not mean that the fault current will stop flowing immediately. The protection schemes not only address the timely disconnection of the generator but also the various approaches to limit generator damage during the short fault duration. These issues will be discussed next.

Phase faults can be of different types. Possible fault types are:

1. Single phase to ground faults

2. Phase to phase faults
3. Double phase to ground faults
4. Turn to turn faults
5. Three phase faults

All these faults have the potential of generating very high fault currents that will damage the generator. For this reason every precaution is taking to limit the possibilities of these faults. The phase to ground fault current can be limited by the grounding system. Thus the common practice is to ground generators with an impedance (typically high impedance) to limit the ground fault current to a low value, for large generator about 5 Amperes. Then we also design generators in such a way that other faults that may involve two or more phases are rare. For example the phase conductors are enclosed in conduit which is grounded and precludes the possibility of a fault from one phase to another. We refer to this as the insulated bus design or iso-bus. These design options and other precautions make the phase to ground fault the most common fault in generators, but also rare, and the possibility of the other faults is diminished.

Generating unit phase faults must be immediately cleared. For this reason, use of percentage differential relaying is applied. Typically, two superimposed differential schemes are applied, one across each of the three phase windings, and another across the entire protection zone (generator and step-up transformer). We refer to these schemes as generator differential (87G) and unit differential (87U).

The differential protection is of the percentage type due to the potentially very high currents during external faults and the possibility of saturating the CTs.

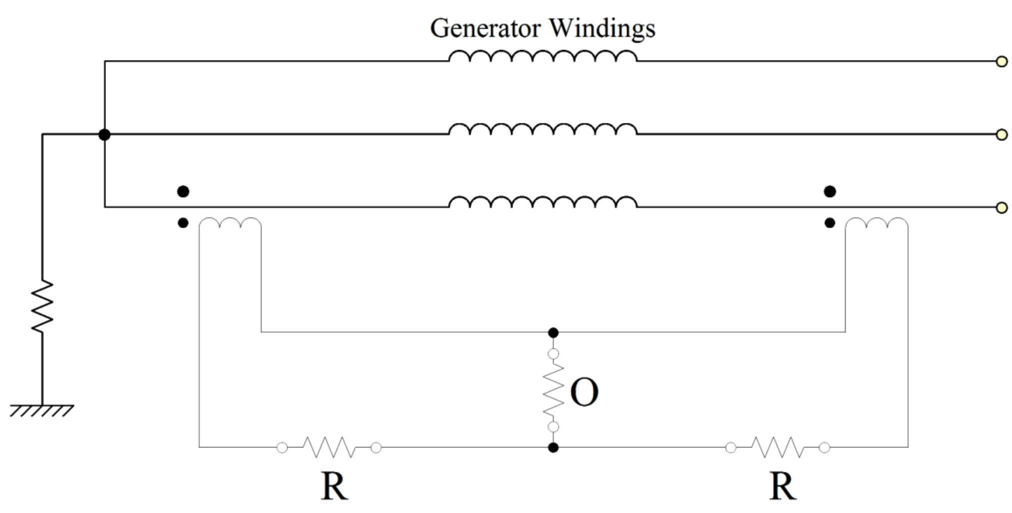


Figure 8.x: Generator Winding Differential Relay Protection (Protection Shown on One Phase Only for Simplicity)

Some generators, due to physical construction, have windings that consist of multiple adjacent turns. It is therefore possible for faults to develop between turns on the same phase (inter-turn faults). These faults are not detected by the stator differential protection, as there is no difference between the neutral and at the terminal currents. Split phase protection may be applied to detect inter-turn faults in the case that the generator is wound with two three-phase windings, each brought separately out of the machine and connected in parallel. The currents in the two windings are compared, any difference indicates an inter-turn fault.

The percentage differential scheme shown in Figure 8.x can protect (detect) faults along most of the generator coil. If the fault is near the generator neutral, the fault may not be detected because it will produce current in the operating coil lower than the typical settings of the percentage differential relay settings. Typically, faults up to 10% of the coil from the generator neutral may not be detected. These leaves the generator unprotected for these faults. There are approaches to provide 100% coil protection. These are discussed next.

8.5 Generator Ground Fault Protection

As has been discussed already, for protection against the most probable fault in generating units, the phase to ground fault, we limit the ground fault current to low values to avoid generator damage. The low fault current also requires more sophisticated methods to detect fault with certainty. It is also important to detect faults anywhere along the windings of the generator.

The primary ground fault detection is performed by monitoring the current in the secondary of the grounding transformer with an instantaneous overcurrent relay and a time overcurrent relay. Note that if there is a ground fault anywhere in the winding of the transformer, the neutral of the generator will be elevated to a voltage and therefore the grounding transformer will be excited by that voltage. For example if the fault is in the middle of the generator coil, the voltage on the primary of the grounding transformer will be half of the phase to neutral rated voltage of the generator. The current will be relatively small, for high impedance grounded generators in the order of 5 Amperes. However the current in the secondary of the grounding transformer will be high. The 50 and 51 relays will see this current and will respond tripping the generator. For security the settings of the 50/51 relays are such that they will trip the generator for faults that are 10 or 20% further from the neutral along the generator windings. This means for faults near the neutral these relays will not respond. In order to provide 100% protection for faults along the entire winding of the generator, typically two other additional relays are used; these are described next.

Protection based on generator harmonics: In order to understand the operation of this relay scheme it is important to consider the normal operation of a generator. The generator because of the way the windings are constructed, it will generate harmonics. The harmonics will be positive sequence, negative sequence and zero sequence. Of these harmonics the zero sequence will sum at the neutral of the generator and they will appear in the grounding transformer. Note that a generator is typically connected to a delta connected transformer. The delta winding of the transformer will block the zero sequence currents. However the generator has some capacitance that will provide a path for the flow of the zero sequence currents. Thus the zero sequence harmonic currents will be circulating between the grounding transformer and the capacitance of the generator. It is notable that the third harmonic generated by the generators is mainly zero sequence. At this frequency the capacitive impedance of the generator is three times lower than the same impedance at 60 Hz. The third harmonic currents will generate a third harmonic voltage at the secondary of the grounding transformer. Note also that in case of a fault near the neutral of the generator, the primary of the grounding transformer will be practically shorted and the third harmonic voltage at the secondary of the transformer will reduce near zero. Therefore an undervoltage relay tuned to the third harmonic connected to the secondary of the transformer can be used as the detector of faults near the neutral of the generator. This is shown in Figure 8.4 as relay 64NG. The setting of this relay is determined by examining the level of the third harmonic for faults near the near of the generator. Specifically, a study is performed to determine the level of third harmonic voltage at the secondary of the transformer for faults over the range of the winding for which we need to protect, for example 10%. Because different generators have different designs and different levels of third harmonic generation, the settings of this relay tyopically require knowledge of the generator design and characteristics of the third harmonics.

Protection based on signal injection: This method in based on the injection of a low frequency signal (20 to 40 Hz) into the neutral of the generator via the grounding transformer. The principle of the method is illustrated in Figure 8.x. A low frequency voltage is applied to the secondary of the grounding transformer. This voltage will appear on the high side of the transformer but it will not generate any current under normal operating conditions because it will be blocked by the delta connection of the step-up transformer. However if there is a fault anywhere in the generator, electric current of same frequency will flow from the grounding transformer to the fault and it will be detected by the overcurrent relay tuned to the same frequency, as it is shown in the figure.

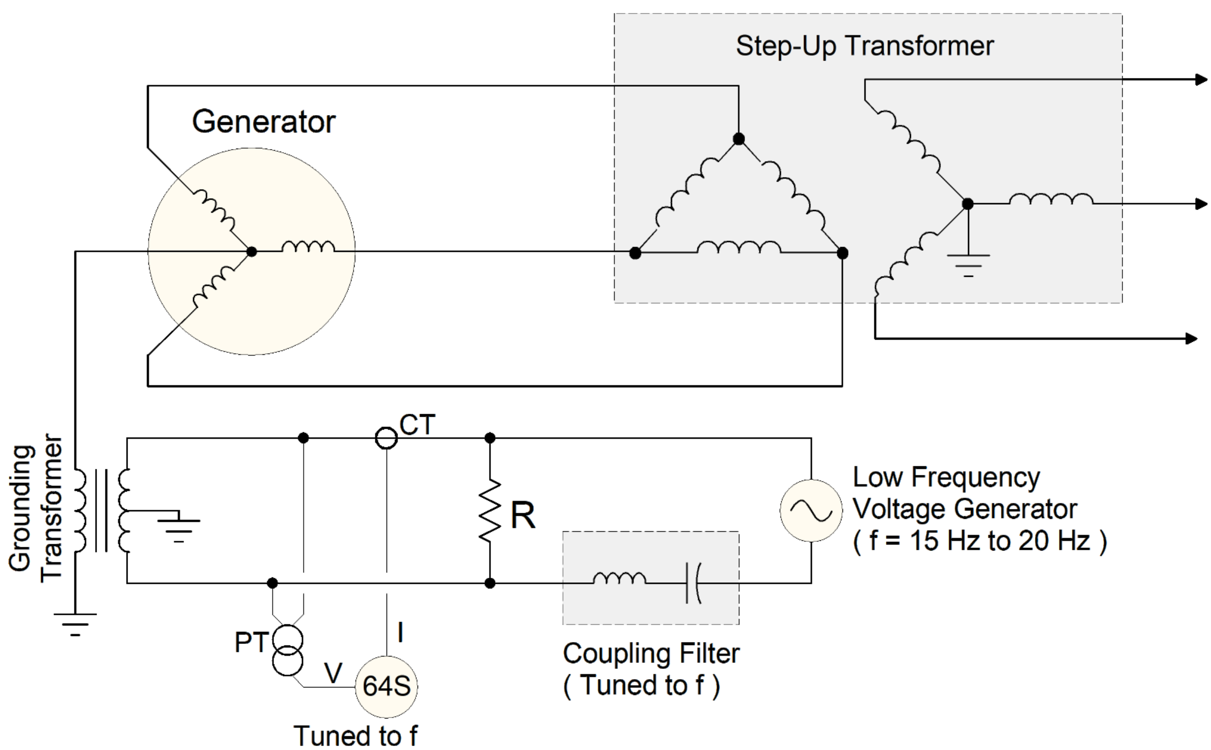


Figure 8.x: Low Frequency Signal Injection into the Generator Neutral

In summary, the combination of methods described above will provide protection for faults anywhere in the windings of the generator, i.e. they will provide 100% protection.

8.5 Generator Unbalance Protection

Asymmetrical faults generate negative sequence currents that flow in the windings of the generator. The generator rotor rotates with approximately synchronous speed and the negative sequence currents generate a rotating magnetic flux with speed equal to the synchronous speed but rotating in opposite direction than the rotor rotation. Thus the relative speed of the negative sequence rotating flux with respect to the rotor is twice the synchronous speed. It follows that the negative sequence currents induce currents in the rotor of frequency 120 Hz. These currents produce excessive ohmic losses in the rotor that raise the rotor temperature and eventually may damage the rotor. Typical generator design is such that negative sequence currents can be tolerated for only a short period of time. Specifically, generators can tolerate negative sequence current for a time duration that meets the following rule:

$$I_2^2 t \le k$$

Where: k is a constant depending on generator design, the negative sequence current is expressed in p.u. on the generator ratings and the time t is expressed in seconds.

Typical values for the constant k provided by manufacturers are given in Table 8.1.

Table 8.1 Typical k Values for Synchronous Generators

Type of Generator	*k*
Salient Pole	40
Synchronous Condenser	30
Cylindrical Rotor	
Indirectly Cooled	20
Directly Cooled (less than 800 MVA)	10
Directly Cooled (greater than 800 MVA)	10-(0.00625)(S-800)
Motors	40

There are many conditions that may result in high negative sequence currents in a generator. Some of them are: (a) unbalanced step-up transformer impedances, especially in case that the step up transformer is made up with three single phase units with unmatched impedances, (b) long fault conditions, (c) single phase tripping, etc. Some of these conditions result in high negative sequence currents (single phase tripping) and other relatively low negative sequence but for a long time. In any case the generator must be protected against the overheating and potential damage from negative sequence currents. The negative sequence relay (46-reverse phase or phase-balance current relay) is typically used to protect against this condition. The operating region of the 46 relay is illustrated in Figure 8.x.

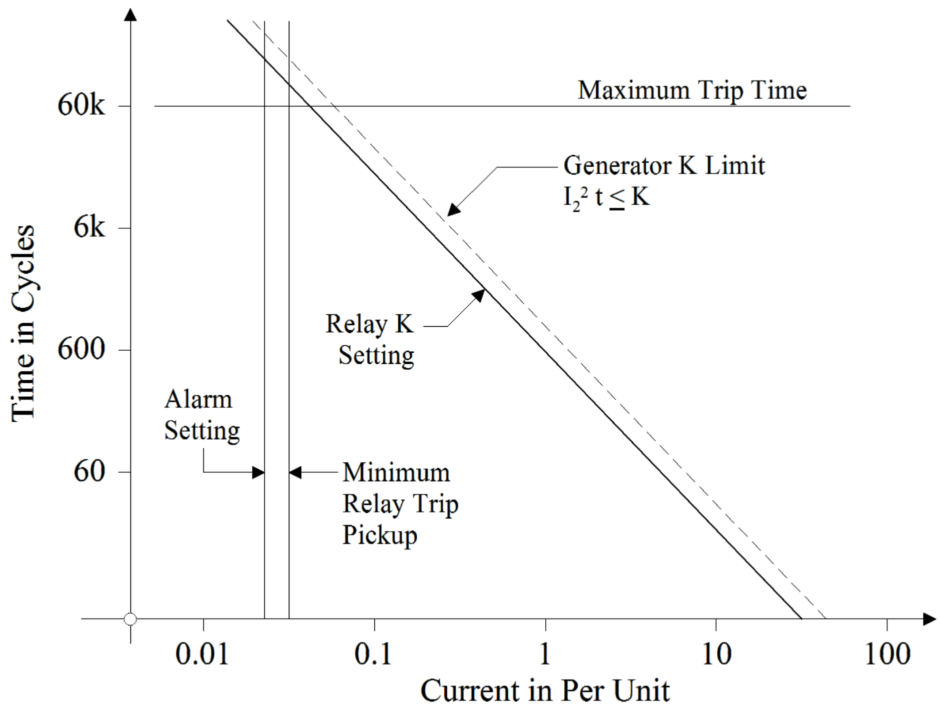


Figure 8.x: Negative Sequence Relay (46) Operating Region

Example E8.x: Consider the synchronous 60 Hz, 375 MVA, 18 kV generator connected to a transmission system as it is illustrated in Figure E8.x. Assume a line to line fault at the indicated location. Further assume that the generator is protected with a negative sequence relay that is set to the value k=6 sec. Assume that the unit does not have any other protective system and relies on the negative sequence relay for protection. What will be the time of operation of the negative sequence relay?

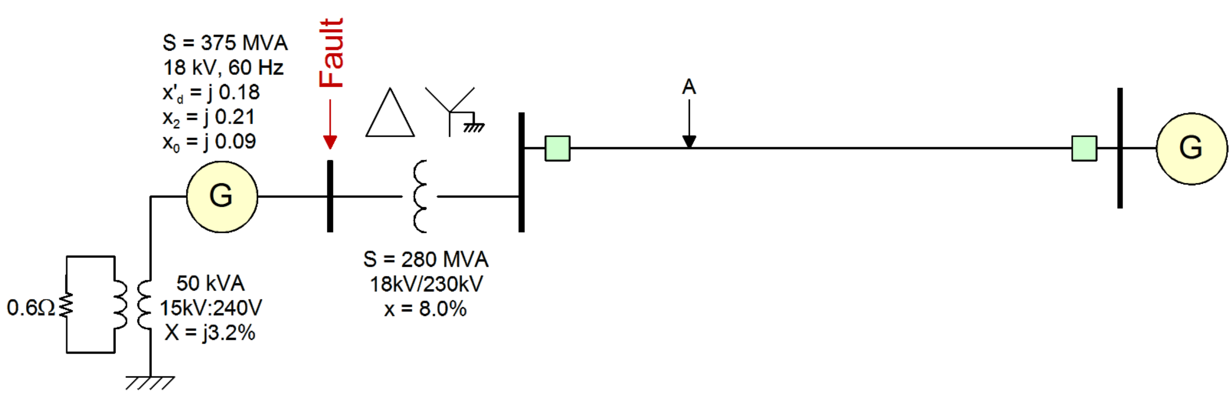


Figure E8.x Example Generator and Step-Up Transformer

Solution: to be continued.

Example E8.x: Consider the electric power system of Figure E8.x. The system consists of a generator and a step-up transformer (delta-wye connected), a generator circuit breaker and a transformer circuit breaker. The transmission line parameters are: positive/negative sequence parameters: 1.0+j12.5 ohms, and zero sequence: 5.0+j32.0 ohms. The short circuit capacity of the equivalent source is 2500 MVA. Assume that breaker pole of phase A of the transformer breaker is stuck open. Prior to this event, the generator phase angle was 13.5 degrees advanced with respect to the equivalent source (not counting the phase shift of the transformer). Further assume that the generator is protected with a negative sequence relay that is set to the value k=6 sec. Compute when the negative sequence relay will alarm and when it will trip.

Solution: to be continued.

8.6 Overload Protection

Overload protection is necessary to protect against heating from prolonged operation at loading conditions above the rating of the generator. For synchronous motors it is tricky because the synchronous impedance is near or above 1.0 pu.

Trying to match the rotating machine thermal limits with electromechanical relays is difficult. Digital relays can be better programmed.

Discuss various problems with start-up.

Discuss difficulty in estimating temperatures from current sensing.

Discuss alternatives with supplemental temperature sensing.

8.7 Rotor Faults

The field circuits of synchronous generators operate ungrounded. In reality this means that the field circuit is only grounded via the high impedance of the instrumentation and control circuits. A typical rotor circuit and its instrumentation is shown in Figure 8.x. Note that the positive and negative poles are grounded via the impedance of the instrumentation circuit.

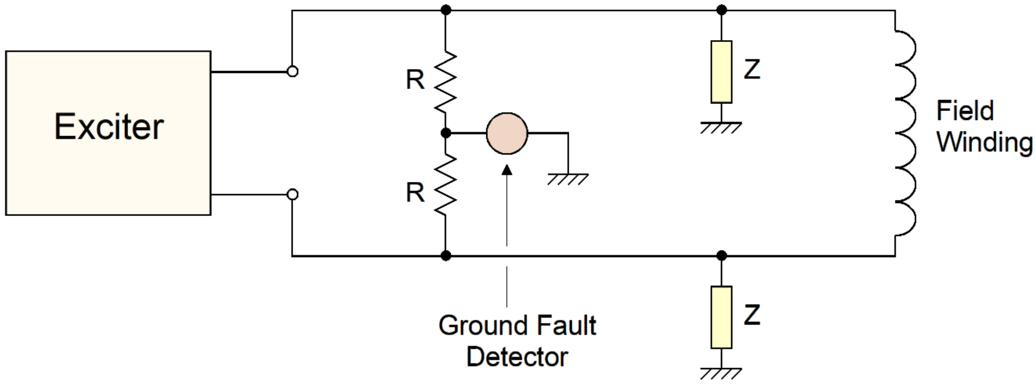


Figure 8.x Grounding of the Rotor Circuit via Instrumentation and Rotor Ground Fault Detector

A ground fault on the rotor circuits will allow normal operation since the circuit is practically ungrounded and the fault becomes the single grounding point of the field circuit. A second fault will cause very high fault currents. Therefore, the first fault must be detected and corrected as soon as possible. Detection schemes to detect fault are relatively simple.

The above method has the disadvantage that for field winding fault near the center of the winding are not detectable. The method of a signal injection (ac or dc) is a more secure method to detect ground fault on the field winding.

In case rotor faults may cause malfunctioning of the field circuit, then this condition can be identified as field loss condition. Protection schemes for loss of field are discussed later in this chapter.

Discuss the limitations for exciters on the generator shaft (brusheless field circuit). Discuss detection of field problems with relays on the generator terminals, such as negative sequence and loss of excitation relay.

Example E8.x: Consider the synchronous 60 Hz, 375 MVA, 18 kV generator connected to a transmission system as it is illustrated in Figure E8.x. Assume a rotor fault. Further assume that the generator is protected with a field relay. Assume that the unit does not have any other protective system and relies on the field relay for protection. What will be the time of operation of the field relay?

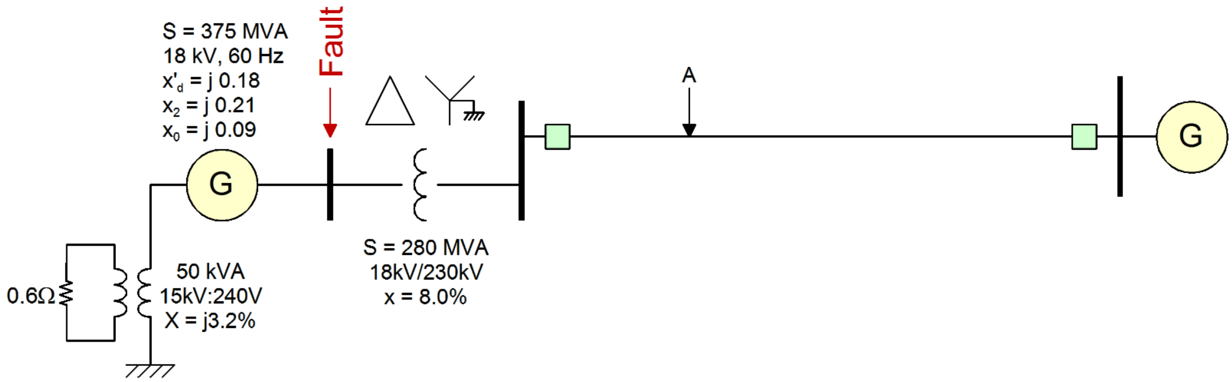


Figure E8.x Example Generator and Step-Up Transformer

8.8 Over/Under Voltage and Over/Under Frequency

A generator exhibits overvoltage when overexcited or overspeeding. Recall that the voltage is related to the magnetic flux linkage by the simple equation (at near sinusoidal conditions):

$$V \approx 2\pi f \Lambda$$

or

$$\frac{V}{f} \approx 2\pi\Lambda$$

At normal operating conditions, the ratio of volts over hertz is constant and known. If the speed of the generator increases while the flux remains the same, the voltage will also increase in such way that the ratio "Volts over Hertz" remains constant. If the frequency drops while the voltage remains constant the ratio "Volts over Hertz" will increase indicating a commensurate increase of the magnetic flux in the generator. In other words, the ratio "Volts over Hertz" indicates the level of the magnetic flux linkage in the generator over the magnetic saturation value. Generators are designed in such a way as to operate near the magnetization knee under normal operating condition. If the magnetic flux linkage in the generator increases, the iron core of the generator will be driven into saturation. In this case, excessive losses in the iron core may increase the temperature of the generator and damage the generator.

We protect against this condition with a V/Hz relay which bear the number 24. This relay monitors the "Volts over Hertz" ratio and will trip when the ratio exceeds the setting of the relay. Typical settings for the 24 relay are:

Generators: 105%
Transformer: 110%

Discuss saturation. If machine is driven to saturation, harmonics are generated, increased heating, etc. In generators, undervoltage is normally a problem for auxiliaries.

For motors, they are protected against undervoltage. This generates a power quality issue. The protection system responds to the voltage sags.

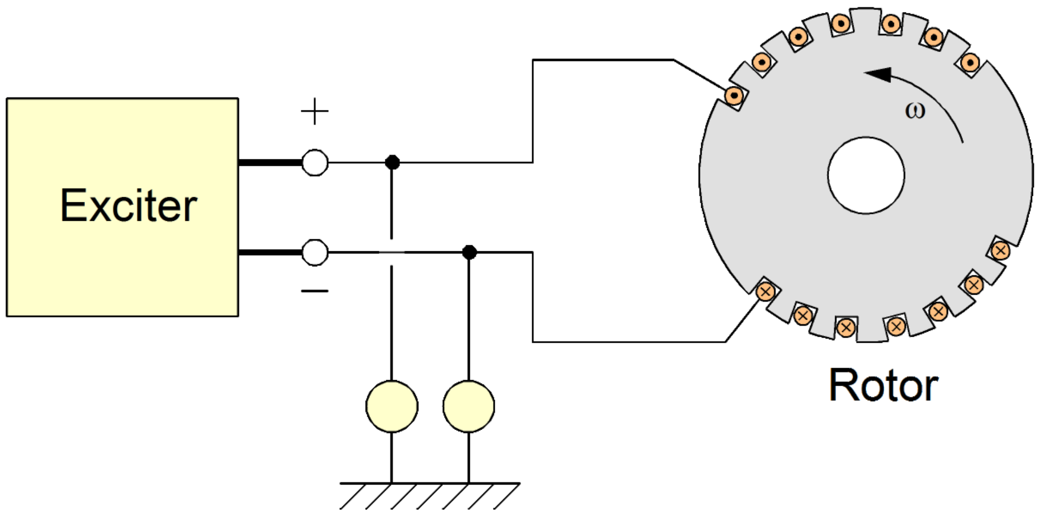
Overfrequency is related to the speed of the rotating machine and therefore it is protected with the overspeed relay. Many times overfrequency protection is provided as backup to overspeed protection.

Underfrequency is important to the turbine. The system is protected against underspeed. Underfrequency is a backup protection. Load shedding is normally used as underfrequency protection.

8.9 Loss of Excitation

A generator normally operates at lagging power factor and occasionally at leading power factor depending on the voltage control requirements for the network. The operating point must be constraint within certain capability curves to avoid damage to the unit. In case that the excitation is lost, a synchronous generator will operate as an induction generator. Immediately after excitation is lost and because the mechanical drive will continue to provide the same mechanical torque, the generator will accelerate rather fast. The acceleration will continue until the governor will limit the speed typically to 3% to 6% over synchronous speed depending on the droop characteristic of the governor. The machine will continue to operate as an induction generator. Typical design of synchronous generators is such that this condition leads to excessive heating of the rotor and the eventual damage of the generator. In general, a generator with damper windings can withstand this condition for relatively longer time than cylindrical rotor generators without damper windings. Manufacturer data must be consulted to determine these times. In any case, it is important that the generator be tripped if this condition persists.

Loss of excitation can be detected by a number of schemes including (a) direct monitoring of the field current, (b) power factor monitoring at the terminals of the machine or (c) impedance monitoring at the terminals of the generator. The most common detection scheme is by considering the impedance “seen” at the terminals of the generator. This requires the use of a mho type distance relay (function 21).



Operation of a synchronous generator without excitation is an undesirable condition which can only be tolerated for a short period of time. For this reason, it is advisable that the first automatic action following the detection of excitation loss is to initiate an alarm. The alarm will attract the attention of an operator who may be able to remedy the situation within a short period of time. If the problem cannot be corrected within an acceptable time period, the unit should be automatically tripped.

The phenomena that are involved in a loss of excitation are complex. Synchronous generators are designed to operate in a specific normal operating region as illustrated in Figure 8.x. The figure illustrates the limiting factors. There are three limiting regions: one determined by the stator windings heating, another determined by the rotor winding heating, and another determined by the heating of the end point stator winding and stator magnetic circuit ends. The generator normally operates within the specified "normal" region of power factor. This operating region can be also defined on the impedance diagram illustrated in Figure 8.x. When loss of field occurs, the impedance seen by the relay moves in the indicated trajectory in Figure 8.x. If the loss of field is partial, a different trajectory will occur.

One can protect against these events with a set of two mho type elements. The first is set to trip whenever the impedance falls within the small circle illustrated in Figure 8.x. Note that the small circle is defined with a diameter on the negative impedance axis starting at $-\frac{x_d^{'}}{2}$ and the other end at about 100% to 125% of the synchronous reactance. The time delay for this zone is normally selected to be about 0.25 seconds. The time delay is necessary to avoid false tripping for stable swings that may take the impedance into the small circle. The larger circle is selected with a larger diameter and a time delay in the order of 1 second. This circle is set to detect partial loss of excitation and to provide backup protection to zone 1.

The relay should be coordinated with the steady state stability limit of the generator, as well as the minimum excitation limit of the unit.

8.9.1 Generator Steady State Limit

The steady state stability limit is computed with the aid of Figure 8.x. Note that the external circuit is represented with a Thevenin equivalent circuit.

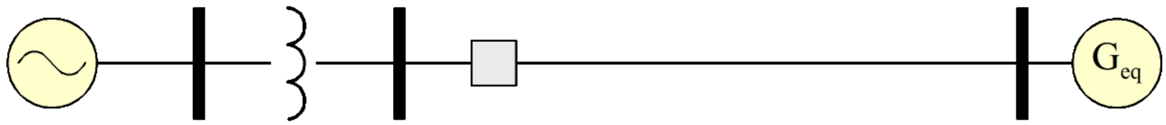


Figure 8.x. Example System for Steady State Stability Limit Evaluation

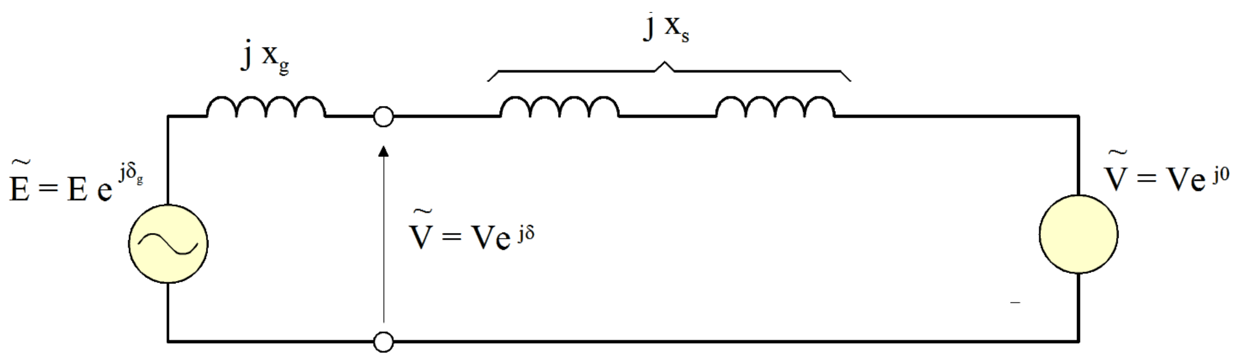


Figure 8.x. Equivalent Circuit of System of Figure 8.x

The real and reactive power transmitted is:

$$P = \frac{1}{2}\left(\frac{V^2}{x_s}\sin\delta + \frac{EV}{x_g}\sin(\delta_g - \delta)\right)$$

$$Q = \frac{1}{2}\left(\frac{V^2}{x_s} - \frac{V^2}{x_s}\cos\delta - \frac{V^2}{x_g} + \frac{EV}{x_g}\cos(\delta_g - \delta)\right)$$

The above equations can be manipulated to yield the following:

$$P^2 + \left(Q - \frac{V^2}{2}\left(\frac{1}{x_s} - \frac{1}{x_g}\right)\right)^2 = \frac{1}{4}V^2\left(\frac{V^2}{x_s^2} + \frac{E^2}{x_g^2} - 2\frac{VE}{x_s x_g}\cos(\delta_g)\right)$$

Maximum steady state transfer occurs when the cosine term becomes zero (or the angle δ_g becomes 90 degrees. This yields the equation:

$$P^2 + \left(Q - \frac{V^2}{2}\left(\frac{1}{x_s} - \frac{1}{x_g} \right) \right)^2 = \frac{1}{4} V^2 \left(\frac{V^2}{x_s^2} + \frac{E^2}{x_g^2} \right) \approx \left(\frac{V^2}{2} \sqrt{\frac{1}{x_s^2} + \frac{1}{x_g^2}} \right)^2$$

This equation represents a circle with center at:

$$P_c = 0, \quad and \quad Q_c = \frac{V^2}{2}\left(\frac{1}{x_s} - \frac{1}{x_g} \right)$$

and radius of:

$$R = \frac{V^2}{2} \sqrt{\frac{1}{x_s^2} + \frac{1}{x_g^2}}$$

8.9.2 Generator Response to Loss of Field

Upon loss of field the generator will start accelerating until it gets to a speed that will support the operation of the machine as an induction generator. In reality, upon loss of field, the generated internal voltage will not immediately collapse to zero. The rotor magnetic flux has some inertia that will cause a gradual decrease of the generated voltage. We will denote the time constant of the decaying flux, τ_a. As the voltage decreases, the generated electromagnetic torque decreases. This will accelerate the machine. Now the machine is running above synchronous speed and therefore it will generate electromagnetic torque by induction. The evolution of the operating point of the machine is characterized with transitions from one near steady state condition to another (quasi steady state operation). At each operating point, the power delivered by the decaying generated voltage and the power produced by induction it will equal the supplied mechanical power.

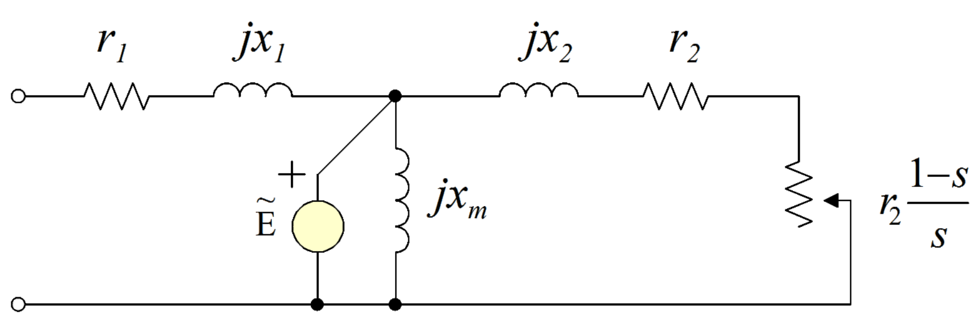


Figure 8.x Equivalent Circuit of a Synchronous Machine When Field is Lost

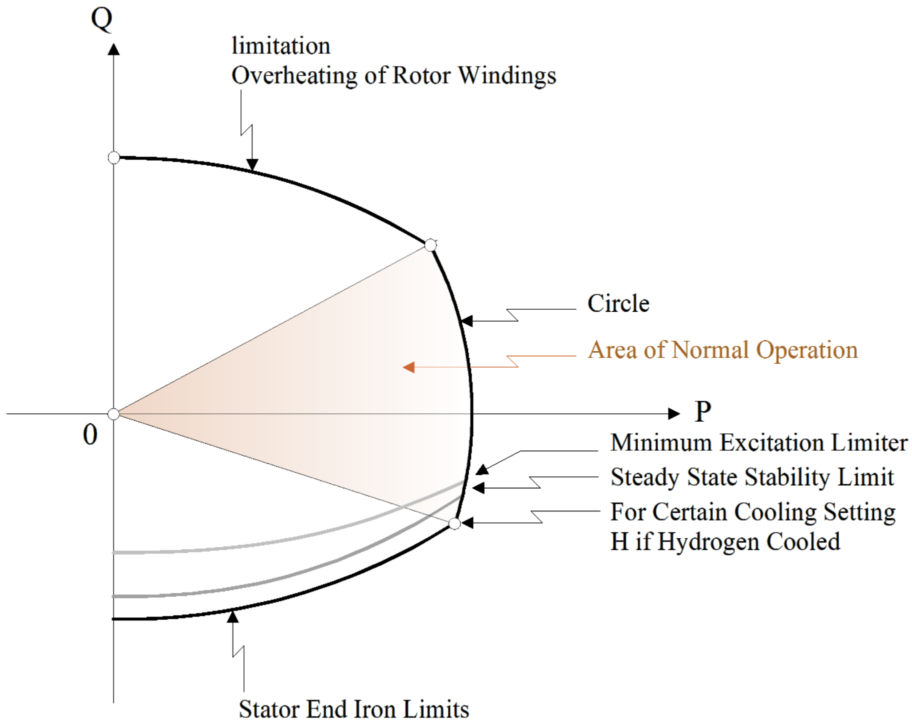
The mathematical model describing each operating point is:

$$P_{mech} = \frac{EE_2}{x_T}\sin\delta(t) + r_2\frac{1-s}{s}\left|\frac{E}{\frac{r_2}{s}+jx_2}\right|^2 = P_{elec}$$

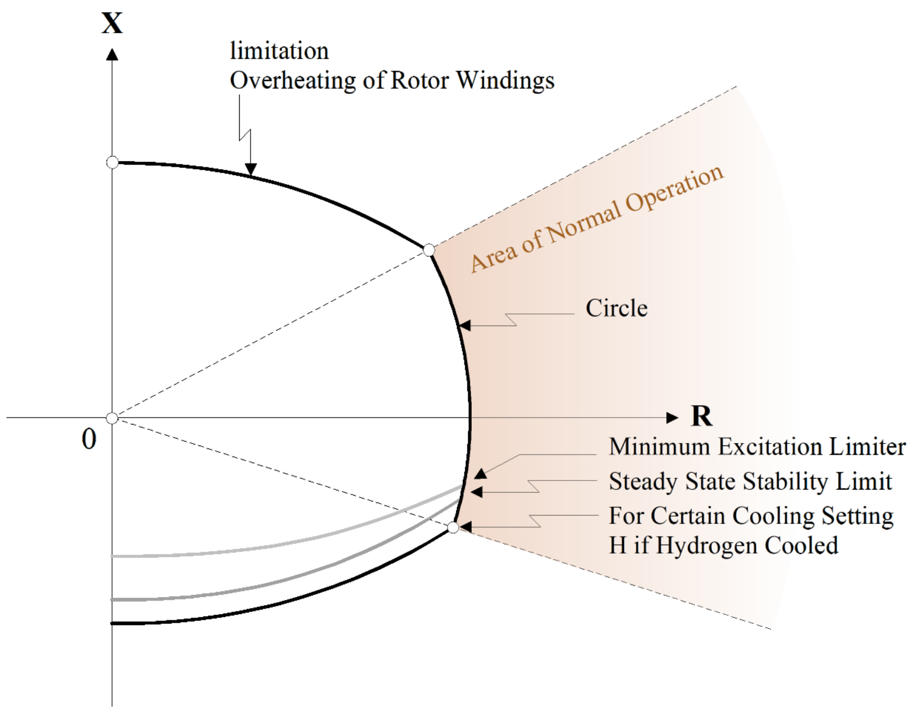
$$s = \frac{\omega_s - \omega}{\omega_s}$$

8.9.3 Generator Operating Limits

The generator operating limits are summarized in Figure 8.x. This figure is converted to Figure 8.x which includes the same information on the R-X diagram.



(a)



(b)

Figure 8.x Generator Capability Curves
(a) P, Q Plane, (b) R, X Plane

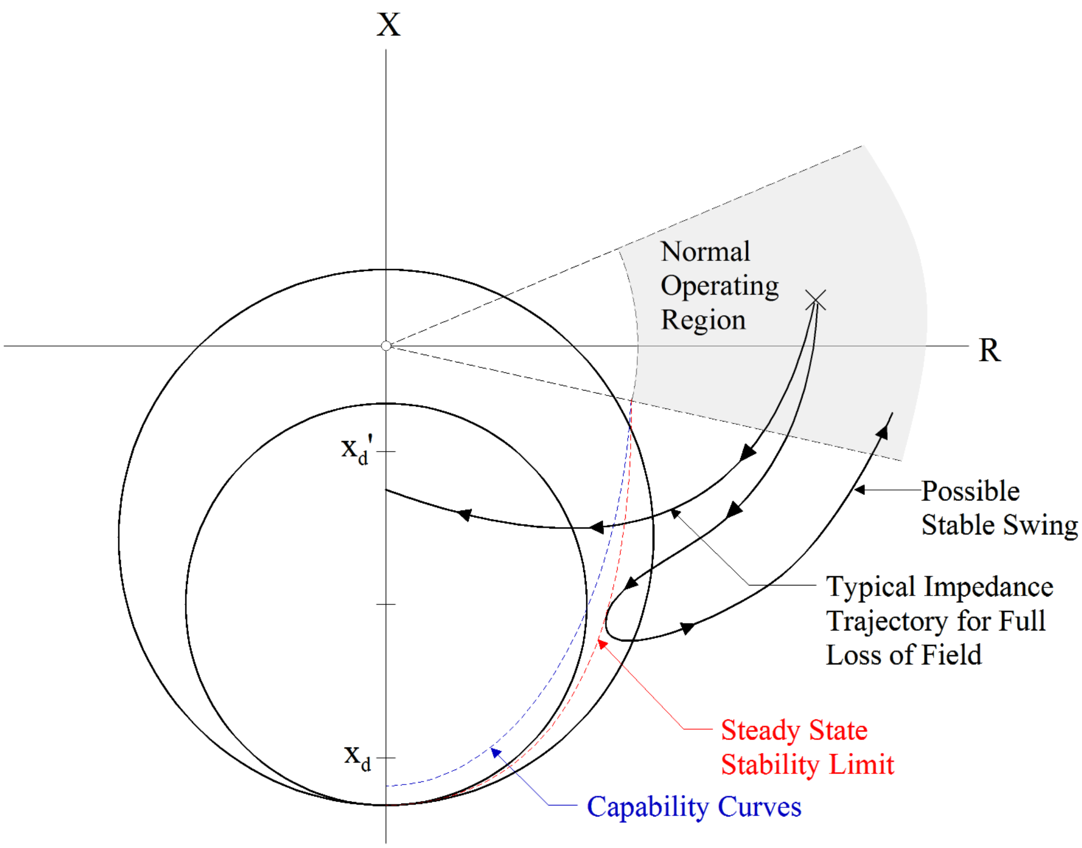


Figure 8.x Loss-of-Field Protection with a Two-Zone Offset Mho Relay

In order to avoid tripping on stable swings, we use a two mho relay characteristics. The first mho relay characteristic has diameter $\left(-j\dfrac{x_d^{'}}{2}, -j1.1x_d\right)$. The second mho relay has diameter $\left(-jx_t, -j1.1x_d\right)$.

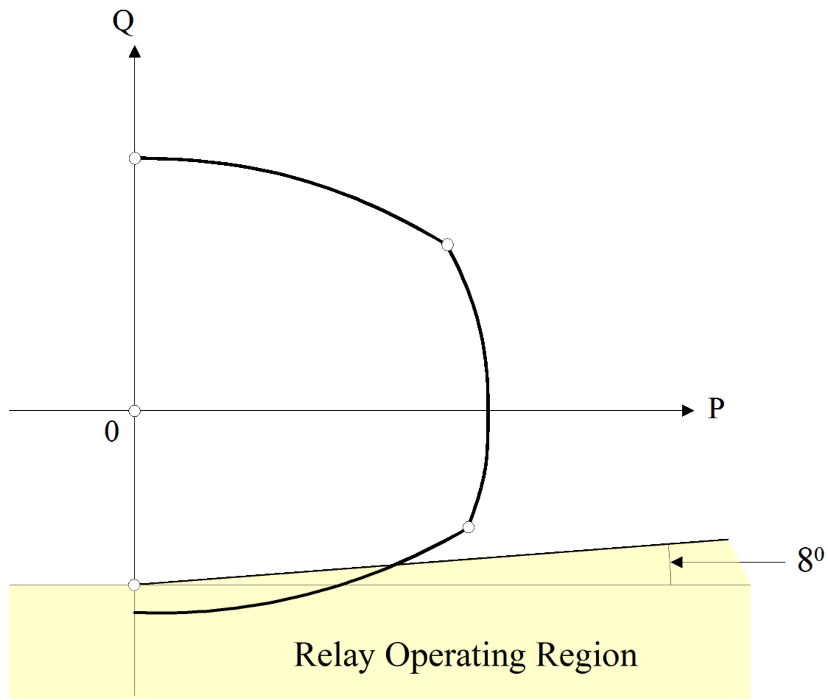
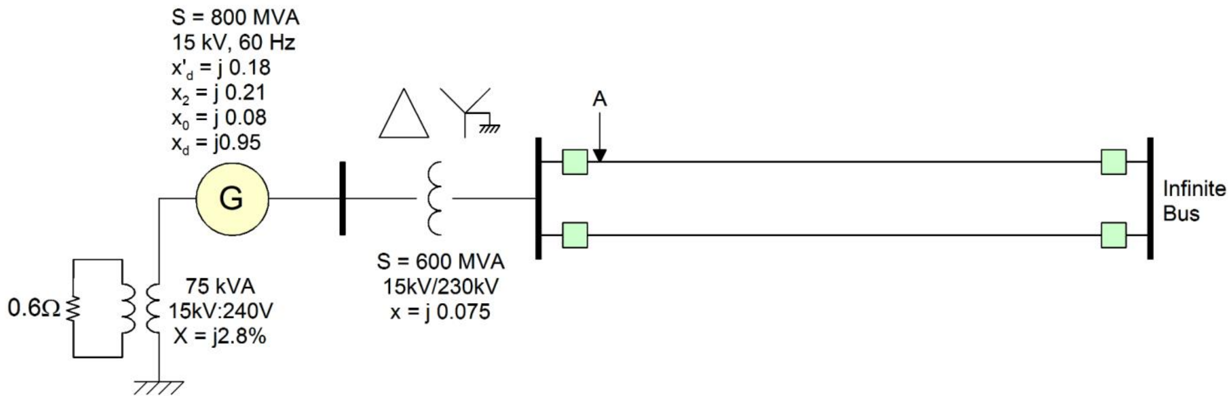


Figure 8.x Loss-of-Field Protection with a VAR Type Relay (typical delay 0.2 sec)

Example E8.x: Consider an 800 MVA, 60 Hz, 15 kV synchronous generator with the following parameters. The generator is to be protected with a two-zone distance relay applied to the terminals of the generator. Select the CT and PT for this application. Then, select the settings of a loss of field excitation relay.



Solution: The protection against loss of field will be accomplished with two mho type distance elements.

The nominal current at the generator is:

$$I_{rated} = \frac{\frac{800\,MVA}{3}}{\frac{15\,kV}{\sqrt{3}}} = 30.79\,kA,$$

Select a CT with ratio 35,000:5

The nominal voltage is:

$$V_{rated} = \frac{15\,kV}{\sqrt{3}} = 8.66\,kV\ ,$$

Select a PT with ratio: 15,000:115

Small circle:

select diameter at $\left(-j\frac{x_d^{'}}{2}, -j1.1x_d\right)$.

Therefore, the center is at

$$z_c = -j0.5675\ pu$$

$$R_c = 0.4775\ pu$$

Converting in ohms on the generator side:

$$z_{c,g} = -j0.1596\,ohms$$

$$R_{c,g} = 0.1343\,ohms$$

Converting on the relay side:

$$z_{c,r} = -j8.5657\,ohms$$

$$R_{c,r} = 7.2\,ohms$$

Select:

-j9.0 and 7.5

Large circle:

select diameter at $jx_T, -j1.1x_d$.

Therefore, the center is at

$$z_c = -j0.4725\,pu$$

$$R_c = 0.5725\,pu$$

Converting in ohms on the generator side:

$$z_{c,g} = -j0.1329\,ohms$$

$$R_{c,g} = 0.1610\,ohms$$

Converting on the relay side:

$$z_{c,r} = -j7.1322\,ohms$$

$$R_{c,r} = 8.6411\,ohms$$

Select: -j7.5 and 9.0

Important: Check to make sure that normal operating conditions do not trip.

The impedance "seen" by relays during operating condition at leading power factor of 0.92 at full load:

$$Z = \frac{8.66\,kV}{30.79\,kA\,e^{j23.07^0}} \frac{PT_{ratio}}{CT_{ratio}} = 15.09e^{-j23.07^0}$$

This operating point is located outside the large circle (see Figure 8.x)

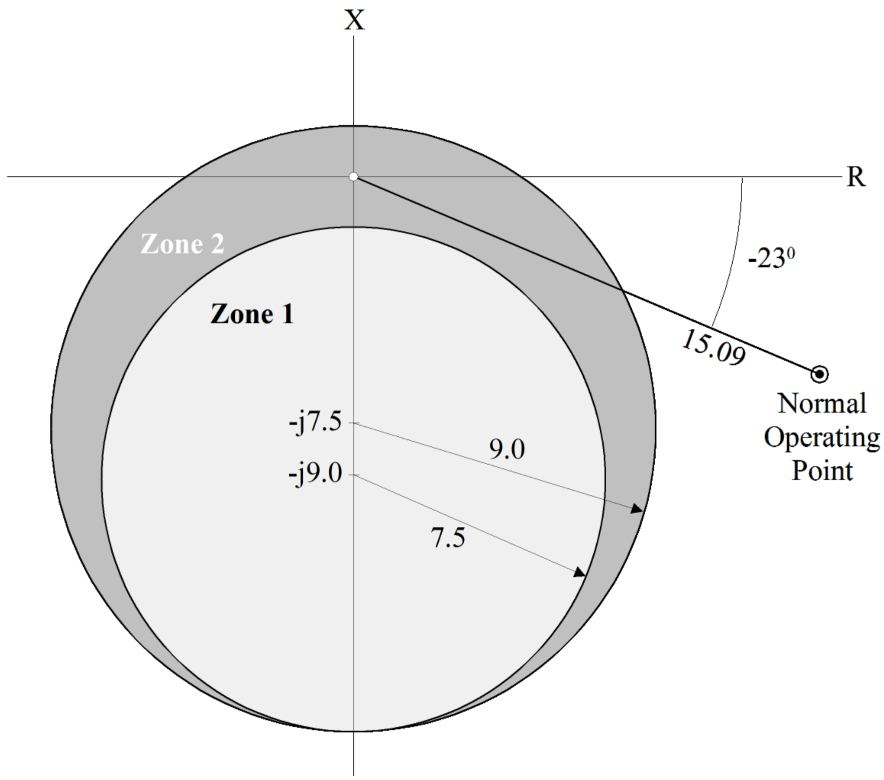


Figure E8.x: Mho Relay Settings for Example E8.x, Values on Relay Side

8.10 Reverse Power Protection (32)

A generator should be protected against motoring, i.e. against operation as a synchronous motor. Prolonged operation as a synchronous motor indicates loss of the prime mover and may result in excessive reactive power and overheating.

8.11 Accidental Energization

A generator may be accidentally energized when it is at standstill or rotating at very low speed. This condition may lead to excessive currents through the generator for a very long time since the machine will operate as a synchronous motor that accelerates very slowly (very small initial torque).

8.12 Out of Step Relaying

The effects of generator instability are serious and therefore the generator should be disconnected from the system if this condition is eminent. This protection of a synchronous machine against instability is normally referred to as out of step protection. This protection scheme comprises a relay that monitors and identifies the condition and then it takes decision to disconnect the unit. Several schemes have been proposed for out of step relaying. The most common scheme today is a scheme that monitors the system impedance trajectory during a swing using impedance relays. Other schemes have been proposed and have been occasionally implemented on a trial basis. We will discuss some of these schemes.

During faults, the balance between the mechanical power input and the electrical power output at one or more generators is disturbed. This causes the generators to accelerate or decelerate resulting in additional system transients. We refer to these disturbances as stability swings or generator swings. During a stability swing, the electric current flowing through various circuits of the system will vary and may acquire high values. If the transients are temporary, the protective system should not respond.

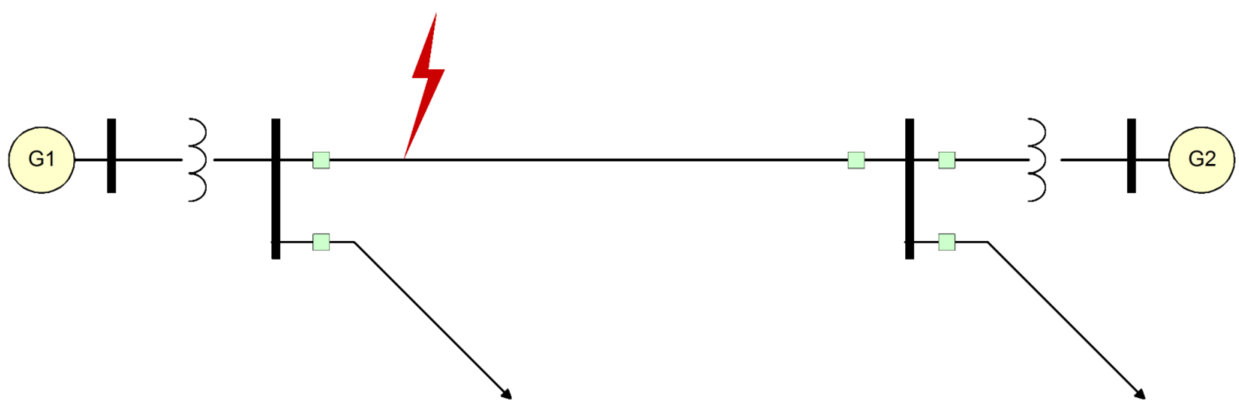


Figure 8.x Initiation of a Stability Oscillation by a Fault

8.12.1 Stable and Unstable Swings

Generator swings can be stable or unstable. Stable swings are such that the phase angles among the generators of the system vary within a narrow range, and eventually settle to constant values. In unstable swings, one or more generator phase angles may increase indefinitely resulting in loss of synchronism. This phenomenon is also referred to as "generator pole slip", or out-of-step condition. The basic objective of out of step relaying is to trip the generator before a pole slip occurs. We shall discuss the phenomena that determine and differentiate a stable swing from an unstable one.

Important considerations:

1. The relaying scheme should be able to distinguish among faults, stable stability swings and unstable stability swings. The discrimination is based on the following observations: (a) faults cause an abrupt change in impedance; (b) the majority of faults are asymmetric, i.e. involve only one or two phases, thus causing highly unbalanced voltages and currents. Conversely, stability swings occur while the system voltages and currents remain balanced. The detection of unstable stability swings is usually implemented using mho relays.

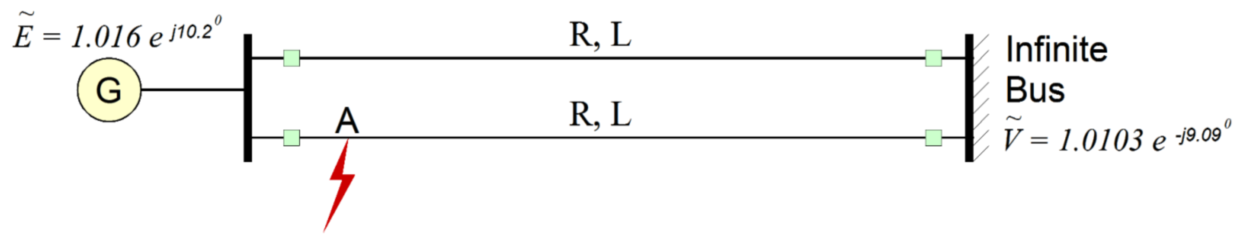
2. The relaying scheme should not trip for stable stability swings.

3. Once an eminent out-of step condition is detected and a decision to trip is taken, the isolated area should be carefully selected (controlled islanding).

4. Consideration should be given to transient recovery voltages (TRV's). It is possible that high TRV' cause multiple restrikes as breakers are opened, thus damaging breakers. The tripping sequence should be designed in such a way as to avoid the high TRVs.

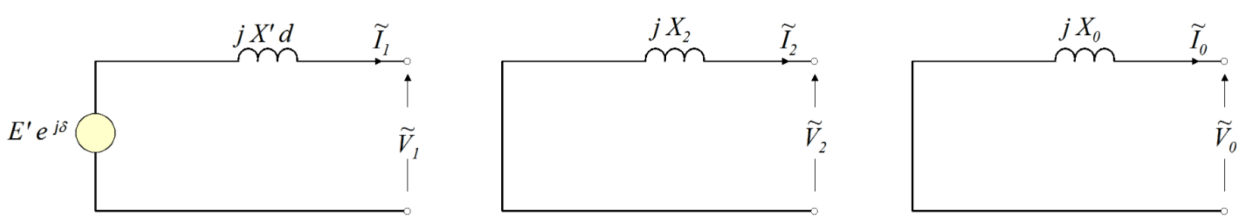
We discuss next the important parameters of:

- critical clearing angle, and
- critical clearing time



The model of the system is:

$$\frac{2H}{\omega_s}\frac{d^2\delta(t)}{dt^2} = P_{mu} - \mathrm{Re}\{\tilde{V}_1\tilde{I}_1^* + \tilde{V}_2\tilde{I}_2^* + \tilde{V}_0\tilde{I}_0^*\}$$



TO BE COMPLETED...

Example E8.x: Consider the electric power system of Figure E8.x. The generator is an 800 MVA, 60 Hz, 15 kV synchronous generator, with a rotor inertia constant H of 2.8 seconds. The generator operates at nominal voltage at the terminals, delivering 0.9 pu power at 0.9 lagging power factor. At time t=0, a three phase fault occurs at point A. The fault is cleared in 0.18 seconds by opening the breakers of the faulted line.

(a) Determine the critical clearing angle for this fault.

(b) Determine the critical clearing time for this fault.

(c) Graph the impedance seen by an out-of-step relay connected at the high side of the step-up transformer. The relay PT and CT have the following ratios: 15,000V:115V and 30,000A:5A. Perform the computations with a time step of 0.06 seconds up to the final time of 0.6 seconds.

(d) Set the out-of-step relaying scheme for this unit using a single mho relay with single blinders.

(e) Determine the maximum transient recovery voltage of the breaker in case of a trip due to out-of-step conditions.

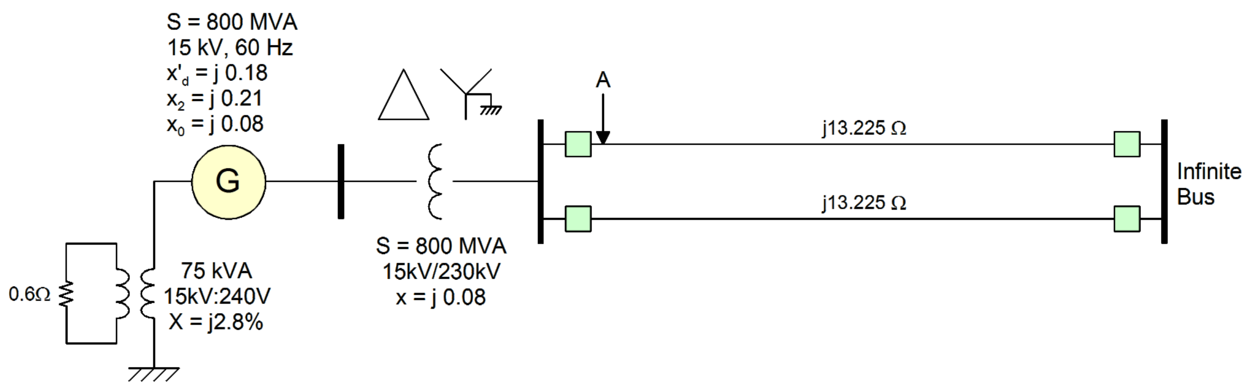


Figure E8.x

Solution: For the solution of this problem we consider the positive sequence equivalent circuit of the system prior to the fault, during the fault, and after the fault is cleared:

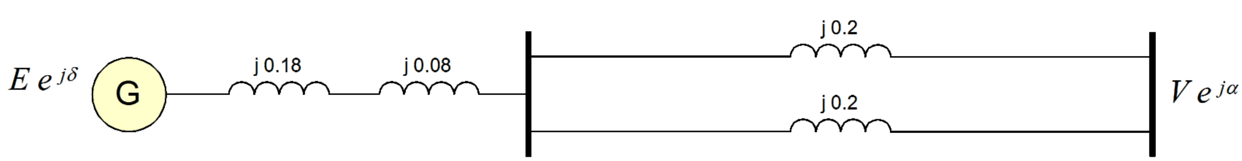


Figure E8.x. Simplified Electric Power System – System Prior to Fault

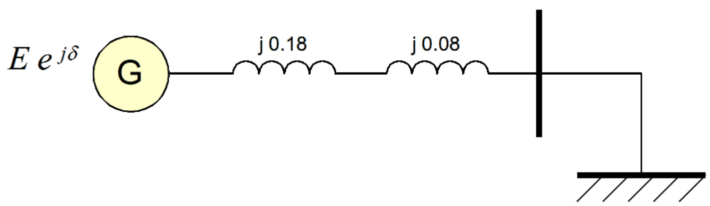


Figure E8.x. Simplified Electric Power System – System During Fault

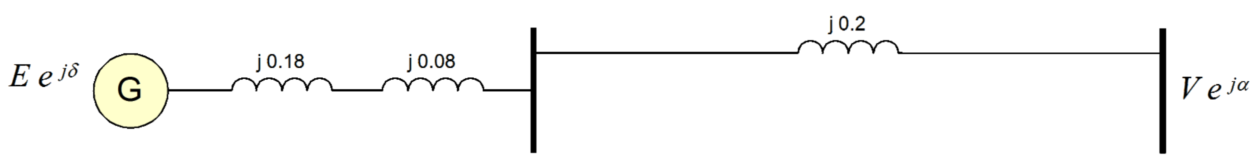


Figure E8.x. Simplified Electric Power System – System After Fault

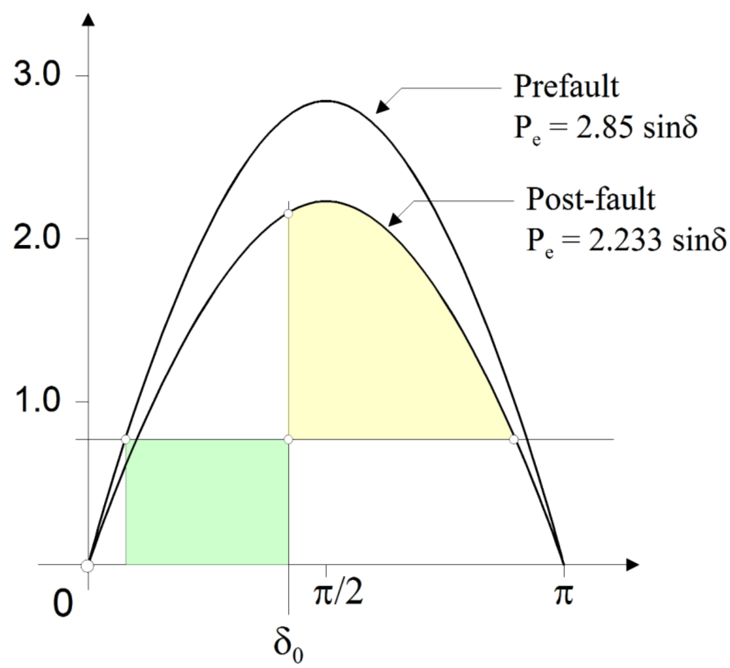


Figure E8.x. Electrical and Mechanical Power versus Angle δ
(Equal Area Criterion)

$$\delta(t=0) = 20.54^0 = 0.3585\, rad$$

$$\delta_1 = 153.39^0 = 2.6772\, rad$$

The critical clearing angle is determined by the equal area criterion:

$$0.9(\delta_c - \delta_0) = \int_{\delta=\delta_c}^{\delta_1} (2.233\sin\delta - 0.9)d\delta$$

Solution of above equation yields:

$$\delta_c = 87.68^0 = 1.5303 rad$$

This is the critical clearing angle. During fault:

$$\delta(t) = 30.294t^2 + 0.3585$$

The time when the angle reaches the critical angle is:

$$t_c = \sqrt{\frac{1.5303 - 0.3585}{30.294}} = 0.1967$$

The trajectory of the system, assuming that the fault is cleared at 0.19 seconds after fault initiation is given in the next three Figures (stable oscillation).

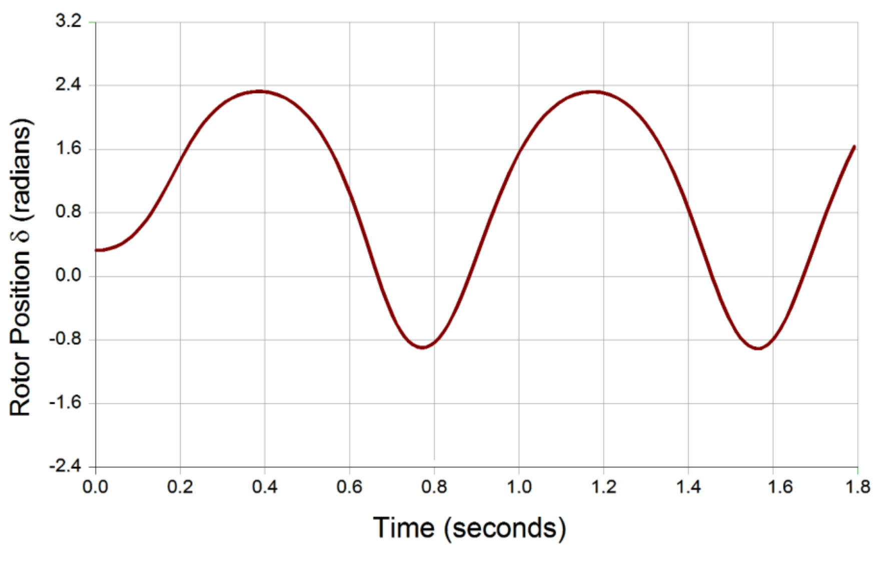


Figure E8.x.

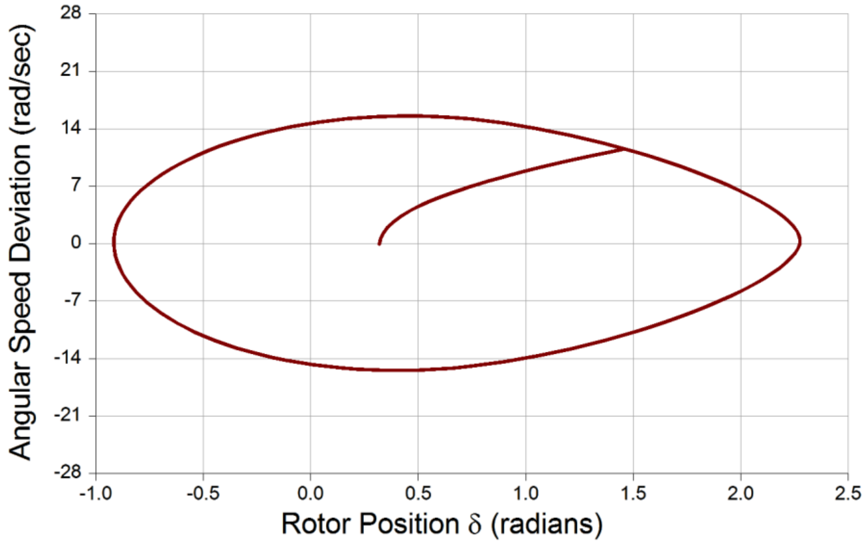


Figure E8.x.

Figure E8.x.

The trajectory of the system when the fault is cleared after 0.2 seconds is given below (unstable oscillation).

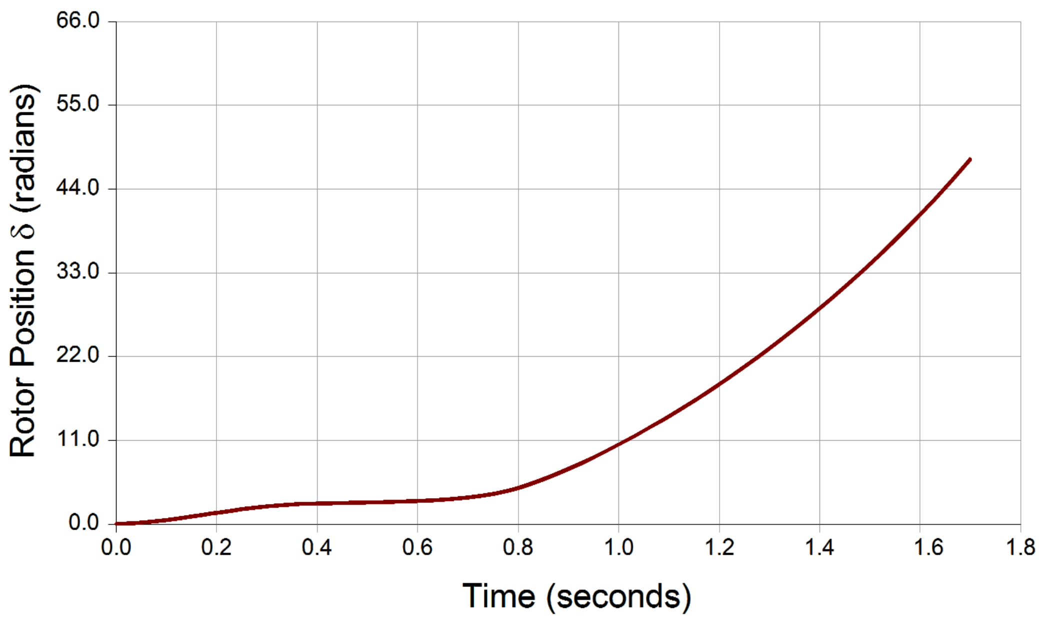


Figure E8.x.

8.12.2 Impedance Diagrams

A frequently used visualization technique for the study of generator swing phenomena as well as mho relay settings is the apparent impedance diagram. An impedance diagram example applicable for a simple generator/transformer/transmission line system is illustrated in Figure 8.x. A point on this diagram represents the apparent impedance seen by an impedance relay at bus B, i.e. the voltage at bus B divided by the current flowing at bus B, from the transformer towards the transmission line:

$$\text{Point B:} \qquad Z = \frac{\tilde{V}}{\tilde{I}}$$

In setting impedance relays, it is useful to plot the trajectory of the apparent impedance on an impedance diagram, during a generator swing. Assume that during a swing the generator source voltage magnitude E_1 and the remote bus voltage magnitude E_2 remain constant, while the phase angle between these sources varies. The locus of the apparent impedance for these conditions is determined as follows.

Plot point **A** at the negative of the impedance looking to the left of the relay, i.e. the negative of generator plus the transformer leakage reactance. This impedance can be also expressed as follows:

$$\text{Point A:} \qquad -X_g - X_T = \frac{\tilde{V} - \tilde{E}_1}{\tilde{I}}$$

Plot point **C** at the impedance looking to the right of the relay, i.e. the transmission line impedance. This impedance can be also expressed as follows:

$$\text{Point C:} \qquad Z_S = \frac{\tilde{V} - \tilde{E}_2}{\tilde{I}}$$

From the above definitions, the vectors $\overline{CB}$ and $\overline{AB}$ on the diagram can be evaluated as:

$$\overline{AB} = \overline{B} - \overline{A} = \frac{\tilde{E}_1}{\tilde{I}}, \qquad \overline{CB} = \overline{B} - \overline{C} = \frac{\tilde{E}_2}{\tilde{I}}$$

Thus, the angle between vectors $\overline{CB}$ and $\overline{AB}$ is the phase angle between voltage phasors $\tilde{E}_1$ and $\tilde{E}_2$, and the lengths of these vectors are proportional to the same phasor magnitudes. From these observations it can be concluded that for equal $\tilde{E}_1$ and $\tilde{E}_2$ magnitudes, the locus of the apparent impedance (point B on the impedance diagram) is the perpendicular bisector of the segment AC. For all other cases the locus of the apparent impedance is a circle whose diameter lies along the direction AC and its end points cut the segment AC to a certain ratio (The fact that this locus is a circle was shown by Apollonius of Perga in the 3d century BC). Two such example circles are shown on the diagram of Figure 8.x, one for a voltage phasor magnitude ratio of 1.25 and one for 0.8.

The impedance value at middle of the segment AC is known as the ***electrical center*** of the system. This is the apparent impedance seen by the relay when the phase angle between the voltages $\tilde{E}_1$ and $\tilde{E}_2$ is 180 degrees.

Another useful observation is that if the phase angle δ is held constant while the voltage phasor magnitude ratio varies, then the locus of the apparent impedance is also a circle. These constant δ circles pass through the points A and B as illustrated in Figure 8.x.

Using these observations, a region can be determined within which the apparent impedance seen by a relay at point B can be expected to lie during normal system operation. This region is enclosed by two constant δ circles and two constant voltage magnitude ratio circles. Obviously, the relay tripping region should not intersect the normal operation region.

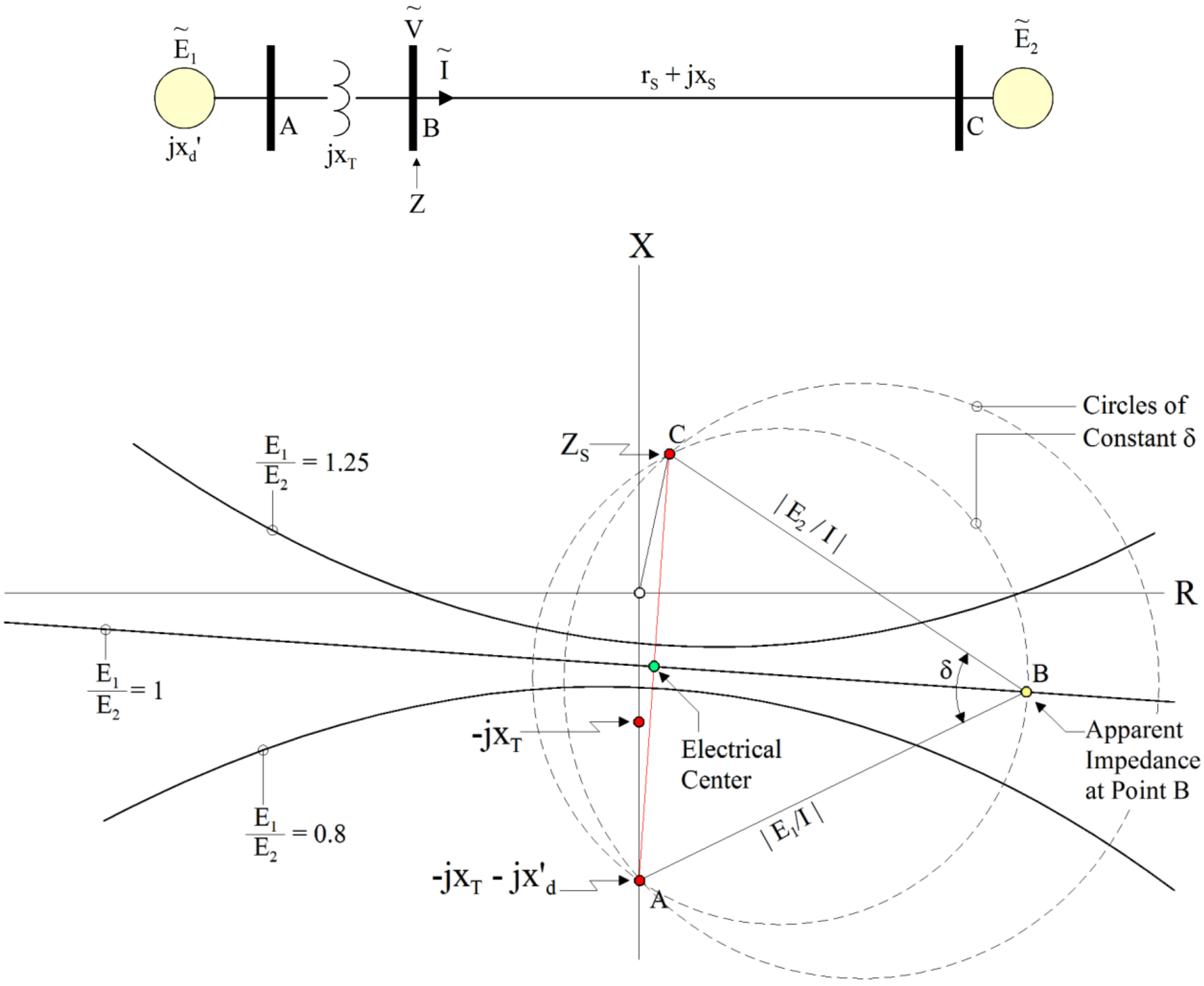


Figure 8.x. Impedance Diagram Illustration

8.12.3 Voltage Collapse Phenomena during Power Swings

Generator swings may generate temporary voltage collapse phenomena that may "fool" distance relays into tripping. Here, we describe the phenomena and discuss methods to avoid false tripping operations.

Example E8.x: Consider the electric power system of Example E8.x. Consider the same conditions as in that example. Compute and graph the voltage magnitude at the terminals of the generator and at the high side of the transformer for a period of up to 0.6 seconds using a time step of 0.06 seconds.

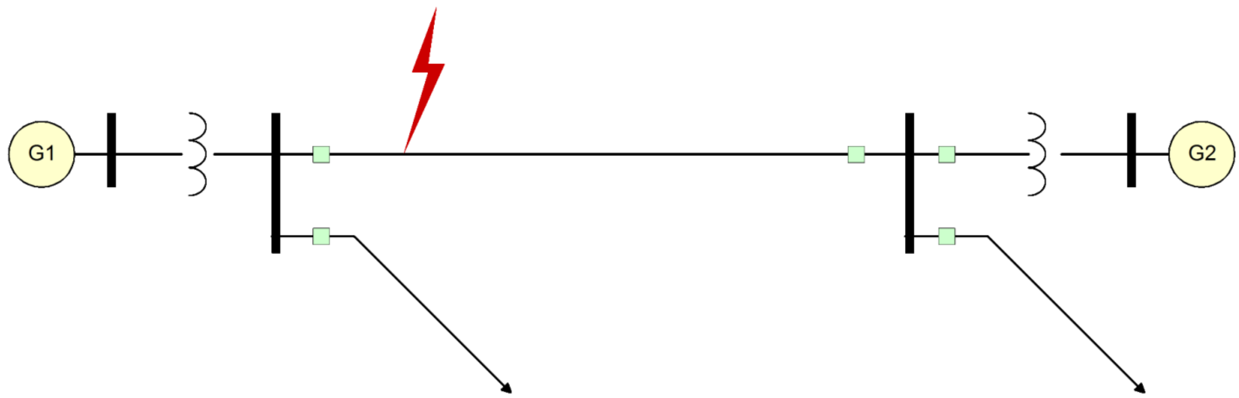


Figure E8.x: Example System for Voltage Collapse at the Center of Oscillation

Solution: For the solution of this problem we consider the operation of the system prior to the fault, during the fault and after the fault is cleared.

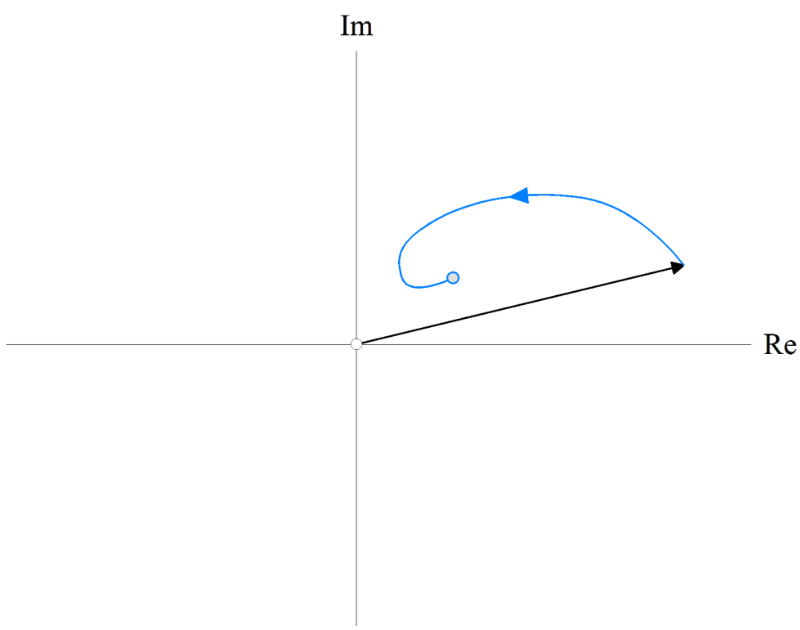


Figure E8.x: Voltage Variation during Stability Swing

To Be Continued.

8.12.3 Transient Recovery Phenomena

Transient recovery voltage on breakers when an out of step relay operates may be much higher than the usual transient recovery voltages. This is explained below. The mathematical model that describes the voltage build up across the plates of a breaker is described with the aid of Figure 5.x. The system illustrates two generating units and a breaker in-between. The indicated capacitors represent the parasitic capacitances of the two systems respectively.

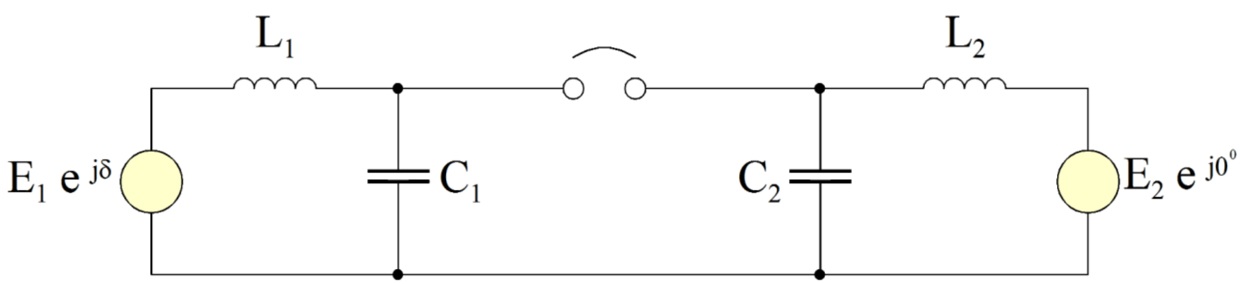


Figure 8.x: Equivalent Circuit for Transient Recovery Voltage Analysis

Assume that during a stability oscillation, the breaker trips at a time when the relative phase angle between the two generators is d. Now consider the period prior to the tripping. This condition is near sinusoidal steady state. The electric current is:

$$\tilde{I} = \frac{\tilde{E}_1 - \tilde{E}_2}{j\omega(L_1 + L_2)}$$

When the current becomes zero and assuming that by that time the plates of the breaker have separated, we will have the following model:

$$e_1(t) = L_1 \frac{di_1(t)}{dt} + v_{c1}(t)$$
$$i(t) = C \frac{dv_c(t)}{dt}$$

and initial conditions: $i_1(t=0) = 0, \quad v_{c1}(t=0) = v_0$.

The solution to this problem is:

$$i_1(t) = \frac{\sqrt{2}E_1}{\omega L_1} \frac{1}{1 - \left(\frac{\omega_1}{\omega}\right)^2} \left(\sin(\omega t + \delta) - \left(\frac{\omega_1}{\omega}\right) \sin(\omega_1 t + \delta) \right), \text{ where } \omega_1 = \frac{1}{\sqrt{L_1 C_1}}$$

and the voltage across the capacitor is:

$$v_{c1}(t)=e_1(t)-L_1\frac{di_1(t)}{dt}=\sqrt{2}E_1\left\{\cos(\omega t+\delta)-\frac{1}{1-\left(\frac{\omega_1}{\omega}\right)^2}\left(\cos(\omega t+\delta)-\left(\frac{\omega_1}{\omega}\right)^2\cos(\omega_1 t+\delta)\right)\right\}$$

A similar solution will exist at the other side. S

Consider the case where the two sources are 180 degrees apart. In this case the transient recovery voltages will be 180 apart. Figure 8.x illustrates the development of the transient recovery voltage. Note that in general, the frequencies may be different and therefore the maxima will occur at different times. This means that one must use numerical simulations to study and identify the worst case scenario.

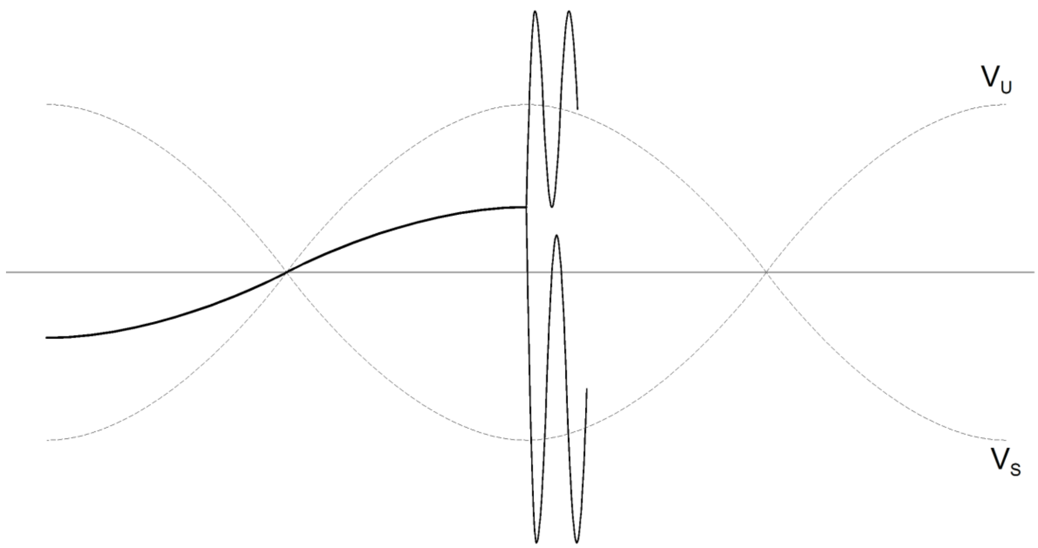


Figure 8.x: Transient Recovery Voltage When Breaker Operates when Systems are 180 degrees Apart

Example E5.x: Consider a two unit system of Example E8.x. Assume that during a stability swing the phase angles of the two units became 180 degrees apart. At this time, the breaker on the high side of the transformer trips. Determine the maximum transient recovery voltage of the breaker. The equivalent parasitic capacitance on the transformer side is 1.25 nanoFarads and the equivalent capacitance on the line side is 14.6 nanoFarads.

Solution: The equivalent circuit is:

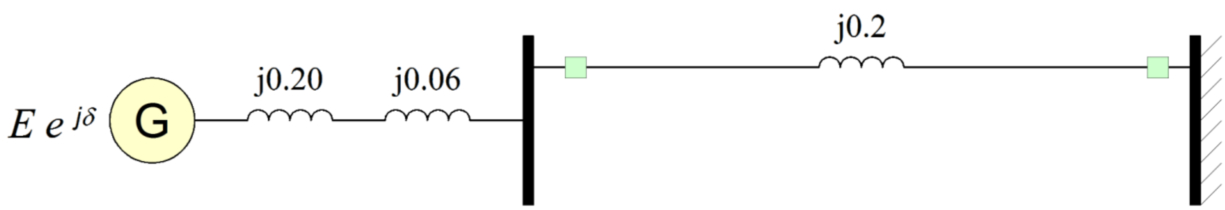


Figure 8.x:

$$Z_B = \frac{V_{L-L}^2}{S^{3\phi}} = 66.12 \quad Ohms$$

$$L_1 = \frac{(0.26)(66.12)}{377} = 45.6\times10^{-3} \quad Henries$$

$$L_2 = \frac{(0.20)(66.12)}{377} = 35.1\times10^{-3} \quad Henries$$

The angular frequencies are:

$$\omega_1 = \frac{1}{\sqrt{LC}} = 132{,}400\ (s^{-1})$$

$$\omega_2 = \frac{1}{\sqrt{LC}} = 44{,}170\ (s^{-1})$$

The voltage across the breaker is:

$$V_b(t) = \sqrt{2}V\left(-\cos\omega t + \frac{1}{1-(\omega_1/\omega)^2}\left(\cos\omega t - \left(\frac{\omega_1}{\omega}\right)^2 \cos\omega_1 t\right)\right) - \sqrt{2}V\left(\cos\omega t - \frac{1}{1-(\omega_2/\omega)^2}\left(\cos\omega t - \left(\frac{\omega_2}{\omega}\right)^2 \cos\omega_2 t\right)\right)$$

$$V_b(t) = \sqrt{2}(132.8\times10^3)(-\cos\omega t + \cos\omega_1 t - \cos\omega t + \cos\omega_2 t)\ (Volts)$$

A graph illustrates the evolution of the voltages. The maximum will occur at the half cycle of the slow waveform:

$$t = \frac{\pi}{\omega_2} = 71.12\times10^{-6}\ (s)$$

$V_b(t)$ is maximum at $t = 71.12\times10^{-6}$. The maximum voltage is:

$$V_b(t) = \sqrt{2}(132.8\times10^3)(4) = 751.23\times10^3 \quad Volts$$

Show the actual waveform for this example.

8.12.4 Out of Step Protection Schemes

An effective protection scheme for out-of-step conditions can be implemented with distance relays that monitor the impedance as seen on the high voltage side of the step up transformer. For this purpose, it is important to consider the impedance trajectory during a stability oscillation. Consider the system of Figure 8.x.

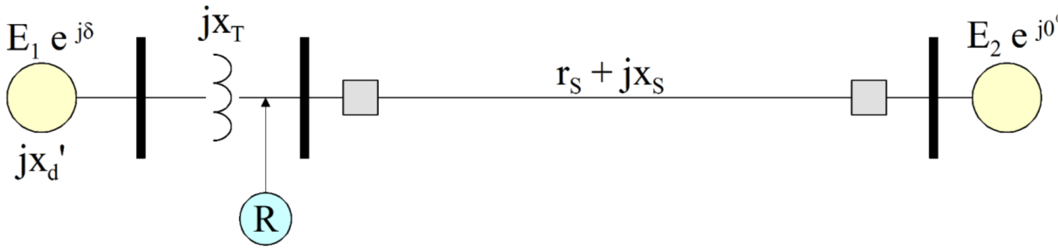


Figure 8.x. A Simplified System of a Unit Connected to a Large System

The impedance "seen" by the relay at the high side of the step-up transformer is:

$$Z_{relay} = \frac{k_1}{k_2}\frac{\tilde{V}}{\tilde{I}} = \frac{k_1}{k_2}\left(-j(x_g + x_t) + j(x_g + x_t + x_s)\frac{\tilde{E}_1}{\tilde{E}_1 - \tilde{E}_2} \right)$$

As the phase angle of the generator 1 changes, the apparent impedance may take the indicated trajectories in the figure.

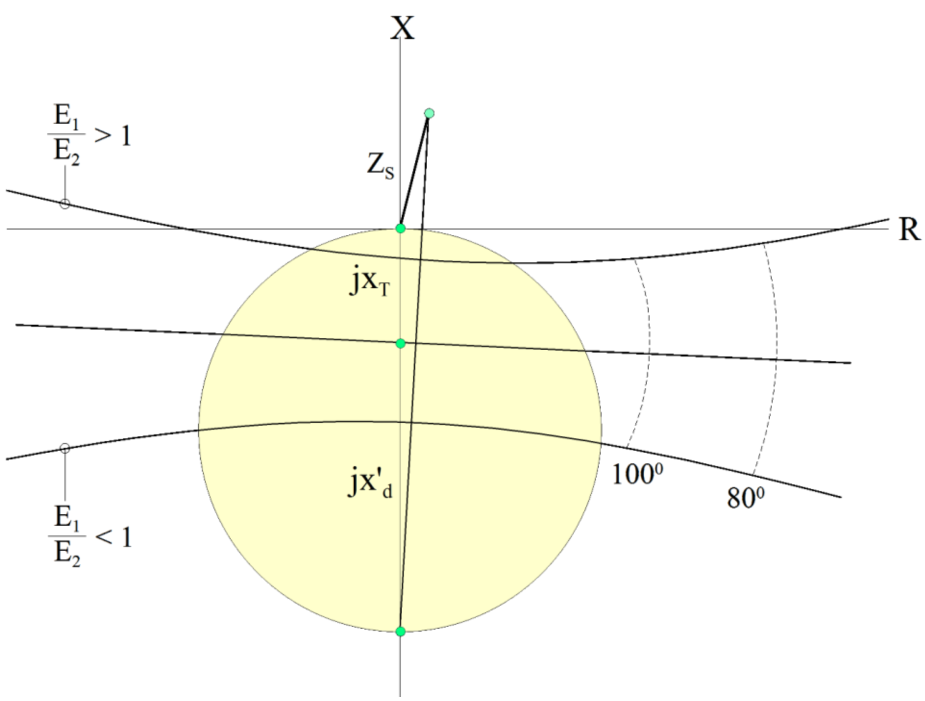


Figure 8.x. Impedance Trajectories for a Simplified System

Following is a discussion of few out-of-step protection schemes.

Single mho relay: This scheme uses a single mho relay at the high side of the step up transformer and set to reach the combine transformer and generator impedance.

Mho relay with single blinder: This scheme….

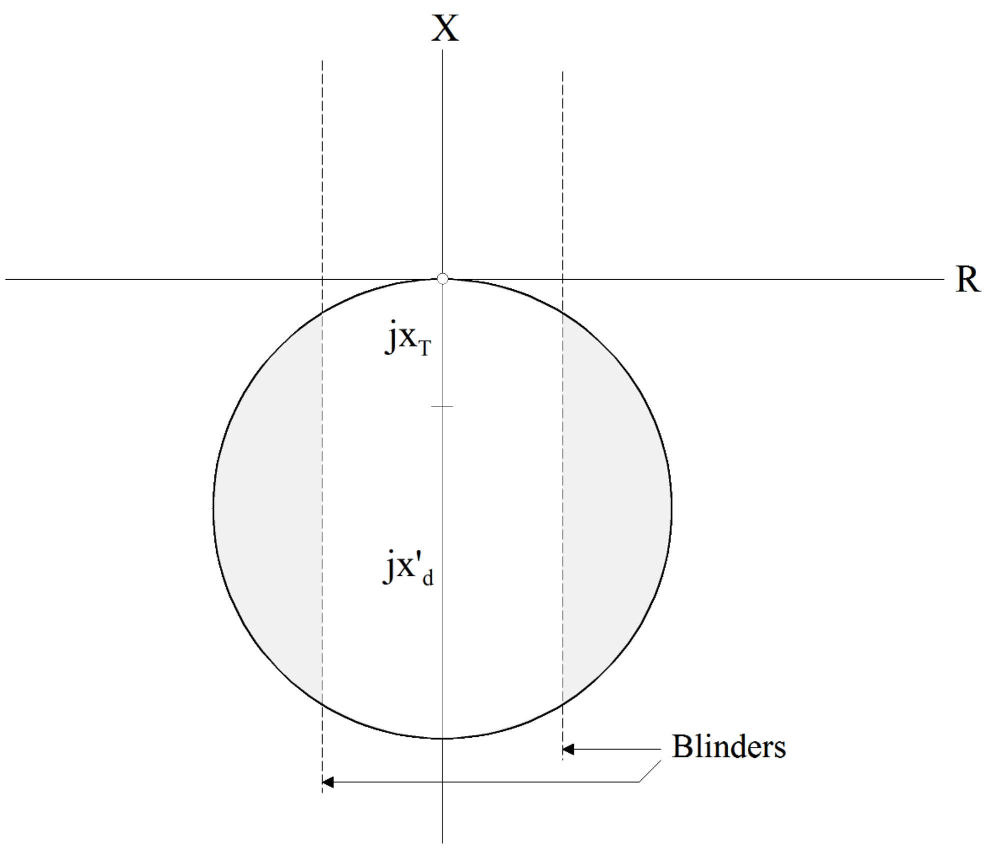


Figure 8.x. Single mho Relay with Single Blinders

Mho relay with double blinders: This scheme….

Single Lens relay: This scheme….

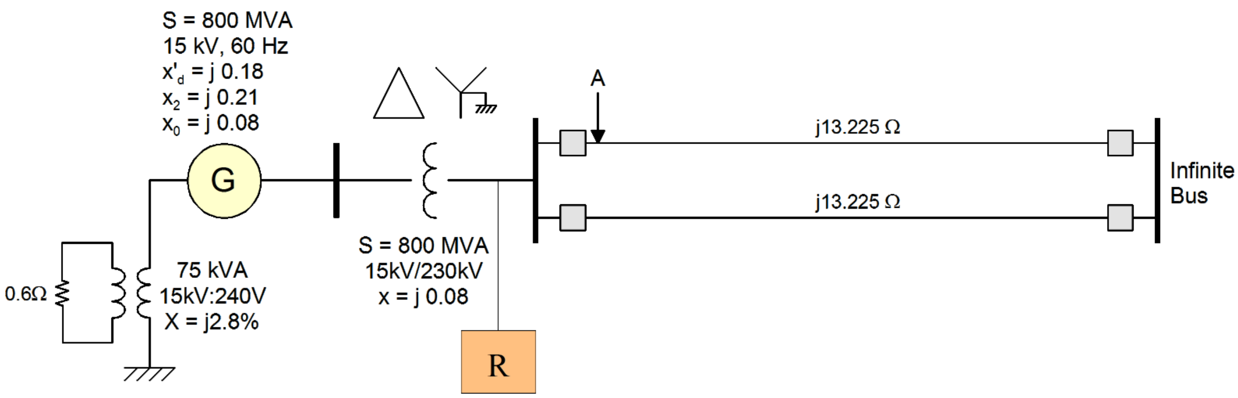
Double Lens relay: This scheme….

Example E8.x: Consider a two-unit system and determine the swing of the system and the impedance that a mho relay will "see" during the swing. Use same example as before.

Solution: The model…. To be completed.

Example E8.x: Consider the electric power system shown below. The generator rotor inertia parameter **H** is **2.8** seconds. The generator operates at nominal terminal voltage delivering **0.90**

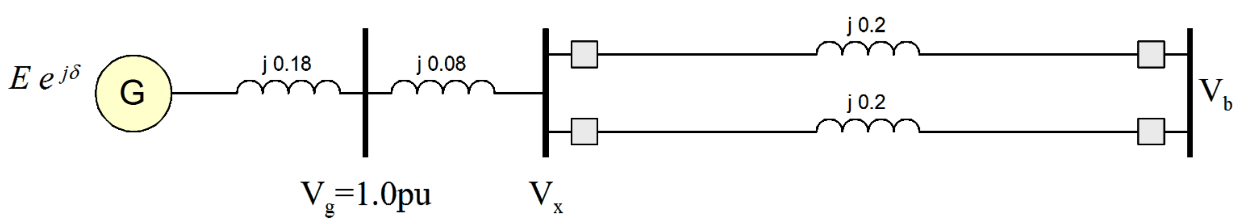
pu power at unity power factor. At time t=0, a three phase fault occurs at point A. The fault is cleared at t=0.19 seconds by opening the faulted transmission line breakers.



(a) Graph the impedance seen by an out-of-step relay on the high side of the transformer. (Assume there is an out of step relay at the high side terminals of the step up transformer. The relay PT and CT have the following ratios respectively: 135,000V:115V and 2,400A:5A. Perform the computations with a time step of 0.03 seconds up to the final time of 0.6 seconds.)

(b) Determine the settings of the out-of-step relaying scheme for this unit using a single mho relay with single blinders

Solution: Refer to the equivalent circuit representing the system before the fault:



Given the specified power flow at the generator terminals, the generator terminal current is:

$$\tilde{I} = 0.9\, e^{-j0}$$

Thus the generator internal equivalent source voltage is:

$$\tilde{E} = 1.0 + (j0.18)(0.9) = 1.0 + 0.162 = 1.013 e^{j9.20}$$

And the remote bus voltage is:

$$\tilde{V}_b = 1.0 - (j0.18)(0.9) = 1.0 - 0.162 = 1.013 e^{-j9.20}$$

Thus the angle δ between the generator equivalent source and the infinite bus is:

$$\delta(0) = 18.4^0 \ \ or \ \ 0.3211 rad$$

The voltage at the high side of the transformer is:

$$\tilde{V}_x = 1.0 - (j0.08)(0.9) = 1.0 - j0.072 = 1.0026 e^{-j4.12}$$

Thus the impedance (in pu) seen by the relay is:

$$Z = \frac{\tilde{V}_x}{\tilde{I}} = \frac{1.0026 e^{-j4.118}}{0.9} = 1.114 e^{-j4.12^0}$$

Recall that the phase angle δ is determined by the differential equation:

$$\frac{2H}{\omega_s}\frac{d^2\delta(t)}{dt^2} = P_{mu} - \mathrm{Re}\{\tilde{V}_1\tilde{I}_1^* + \tilde{V}_2\tilde{I}_2^* + \tilde{V}_0\tilde{I}_0^*\}$$

During the 3-phase fault the electrical power collapses to zero yielding:

$$\frac{2H}{\omega_s}\frac{d^2\delta(t)}{dt^2} = P_{mu}$$

The solution of the above is:

$$\delta(t) = \frac{\omega_s P_{mu}}{4H} t^2 + \delta(0)$$

Substituting the appropriate coefficients:

$$\delta(t) = 30.29 t^2 + 0.3211$$

$$\frac{d\delta(t)}{dt} = 60.58 t$$

Thus at t=0.19 sec:

$$\delta(0.19) = 1.4146 rad \text{ or } 81.05 \text{ degrees}$$

$$\frac{d\delta}{dt}(0.19) = 11.51 \mathrm{rad/sec}$$

Once the fault clears, the phase angle δ is determined by the differential equation:

$$\frac{2H}{\omega_s}\frac{d^2\delta(t)}{dt^2} = P_{mu} - \frac{EV_b}{x}\sin(\delta)$$

where

x = 0.18 + 0.08 + 0.20 = 0.46 pu, (one line is now disconnected)
H = 2.8 sec
ω_s = 377 rad/sec
P_{mu} = 0.9 pu
E = 1.013 pu
Vb = 1.013 pu

Thus, the above equation becomes

$$\frac{d^2\delta(t)}{dt^2} = 60.589 - 150.19\sin(\delta)$$

The above equation is integrated using the modified Euler method from t=0.19 through t=0.34 seconds with a time step of 0.01 seconds, and initial conditions:

$\delta = 1.4146 rad$, $\frac{d\delta}{dt} = 11.51$rad/sec

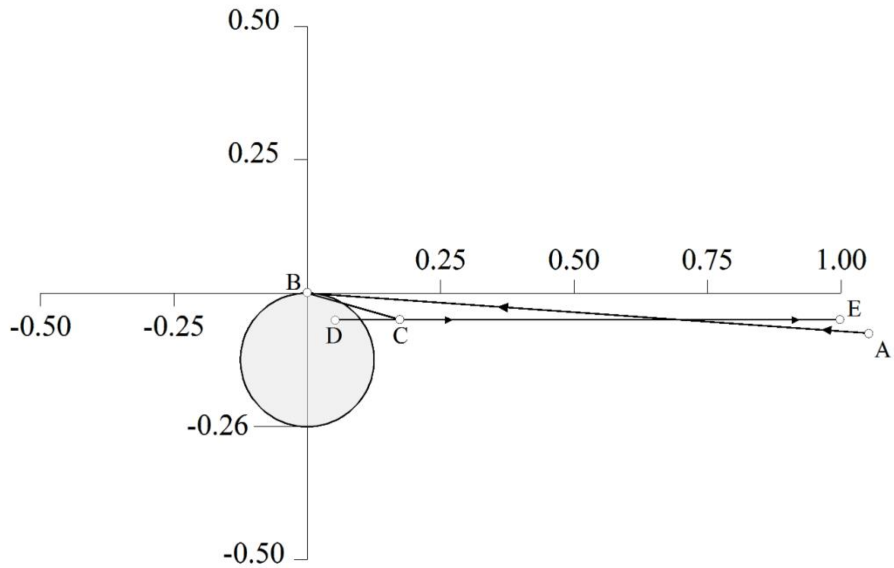
Given the angle δ at every iteration, the apparent impedance seen by the relay is computed as follows:

$$\tilde{I} = (1.013e^{j\delta} - 1.013) / j0.46$$
$$\tilde{V} = 1.013 - j0.26\tilde{I}$$
$$Z = \tilde{V} / \tilde{I}$$

A subset of the results is given below (reported once every five time steps):

Time	δ	dδ/dt	Re{Z}	Im{z}
0.200	1.525	10.621	0.241	-0.030
0.250	1.946	6.321	0.157	-0.030
0.300	2.170	2.786	0.121	-0.030
0.350	**2.234**	**-0.185**	**0.112**	**-0.030**
0.400	2.151	-3.197	0.124	-0.030
0.450	1.904	-6.836	0.164	-0.030
0.500	1.455	-11.194	0.258	-0.030
0.550	0.794	-14.874	0.548	-0.030
0.600	0.030	-14.780	15.56	-0.030

Note that the largest angle reached is 2.234 radians at 0.35 seconds. At that time the apparent impedance is closest to the origin at 0.112 - j0.03 pu. Figure 7E.x illustrates the impedance swing and the relay characteristic.



Blinders at $\pm$0.05 pu

Converting to ohms at the relay level:

$$Z_{line} = 0.26 \frac{230kV^2}{800MVA} = 17.19\,\Omega$$

$$Z_{relay} = 17.19\,\Omega \frac{PT}{CT} = 17.19\,\Omega \frac{115/135000}{5/2400} = 7.029\,\Omega$$

$$\text{Blinders at: } Z_{relay} = 0.05 \frac{230^2}{800} \frac{115/135000}{5/2400} = 1.35\,\Omega$$

8.12.5 Other Protection Schemes for Out-of-Step Conditions

For very important generators, several other schemes have been developed and utilized to avoid out of step conditions of the generator during a fault that should be cleared by another protection zone. Examples of these schemes are: (a) Fast valving, (b) dynamic braking, (c) reclosing, etc.

8.12.6 Discussion

When the out of step condition is detected via blinders, the system is close to 180 degrees. In this case TRV will be at about 4 times rated voltage. To avoid risk of restrike some schemes wait until the angle becomes smaller before tripping. This delay tripping increases the risk of damage to the synchronous machine.

If possible controls should be applied to minimize tripping and maintain grid balance.

Tripping at small angles to minimize TRV stress on breakers.

Other helpful items:

1. high speed relays
2. breaking resistors
3. fast valving
4. independent pole tripping

8.13 Reclosing and Synchronizing

Restoration of a tripped generator will require resynchronizing the generator with the system prior to connecting the unit to the network.

The typical precautions to be taken are:

- Transient Currents
- Inrush Currents
- Winding Forces
- Required Delays (i.e. Deionization: 10.5+kV/34.5 cycles)
- Synchronism Check
- Automatic Synchronizing
  - Voltage
  - Frequency
  - Phase

8.14 Summary and Discussion

In this chapter we have discussed protection schemes for various generator fault conditions. The generator is a very important and expensive asset. For this reason, the protection schemes developed for generators are very sophisticated, reliable and secure. The philosophy is to trip the generator before any damage occurs to the generator.

8.15 Problems

Problem P8.1: At a certain location of three phase system, an engineer measures the following phase currents and phase to neutral voltages:

Problem P8.2: An 800 MVA, 18 kV, 60 Hz generator is resistance grounded. The maximum fault current during a phase to ground fault is 200 Amperes. The generator impedances are 18%, 21% and 9% for the positive, negative and zero sequence impedances respectively. The generator is protected with a differential scheme across each phase winding and ground fault protection at the terminals of the generator.
The relay settings are as follows:
Differential: 0.1 Amperes minimum pickup, 10% slope, the CTs are 30000:5A.
Ground relay: instantaneous: 50% of the 200 Amperes (on the generator side), pickup 8% of the 200 Amperes (on the generator side).
Consider a ground fault in one coil at a location about 5% from the neutral. Determine whether the differential relay or the ground fault relay will pickup.

Solution: The ground resistor is: $R_g = \dfrac{18kV/\sqrt{3}}{200A} \cong 52.0\,Ohms$

The fault current for a ground fault at 5% of the coil will be:

$$I_f \cong \frac{(0.05)(18kV)/\sqrt{3}}{52.0\,Ohms} \cong 10\,A$$

Differential Relay:

Figure

Ground fault relay

NO TRIP

Problem P8.3: Consider the electric power system of Figure P8.3a. The generator is equipped with a loss of field relaying scheme that is based on an impedance relay "looking" at the generator terminals. Assume that the generator operates under nominal terminal voltage, delivering 0.9 pu real power with power factor 0.97 current lagging when suddenly the field circuit is opened. Graph the trajectory of the impedance for 2.0 seconds following the loss of field. For simplicity assume that the model of the generator upon loss of the field is illustrated in Figure P8.3b. The parameters of the equivalent circuit of the generator when the field is lost are:

$$r_1 = 0.0005\,pu, \quad r_2 = 0.01\,pu, \quad x_1 + x_2 = 0.195\,pu, \quad x_m = 3.56\,pu, \quad \tau_a = 1.2\,\text{seconds}$$

Hint: Simulate the operation of the system with a time step of 0.1 seconds. At each point, compute the decayed voltage source and the speed of the machine. Subsequently compute the impedance as "seen" at the terminals of the machine.

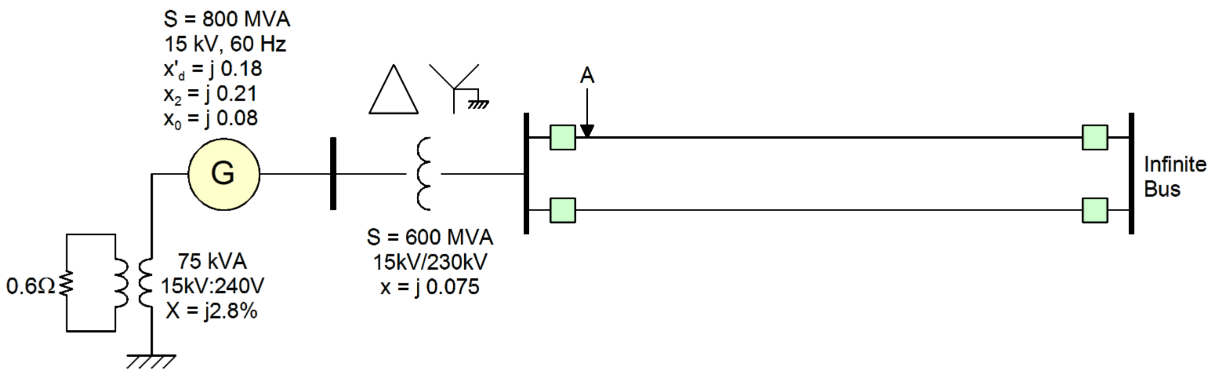


Figure P8.3a

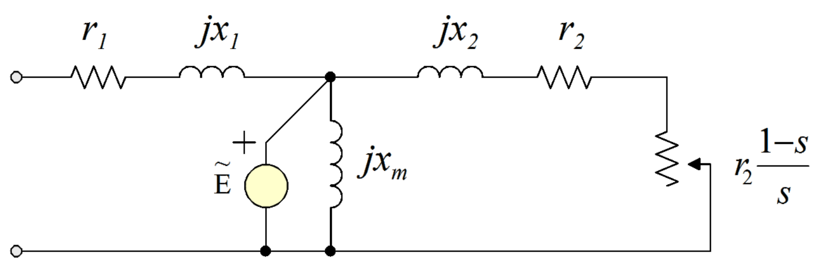


Figure P8.3b

Problem P8.4: Consider a 625 MVA, 60 Hz, 18 kV synchronous generator with the parameters indicated in Figure P8.4. The generator operates at nominal voltage at the terminals, delivering rated MVA power at power factor 0.98 current lagging. At time t=0, a three phase fault occurs at point A. The fault is cleared in 0.18 seconds by opening the breakers of the line.

Assume there is an out of step relay at the high side terminals of the step up transformer. The relay PT and CT have the following ratios respectively: 135,000V:115V and 600A:5A. Compute and graph the impedance "seen" by the relay. Perform the computations with a time step of 0.03 seconds up to the final time of 0.6 seconds.

The per-unit inertia constant of the generator is 2.2 seconds. The impedance of each transmission line is j13.225 ohms.

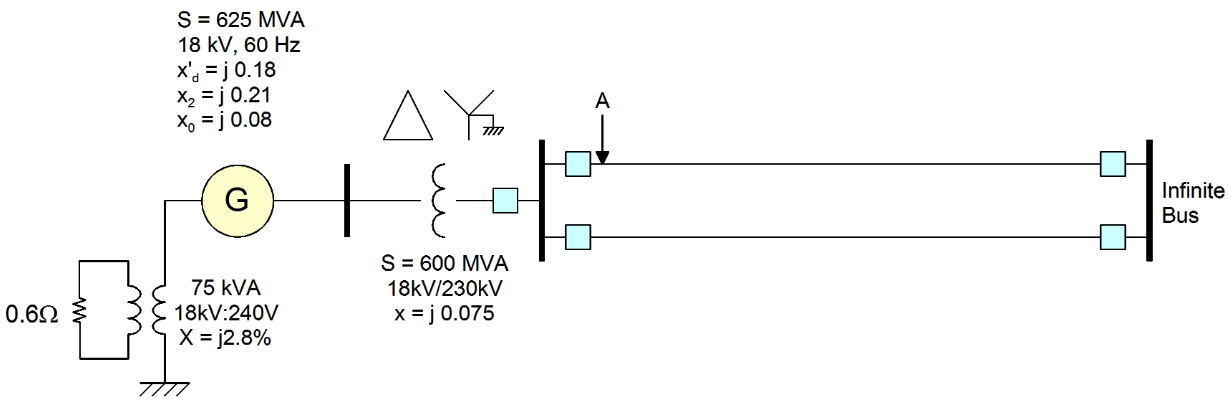


Figure P8.4

Solution: The pre-fault conditions equivalent circuit (in pu, 625 MVA) is:

The sources are:

$$\tilde{E} = 1.0 + j0.18e^{-j11.48^0} = 1.0507e^{j9.66^0}$$
$$\tilde{V} = 1.0 - j0.1563e^{-j11.48^0} = 0.9809e^{-j8.98^0}$$

During fault:

$$\frac{2H}{\omega}\frac{d^2\delta(t)}{dt^2} = 0.98,$$

Solution yields:

$$\delta(t) = 0.3253 + 41.98t^2$$

At fault clearing, t_c = *0.18 sec*, Thus: $\delta(t_c) = 1.6855$

After fault clearing:

$$P_e = \frac{(1.0507)(0.9809)}{0.4144}\sin\delta(t) = 2.487\sin\delta(t)$$

Problem P8.5: Consider a 625 MVA, 60 Hz, 18 kV synchronous generator with the parameters indicated in Figure P8.5. It is desirable to apply a two-zone loss of field relay at the terminals of the generator. The relay PT and CT have the following ratios respectively: 18,000V:115V and 20,000A:5A.

(a) Select the settings for the loss of field relay. Other data are as follows: the generator synchronous reactance is 2.1 pu.

(b) Assume that the generator operates at rated voltage at its terminals, rated MVA power at power factor 0.90 current leading. Determine whether the loss of field relay will operate on this condition.

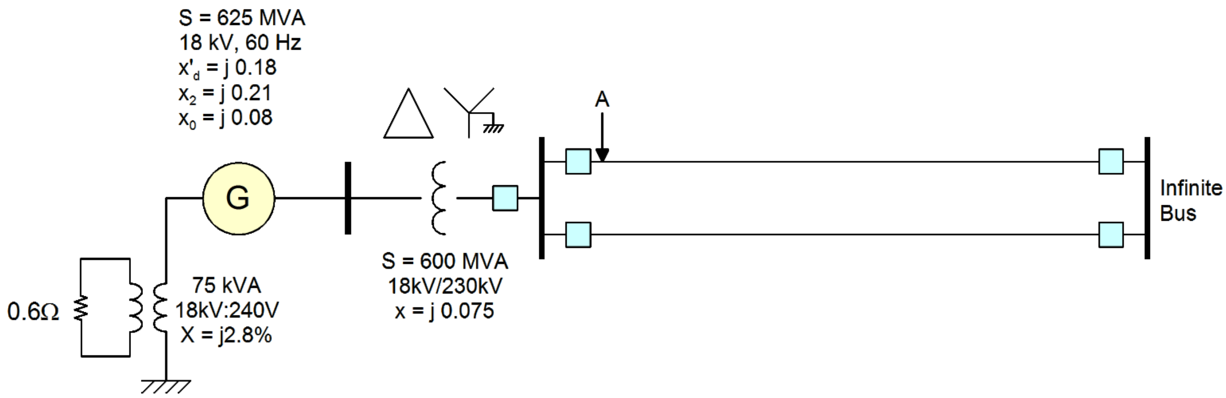


Figure P8.5

Problem E8.6: Consider a 625 MVA, 60 Hz, 18 kV synchronous generator with the parameters indicated in the figure. The generator operates at nominal voltage at the terminals, delivering rated MVA power at power factor 0.98 current lagging.

(a) Compute the generated voltage of the generator.

(b) It is desirable to apply a negative sequence current relay. The CTs have the following ratio 20,000A:5A. Select the settings of the negative sequence relay. Other data are as follows: the generator k constant is 6.25. Hint: use 10% margin, i.e. select the setting of the relay to correspond to 90% of the k constant of the generator.

(c) Assume a line to line fault at the high side of the transformer. Determine the time that the negative sequence relay will trip the generator assuming your settings from part (a). Assume that the operating condition of the generator is the one defined in part (a). For simplicity, neglect the transmission network.

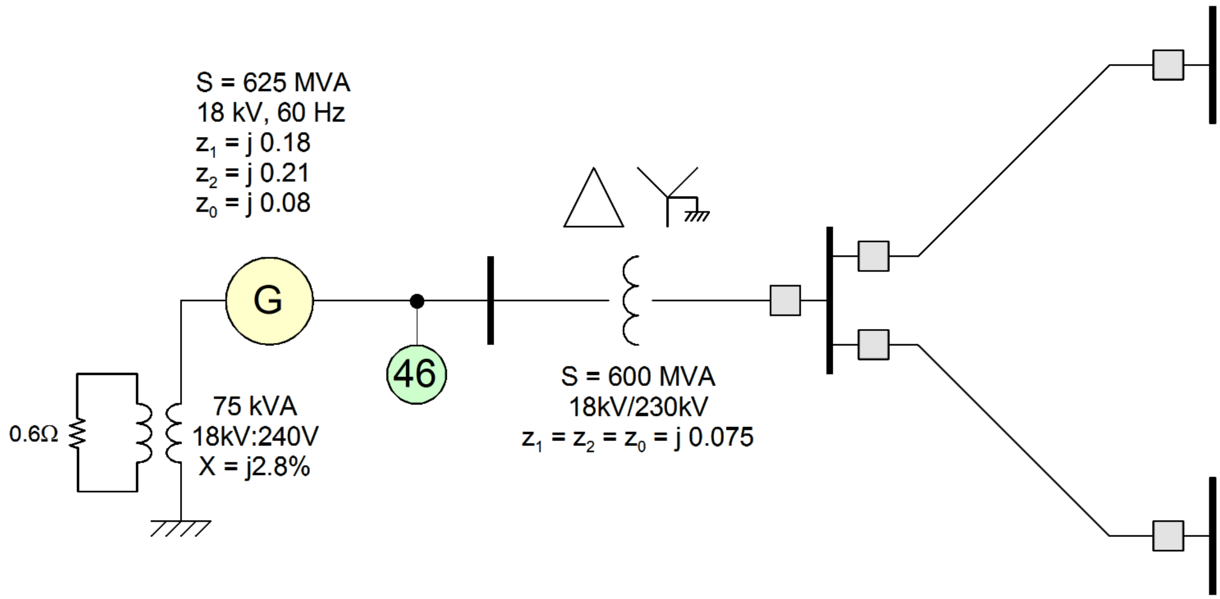


Figure P8.6

Solution: (a) the generated voltage is computed from:

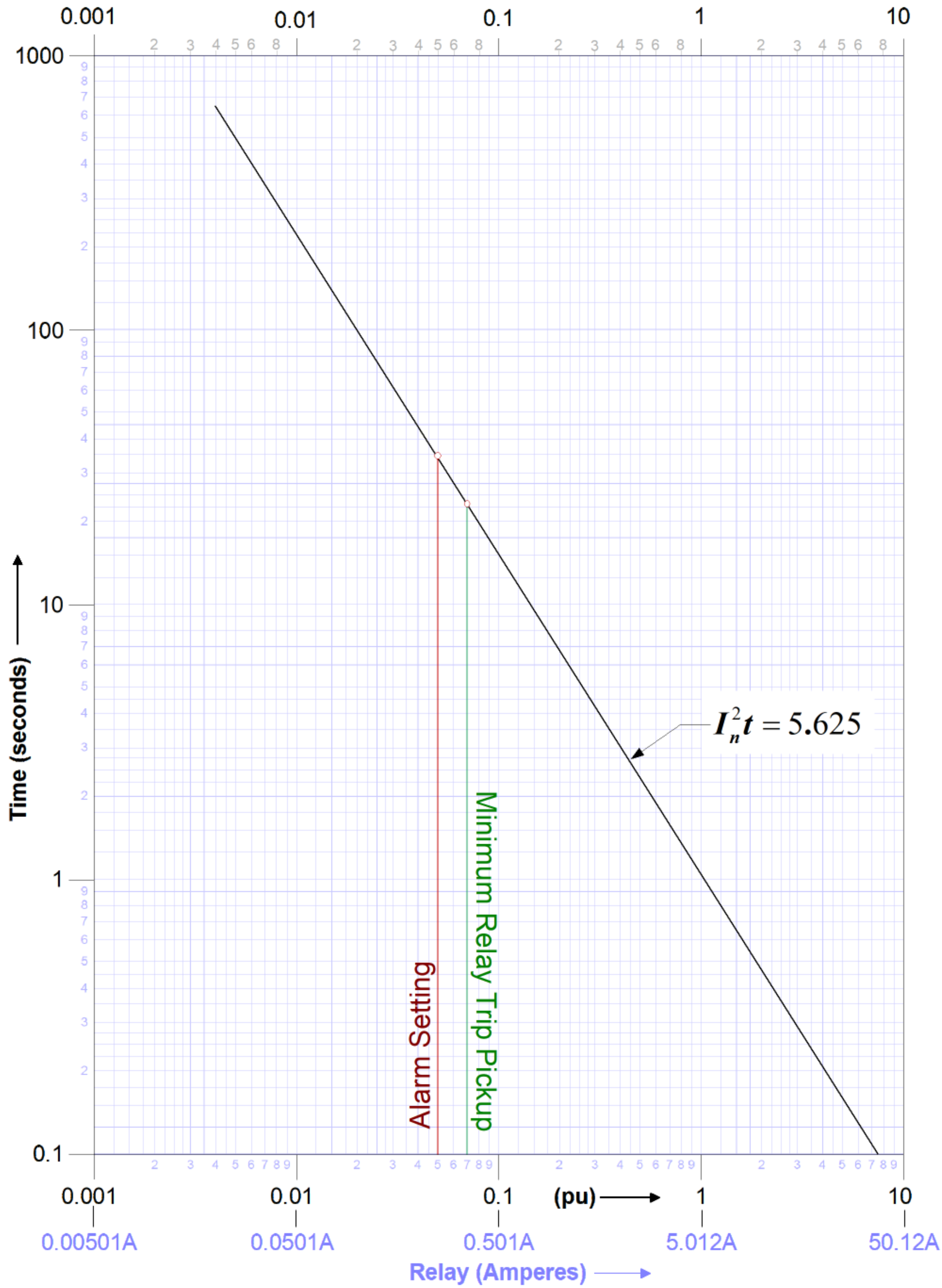
The generator current is (assuming terminal voltage phase is zero): $\tilde{I}_g = 1.0e^{-j11.48^0}$ pu

The generator generated voltage is: $\tilde{E} = 1.0 + (j0.18)(1.0e^{-j11.48^0}) = 1.0507e^{j9.665^0}$ pu

(b) The settings of the negative sequence relay are shown in the figure below.

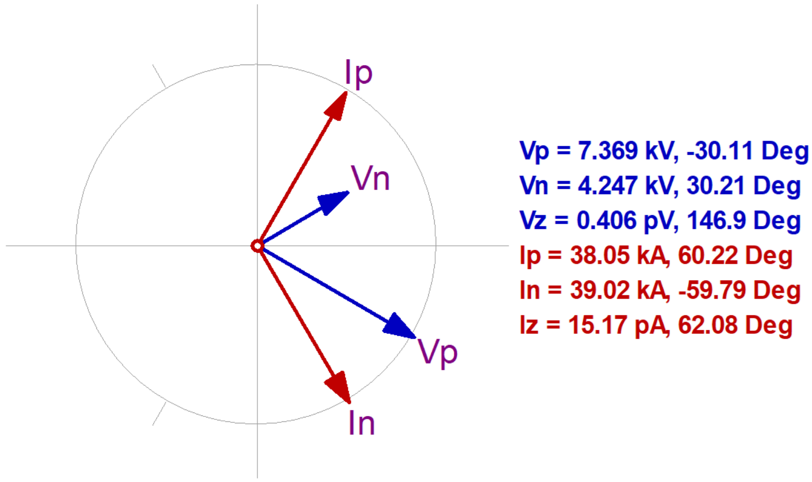
Alarm at 5% of current or 1,002 Amperes on generator side or 0.25 Amperes on relay side.
Minimum relay trip pickup at 7% of nominal current or 1,403 Amperes on generator side or 0.35 Amperes on relay side. See graph.
Trip characteristic:

$I_n^2 t \geq 5.625$, in pu, OR $I_{n,relay}^2 t \geq 231.48$ on relay side, relay current in Amperes.



0.001
0.01
0.1
1
10
1000
100
10
1
0.1
Time (seconds)
$I_n^2 t = 5.625$
Alarm Setting
Minimum Relay Trip Pickup
(pu)
0.00501A
0.0501A
0.501A
5.012A
50.12A
Relay (Amperes)

(c) For this part, the model of this system is constructed in WinIGS format and the currents under the specified condition are computed as in the figure below.



The negative sequence current in pu is:

$$\tilde{I}_n = \frac{39.02}{20.047} = 1.9464\, pu$$

The time to trip is given by:

$$t_{trip} = \sqrt{\frac{5.625}{1.9464^2}} = 1.2184 seconds$$

Problem P8.7: Consider a 625 MVA, 60 Hz, 18 kV synchronous generator with the following parameters. Differential relay setting.

Problem P8.8: Consider a 625 MVA, 60 Hz, 18 kV synchronous generator with the following parameters. Grounding system design.

Problem P8.9: A synchronous 60 Hz generator delivers 1.0 per unit real power to an infinite bus through a series capacitor compensated transmission line as it is illustrated in Figure P10.3.

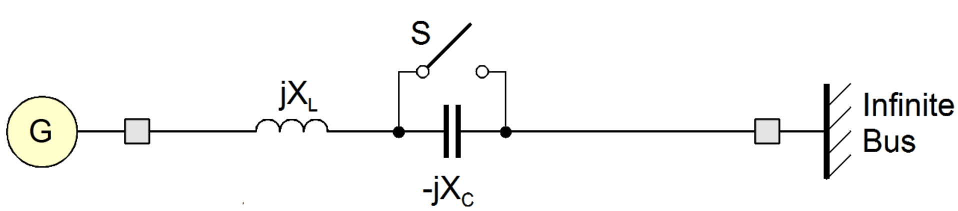


Figure P10.3

The power factor at the terminals of the generator is 1.0 and the voltage is 1.0 pu. The generator transient reactance is 0.25 per unit. The generator per unit inertia constant is H=2.8 seconds and the impedance of the transmission line is $z_L = j0.55\,pu$. All quantities are expressed on the same basis. Consider the possibility of a fault in the series capacitor. Assume that upon the occurrence of this event, the protective system will act immediately, the switch S will be closed and the capacitor will be bypassed.

Assume $z_c = -j0.45\,pu$. Determine whether system is stable for the specified fault and protective system response. Compute the new equilibrium point, i.e. compute the steady state position of the rotor δ. Consider an out-of-step relay at the terminals of the generator. Compute and graph the impedance seen by the relay during this disturbance.

Assume $z_c = -j0.70\,pu$ per unit. Determine whether system is stable for the specified fault and protective system response. Consider an out-of-step relay at the terminals of the generator. Compute and graph the impedance seen by the relay during this disturbance.

Problem P8.10: Consider the electric power system of Figure P8.10. The system consists of a generator, a delta-wye connected transformer and a three phase line. The point A of the line is located 15 miles from the transformer.

A negative sequence relay (electric current) is connected to the terminals of the generator via three identical 12,000A:5A current transformers (CTs). Compute the negative sequence current that the relay will "see" during a single phase to ground fault at location A. Use symmetrical component theory in the computations. System data are as follows:

Generator (350 MVA, 15kV): $z_1 = j0.175\,pu$, $z_2 = j0.25\,pu$, $z_0 = j0.08\,pu$

Transformer (350 MVA, 15/230 kV): $z_1 = j0.08\,pu$, $z_2 = j0.08\,pu$, $z_0 = j0.08\,pu$ (on transformer rating)

Transmission line: $z_1 = z_2 = 0.3 + j0.72\,ohms/mi$, $z_0 = 0.65 + j1.75\,ohms/mi$

Transformer shunt impedance and transmission line capacitive shunt impedance are to be neglected. Neglect system beyond the fault.

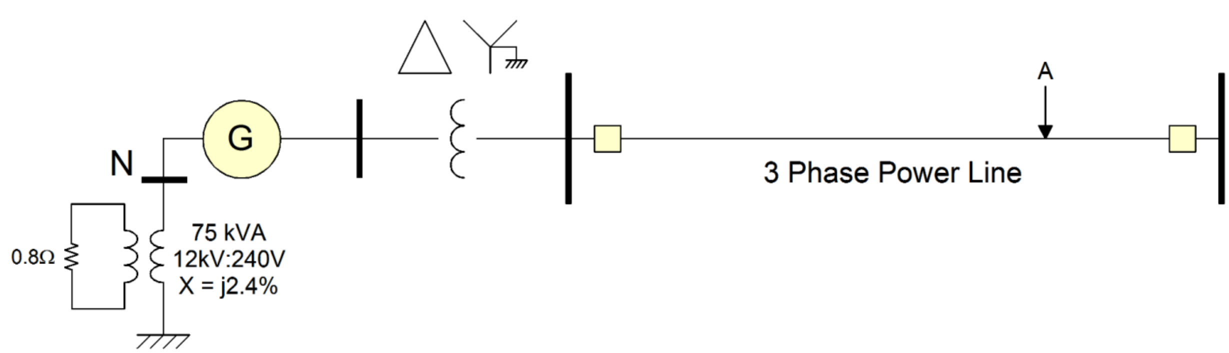
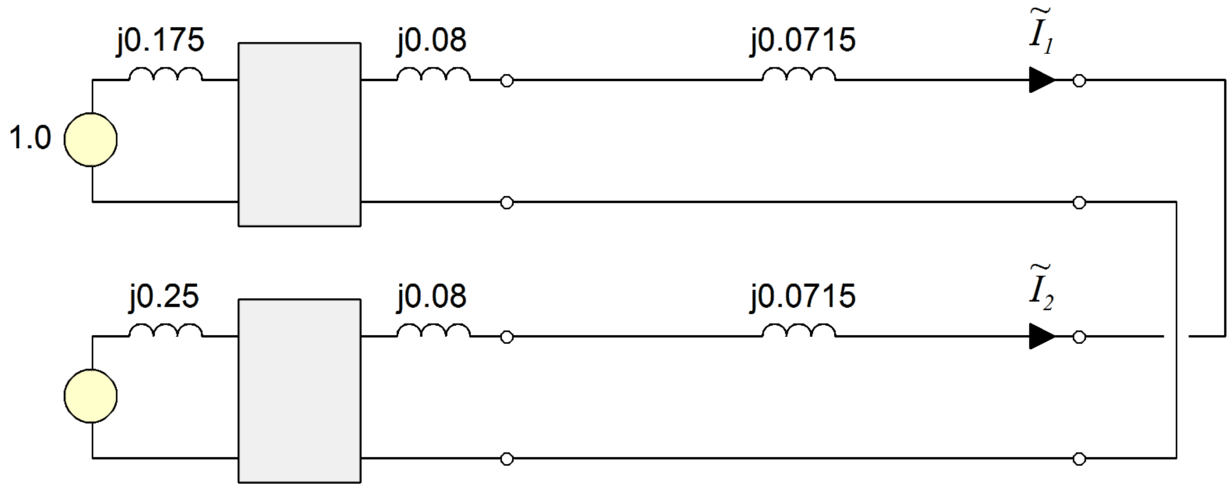


Figure P8.10: A Simplified Power System

Solution: The fault currents are:



$$\tilde{I}_1 = -\tilde{I}_2 = \frac{1.0}{j0.72} = -j1.3736\, pu$$

Negative sequence current at generator side:

$$\tilde{I}_{2,g} = 1.3736 \frac{350}{15\sqrt{3}}\, kA = 18.5045\, kA$$

Negative sequence current at relay side:

$$\tilde{I}_{2,relay} = 18.5045 \frac{5}{12{,}000}\, kA = 7.71\, A$$

Problem P8.x: Consider a 625 MVA, 60 Hz, 18 kV synchronous generator with the parameters indicated in Figure P5. The generator operates at nominal voltage at the terminals, delivering rated MVA power at power factor 0.90 current lagging. At time t=0, a three phase fault occurs at point A. The fault is cleared in 0.20 seconds by opening the breakers of the faulted line. The other line remains energized.

Assume there is an out of step relay at the generator terminals. The relay PT and CT have the following ratios respectively: 18,000V:115V and 20,000A:5A. Compute and graph the impedance “seen” by the relay. Perform the computations with a time step of 0.05 seconds, starting at time t=0 (fault initiation) and continue to the final time of t=1.0 seconds.

The per-unit inertia constant of the generator is 3.2 seconds. The impedance of each transmission line is z=j13.225 ohms.

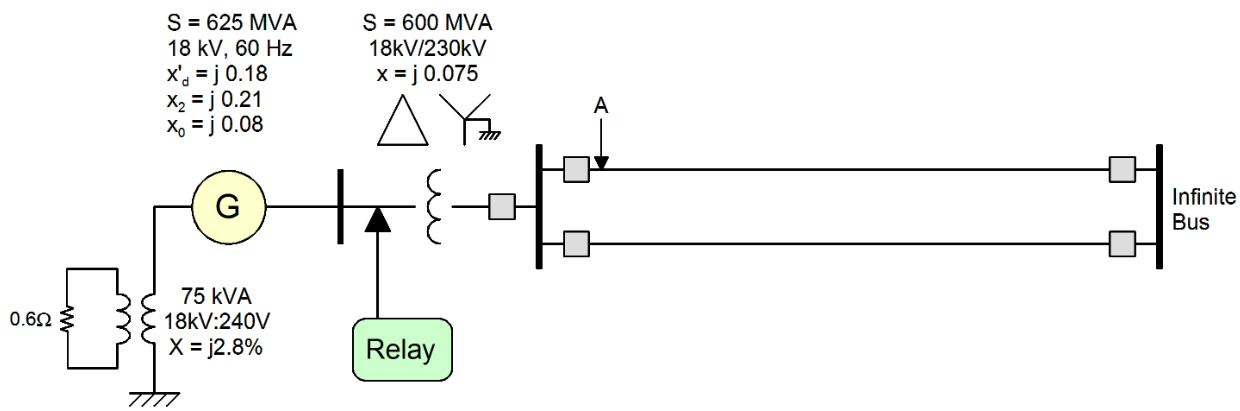


Figure P5

Solution: System information is as follows,
Generator: 625MVA, 60Hz, 18kV, and $x'_d = j0.18$pu.
Transformer: 18kV/230kV, $z_1 = z_2 = z_0 = j0.075$pu.
Each line: 230kV, $z = j13.225$ ohms @ 600MVA and 230kV.

Transform to 600MVA and 230kV base.

$$Z_B = \frac{V_B^2}{S_B} = \frac{230^2}{600} = 88.1667\Omega$$

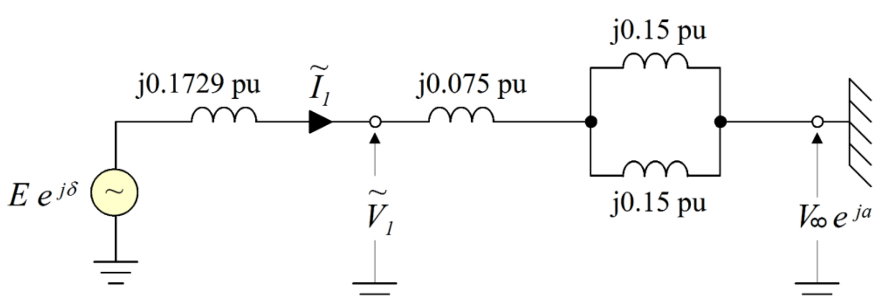
Generator:

$$x'_d = j0.18 \times \frac{600}{625} = j0.1728\,pu$$

Each line:

$$z_l = \frac{j13.225}{88.1667} = j0.15\,pu$$

The system of phase 1 (per fault) can be represented as the following



The voltage and current at the generator output terminal are

$\tilde{V}_1 = 1.0\,pu$, $\tilde{I}_1 = 1.0e^{-j\arccos 0.9} = 1.0e^{-j25.8419^\circ}\,pu$

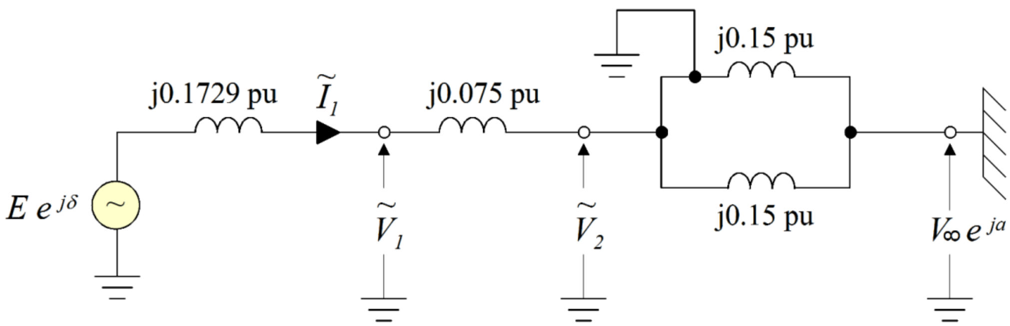
The direction of the current is out of the generator.

$$\tilde{E}_g = \tilde{V}_1 + \tilde{I}_1 x'_d = 1.0 + 1.0e^{-j25.8419^\circ} \times j0.1728 = 1.0753 + j0.1555 = 1.0865e^{j8.2294^\circ}$$

$\tilde{V}_\infty = \tilde{V}_1 - \tilde{I}_1(x_{xfm} + z_l / 2) = 1.0 + 1.0e^{-j25.8419^\circ} \times (j0.075 + 0.15/2) = 0.9346 - j0.135 = 0.9443e^{-j8.2192^\circ}$

Assume $\tilde{V}_\infty = 0.9443e^{j0^\circ}$, $\tilde{E}_g = 1.0865e^{j16.4486^\circ} = 1.0865e^{j0.2871}$

During fault (the fault is located at the beginning of the line):



$\tilde{V}_2 = 0$ and $P_e = \mathrm{Re}(\tilde{V}_2 \tilde{I}_1^*) = 0$.

Swing equation:

$$\frac{2H}{\omega_s}\frac{d\omega(t)}{dt} = P_m - P_e$$

Substitute values into the swing equation

$$\frac{2 \times 3.2}{2\pi \times 60}\frac{d\delta^2(t)}{dt^2} = 0.9$$

$$0.017\frac{d\delta^2(t)}{dt^2} = 0.9$$

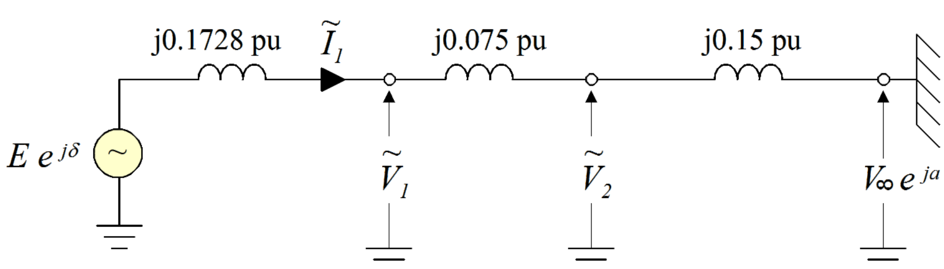
Initial condition: $\delta(0) = 0.2871$, $\delta'(0) = 0$.

Therefore, $\delta(t) = \dfrac{\omega_s P_{mu}}{4H} t^2 + \delta(0) = 26.5072t^2 + 0.2871$

$\delta(0.2) = 26.5072 \times 0.2^2 + 0.2871 = 1.3474 rad$

$\dfrac{d\delta(0.2)}{dt} = 2 \times 26.5072 \times 0.2 = 10.6029 rad/\sec$

Post fault:



$$\frac{2H}{\omega_s}\frac{d\delta^2(t)}{dt^2} = P_m - P_{post}(\delta(t)) = P_m - \frac{|E||V_\infty|}{z_{total}} sin\delta(t)$$

Substitute values into the swing equation

$$\frac{2\times 3.2}{2\pi\times 60}\frac{d\delta^2(t)}{dt^2} = 0.9 - \frac{1.0865\times 0.9443}{0.1728+0.075+0.15}sin\delta(t)$$

$$0.017\frac{d\delta^2(t)}{dt^2} = 0.9 - 2.5791 sin\delta(t)$$

The differential equation system to be solved:

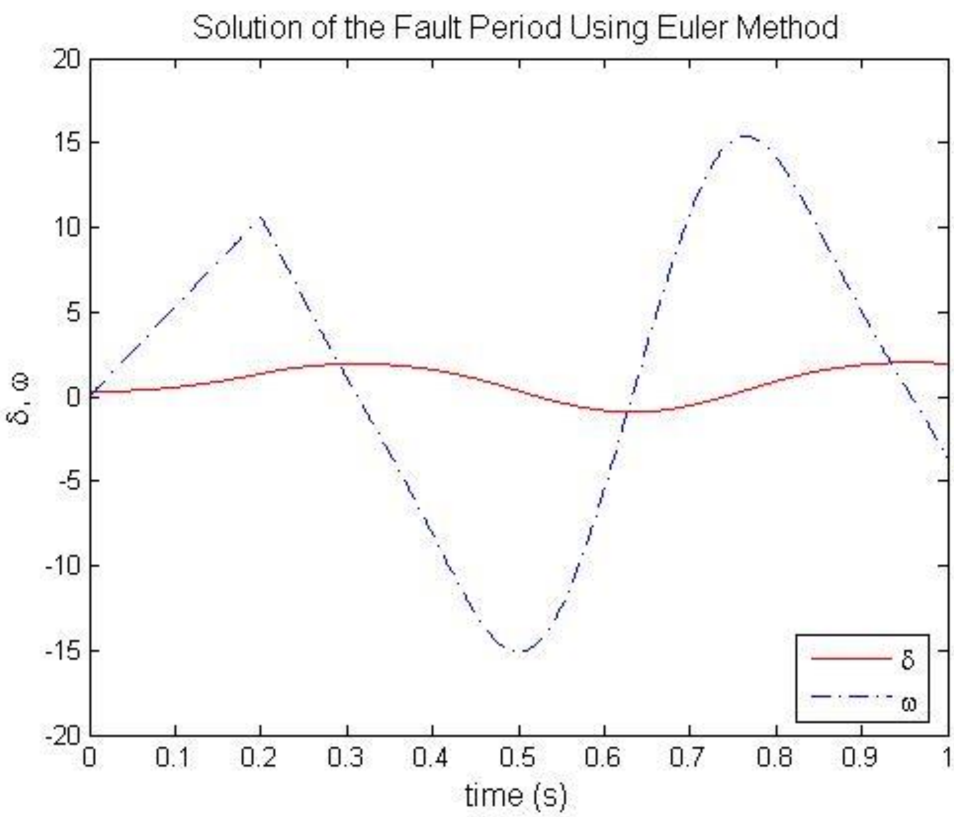
$$\begin{cases} \dfrac{d\delta(t)}{dt} = \omega(t) \\ \dfrac{d\omega(t)}{dt} = a - bsin\delta(t) \end{cases}$$

Euler Method:

The algorithm is

$$\begin{bmatrix}\delta(t)\\ \omega(t)\end{bmatrix} = \begin{bmatrix}\delta(t-h)\\ \omega(t-h)\end{bmatrix} + h\begin{bmatrix}\omega(t-h)\\ a - bsin\delta(t-h)\end{bmatrix}$$

Post fault case (the computation from during fault case using Euler method):
Initial condition: $\delta(0.2) = 1.3474 rad$, $\omega(0.25) = 10.6029 rad/sec$.
$a = 52.9412$, $b = 151.7118$.
The solution of Euler method is shown as follows,



The impedance "seen" by the relay is summarized as follows,

Pre-fault:, $Z = \frac{\tilde{V}_1}{\tilde{I}_1} = \frac{1.0}{1.0e^{-j25.8419^\circ}} = 1.0e^{j25.8419^\circ}\,pu = 0.9 + j0.4359\,pu$

During fault (t= 0-0.2 sec): $Z = z_{xfm} = j0.075\,pu$

Post-fault ($t \geq 0.2$ sec):

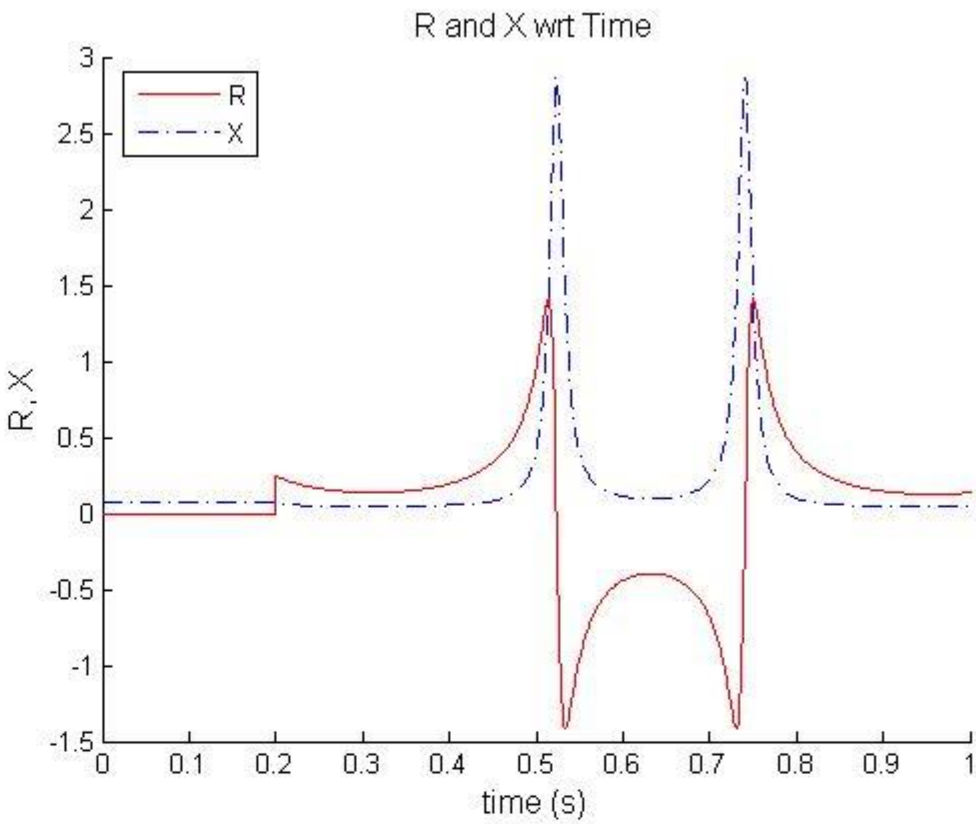
$$\tilde{I}_1 = \frac{1.0865e^{j\delta} - 0.9443}{j0.3978}$$

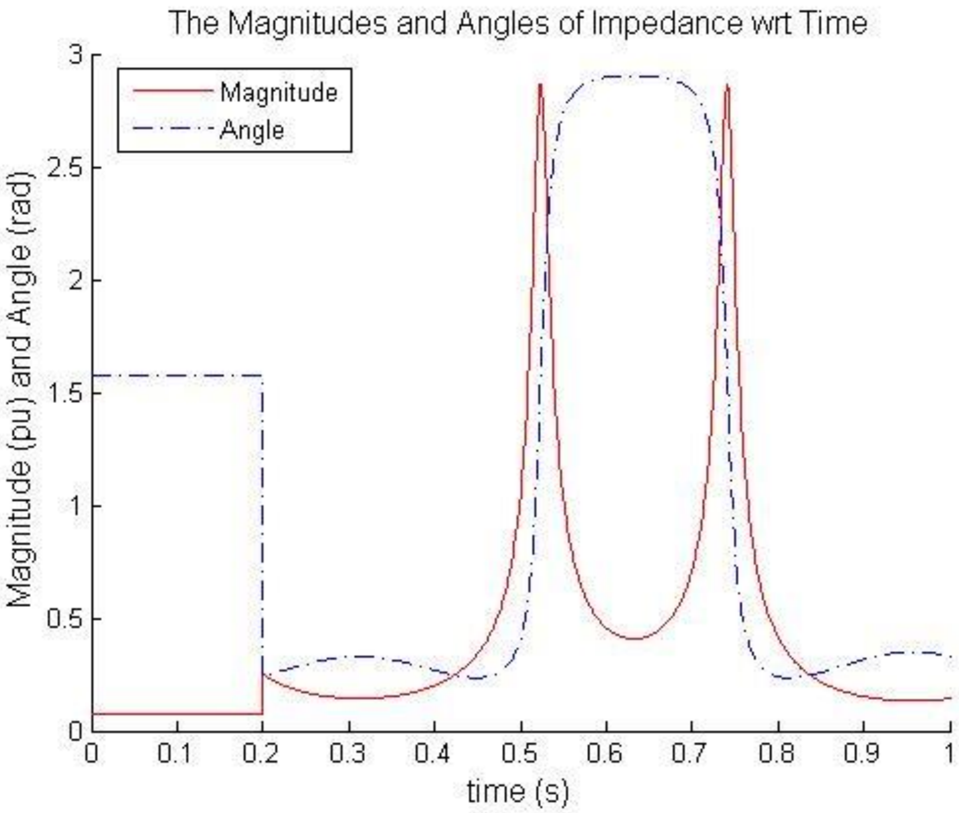
$$\tilde{V}_1 = 1.0865e^{j\delta} - j0.1728\tilde{I}_1$$

$$Z = -j0.1728 + \frac{j0.3978e^{j\delta}}{e^{j\delta} - 0.86912}$$

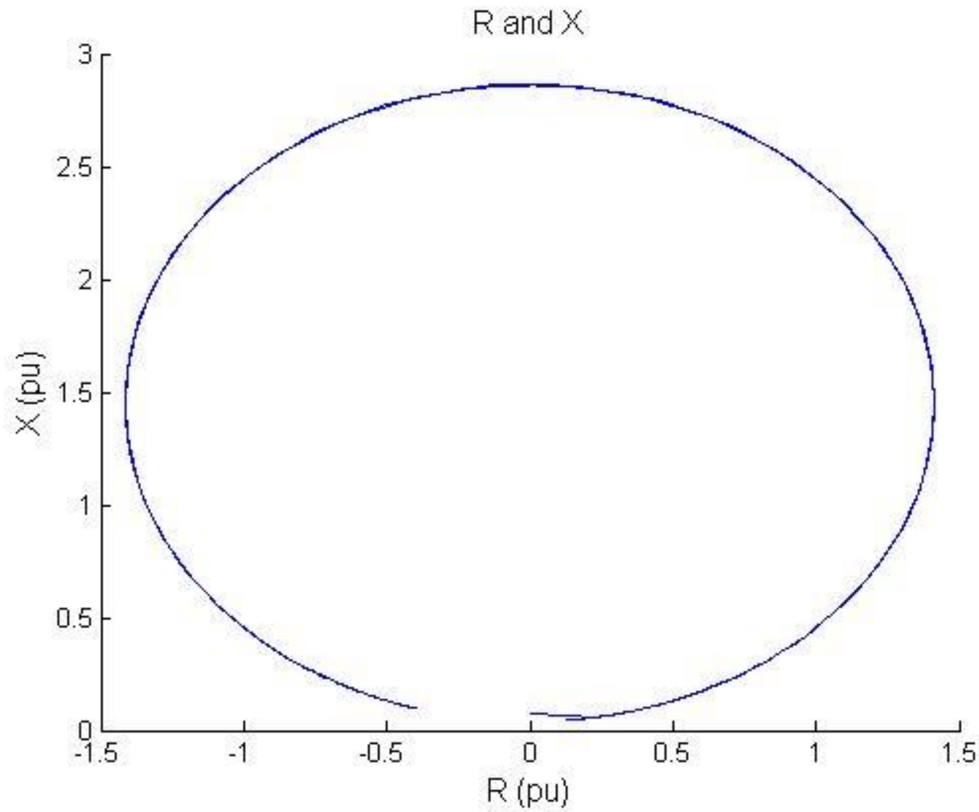
The graphs below must be recomputed

The plot of impedance is





R and X of Z is shown as follows,


R and X
3
2.5
2
1.5
1
0.5
0
X (pu)
-1.5
-1
-0.5
0
0.5
1
1.5
R (pu)

DRAFT and INCOMPLETE

Table of Contents

From

A. P. Sakis Meliopoulos and George J. Cokkinides

Power System Relaying, Theory and Applications

Chapter 09
Transmission Line Protection

9.1 Introduction

This chapter presents protection schemes for transmission lines. Most transmission lines are two terminal devices that are connected to a network. There are however many multi-terminal lines or lines with taps (loads, substations, etc.) along the length of the line. Transmission lines can be overhead or underground cable circuits (UG). In general the power flow in transmission circuits is bi-directional, i.e. throughout the duration of a day the direction of power flow may change.

The parameters that affect protection options, criteria and requirements (i.e. selection of protection functions and settings) are: (a) length of the circuit, (b) the strength of the network connected to each of the terminals of the transmission circuit expressed in terms of the available fault current, and (c) whether the line is near a generating unit or in general the importance of the line. The last case normally requires consideration of system stability as well as the stability of nearby generating units in case of a fault in the line. In this case the clearing of a line fault should be coordinated with the protection of the generator(s) and should be fast enough to avoid instability of the generating unit(s). Generation stability has been addressed in Chapter 8.

We will examine the protection schemes for overhead transmission circuits as well as underground (UG) transmission circuits (cables). Depending on the importance of the transmission circuit the protection scheme may be simple or sophisticated and complicated requiring communications between the two ends of the line, which are typically far apart. In general we classify the protection schemes of transmission circuits as non-pilot and pilot schemes. Non-pilot schemes employ relays and the two ends of the circuit without communication between them. Pilot schemes require communications between the relays at the two ends of the circuit. We will examine non-pilot and pilot technologies that enable accurate, selective and fast protection schemes. Recent technologies and especially communication technologies (fiber) and GPS synchronization have opened the way for more advanced protection methods. We will examine these technologies.

A substantial percentage of transmission lines exist that may have taps or they may have three or more terminals. The special requirements for these lines will be also examined.

The protection of transmission circuits is typically different than distribution circuits. In general, distribution circuits (operating typically at voltage levels up to 35 kV) are radial, i.e. the power flows in one direction only and they have many components (loads, capacitors, etc.) connected along the length of the distribution circuit. These characteristics result in simpler protection schemes. The basic approach in the design of distribution circuit protection assumes that the circuit operates radially. It is important to note that today distributed energy resources (DERs) are commonplace in distribution circuits (customer owned resources, PV systems, wind turbine systems, standby generation, storage, microgrids, etc.) and they are changing the paradigm of radial distribution circuits. Traditional distribution circuit protection schemes can be compromised as the penetration of DERs increases. Specifically, power flow in one direction only cannot be guaranteed anymore. It is clear that new approaches are necessary. These issues will be discussed in Chapter 10.

9.2 Transmission Line Protection Requirements

The protection requirements for a transmission circuit are: (a) in case of any fault, low or high impedance fault, anywhere along the entire length of the line, the line should be tripped, (b) for any operating condition that may bring the line outside the design limits, the line should be tripped - this may include overloading, overheating line conductors that may cause excessive sag in overhead circuits or insulation damage to underground circuits (cables), etc., (c) the line should be disconnected as simultaneously as possible from both ends of the line (all from all terminals in case of multi-terminal lines), and (d) due to other considerations (such as effects on nearby equipment, i.e. generators, transformers, etc. as well as stability considerations) we may have additional requirements such as the fault should be cleared in less than a certain time to avoid generation or system instabilities, excessive forces in the coils of nearby transformers or generators, unsafe voltages in grounded equipment and others. In this chapter, we will assume that these additional requirements are given.

For a secure protection system, the line should not be tripped for any faults or abnormal conditions generated by events outside the transmission line, unless these conditions persist due to the failure of the protection system for the other components. For this reason, most protection systems for transmission lines are also designed to provide backup protection functions for faults that may occur in neighboring power components in case the protection system for these components fails.

9.3 Basic Line Protection Schemes

Protection schemes for transmission circuits may be simple or quite complicated depending on the importance of the circuit. In general, the protection approach for transmission circuits may be based on relays that do not communicate with the other relays at the other terminal of the circuit. We refer to these schemes as non-pilot relaying schemes. Non-pilot protection schemes include directional overcurrent (instantaneous or inverse time), stepped distance protection, and various options of these functions. Communications among the relays at the two ends (or multiple ends in case or multiple terminal lines) can be added to increase the reliability (dependability and security) of the protection schemes as well as the capability to trip the line from both ends simultaneously. We refer to these schemes as pilot relaying. Pilot relaying may include line differential and various legacy relaying schemes that include phase comparison, direct transfer trips, blocking schemes, and others. Pilot relaying uses various technologies for communicating between the terminals of the transmission circuit. Figure 9.1 illustrates typical protection schemes for a two terminal overhead transmission circuit that includes both non-pilot and pilot functions. A similar protection system applies to UG cable transmission circuits. In general, a typical protection scheme may include combination of non-pilot and pilot protection schemes.

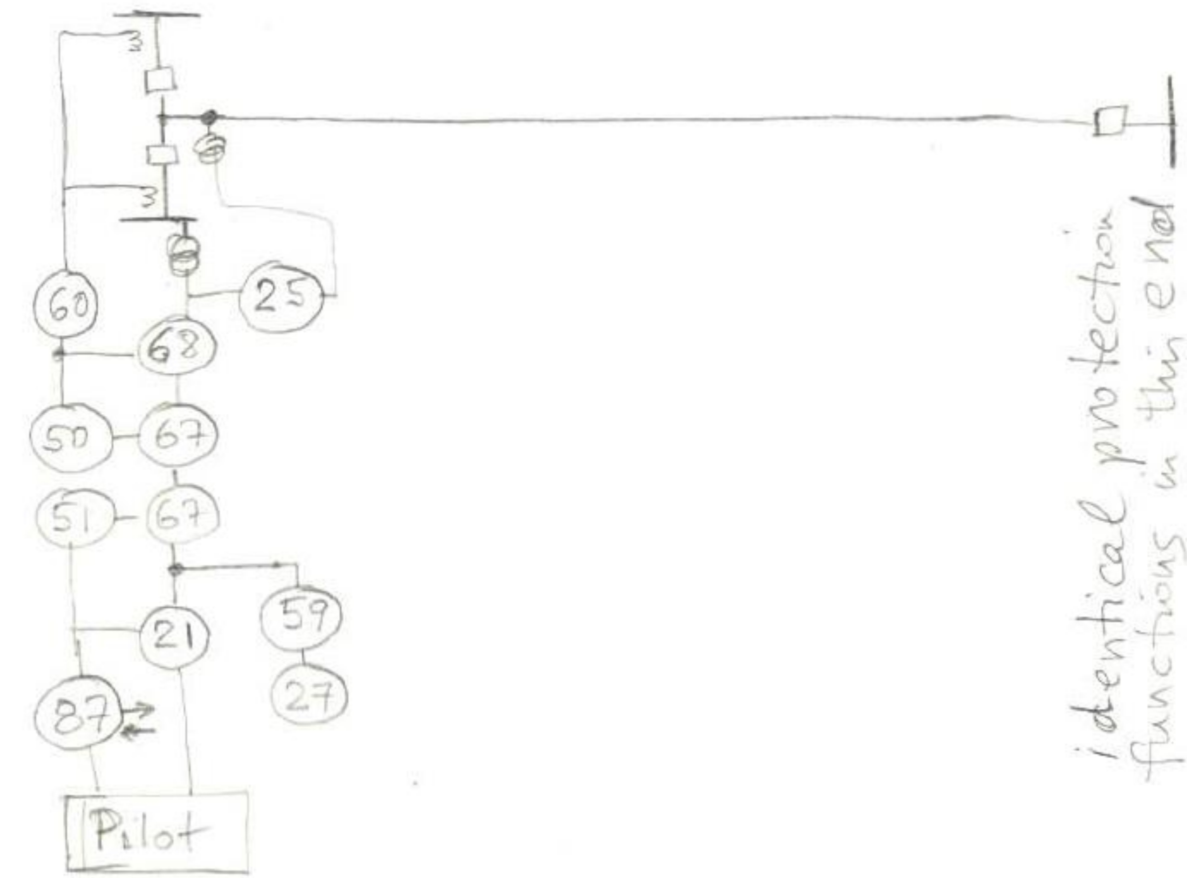


Figure 9.1: Typical Protection Scheme for Overhead Transmission Circuits

Note in above figures that the transmission circuit is protected for the following conditions:

1. Directional instantaneous overcurrent (50), (67)
2. Directional inverse time delay overcurrent (51), (67)
3. Distance (21)
4. Differential (87)
5. Synchronizing relay (25)
6. Undervoltage relay (27)
7. Overvoltage relay (59)
8. Pilot

Provide a summary of these functions.

9.4 Non Pilot Transmission Circuit Protection

Non pilot protection schemes are based on relays that use only local information, i.e. the information that is available at the terminals of the circuit to where they are installed. Non pilot schemes for transmission circuits may include:

1. Directional instantaneous overcurrent
2. Directional inverse time delay overcurrent
3. Distance
4. Synchronizing

The application of these functions for transmission circuit protection will be examined next.

9.4.1 Transmission Circuit Directional Overcurrent Protection

Overcurrent protection includes instantaneous and inverse time delay with directional elements. As always the objective of these functions is to clear a fault on the line and not to trip on any faults outside the line. However, depending on the parameters of the power line, this scheme may not be as selective as desired. For example if the power line is short and the fault current level at the two terminals of the power line is approximately equal, then the directional overcurrent relay may not be able to differentiate between a fault on the line and a fault beyond the line. In addition, as system configurations and operating conditions change (number of generating units on line, etc.) the fault current levels may change making it difficult to select the optimal settings. In any case, we will consider the typical approach for selecting the settings of these functions and then we will discuss the advantages and limitations. The settings for these functions depend on many parameters. In general the following basic rules will apply.

Directional inverse time delay overcurrent: The pickup current of the directional inverse time overcurrent should be higher or about twice the maximum load current of the circuit. This will provide some safety margin against tripping on small overcurrent and other events that lead to temporary overcurrents. The time delay should be so selected as to provide reasonable clearing times for faults along the entire length of the circuit.

Directional instantaneous overcurrent: The settings for the directional instantaneous overcurrent should be set in such a way that it operates only for faults on the line. To determine the settings it is necessary to perform a comprehensive fault analysis study, under various conditions of the electric power grid. The study should determine the maximum fault current that the relay will "see" for faults on the line under protection and the maximum fault current that the relay will "see" for faults that are outside the protection zone of the line. The setting of the relay should be above the last maximum fault current and below the maximum fault current for faults in the line. The study should consider all possible configurations and operating conditions.

Polarizing Quantity for Directional Overcurrent: The directional function requires a polarizing quantity which is typically a voltage that provides a reliable phase reference during the fault conditions. The directional element computes the phase angle of the fault current relative to the polarizing quantity to determine the direction of the current flow. It should be apparent if the polarizing quantity (voltage) collapses or becomes very small during the fault, the computation of the relative phase will be laced with large errors and may lead to mis-operation. For this reason, it is important to select a polarizing quantity that will not collapse or become very small during faults. It is unwise to use the voltage of the faulted phase as a polarizing quantity. In general the polarizing quantity is selected to be a quantity (voltage or current) that is not the same as the faulted phase. For a phase fault, one popular selection is the line to line voltage of the other two phases, for example for comparing the phase A fault current, the voltage B-C is used as the polarizing quantity. Figure 9.3 illustrates the wiring for the above selection. For ground faults a popular selection of polarizing quantity is the negative sequence voltage or the zero sequence voltage; the zero sequence voltage is always shifted by about 90 degrees from the faulted phase to ground current. Figure 9.4 illustrates the wiring for this selection.

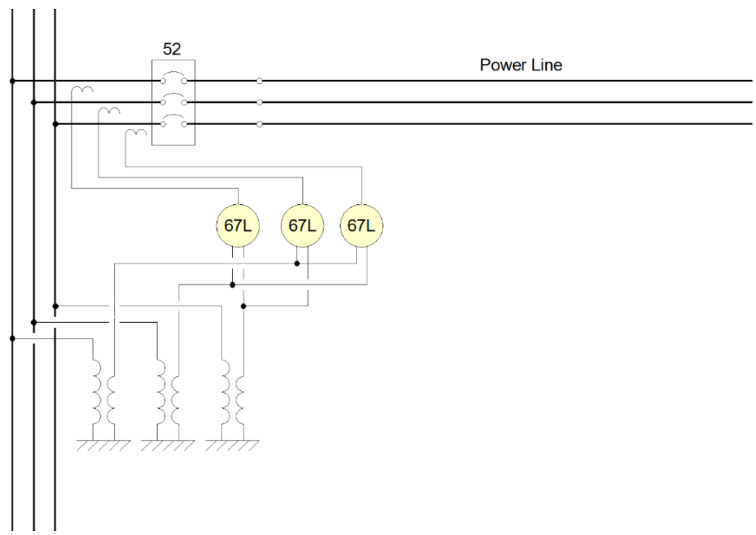


Figure 9.3: Usual Polarizing Voltage for Line Faults Directional

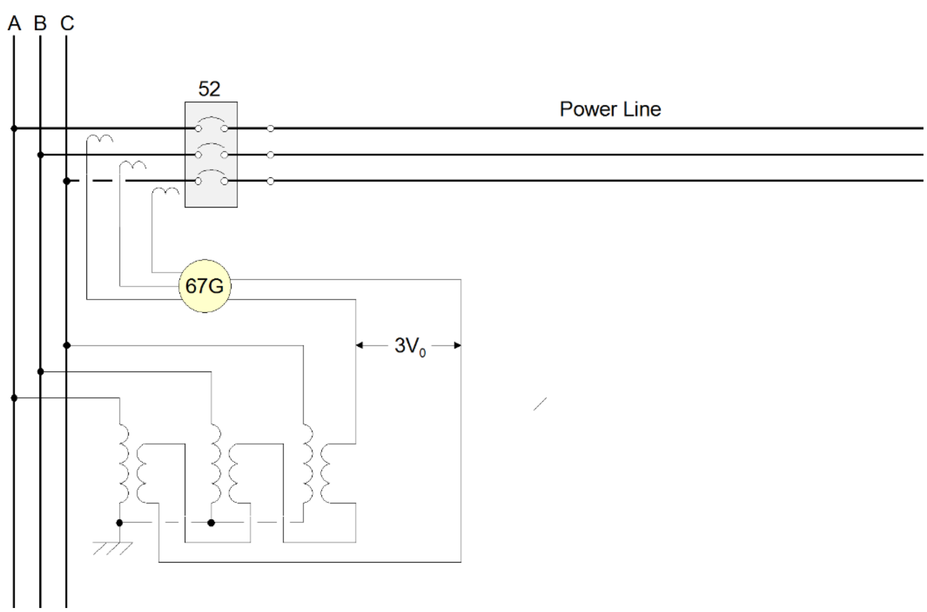


Figure 9.4: A Popular Polarizing Voltage for Ground Fault Directional

It is important to recognize that for power line protection, it is typical practice to use a directional element to supervise the overcurrent protection (instantaneous and time) so that only faults in the direction of the power line will be tripped. It is also important to recognize that overcurrent protection for power lines is not as secure as other protection approaches for power circuits that will be discussed later. For this reason, in general we approach overcurrent protection for power circuits as a backup protection in case the other schemes fail.

The overall procedure of selecting the settings of overcurrent protection for power lines will be illustrated with an example.

Example E9.1: Consider the power circuit of Figure E9.1. The figure illustrates two transmission lines, one terminal is connected to a double breaker arrangement and the other terminal connected to a straight breaker arrangement. The lines are 25.6 (line on left hand side) and 13.4 miles long respectively. The remaining system is represented with its equivalent expressed in terms of the two sources S1 and S2 and the equivalent line shown with the dashed line.

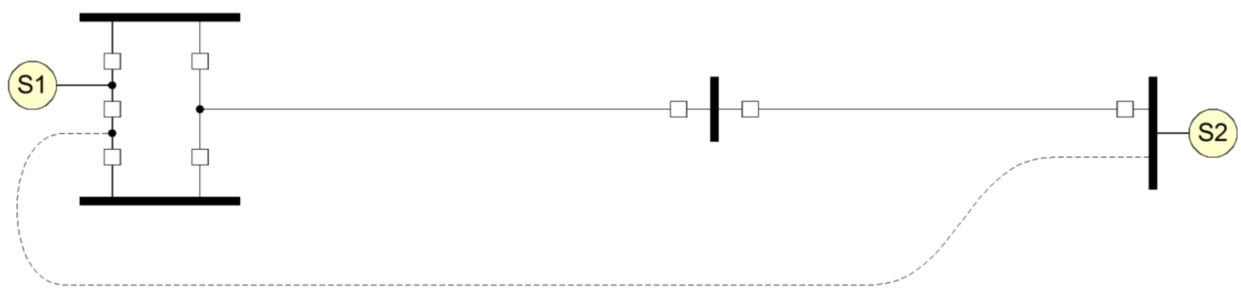


Figure E9.1: Example Two Power Line Transmission System
Add another source

The parameters of the sources and equivalent line are:

Source S1: $115kV$, $z_1 = j9.15ohms$, $z_2 = j8.75ohms$, $z_0 = j6.15ohms$
Source S2: $115kV$, $z_1 = j9.15ohms$, $z_2 = j8.75ohms$, $z_0 = j6.15ohms$
Source S3: $115kV$, $z_1 = j9.15ohms$, $z_2 = j8.75ohms$, $z_0 = j6.15ohms$
Line under Protection: $115kV$, $23.25miles$, $z_1 = z_2 = j16.35ohms$, $z_0 = j46.15ohms$
Equivalent Line 1:
Equivalent Line 2

For this line, select the settings of the instantaneous overcurrent (directional) and inverse time delay overcurrent (directional) and graph the trip times for (a) line-to-line faults along the length of the two lines, and (b) line to ground faults along the length of the two lines.

Solution: First a comprehensive fault analysis is performed.

to be continued...

9.4.2 Transmission Line Distance Protection

Distance relaying for transmission lines provide a more secure protection scheme as compared with directional overcurrent based protection schemes discussed in the previous section. Distance relays (modified mho relays) have been discussed in Chapter 5. They simply trip when the impedance they "see" falls within the characteristic of the relay. When applied to transmission lines, depending on the fault type, the equivalent per unit length impedance may vary. For example for a three phase fault the per unit length impedance of the line equals the positive sequence impedance of the line. For a single line to ground fault the equivalent per unit impedance is approximately equal to the average of the positive, negative and zero sequence impedance of the line.

For the purpose of standardizing the distance relay design for three phase circuits, the relays should be so designed as to "see" an equivalent impedance that is approximately equal to the positive sequence impedance of the circuit per unit length times the distance to the fault. This is easily achieved with numerical relays by providing appropriate algorithms. For electromechanical relays, one can have multiple relays that will determine the distance to the fault for various fault types and then have logic to select the correct answer. We shall discuss the design of a distance relay that uses two types for distance relays to cover the majority of fault types in a circuit.

9.4.3 Distance Relay Elements for Phase Faults

Distance relays for phase faults are designed so that they "see" an apparent impedance equal to the distance to the fault times the per unit length positive sequence impedance of the circuit, i.e. $Z = \ell z_1$, for any phase fault in the line, i.e. three phase faults, and line to line faults. This objective is achieved with the relay design of Figure 9.5. Note that there are three elements, i.e. three relays and each relay has three coils, two current coils and one voltage coil. The coils of the first relay are supplied with the following inputs: voltage $\tilde{V} = k_1(\tilde{V}_a - \tilde{V}_b)$, current $\tilde{I}_{c,coil,1} = k_2 \tilde{I}_a$, and current $\tilde{I}_{c,coil,2} = -k_2 \tilde{I}_b$, respectively. The coils of the second relay are supplied with the following inputs: voltage $\tilde{V} = k_1(\tilde{V}_b - \tilde{V}_c)$, current $\tilde{I}_{c,coil,1} = k_2 \tilde{I}_b$, and current $\tilde{I}_{c,coil,2} = -k_2 \tilde{I}_c$, respectively. And the coils of the third relay are supplied with the following inputs: voltage $\tilde{V} = k_1(\tilde{V}_c - \tilde{V}_a)$, current $\tilde{I}_{c,coil,1} = k_2 \tilde{I}_c$, and current $\tilde{I}_{c,coil,2} = -k_2 \tilde{I}_a$, respectively.

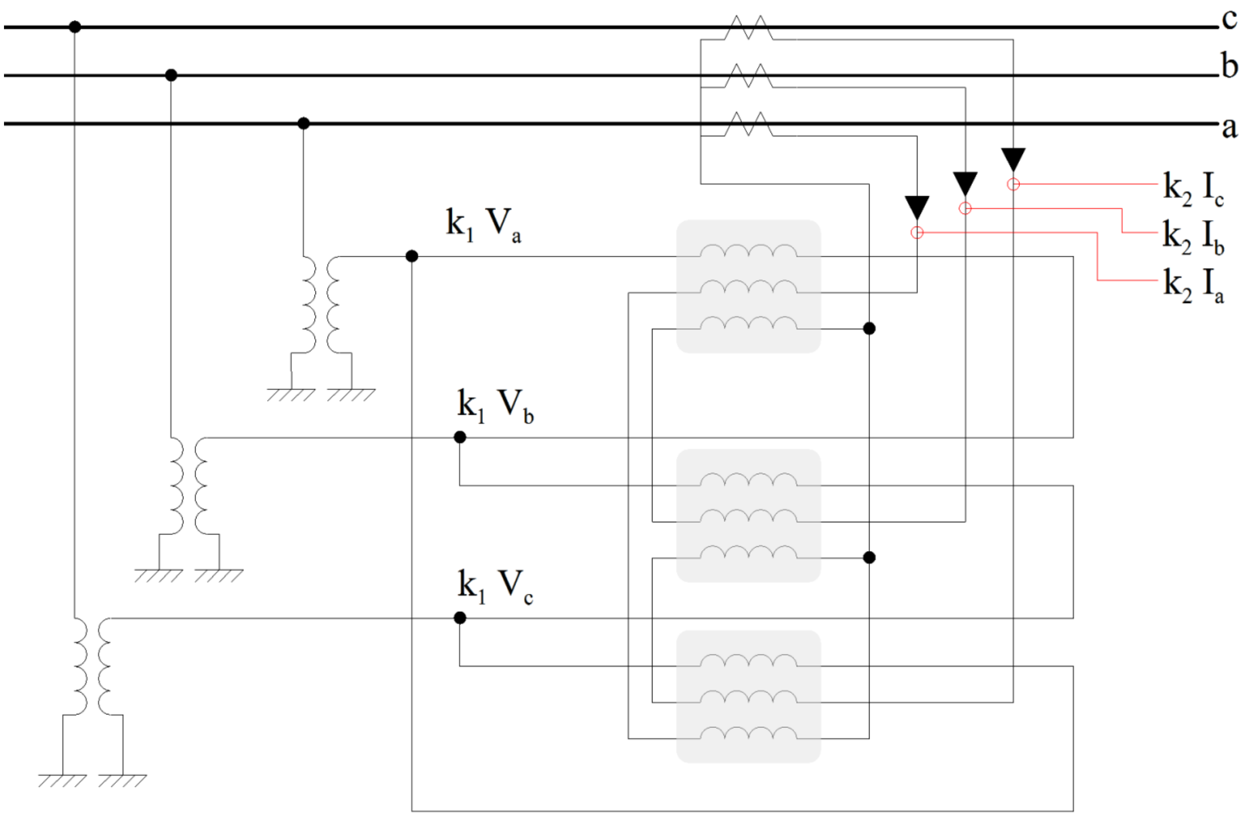


Figure 9.5: Three Phase Distance Relay for Phase to Phase Faults
(Indicate Location of Breaker to the right of CTs)

This means that the three relays will see an impedance which is proportional to $\frac{k_1(\tilde{V}_a - \tilde{V}_b)}{k_2(\tilde{I}_a - \tilde{I}_b)}$, $\frac{k_1(\tilde{V}_b - \tilde{V}_c)}{k_2(\tilde{I}_b - \tilde{I}_c)}$, and $\frac{k_1(\tilde{V}_c - \tilde{V}_a)}{k_2(\tilde{I}_c - \tilde{I}_a)}$, respectively. Examination of the value of these impedances during three phase faults and line to line faults is provided below. As an example, consider relay 2. Figure 9.6 illustrates the modified impedance implementation of this relay and the inputs into the relay.

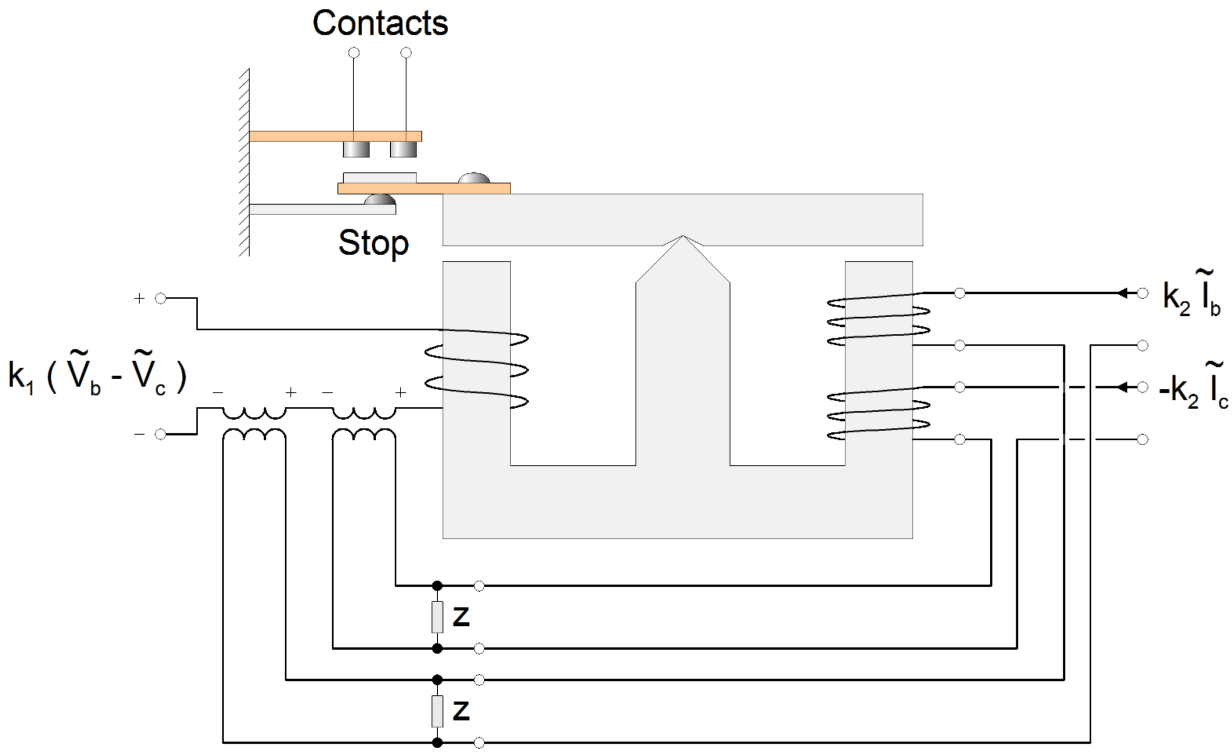


Figure 9.6: Balancing Beam Implementation of Relay 2 of Figure 9.5

Three phase faults: For a three phase fault, and making the simplifying assumption that the line is symmetric, the three relays will see the following impedances:

Relay 1: $z_{relay1} = \dfrac{k_1(\tilde{V}_a - \tilde{V}_b)}{k_2(\tilde{I}_a - \tilde{I}_b)} = \dfrac{k_1}{k_2}\ell z_1$

Relay 2: $z_{relay2} = \dfrac{k_1(\tilde{V}_b - \tilde{V}_c)}{k_2(\tilde{I}_b - \tilde{I}_c)} = \dfrac{k_1}{k_2}\ell z_1$

Relay 3: $z_{relay3} = \dfrac{k_1(\tilde{V}_c - \tilde{V}_a)}{k_2(\tilde{I}_c - \tilde{I}_a)} = \dfrac{k_1}{k_2}\ell z_1$

Line to Line faults: We should consider three specific cases: Phase A to Phase B fault, Phase B to Phase C fault and Phase C to Phase A fault. To avoid lengthy descriptions, we shall discuss the case of a Phase B to Phase C fault.

Phase B to Phase C Fault: For this fault, the three relays will "see" the following impedances:

$$\text{Relay 1: } z_{relay1} = \frac{k_1(\tilde{V}_a - \tilde{V}_b)}{k_2(\tilde{I}_a - \tilde{I}_b)}$$

$$\text{Relay 2: } z_{relay2} = \frac{k_1(\tilde{V}_b - \tilde{V}_c)}{k_2(\tilde{I}_b - \tilde{I}_c)}$$

$$\text{Relay 3: } z_{relay3} = \frac{k_1(\tilde{V}_c - \tilde{V}_a)}{k_2(\tilde{I}_c - \tilde{I}_a)}$$

Upon substitution of the phase voltages and currents with the sequence voltages and currents (note that in this case, assuming that the line is symmetric, the zero sequence voltage and current will be zero), and simplifying the expressions:

$$\text{Relay 1: } z_{relay1} = \frac{k_1(\tilde{V}_a - \tilde{V}_b)}{k_2(\tilde{I}_a - \tilde{I}_b)} = \frac{k_1}{k_2}\frac{(1-a^2)\tilde{V}_1 + (1-a)\tilde{V}_2}{(1-a^2)\tilde{I}_1 + (1-a)\tilde{I}_2} = \frac{k_1}{k_2}\frac{\tilde{V}_1 - a\tilde{V}_2}{\tilde{I}_1 - a\tilde{I}_2}$$

$$\text{Relay 2: } z_{relay2} = \frac{k_1(\tilde{V}_b - \tilde{V}_c)}{k_2(\tilde{I}_b - \tilde{I}_c)} = \frac{k_1}{k_2}\frac{(a^2-a)\tilde{V}_1 + (a-a^2)\tilde{V}_2}{(a^2-a)\tilde{I}_1 + (a-a^2)\tilde{I}_2} = \frac{k_1}{k_2}\frac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2}$$

$$\text{Relay 3: } z_{relay3} = \frac{k_1(\tilde{V}_c - \tilde{V}_a)}{k_2(\tilde{I}_c - \tilde{I}_a)} = \frac{k_1}{k_2}\frac{(a-1)\tilde{V}_1 + (a^2-1)\tilde{V}_2}{(a-1)\tilde{I}_1 + (a^2-1)\tilde{I}_2} = \frac{k_1}{k_2}\frac{\tilde{V}_1 - a^2\tilde{V}_2}{\tilde{I}_1 - a^2\tilde{I}_2}$$

Considering the sequence network connections of Figure 9.7, we obtain:

Consider the sequence network connections for a Phase B to Phase C fault of Figure 9.7.

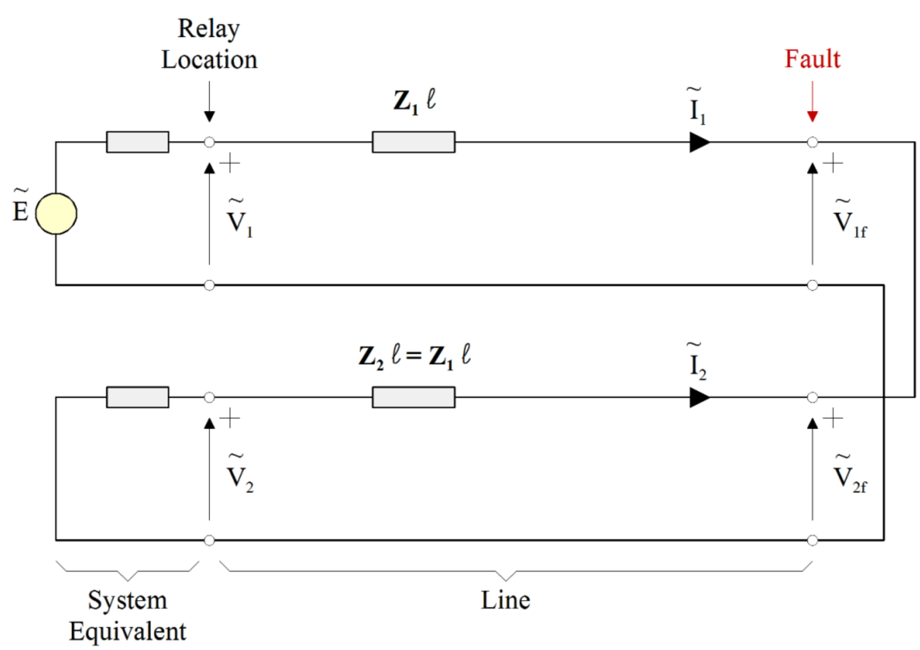


Figure 9.7: Equivalent Sequence Network for Phase to Phase Faults

Note that the following are valid:

$$\tilde{V}_1 = z_1 \ell \tilde{I}_1 + \tilde{V}_{1f}$$
$$\tilde{V}_2 = z_1 \ell \tilde{I}_2 + \tilde{V}_{2f}$$
$$\tilde{V}_{1f} = \tilde{V}_{2f}$$

Upon substitution into the equations for the impedance "seen" by the relays, one obtains:

Relay 1: $$z_{relay1} = \frac{k_1(\tilde{V}_a - \tilde{V}_b)}{k_2(\tilde{I}_a - \tilde{I}_b)} = \frac{k_1}{k_2} \frac{(1-a^2)\tilde{V}_1 + (1-a)\tilde{V}_2}{(1-a^2)\tilde{I}_1 + (1-a)\tilde{I}_2} = \frac{k_1}{k_2} \frac{\tilde{V}_1 - a\tilde{V}_2}{\tilde{I}_1 - a\tilde{I}_2} = \frac{k_1}{k_2} z_1 \ell + \frac{k_1}{k_2} \frac{\tilde{V}_{1f} - a\tilde{V}_{2f}}{\tilde{I}_1 - a\tilde{I}_2}$$

Relay 2: $$z_{relay2} = \frac{k_1(\tilde{V}_b - \tilde{V}_c)}{k_2(\tilde{I}_b - \tilde{I}_c)} = \frac{k_1}{k_2} \frac{(a^2-a)\tilde{V}_1 + (a-a^2)\tilde{V}_2}{(a^2-a)\tilde{I}_1 + (a-a^2)\tilde{I}_2} = \frac{k_1}{k_2} \frac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2} = \frac{k_1}{k_2} z_1 \ell$$

Relay 3: $$z_{relay3} = \frac{k_1(\tilde{V}_c - \tilde{V}_a)}{k_2(\tilde{I}_c - \tilde{I}_a)} = \frac{k_1}{k_2} \frac{(a-1)\tilde{V}_1 + (a^2-1)\tilde{V}_2}{(a-1)\tilde{I}_1 + (a^2-1)\tilde{I}_2} = \frac{k_1}{k_2} \frac{\tilde{V}_1 - a^2\tilde{V}_2}{\tilde{I}_1 - a^2\tilde{I}_2} = \frac{k_1}{k_2} z_1 \ell + \frac{k_1}{k_2} \frac{\tilde{V}_{1f} - a^2\tilde{V}_{2f}}{\tilde{I}_1 - a^2\tilde{I}_2}$$

It is noted that the smallest impedance will be seen by the second relay (the other terms that appear in relays 1 and 3 are additive).Thus the second relay will provide the smallest value and it is equal to the positive sequence impedance of the line from the relay to the fault location.

Similar analysis can be performed for other line to line faults. For example, for the case of a Phase A and Phase B fault, the first relay will "see" the smallest impedance value and it will be equal to the positive sequence impedance of the line from the relay to the fault location.

9.4.4 Distance Relay Elements for Ground Faults

Distance relays for ground faults are designed so that they "see" an apparent impedance equal to the distance to the fault times the per unit length positive sequence impedance of the circuit, i.e. $Z = \ell z_1$, for any ground fault in the line, i.e. single phase to ground faults, and line to line to ground faults. This objective is achieved with the relay design of Figure 9.8. Note that there are three elements, i.e. three relays and each relay has three coils, two current coils and one voltage coil. The coils of the first relay are supplied with the following inputs: voltage $\tilde{V} = k_1 \tilde{V}_a$, current $\tilde{I}_{c,coil,1} = k_2 \tilde{I}_a$, and current $\tilde{I}_{c,coil,2} = -k_2(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)$, respectively. The coils of the second relay are supplied with the following inputs: voltage $\tilde{V} = k_1 \tilde{V}_b$, current $\tilde{I}_{c,coil,1} = k_2 \tilde{I}_b$, and current $\tilde{I}_{c,coil,2} = -k_2(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)$, respectively. And the coils of the third relay are supplied with the following inputs: voltage $\tilde{V} = k_1 \tilde{V}_c$, current $\tilde{I}_{c,coil,1} = k_2 \tilde{I}_c$, and current $\tilde{I}_{c,coil,2} = -k_2(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)$, respectively.

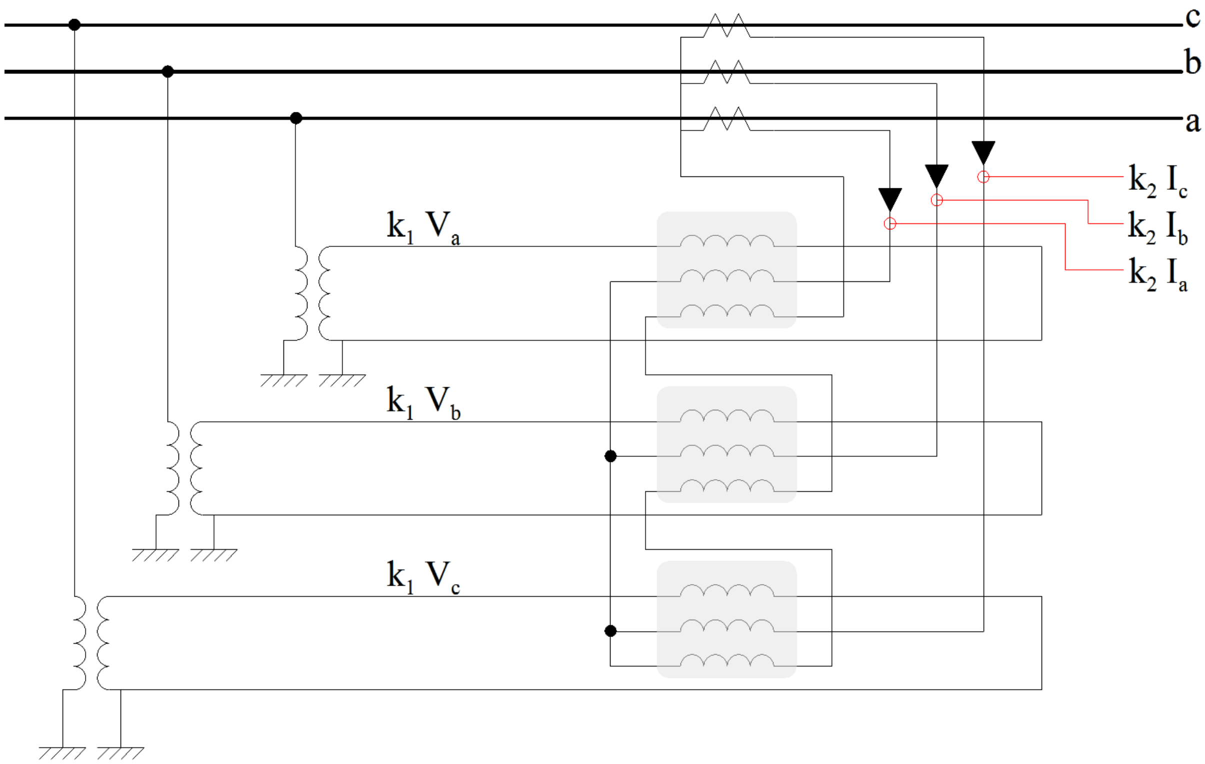


Figure 9.8: Three Phase Distance Relay for Phase to Ground Faults
(Indicate Location of Breaker to right of CTs)

Consider the first relay of Figure 9.8. The inputs to this relay are: $k_1\tilde{V}_a, k_2\tilde{I}_a, k_2\tilde{I}_0$. An electromechanical realization of a modified impedance relay is shown in Figure 9.x.

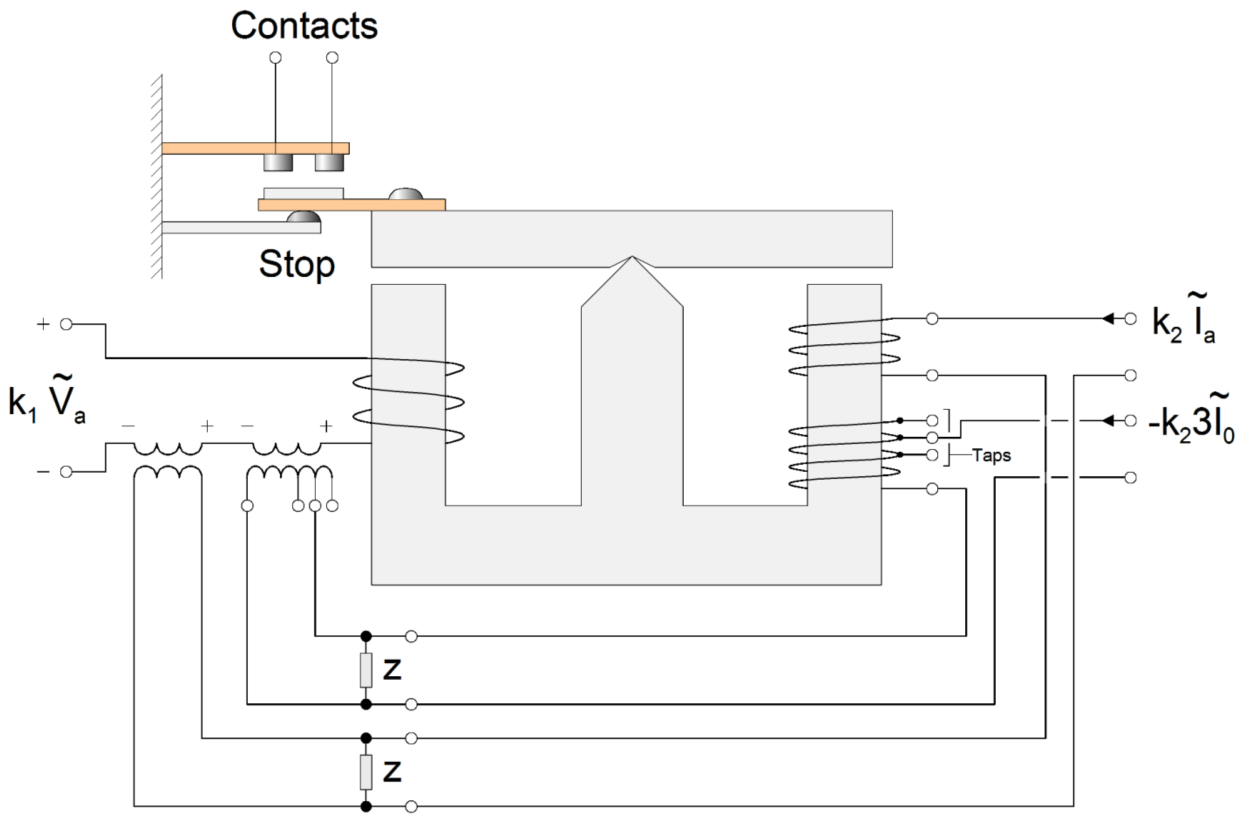


Figure 9.9: Balancing Beam Implementation of Relay 1 of Figure 9.8

For a phase A to ground fault, the phase A relay will "see" the following impedance:

$$\frac{k_1\tilde{V}_a}{k_2\tilde{I}_a + k_2 m\tilde{I}_0} = \frac{k_1}{k_2}\left(\frac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0}\right)$$

By appropriate selection of the constant m it can be shown that the relay will "see" an equivalent impedance equal to the positive sequence impedance of the circuit from the relay location to the fault. Consider the equivalent circuit for a single line to ground fault. The following hold:

$$\tilde{V}_1 = z_1\ell\tilde{I}_1 + \tilde{V}_{1f}$$

$$\tilde{V}_2 = z_1\ell\tilde{I}_2 + \tilde{V}_{2f}$$

$$\tilde{V}_0 = z_0\ell\tilde{I}_1 + \tilde{V}_{0f}$$

By adding all above equations:

$$\tilde{V}_a = \ell(2z_1 + z_0)\tilde{I}_1 = 3z_1\ell\tilde{I}_1 + (z_0 - z_1)\ell\tilde{I}_0 = z_1\ell\tilde{I}_a + (z_0 - z_1)\ell\tilde{I}_0$$

$$\frac{\tilde{V}_a}{\tilde{I}_a + \left(\frac{z_0 - z_1}{z_1}\right)\tilde{I}_0} = z_1 \ell$$

Note by comparing above equation to the earlier one, in order for the relay to "see" the positive sequence impedance, the factor m should be selected as:

$$m = \frac{z_0 - z_1}{z_1}$$

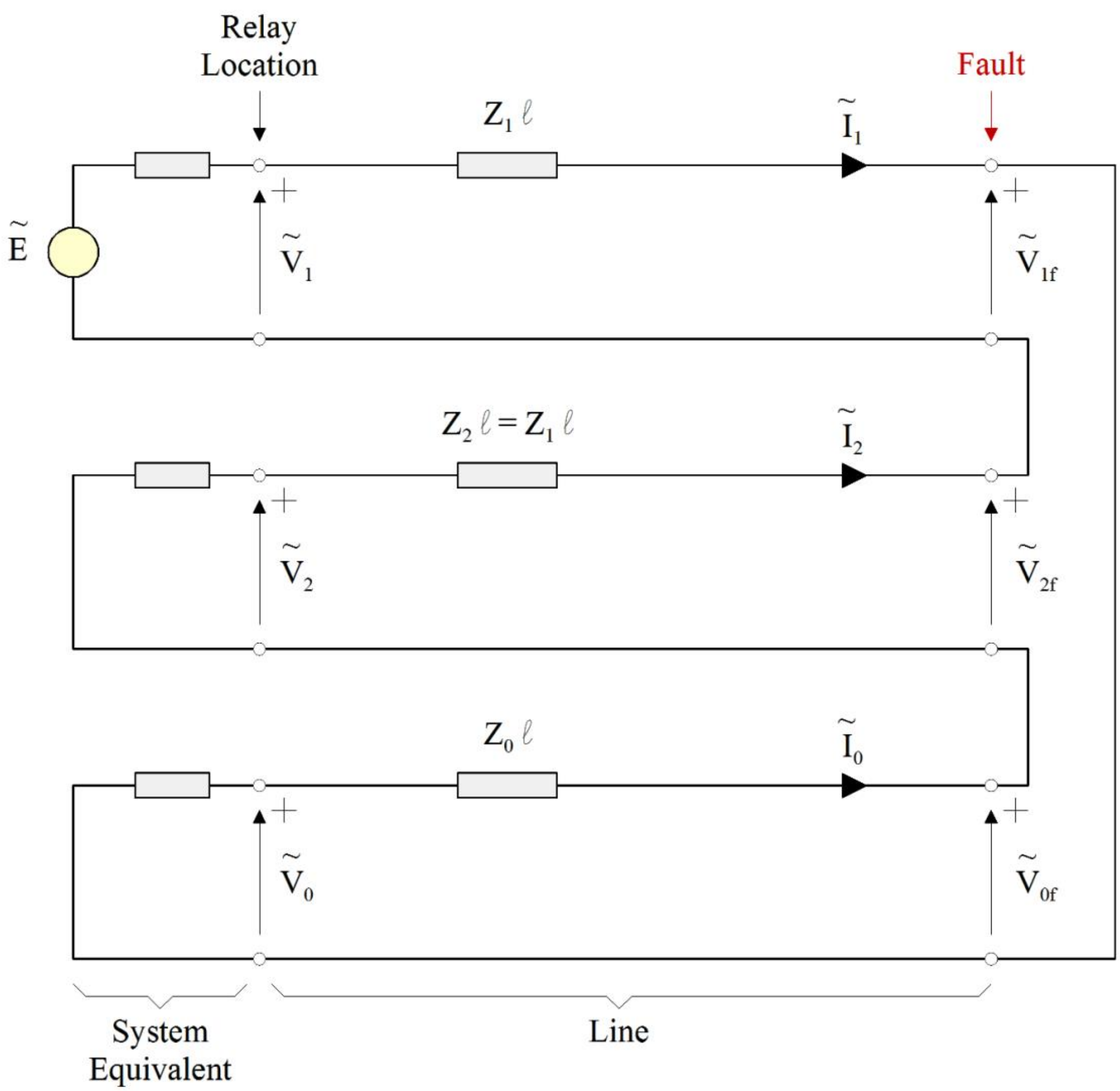


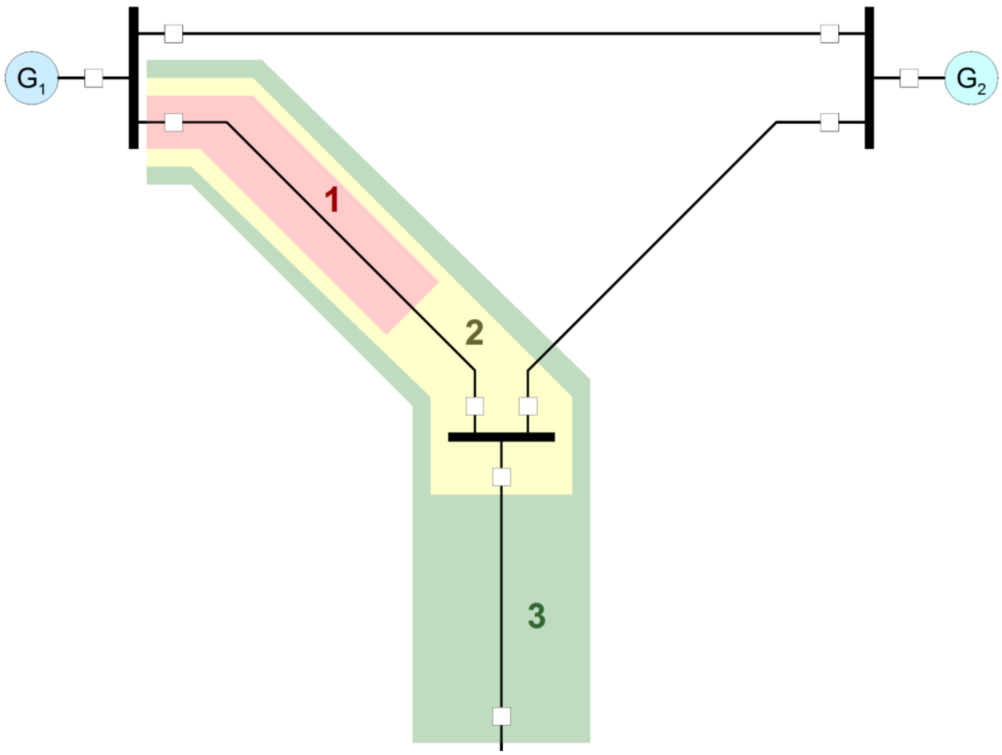
Figure 9.10: Equivalent Sequence Network for Phase to Ground Faults

The application of the above described three phase distance relays will cover most of the faults in a three phase line. In case of lines on the same tower/pole then additional types of faults may occur. In this case, the performance of distance relays must be carefully evaluated.

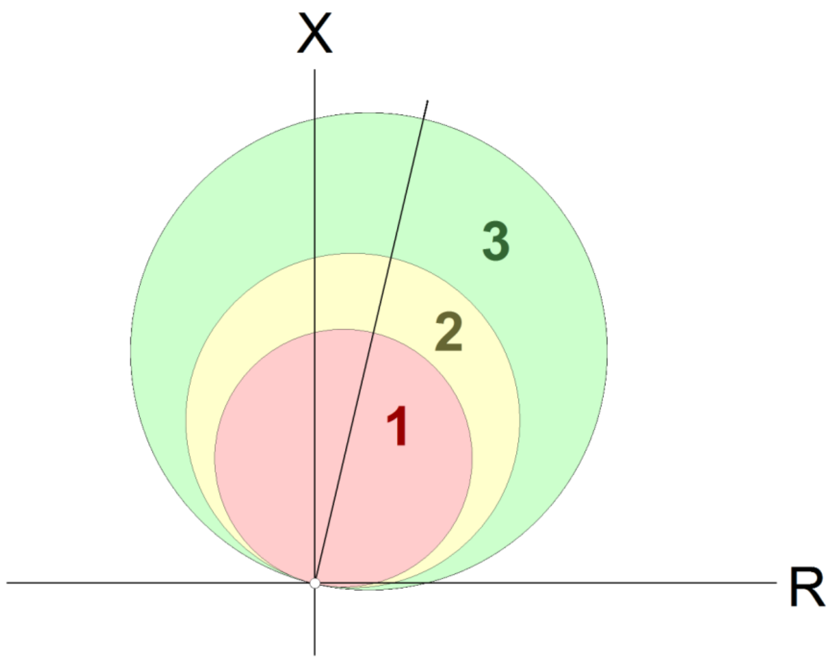
9.4.5 Typical Practice for Line Protection

The typical distance relay includes the following functions: line distance protection and ground distance protection. For each one three zones are typically provided with some manufacturers also providing additional zones of protection (i.e. zone 4). We will discuss the typical practice for zones 1, 2 and 3. A typical practice is to set the zone 1 of the distance relay to about 80% of the line impedance (i.e. to reach 80% of the length of the line). This operation is fast with just a small delay (two to three cycles) to avoid tripping on transients. This practice allows line protection for the majority of the faults along the line. The 80% figure is selected to make sure that zone 1 operation (which normally does not have any appreciable time delay) does not operate on faults past the line. In other words we have a safety margin of 20%. Zone 2 is typically set to reach 125% of the line length. Time delays are moderate in the order of 10 to 20 cycles to coordinate with other fast tripping schemes. And zone 3 is typically set to reach 100% of the line plus 150% of the next line. The time delays for zone 3 are typically 30 or more cycles. These selections are illustrated in Figure 9.11. These selections provide primary line protection (zone 1 and 2) as well as backup protection (zones 2 and 3).

The advantages of the above scheme is simplicity and reliance on local information only. There are however certain disadvantages: (a) it is possible and most likely that the breakers at the two terminals of the line will not trip simultaneously, especially if the relays at the two terminals of the line "see" the fault at different zones, (b) it is difficult to apply this scheme on lines with series capacitor compensation, and (c) under certain conditions, the impedance of the line as "seen" by the relays may enter the zone 3 characteristic - for example heavy loading of the line and possibly reduced voltage. In this case the relay may trip even if there is no fault on the line. We refer to this event as "load encroachment". Later on we will discuss pilot schemes that address some of these issues.



(a)



(b)

Figure 9.11: Illustration of a Typical Distance Relay Application
(a) Geographical Coverage of the Zones
(b) Relay Characteristic of the Three Zones

The application of distance relaying for power lines will be illustrated with examples.

Example E9.x. A distance relay is used to protect a 115 kV transmission line with the following parameters:

$$Z_1 = 5.710 + j39.052\, ohms, \quad Z_2 = 5.710 + j39.052\, ohms, \quad Z_0 = 35.152 + j120.446\, ohms$$
$$Z_1^{'} = 2.871 - j128050.0\, ohms, \quad Z_2^{'} = 2.871 - j128050.0\, ohms, \quad Z_0^{'} = 17.757 - j205008\, ohms$$

The line is 115 kV (L-L), 53.5 miles long. The CT ratio of the distance relay is 2000:5 and the PT ratio is 69kV:115V (the PT high side is connected phase to ground). The distance relay configuration is set for ground faults, i.e. it monitors the phase to ground voltage and a compensated current equal to phase current plus a factor m times the zero sequence current.

It is desired to set this relay to reach 75% of the line. Select the compensation factor m and the impedance setting of the relay in ohms (on the relay side NOT the line side). The compensation factor should be rounded to the first decimal point and the impedance setting rounded to ohms.

Solution: The compensation factor is:

$$m = \frac{120.446 - 39.052}{39.052} = 2.0836 \quad \Rightarrow \quad m = 2.1$$

The impedance setting on line side

$$Z_R^{Line} = (0.75)(5.71 + j39.052) = 4.28 + j29.29\, ohms$$

The impedance setting on relay side:

$$Z_R^{relay} = \frac{\dfrac{115}{69000}}{\dfrac{5}{2000}} Z_R^{Line} = (0.644)(4.28 + j29.29) = 2.758 + j18.861\, ohms = 19.06 e^{j81.9^0}\ ohms$$

Select: 19 ohms at 80 degrees.

Example E9.x. Consider the electric power system of Figure E9.1 and the illustrated power line. A distance relay is installed at the terminals of the line and it is desired to set the zone 1 of the relay to reach 80% of the line length. The CT and PT ratios are 3000:5 and 69,000:115 respectively. Select the settings of the relay. Express the settings on the relay side.

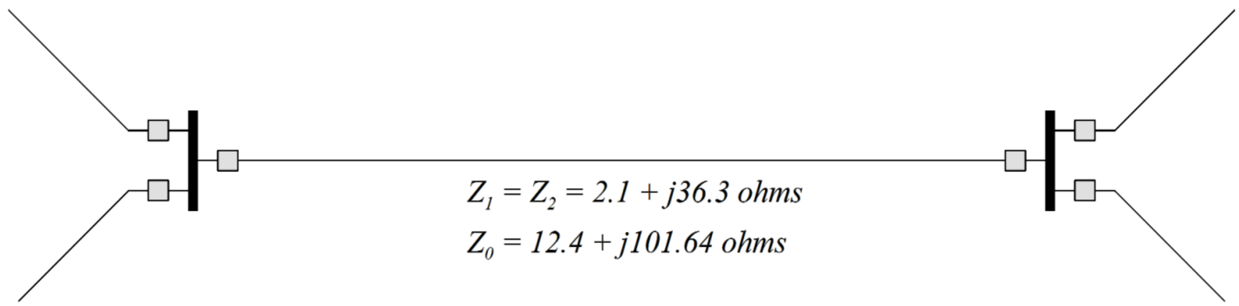


Figure E9.x: A Power Line

Solution: To determine the settings of the relay it is necessary to compute the positive sequence impedance of the line as well as the zero sequence impedance of the line.

To be completed.

Example E9.x: Consider a 53.7 mile long line protected with distance relaying at both ends. A distance relay is protecting this line. The CTs and VTs feeding this relay are: 2000:5A, 86,000:69V. The settings of the relay are:

$$z_{relay,zone1} = 10.220e^{j83.12^0} \ ohms, \quad k = 1.95e^{-j5}, \quad delay: 0.03s$$

$$z_{relay,zone2} = 17.252e^{j83.12^0} \ ohms, \quad k = 1.95e^{-j5}, \quad delay: 0.15s$$

$$z_{relay,zone3} = 25.204e^{j83.12^0} \ ohms, \quad k = 1.95e^{-j5}, \quad delay: 0.5s$$

(a) Assume a single line to ground fault at point A which is located 5.1 miles from terminal T1 of the line. Estimate the tripping times of the two breakers CB1 and CB2. (note provide circuit with parameters).

(b) Assume a single line to ground fault at point B which is located 16.2 miles from terminal T1 of the line. Estimate the tripping times of the two breakers CB1 and CB2. (note provide circuit with parameters).

The WinIGS model of this line and the interconnected system are provided, file: xxxxxxxxxxx. This example indicates that with distance relay we may end up allowing the fault on the line for a long time.

Solution: First a fault analysis will be performed to determine what the two relays at the terminals of the line will "see".

9.4.6 Discussion on Line Non-Pilot Distance Protection

Line distance protection is an effective and selective scheme for the protection of a transmission line. Yet there are cases for which it may not provide the required protection and with a desirable speed. We discuss a number of cases that result in delayed operation of distance protection, or failure to trip. The various protection schemes that have been described so far have two basic limitations when applied to transmission lines: (a) they are not capable to determine with absolute certainty that the fault is on the line under protection and therefore they must rely on coordination for the proper clearing of the fault, and (b) they lack the ability to clear the fault on a line simultaneously at the two ends of the line. Because of these limitations, fault detection and clearing cannot be fast.

Case 1: Consider a high impedance line fault. It is possible that the distance relay may "see" this fault beyond zone 1 and even zone 2 in which case it may trip after a long delay or it may never trip.
Case 2: The distance relays at the two ends "see" the fault at different zones.

The issues discussed above are limitations of line distance protection. The end result is that reliance on non-pilot distance protection only cannot accomplish fast fault detection and tripping of the line simultaneously from the two ends.

For above reasons, pilot relaying schemes have been developed to enable reliable and fast fault detection and tripping of the line. These schemes will be discussed next.

9.5 Pilot Power Line Relaying

Pilot relaying can be viewed as the poor man's differential protection scheme. Pilot relaying requires communication between the relays at the terminals of the line. In general, pilot relaying schemes do not send complete information from one terminal to another, but limited or processed information, as it is possible by the technology used. Figure 9.12 illustrates the pilot relaying scheme in a conceptual manner. It is important to note that the various pilot relaying schemes utilize different information between the two ends of the line. In general, any pilot relaying scheme consists of three components: (a) the communications media, (b) the information communicated and the logic used to process this information, and (c) the trip action. Presently, the utilized options with respect to these three components are shown in Table 9.x. A discussion of these items follow.

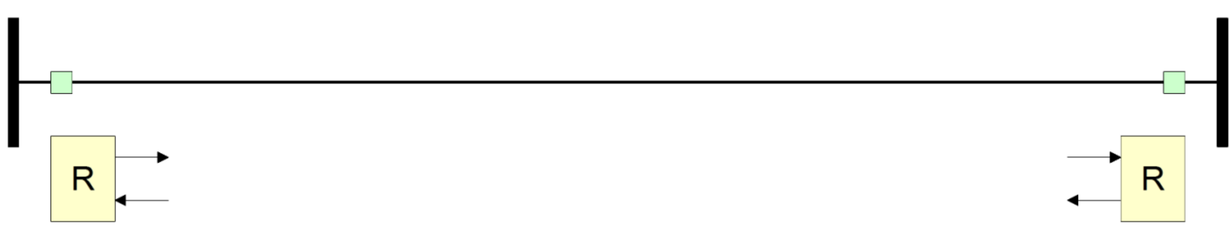


Figure 9.12: Pilot Relaying – Conceptual Arrangement

Table 9.x: Pilot Relaying Media, Logic and Trip Options

Comm Media	Logic	Trip
Pilot Wire Telephone Line (Audio Tones)	Directional Phase	Transfer Trip Blocking

Power Line Carrier (30-300kHz) Microwave (2-12GHz) Fiber Optic	Analog Differential Digital Differential	Synchronized

9.5.1 Communication Media

This section describes the various options for communication media. There are two issues: (a) the information to be communicated and (b) the physical media to be utilized for the transmission. Both will be discussed briefly.

Communication messages: In general, pilot relaying schemes will transmit information from one relay to another relay. The information is typically a voltage or current or both that are typically sinusoids, i.e.

$$\sqrt{2}A\cos(\omega t+\varphi)$$

Note that a voltage or current may be described in terms of three variables, magnitude, frequency and phase.

Many times, and in order to minimize cost, the currents and voltages at the three phase system is used to form one single variable, typically a linear combination of the positive, negative and zero sequence quantities. As examples, Figures 9.13, 9.14 and 9.15 illustrate three different systems, the first illustrates the transmission of a specific signal and the lack of the signal. This can be considered a system that sends only two distinct pieces of information: ON or OFF. An obvious disadvantage of this system is that when the OFF signal is transmitted, there is no way to know whether the intention is to send a zero signal or the system is down. The second system also sends only two pieces of information by switching to different frequencies. In general we refer to these frequencies as "guard" and "trip" since we may use these signal for simply verifying that the relay is in operational mode or the relay is communicating that it is ok to trip. The third scheme provides additional information by modulating the transmitted signal. With the modulation on can send additional information.

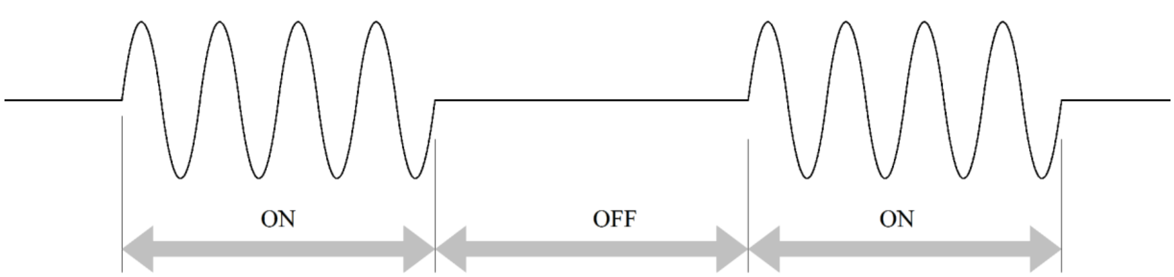


Figure 9.13 ON-OFF Communications Signal

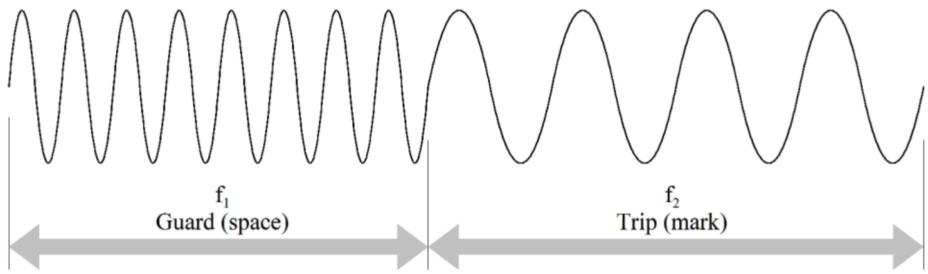


Figure 9.14 Guard/Trip Communications Signal (FSK: Frequency Shift Keying)

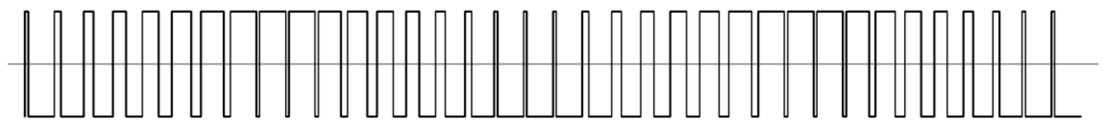


Figure 9.15: Pulse-Period Modulated Communication Signal

Pilot Wire: Pilot wire is used for short lines. Typically, a 15 kV cable, appropriately shielded is used. Typical size is #19, twisted pair. Bandwidth 0 to 4 kHz. The pilot wire transmits an analog signal that is proportional to the electric current in the line or proportional to the differential current of the line between the two terminals of the line. These two options, i.e. (a) pilot wire carries currents under normal operating conditions, and (b) pilot wire does not carry current, are shown in Figures 9.16 and 9.17 respectively. Note that the advantage of the pilot wire approach is that the faulted line is disconnected from the network via a simultaneous tripping of the breakers at the two ends, since the two relays will see the same operating current and restraining currents.

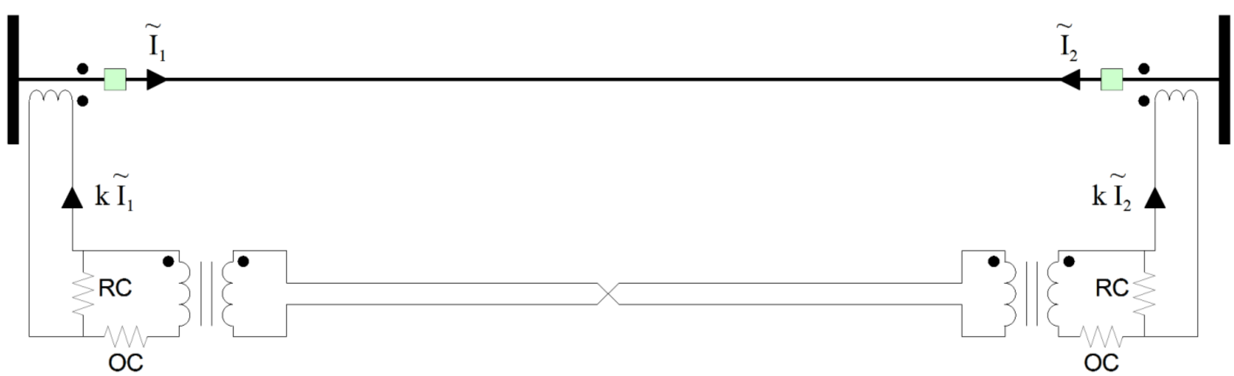


Figure 5.16: Schematic for Tripping Pilot-Wire System - Pilot Wire Does Not Carry Current Under Normal Operating Conditions

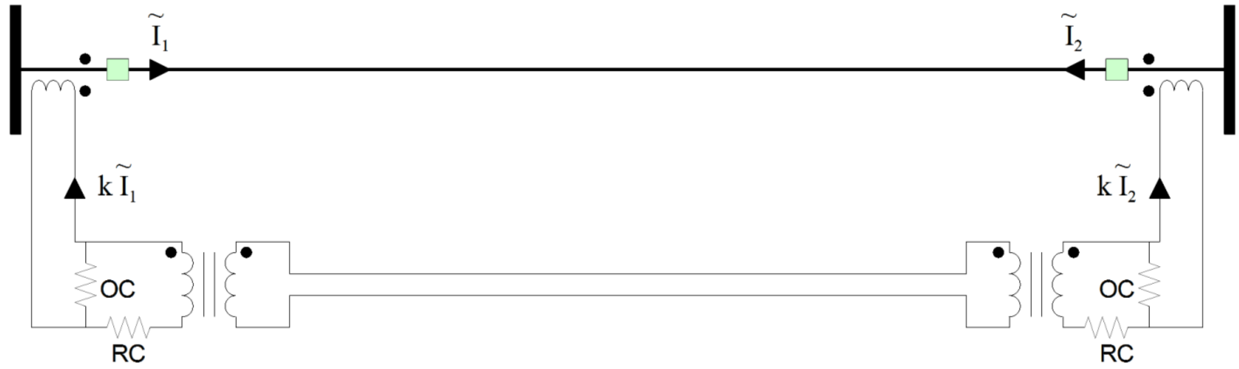


Figure 9.17: Schematic for Blocking Pilot-Wire System - Pilot Wire Carries Current Under Normal Operating Conditions

Pilot wire is typically used for relatively short lines, for example less than 5 miles. Even for short length transmission lines, one has to consider the substantial induced voltages along the pilot wire during a fault. An example will illustrate the levels of the induced voltages on the pilot wires.

The pilot wire deferential scheme applies equally well to two terminal and multi terminal lines.

Example E9.x: Consider the power line of Figure E9.x and the pilot wire along the line. The pilot wire is grounded as it is indicated in the figure. Assume a single phase to ground fault at point A of the line.
(a) Compute the induced voltage on the pilot wire.
(b) Assume that a neutralizing transformer is placed in the location B. Compute the induced voltages on the pilot wire.
(b) Assume a mutual drainage reactor installed at the location C and grounded as indicated in the figure. Compute the maximum transfer voltage on the pilot wire.

Solution: The WinIGS model for this system is shown in Figure E9.xa.

To be completed.

Power Line Carrier: The power line carrier uses the power conductors to transmit information. To confine the information in the line under protection, wave traps, coupling capacitors and RF chokes are used. A typical arrangement is shown in Figure 9.18. A photograph of a typical installation is shown in Figure 9.19. The frequency of the signals used are in the range 50 to 490 kHz. Typical power is 10 Watts which is good for distances up to 100 miles. For longer distances 100 Watts transmitters are used. Usually only one 4 kHz bandwidth is used for protection.

Power line carrier is used in various arrangements. Most common is to equip only one phase with a wave trap, coupling capacitor and RF choke. In this case, typically the middle phase is used since the middle phase is most likely to experience the least probability of a fault on that phase. There are however systems that use two phases for greater reliability, as the one shown in Figure 9.19. There are also systems that use all three phases.

Discuss problems of transmission when a fault occurs in the phase conductor used for transmission.

Figure 9.18: Illustration of the Power Line Carrier System

Figure 9.19: Line Carrier System on a 500 kV Using Two Phases
(Courtesy of TVA)

Microwave: Operates at frequencies 150 MHz to 20 GHz. Needs “optical vision” path. Provides many 4 kHz channels. The extra capacity is used for other communications. It is used when transmission of data is required.

Fiber Optic: This is becoming the communication link of choice. Provides several thousands of 4 kHz communication channels. Extra capacity is used for other communications and can be leased for extra income. Several different implementations have been used. Presently, the use of composite shield wire has become the standard. It is used when transmission of data is required.

9.5.2 Trip

This section describes the various options for generating the relay trip signal.

Tripping versus Blocking: Pilot relaying is classified as a *transfer trip system* if a communication signal is necessary for tripping. A blocking system is one in which the presence of a transmitted signal prevents the tripping of the breaker.

The selected option depends on the selection of the communication media and the reliability of the communication channel. As an example consider the use of PLC. In this case, a fault in the line may affect the communication channel. A blocking scheme will be preferable.

For redundancy, two systems may be used. In this case, one may be transfer trip and the other blocking.

Transfer trip can be (a) direct, (b) permissive or (c) redundant.

9.5.3 Logic

This section describes the various options for relaying logic.

Directional Comparison Blocking: The fundamental principle of this scheme is as follows: direction of fault can be easily determined at any terminal by distance relays or by directional overcurrent. The blocking signal is transmitted only when a fault occurs in reverse direction at one end. The scheme is illustrated in Figure 9.20.

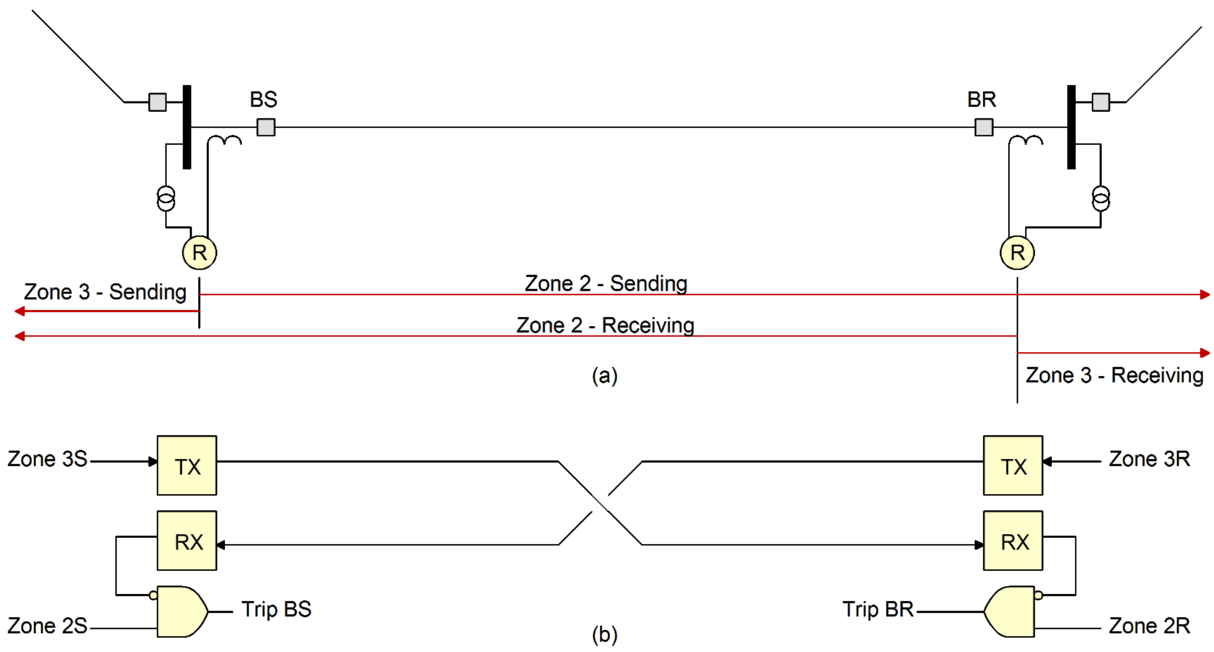


Figure 9.20: Basic Directional Comparison Blocking (DCB) Scheme

The scheme can be implemented with a variety of relays, it has been mentioned that distance relays or directional overcurrent relays can be used. In case of mho relays: set forward relay to 175% to 200% of line. Set reverse relay for 125% to 160%.

Example E9.x: Consider the electric power system of Figure E5.x. Determine the settings for the protection of the line 10-20.

To be completed

Directional Comparison UnBlocking: The fundamental principle of this scheme is: During normal operating conditions send a blocking signal of different frequency. The presence of this signal indicates that equipment is functional.

Example E9.x: Consider the simplified electric power system of Figure E5.x. Determine the settings for the protection of the line 10-20.

To be completed

Direct Underreaching Transfer Trip: The fundamental principle of this scheme is: If fault is detected in line to be protected (zone 1 underreaching, typically 80% of line length), the line is tripped without intentional delay. Requires communication channel independent of line to be protected.

The scheme can be implemented with: a guard signal which changes frequency for trip. The guard frequency monitors the integrity of the communication channel.

Typical settings: use of a distance relay and set it for 80% of then line length.

If line is too short, this scheme may not work well.

Example E9.x: Consider the simplified electric power system of Figure E5.x. Determine the settings for the protection of the line 10-20. Transfer trip can be (a) direct, (b) permissive or (c) redundant.

To be completed

Communication channel issues:

(a) channel integrity
(b) noise
(c) delays

Permissive Overreaching Transfer Trip (POTT): The fundamental principle of this scheme is: Use distance overreaching relays and transfer trip of different frequency for each end. Fault in line will be seen by both relays in forward direction. The term permissive indicates in general that the cooperation of two or more relays is needed for tripping action. The basic logic for this scheme is show in Figure 9.21.

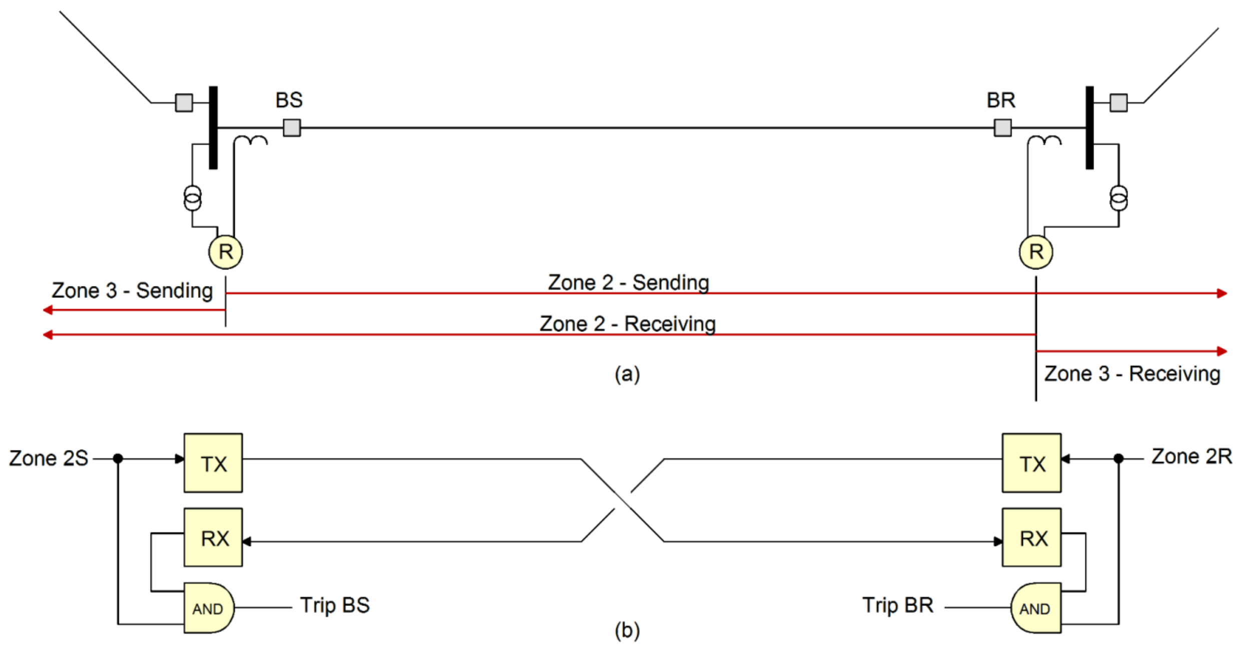


Figure 9.21: Basic Permissive Overreaching Transfer Trip (POTT) Scheme

The scheme can be implemented with: Typical distance settings: 150% of line impedance.

Example E9.x: Consider the simplified electric power system of Figure E5.x. Determine the settings for the protection of the line 10-20.

Solution: For the POTT scheme we will select a setting of 150% of the line impedance.

To be completed

Example E9.x: A transmission line has zero sequence impedance which is 3.5 times its positive sequence impedance. Compute the recommended overreach for the POTT scheme.

Solution: The impedance seen by the relay for a single-phase to ground fault at the end of the line will be:

$$Z = \frac{z_1 + z_2 + z_0}{3}\ell = \frac{2z_1 + 3.5z_1}{3}\ell = 1.83z_1\ell$$

or 183 % of the line impedance. Therefore in order to reach the end of the line for single phase faults, select a setting of 190% or 200%.

Permissive Underreaching Transfer Trip (PUTT): The fundamental principle of this scheme is: Use an underreaching and an overreaching relay at both ends. The overreaching relay performs the permissive function. The underreaching relay initiates a transfer trip.

The relay trips when the overreaching relay “sees” the fault (permissive function) and a transfer trip has been received. Note that the scheme requires two relays at each end. The control circuit is shown below.

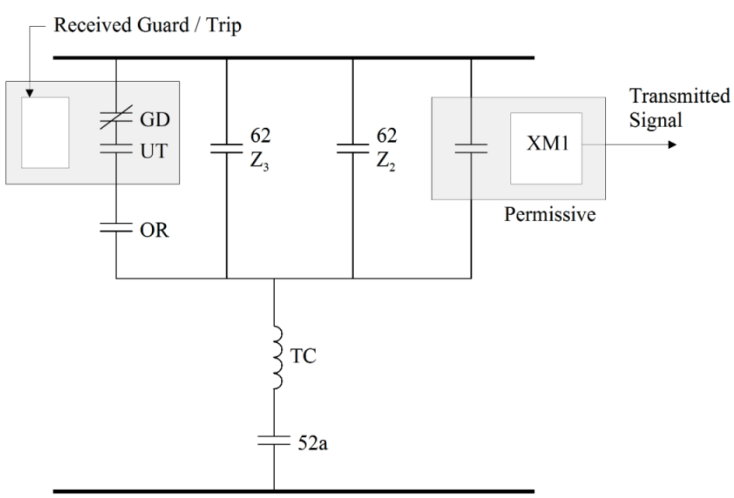


Figure 9.22: Control Circuit of a Permissive Underreaching Transfer Trip Relay

Example E9.x: Consider the simplified electric power system of Figure E5.x. Determine the settings for the protection of the line 10-20.

To be completed

Phase Comparison: The fundamental principle of this scheme is: If currents at both ends are in phase, fault is in the line to be protected. If fault is outside, currents are 180 degrees out of phase.

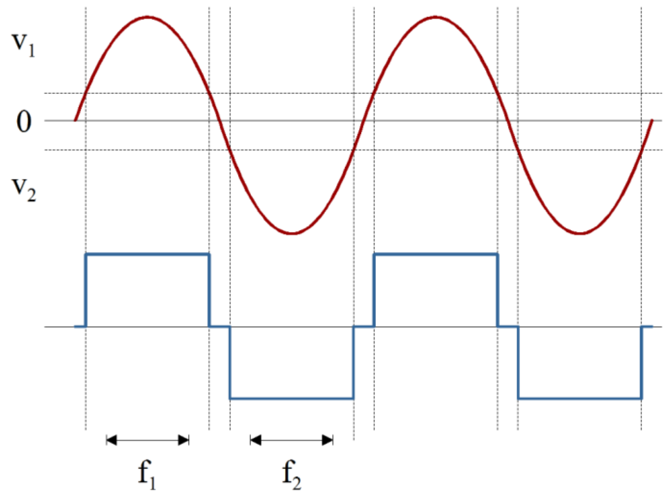


Figure 9.23: Phase Comparison Signal Generation

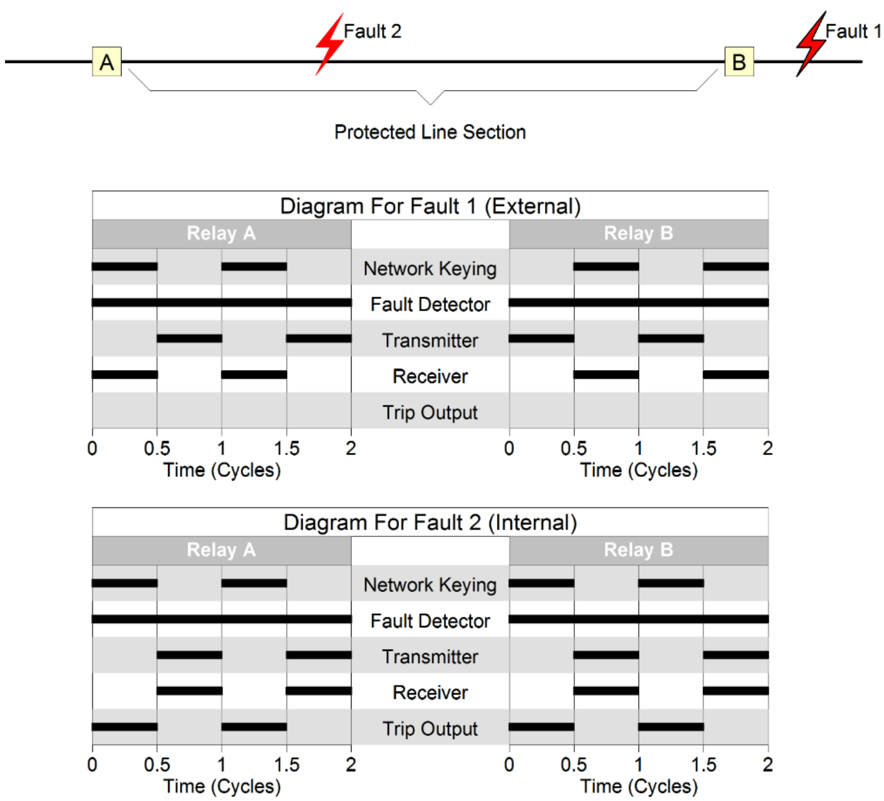


Figure 9.24: Example Phase Comparison Line Protection

The scheme can be implemented with: A signal of certain frequency is send during the positive half cycle of the current. (or a signal is send for both half cycles of different frequency). In reality the phase relationship is not exactly 180 degrees. Generate examples.

Implementation can be with non-segregated phase comparison or with segregated phase comparison. Explain advantages, disadvantages.

Discuss transmission related phase shifts and delays. If cable uses insulation with epsilon dielectric, transmission time over long lines can introduce substantial phase shift. Example, 150 mile long line.

Advantage: it operates only on current and therefore it does not need voltage (cost advantage).

Ideal for short lines.

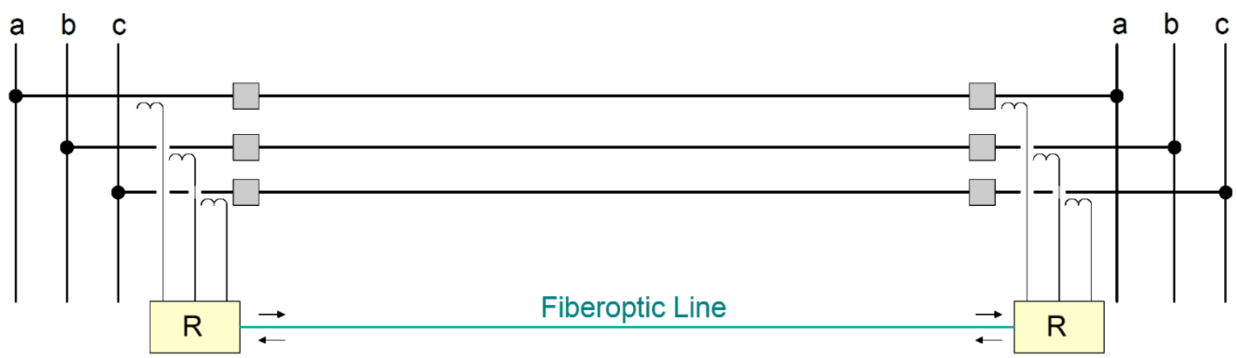
9.6 Line Differential Protection

The various protection schemes that have been described so far have two basic limitations when applied to transmission lines: (a) they are not capable to determine with absolute certainty that the fault is on the line under protection and therefore they must rely on coordination for the proper clearing of the fault, and (b) they lack the ability to clear the fault on a line simultaneously at the two ends of the line. Because of these limitations, fault detection and clearing cannot be fast. Fast protection and simultaneous tripping of the line from both terminals can be achieved with differential protection and communications between the relays at the two terminals of the line. To a lesser extent, similar objectives can be achieved with pilot relaying. We will examine differential protection first and then pilot relaying.

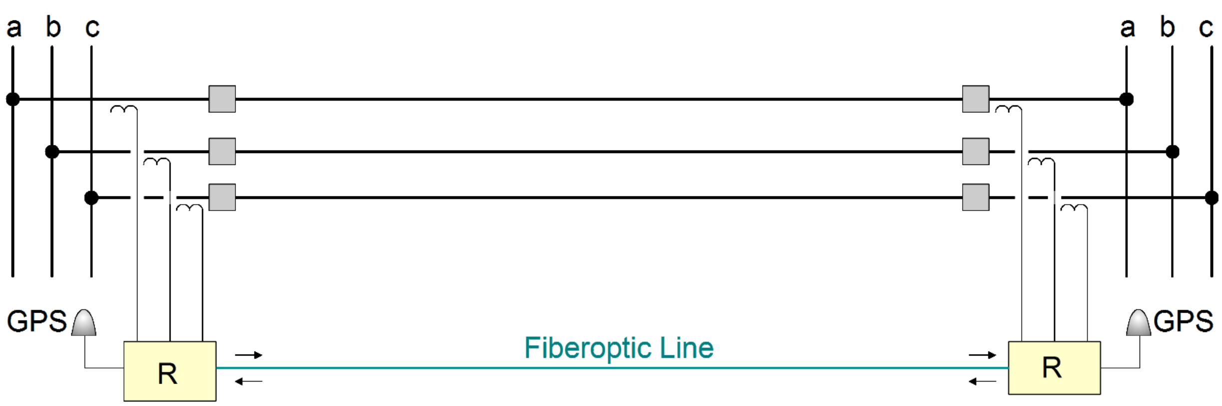
Differential protection for transmission lines presents challenges because of the distance between the two terminals of the line. Direct application of differential protection will require wires from one end of the line to the other. These wires will parallel the power line and during faults may experience induced voltages.

Differential protection schemes do provide more certainty in identifying whether the fault is within the zone of protection. However, because of the geographic extent of transmission lines, differential relaying cannot be applied easily, except for relatively short lines. This can be easily understood since for a differential scheme the currents should be measured at the two ends of the line simultaneously and be brought to the location of the differential relay. The wires that are required by the traditional differential protection will present additional technical, logistical and cost issues. During faults the induced voltages on these wires may be excessive and should be mitigated by appropriate designs. Without going into more discussion of the issues, it should be understood that conventional differential protection for transmission lines is not practical or economical.

Differential protection schemes have attractive advantages. For this reason, approaches and systems have been developed over the years to enable some of these advantages for the protection of transmission lines. Since the basic obstacle in providing differential protection for lines is the communication of the voltages and currents from the terminals of the line to the relay, the developments were along the lines of how much information can be communicated from one end of the line to another, with what communication means and how this information will be utilized. We refer to all these developments (that will be introduced later) as pilot relaying. Recently, the introduction of GPS synchronized measurements and fiber optic communications provided the enabling technology for true differential protection of transmission lines. Specifically, GPS synchronized measurements enable the simultaneous measurement of voltages and currents at remote terminals of a line. These measurements are time tagged and can be communicated via fiberoptic to the location of the relay. The relay can time align the measurements and perform a true differential protection function. An illustration of this approach is given in Figure 9.25. The fiberoptic communication can be provided in the shield wires of an overhead line, or by fiberoptic lines embedded in the cable for UG transmission. This approach provides a secure protection method, as long as the GPS signal is available and the fiberoptic communication is operational. The reliability of these systems can be very high, for example for GPS availability, one can provide low cost equipment that will maintain GPS time for substantial periods in case of lost satellite signals and one can use redundant fiberoptic lines.



(a)



(b)

Figure 9.25: Illustration of Line Differential Protection
(a) Line Differential with Fast Communications
(b) GPS Synchronized Measurements

Line differential protection is a relatively recent development. On the other hand, there are many legacy pilot relaying schemes and many more being installed. A review of these schemes will be given below.

To further understand the impact of non-synchronization, a simple example is considered.

Example E9.x: Consider a transmission line with a fiberoptic cable between the two terminals of the line. Assume a line differential scheme is applied for the protection of the line. Assume a timing error of 0.1 msec. Compute the current in the operating coil of the differential relay when the line carries 1,500 A.

Solution: for simplicity we will not consider any other issues. We assume that current is the same at both terminals of the line and the error is only due to the timing error. A time error of 0.1 msec means a phase error of 2.16 degrees. Thus the differential current will be:

$$\tilde{I}_o = 1,500.0 - 1,500 e^{j2.16^0} = 1.065 - j56.5353 = 56.545 e^{-j88.92^0}$$

9.6.1 Pilot Wire Differential Protection

This scheme is illustrated in Figures 9.16 and 9.17.

9.6.2 Alpha Plane Line Differential Protection

This method is based on transmitting the measured current phasor from one end of the line to the other and taking the ratio of the current phasors at the two ends of the line. For an ideal line and neglecting the capacitive current of the line, this ratio will be exactly -1.0. If there is an internal fault in the line the ratio of the current phasor will be different than -1.0. For an external fault the ratio will be close to -1.0. For a three phase line, the method uses three-phase currents at both sides of the line.

Because of the capacitive current of the line, the ratio of the currents will deviate from -1.0. The longer the line is the higher the deviation will be. In addition the ratio will vary depending on the loading of the line. In lighter loaded line the ratio of the currents at the two ends may deviate much more than the -1.0 value. Another issue that affect the accuracy of the ratio is the time synchronization of the measurements. Any time errors will cause the ratio to deviate from the -1.0 value. For all of these reasons, the relay is set to operate only when the ratio has a value that practically guarantees that an internal fault is causing the deviation. Based on many studies, the recommended relay characteristic is shown in Figure 9.26.

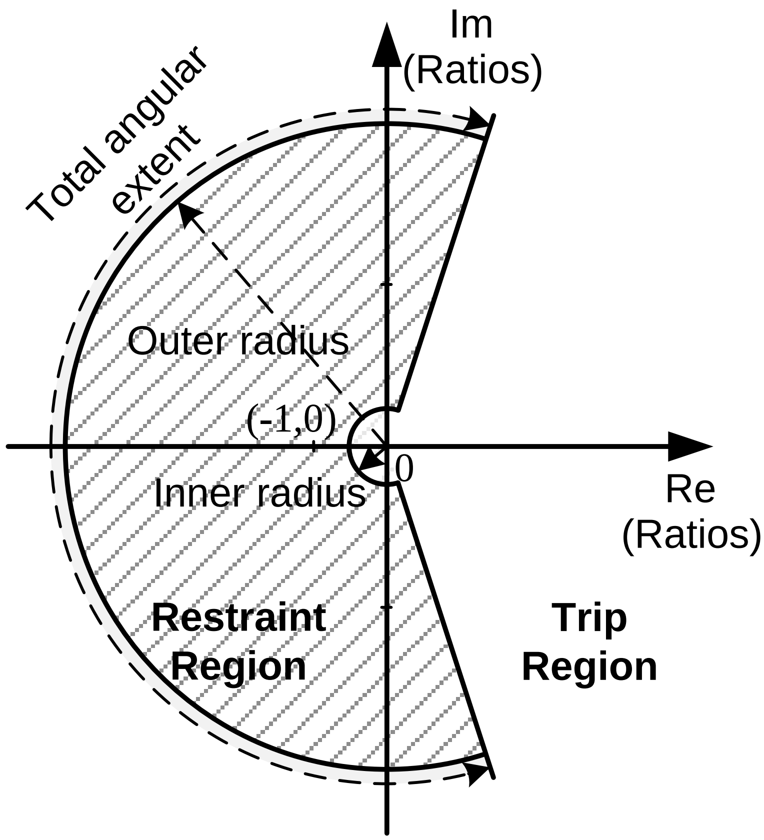


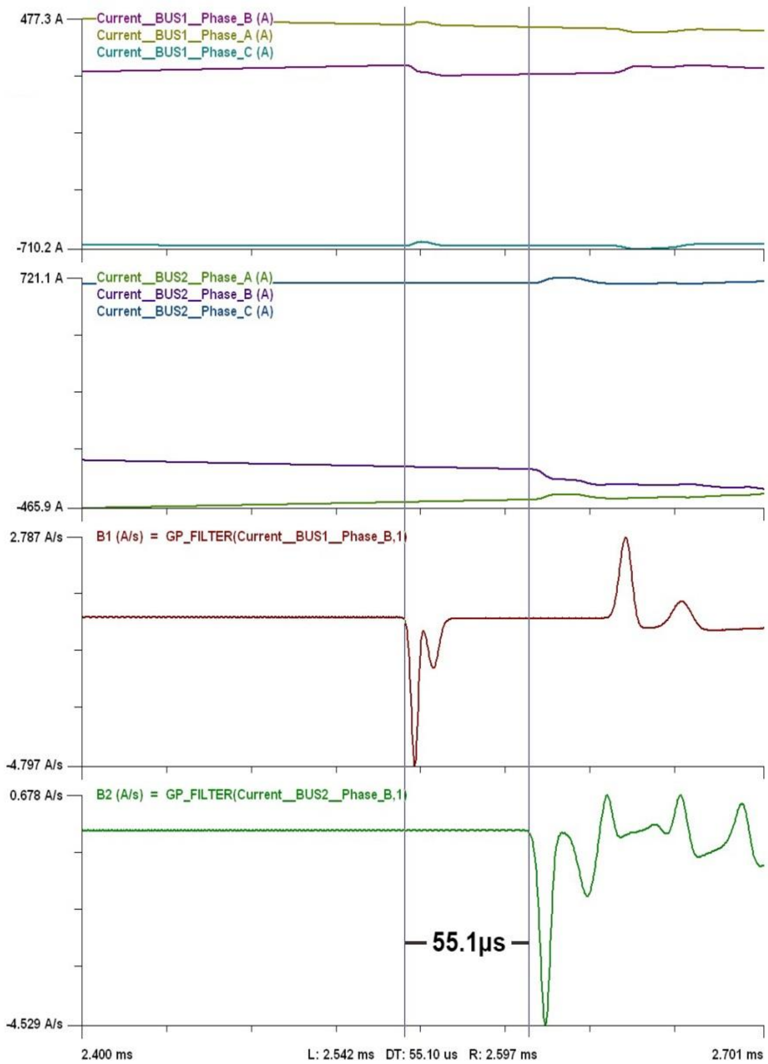
Figure 9.26: Line Differential Protection Characteristic in alpha-plane

9.7 Traveling Wave Protection of Lines

When a fault occurs on a transmission line it generates transients that propagate through the transmission lines at very fast speeds. We refer to these as traveling waves. The speed of propagation varies depending on the mode of the transient. For example line to line faults typically generate transients that they travel at a speed approximately equal to the speed of light. Ground faults initiate two types of traveling waves; one is traveling with the speed of light (line mode) and another traveling at reduced speed, typically 50 to 70% of the speed of light (ground mode). Traveling waves have been explored for transmission line protection for many decades. Technology advancements have enabled the development of relays that use traveling waves (TWs) to detect and locate faults in a speedy, cost effective and relatively reliable manner. An advantage of TW relays is that they are insensitive to fault impedance.

A fault initiates both current traveling waves as well as voltage traveling waves. Thus in theory, one can detect voltage traveling waves or current traveling waves. An example below illustrates the operation of an ideal traveling wave relay.

Example E9.x: Consider a 27.5 Mile Long Overhead 115 kV Line
Ideal Current Transformers
Sampling Rate 2 Ms/s
Top two sets of traces: three phase currents at two ends of the line respectively
Lower two traces: Filtered waveforms of faulted phases. Time of wave arrivals at the two ends are clear.
Fault location:
Fault location:



$$l_f = \left(\tau_{line} - \tau_s + \tau_r\right) L / \left(2\tau_{line}\right)$$

27.5 Mile Long Overhead 115 kV Line

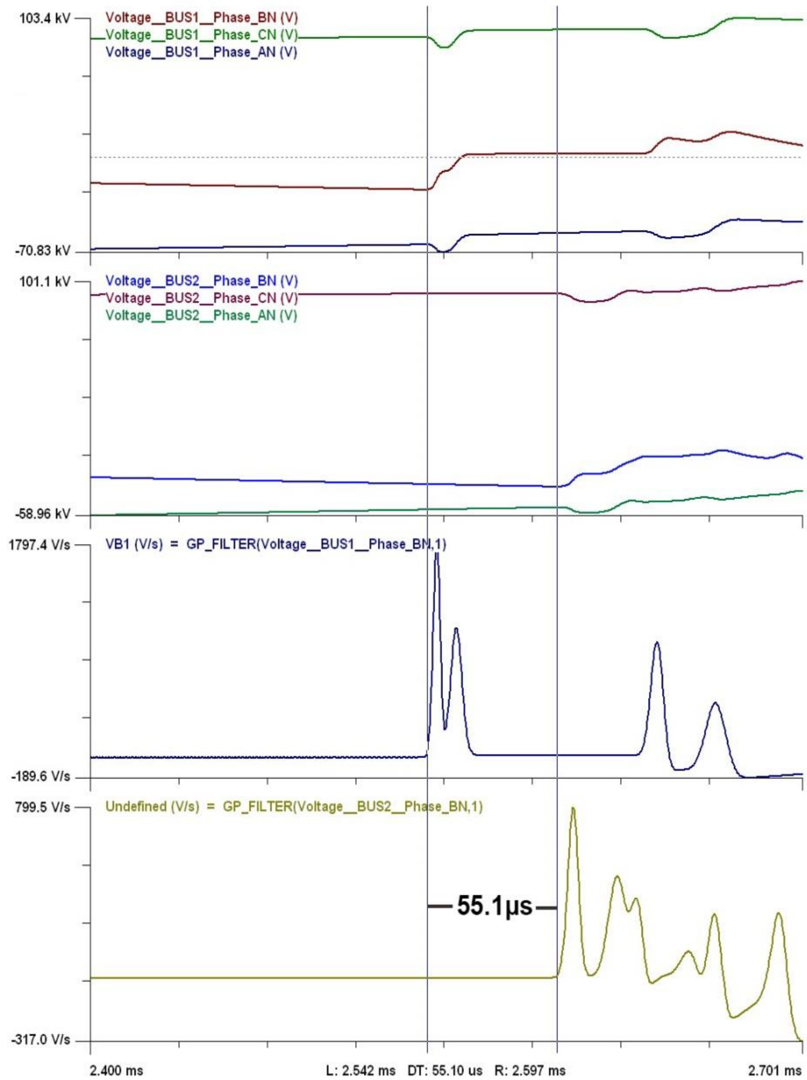
Ideal Voltage Transformers

Sampling Rate 2 Ms/s

Top two sets of traces: three phase voltages at two ends of the line, respectively

Lower two traces: Filtered waveforms of faulted phase. Time of wave arrivals at the two ends are clear.

Fault location:



This example illustrates that, in theory, both voltage and current traveling waves can be used for detecting the arrival time of voltage or current waves and thus compute the location of the fault. It should be also noticed that the use of GPS synchronized measurements is very important for this relay.

When these relays are applied with conventional or special design CTs or VTs, the characteristics of the instrument transformers must be considered. Note that these relays depend on detecting the fast rise of the traveling wave front end which in general comprises high frequencies. Thus, instrument transformers should allow the passage of high frequencies as much as possible. From this point of view, current transformers represent a better choice. This is the reason that many manufacturers prefer to use current as the sensing quantity [7] [8].

The example indicated a relay that uses the traveling wave arrival times at both ends of the line (double-ended method). It is possible to design the relay to rely on traveling wave arrival times at one end only. In this case the arrival time of reflected waves are measured for the purpose of locating the fault. Because as traveling waves are reflected and refracted as they encounter discontinuities along the line, the reflected waves are not as strong and generate an issue of reliability.

9.7 Protection of Mutually Coupled Lines

Many power lines may share the right of way (or towers/poles) with other lines for part of their length or for the entire length. In this case, mutual coupling exists among the lines. The mutual coupling induces voltages on the faulted and unfaulted lines and may alter what the relays "see" with the possibility of altering the relay decision. These effects will be discussed with an example. Example to be added.

For mutually coupled lines, the challenge is brought by the magnetic mutual coupling which affects mainly the zero-sequence networks. Figure 9.27 shows an example mutually coupled transmission line system. Note that in general the mutual coupling may occur on only one section of the lines.

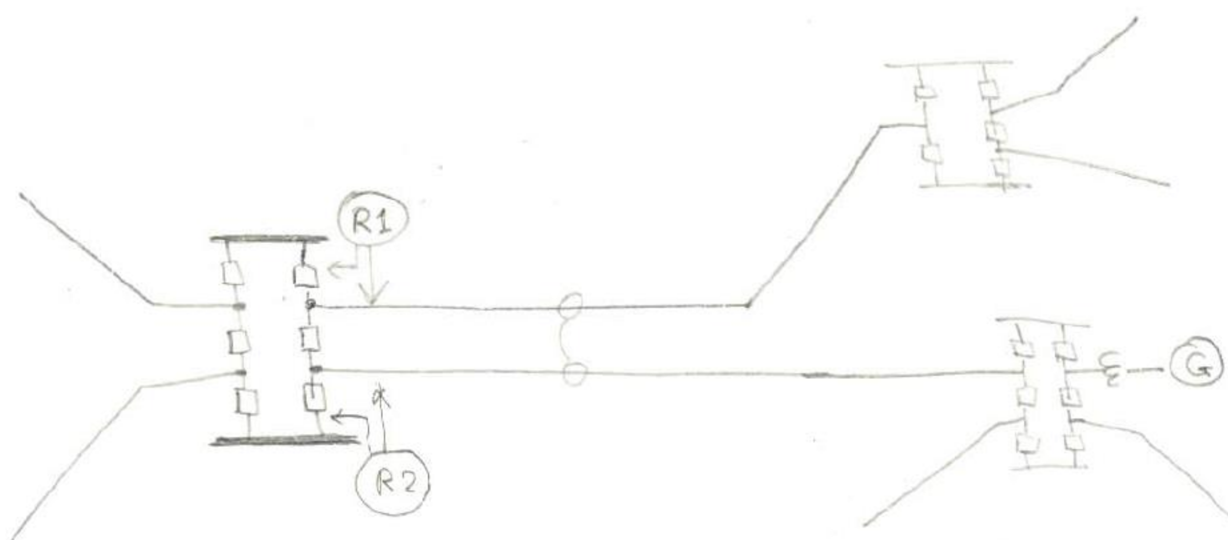


Figure 9.27: Mutually Coupled Transmission Lines

One approach to overcome the influence of zero-sequence coupling and induced voltage/current in adjacent lines is by providing compensation to distance relays. The relays receive input of the zero-sequence current of adjacent lines to calculate apparent impedance. It is based on the assumption that the whole length of the protected line is under the same influence of the zero-sequence current I_0. The main disadvantages of this approach is that the induced voltage in the line under protection is not only influenced by zero-sequence current of the adjacent line, but also by positive- and negative-sequence current as well (normally 5%-7%). Thus, even in perfect conditions, the compensation method suffers with systematic errors.

Line differential protection scheme performs well in mutually coupled lines [12].

Example E9.x: Consider the electric power system of Figure E9-X. The parameters of the various system components are given in the Figure. The sequence parameters of the two lines are given in Figures E9-xa and E9-xb (the faulted line is the one shown in Figure E9-X). The mutual zero sequence impedance between the two lines is j27.5 ohms. The mutual positive/negative sequence impedance is zero.

Consider a line to ground fault at location A indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. Assume there is a distance relay located at the infinite bus side of the faulted line. Further assume that the zone one of this relay is set for 54 ohms at an angle of 80 degrees (on the line side) and the compensating factor m of this relay is set to 1.5.

Determine whether the relay will trip on zone 1. Show all your computations.

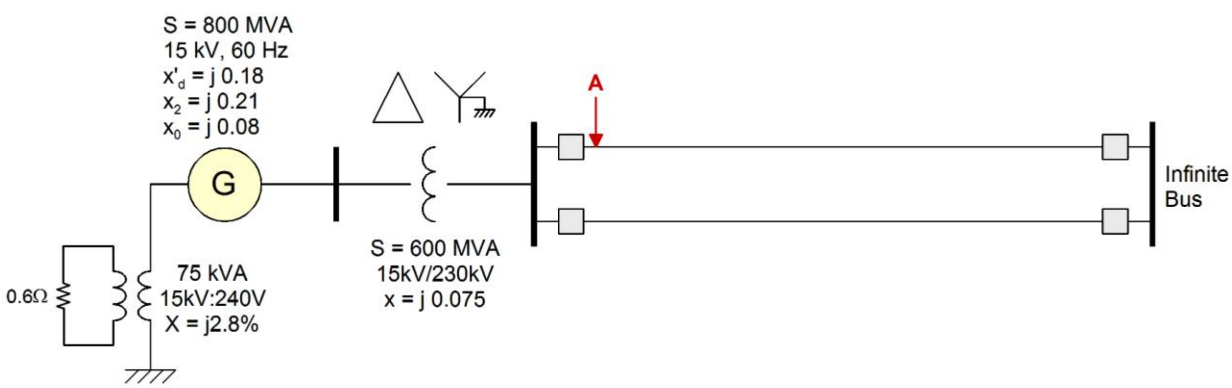


Figure E9-x: Example of Mutually Coupled Transmission Lines

Total Line Series Impedance & Shunt Admittance (Base 100.0 MVA)

	Real / React (%)	Real / React	Magn / Phase
Z1 =	1.4817 / 10.1649	7.8381 / 53.7722 Ohms	54.3405 Ohms / 81.71 Deg
Z0 =	3.9587 / 25.6432	20.9413 / 135.6525 Ohms	137.2594 Ohms / 81.22 Deg
Y1 =	0.0032 / 11.3560	0.0001 / 0.2147 mMhos	0.2147 mMhos / 89.98 Deg
Y0 =	0.0041 / 6.0573	0.0001 / 0.1145 mMhos	0.1145 mMhos / 89.96 Deg

Surge Impedance

	Magn/Phase
Z1 =	503.13 Ohms / -4.14 Deg
Z0 =	1094.86 Ohms / -4.37 Deg
Surge Imp. Loading (MVA) :	105.14
Load Carrying Capability (A) :	890.0

Other Parameters

Computed at Frequency (Hz):	60.00
Span Length (miles):	0.10
Soil Resistivity (ohm-meters):	200.00
Tower Ground Resistance (ohms):	25.00

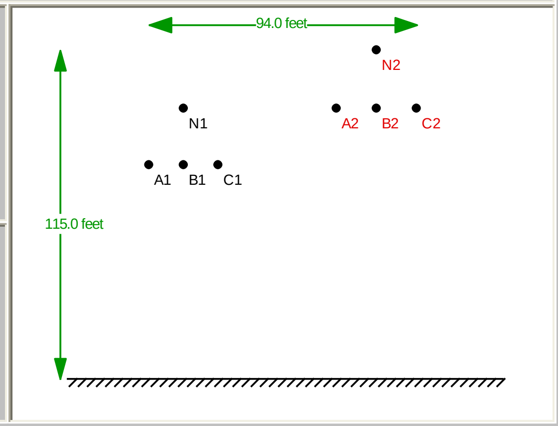


Figure E9-Xa: Sequence Parameters and Topology of the Example Mutually Coupled Lines – Line 2

Total Line Series Impedance & Shunt Admittance (Base 100.0 MVA)

	Real / React (%)	Real / React	Magn / Phase
Z1 =	1.7698 / 10.0468	9.3623 / 53.1478 Ohms	53.9661 Ohms / 80.01 Deg
Z0 =	3.6035 / 26.6774	19.0625 / 141.1234 Ohms	142.4050 Ohms / 82.31 Deg
Y1 =	0.0039 / 11.5113	0.0001 / 0.2176 mMhos	0.2176 mMhos / 89.98 Deg
Y0 =	0.0038 / 6.0316	0.0001 / 0.1140 mMhos	0.1140 mMhos / 89.96 Deg

Surge Impedance

	Magn/Phase
Z1 =	498.00 Ohms / -4.99 Deg
Z0 =	1117.57 Ohms / -3.83 Deg
Surge Imp. Loading (MVA) :	106.23
Load Carrying Capability (A) :	890.0

Other Parameters

Computed at Frequency (Hz):	60.00
Span Length (miles):	0.10
Soil Resistivity (ohm-meters):	200.00
Tower Ground Resistance (ohms):	25.00

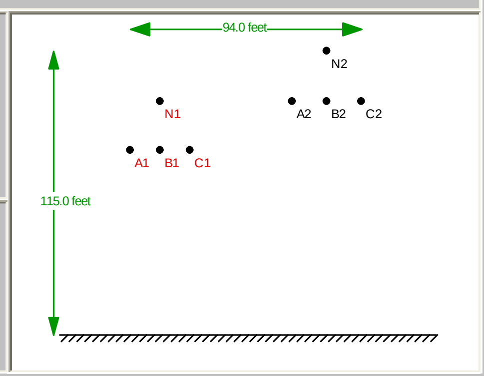
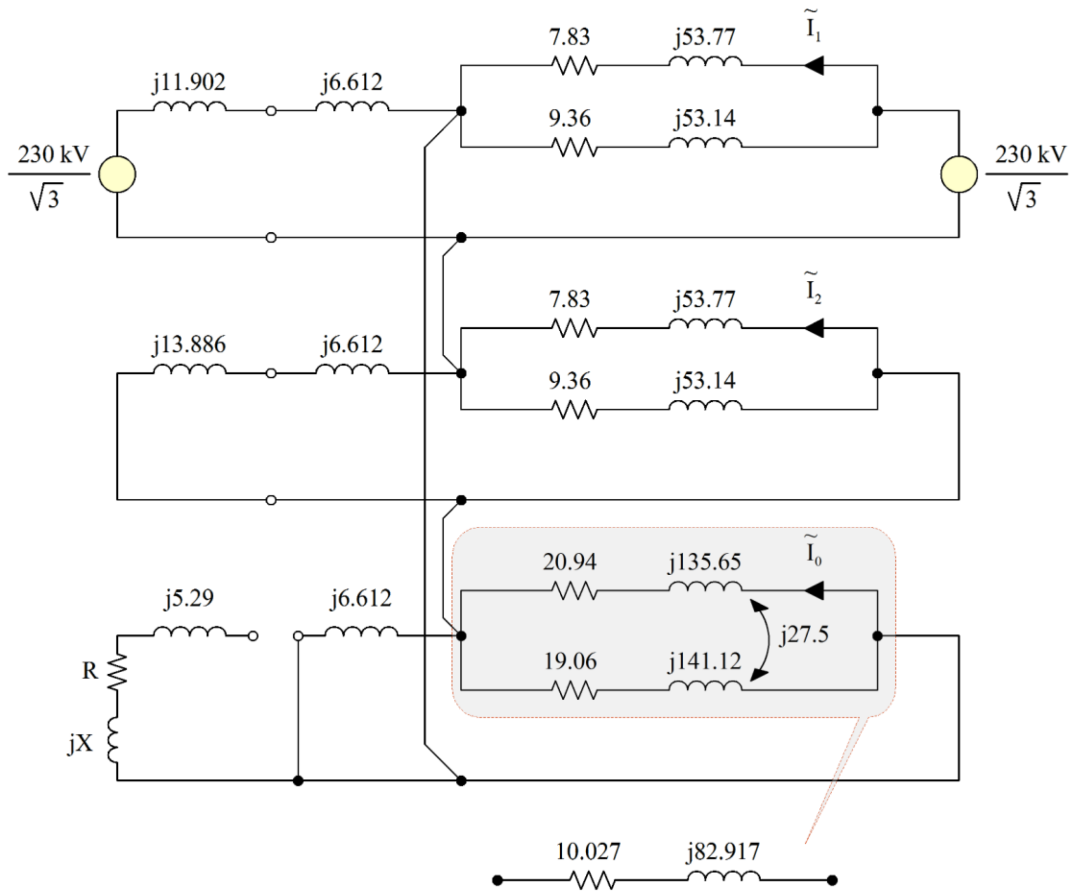


Figure E9-Xb: Sequence Parameters and Topology of the Example Mutually Coupled Lines – Line 1

Solution: The equivalent circuit for the specified fault condition is:



The short circuit analysis of above circuit yields:

$$\tilde{I}_1 = 933.6e^{-j82.31^0}$$

$$\tilde{I}_2 = 990.6e^{-j82.53^0}$$

$$\tilde{I}_0 = 172.8e^{-j79.79^0}$$

$$\tilde{I}_a = 933.6e^{-j82.31^0} + 990.6e^{-j82.53^0} + 172.8e^{-j79.79^0} = 2{,}096.8e^{-j82.20^0}$$

$$\tilde{V}_1 = 132.77e^{-j0^0}$$

$$\tilde{V}_2 \cong 0$$

$$\tilde{V}_0 \cong 0$$

The relay will see the following impedance:

$$Z = \frac{\tilde{V}_a}{\tilde{I}_a + 1.5\tilde{I}_0} = 56.36 e^{j81.94^0} \ ohms$$

The relay will "see" the fault outside its zone.

Example E9.y: Consider the electric power system illustrated in Figure E9-Ya. The construction of the lines is illustrated in Figure E9-Yb. The phase conductors of the line have the following parameters: r=0.08 ohms/mile, radius=0.5 inch, GMR=0.0325 ft. For simplicity neglect the shield wires. Consider a three phase fault at point A (70% length of the line). Compute the impedance "seen" by the indicated relay in the figure. The length of the lines shown horizontally in the Figure is 83 miles, while the length of the lines shown vertically is 12 miles. All lines are 230 kV lines. The three equivalent sources are identical with the following parameters: Z_1=Z_2=j12.2 ohms and Z_0=j8.9 ohms. Prior to the fault the system operates with nominal voltages and unloaded. For simplicity, neglect capacitive currents.

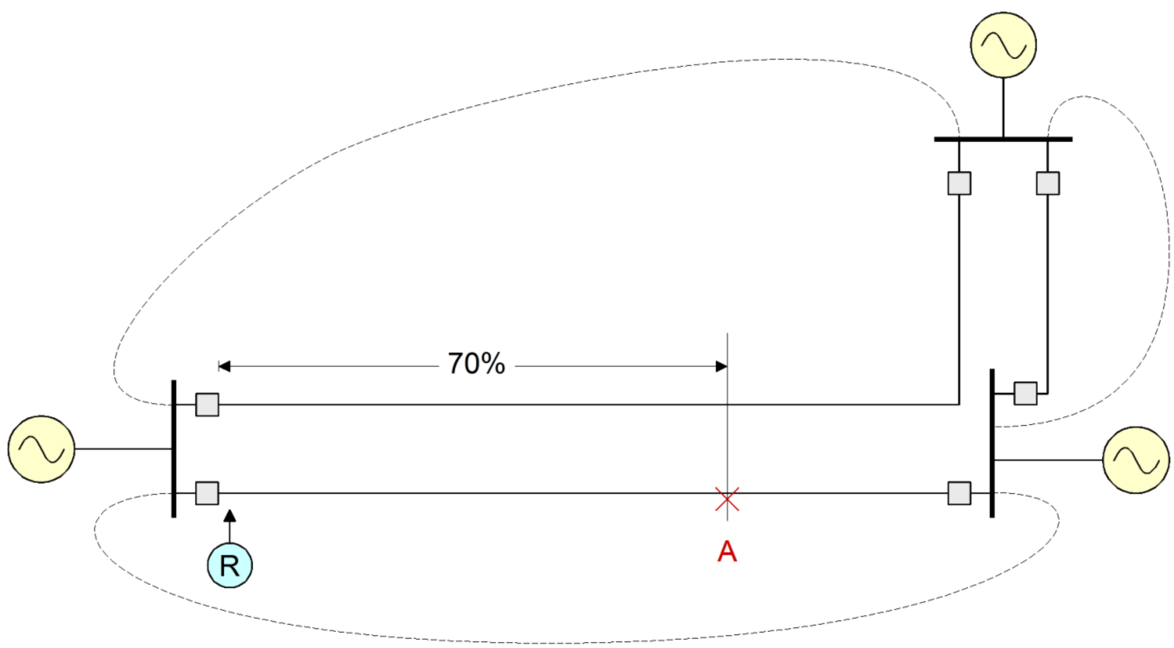


Figure E9-Ya: Example of Partially Coupled Transmission Lines

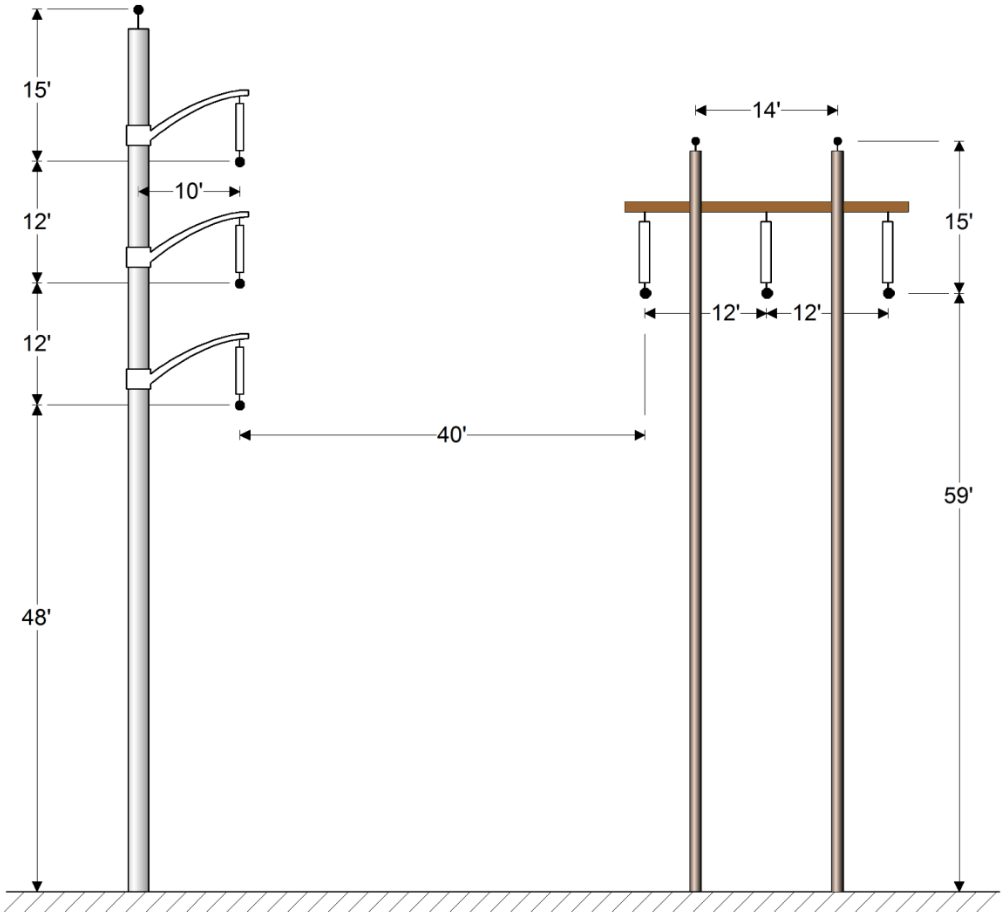
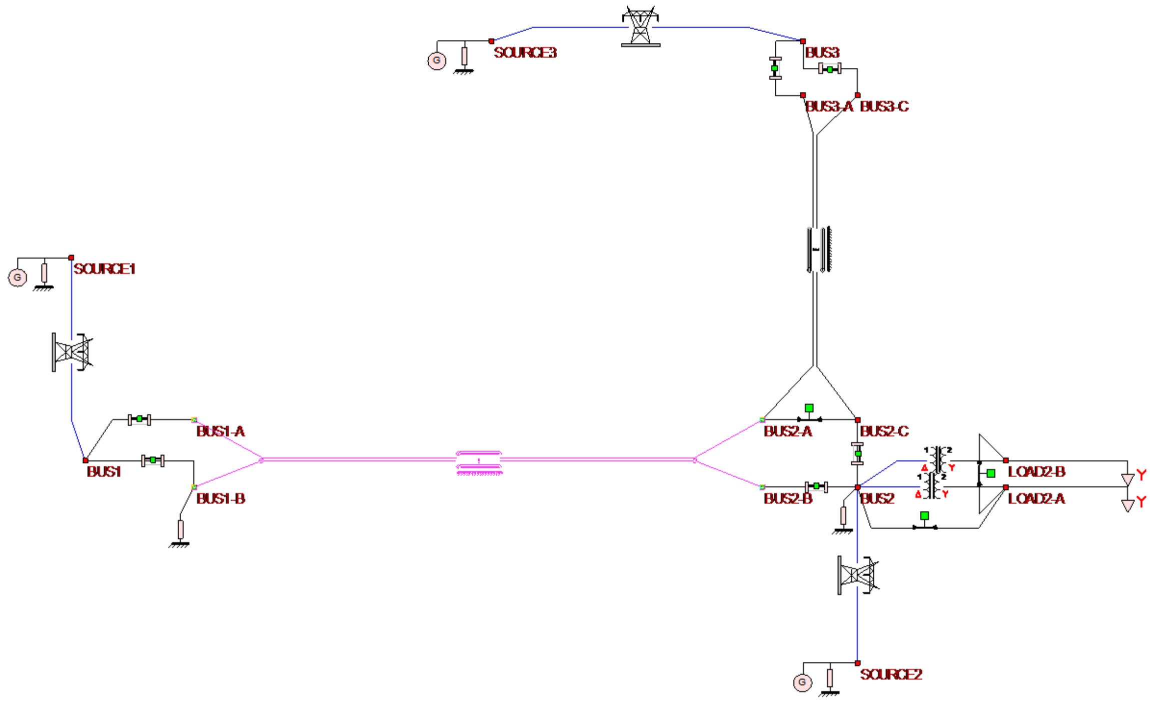
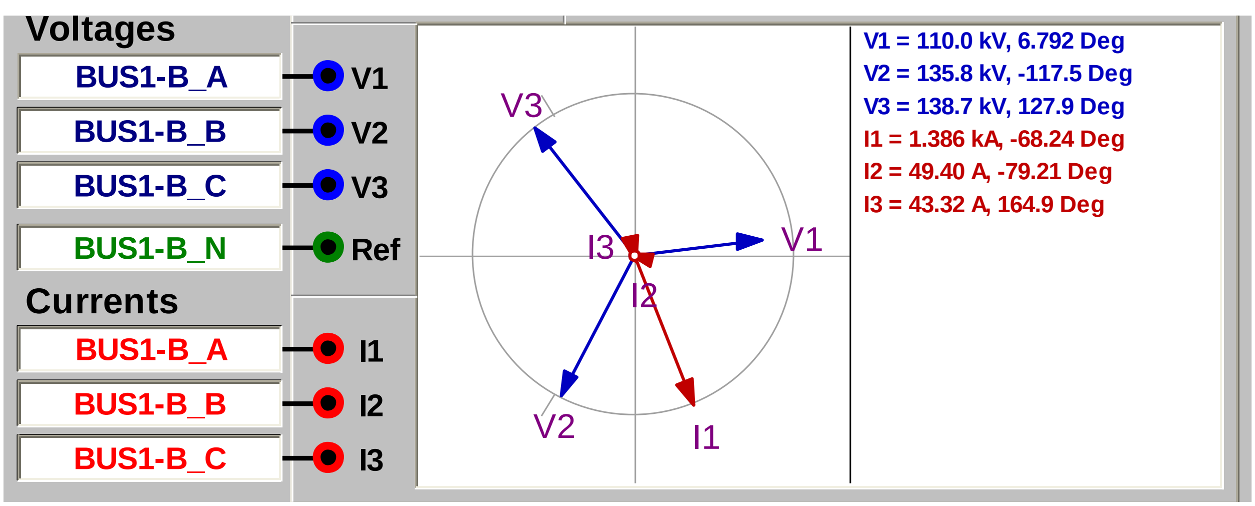
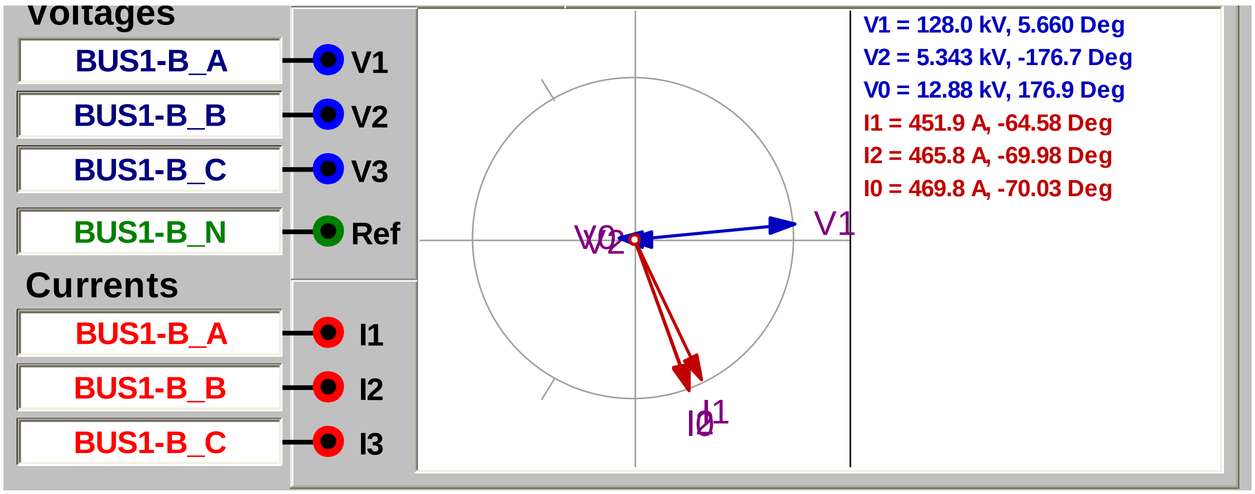


Figure E9-Yb: Topology of Mutually Coupled Transmission Lines


SOURCE3
BUS3
BUS3-A
BUS3-C
SOURCE1
BUS1-A
BUS1
BUS1-B
BUS2-A
BUS2-C
BUS2-B
BUS2
LOAD2-B
LOAD2-A
SOURCE2


Voltages
BUS1-B_A
V1
BUS1-B_B
V2
BUS1-B_C
V3
BUS1-B_N
Ref
Currents
BUS1-B_A
I1
BUS1-B_B
I2
BUS1-B_C
I3
V1 = 110.0 kV, 6.792 Deg
V2 = 135.8 kV, -117.5 Deg
V3 = 138.7 kV, 127.9 Deg
I1 = 1.386 kA, -68.24 Deg
I2 = 49.40 A, -79.21 Deg
I3 = 43.32 A, 164.9 Deg
V3
V1
I3
I2
V2
I1



Total Line Series Impedance & Shunt Admittance (Base 100.0 MVA)

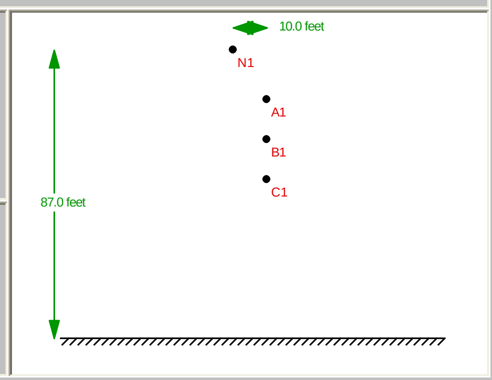
	Real / React (%)	Real / React	Magn / Phase
Z1 =	5.0292 / 43.4893	6.6512 / 57.5145 Ohms	57.8978 Ohms / 83.40 Deg
Z0 =	26.7548 / 159.0802	35.3833 / 210.3836 Ohms	213.3383 Ohms / 80.45 Deg
Y1 =	0.0010 / 3.3801	0.0001 / 0.2556 mMhos	0.2556 mMhos / 89.98 Deg
Y0 =	0.0013 / 1.7097	0.0001 / 0.1293 mMhos	0.1293 mMhos / 89.96 Deg

Surge Impedance

	Magn/Phase
Z1 =	475.96 Ohms / -3.29 Deg
Z0 =	1284.60 Ohms / -4.75 Deg
Surge Imp. Loading (MVA) :	27.79
Load Carrying Capability (A) :	1175.0

Other Parameters

Computed at Frequency (Hz):	60.00
Span Length (miles):	0.10
Soil Resistivity (ohm-meters):	100.00
Tower Ground Resistance (ohms):	25.00



Total Line Series Impedance & Shunt Admittance (Base 100.0 MVA)

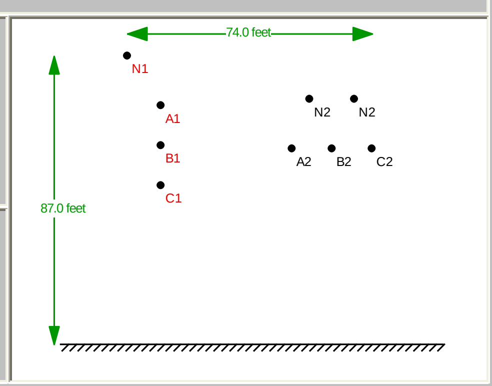
	Real / React (%)	Real / React	Magn / Phase
Z1 =	5.0299 / 43.4828	6.6520 / 57.5060 Ohms	57.8895 Ohms / 83.40 Deg
Z0 =	12.8346 / 110.2717	16.9738 / 145.8343 Ohms	146.8188 Ohms / 83.36 Deg
Y1 =	0.0010 / 3.3808	0.0001 / 0.2556 mMhos	0.2556 mMhos / 89.98 Deg
Y0 =	0.0013 / 1.8091	0.0001 / 0.1368 mMhos	0.1368 mMhos / 89.96 Deg

Surge Impedance

	Magn/Phase
Z1 =	475.87 Ohms / -3.29 Deg
Z0 =	1036.00 Ohms / -3.30 Deg
Surge Imp. Loading (MVA) :	27.79
Load Carrying Capability (A) :	1175.0

Other Parameters

Computed at Frequency (Hz):	60.00
Span Length (miles):	0.10
Soil Resistivity (ohm-meters):	100.00
Tower Ground Resistance (ohms):	25.00



Total Line Series Impedance & Shunt Admittance (Base 100.0 MVA)

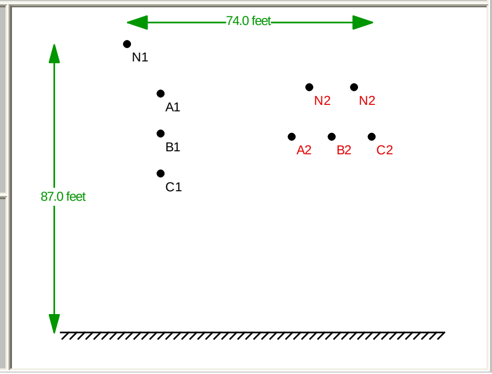
	Real / React (%)	Real / React	Magn / Phase
Z1 =	6.6630 / 44.4818	8.8118 / 58.8272 Ohms	59.4835 Ohms / 81.48 Deg
Z0 =	17.0545 / 109.3824	22.5545 / 144.6582 Ohms	146.4060 Ohms / 81.14 Deg
Y1 =	0.0013 / 3.2914	0.0001 / 0.2489 mMhos	0.2489 mMhos / 89.98 Deg
Y0 =	0.0046 / 1.9441	0.0003 / 0.1470 mMhos	0.1470 mMhos / 89.87 Deg

Surge Impedance

	Magn/Phase
Z1 =	488.88 Ohms / -4.25 Deg
Z0 =	997.96 Ohms / -4.36 Deg
Surge Imp. Loading (MVA) :	27.05
Load Carrying Capability (A) :	1175.0

Other Parameters

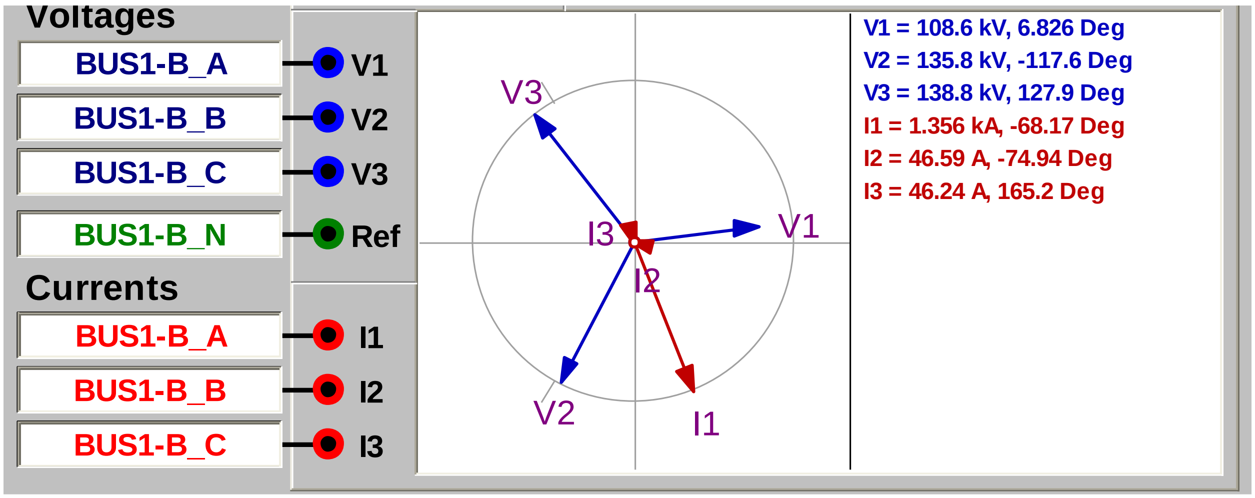
Computed at Frequency (Hz):	60.00
Span Length (miles):	0.10
Soil Resistivity (ohm-meters):	100.00
Tower Ground Resistance (ohms):	25.00

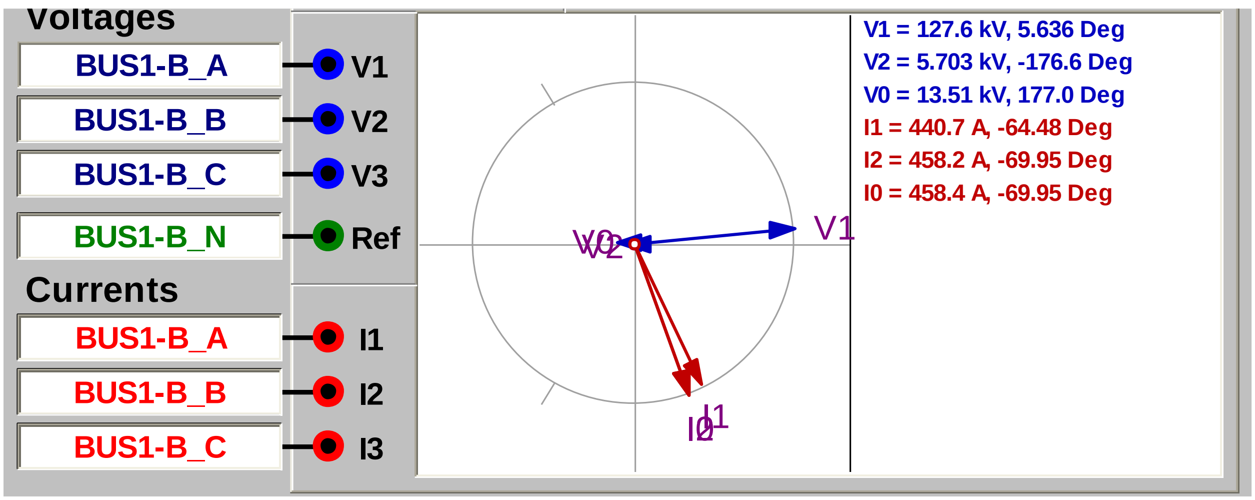


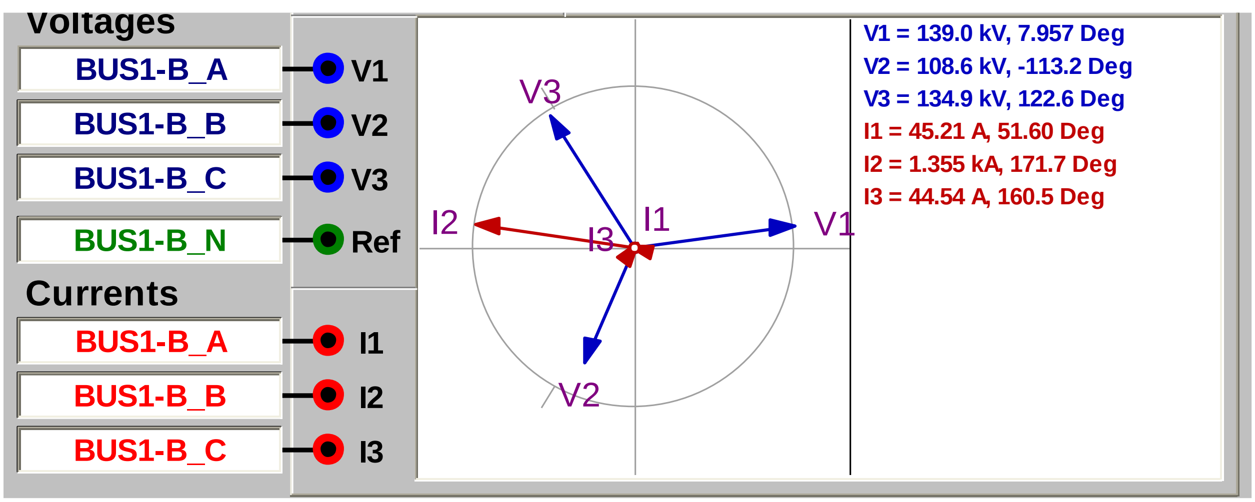
Total Line Zero Sequence Series Impedance & Shunt Admittance (Bases: 115.0 kV 100.0 MVA)

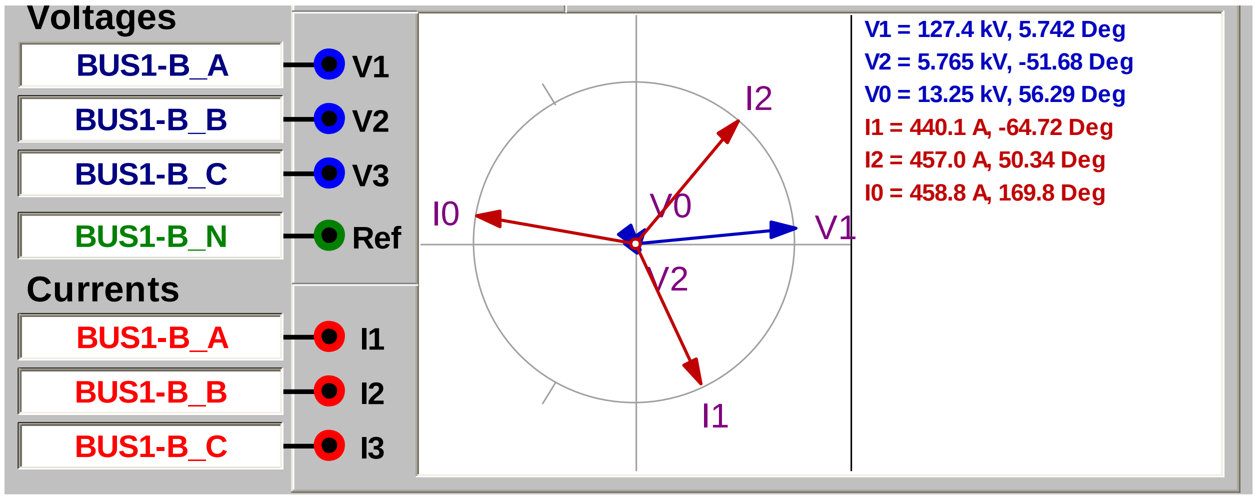
	Real / React (%)	Real / React	Magn / Phase
Z0 =	-0.2388 / -205.8164	-0.3158 / -272.1923 Ohms	272.1924 Ohms / -90.07 Deg
Y0 =	0.0014 / -0.2767	0.0001 / -0.0209 mMhos	0.0209 mMhos / -89.71 Deg

Other Parameters Computed at 60.00 Hz Soil Resistivity 100.00 Ohm-m


Voltages
BUS1-B_A
BUS1-B_B
BUS1-B_C
BUS1-B_N
V1
V2
V3
Ref
Currents
BUS1-B_A
BUS1-B_B
BUS1-B_C
I1
I2
I3
V1 = 108.6 kV, 6.826 Deg
V2 = 135.8 kV, -117.6 Deg
V3 = 138.8 kV, 127.9 Deg
I1 = 1.356 kA, -68.17 Deg
I2 = 46.59 A, -74.94 Deg
I3 = 46.24 A, 165.2 Deg


Voltages
BUS1-B_A
BUS1-B_B
BUS1-B_C
BUS1-B_N
V1
V2
V3
Ref
Currents
BUS1-B_A
BUS1-B_B
BUS1-B_C
I1
I2
I3
V1 = 127.6 kV, 5.636 Deg
V2 = 5.703 kV, -176.6 Deg
V0 = 13.51 kV, 177.0 Deg
I1 = 440.7 A, -64.48 Deg
I2 = 458.2 A, -69.95 Deg
I0 = 458.4 A, -69.95 Deg


Voltages
BUS1-B_A
BUS1-B_B
BUS1-B_C
BUS1-B_N
V1
V2
V3
Ref
Currents
BUS1-B_A
BUS1-B_B
BUS1-B_C
I1
I2
I3
V1 = 139.0 kV, 7.957 Deg
V2 = 108.6 kV, -113.2 Deg
V3 = 134.9 kV, 122.6 Deg
I1 = 45.21 A, 51.60 Deg
I2 = 1.355 kA, 171.7 Deg
I3 = 44.54 A, 160.5 Deg



9.8 Protection of Series Compensated Transmission Lines

The application of impedance relays may be compromised in lines with series compensation.

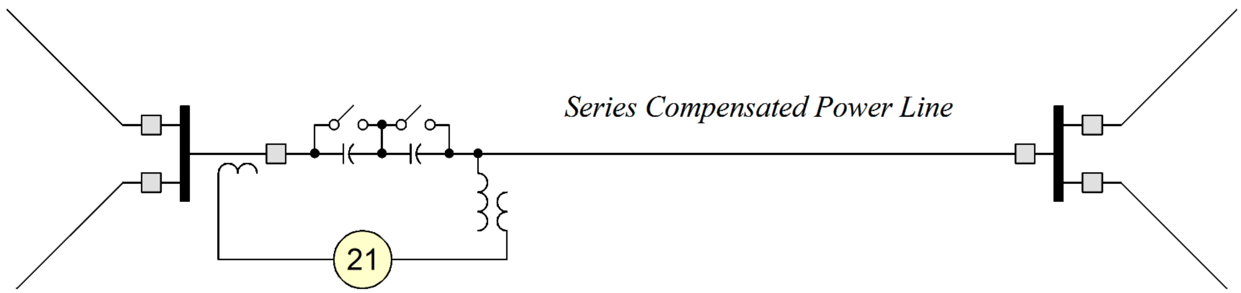


Figure 9.28: Example Series Compensated Transmission Line

Transmission lines are important components of modern power systems. With ever larger demand for power transmission, transmission systems are evolving with more complexity. One way to increase transmission capability and improve stability is to add Series Capacitors (SCs) to form series compensated lines. Appropriately designed, they can also reduce losses and damp sub-synchronous oscillations [1-2]. Because of limited right of ways, many transmission circuits share the right of way creating mutually coupled lines. The coupling can involve multiple circuits with different lengths of coupling for each circuit.

In series compensated lines, protection challenges are caused by the capacitive impedance of the series connected capacitors that interrupt the continuous inductive impedance of transmission lines. Figure 9.28 depicts an example series compensated transmission system, with SCs at one side of the line. Consider faults F_1 and F_2 at the two terminals of the series capacitor. A distance protective relay located at the right side terminal of the line will "see" these faults at different locations. For example, if the line is 50% compensated, the relay will see the fault F1 at the middle of the line and it will incorrectly trip the line while it will see the fault F2 at the end of the line. To avoid this type of mis-operation, the distance relay settings will be shortened thus desensitizing the relay and reducing protection.

Pilot distance protection scheme cannot properly protect series compensated lines for the following reasons that non-pilot distance protection has to be desensitized and pilot schemes depend on distance relays. Direction comparison scheme suffer from similar current inversion problems.

Line differential protection is one of the most effective protection schemes. However, there still exist some deficits. (a) current inversion in some situations will lead to mis-operation of differential protection; (b) series compensated EHV transmission lines are typically long with substantial charging currents that violate the differential protection principle.

An effective and reliable method to protect series compensated lines is the dynamic state estimation based protection.

To be completed.

9.9 Protection of Multi-Terminal Lines

To be completed.

9.9.1 In-Feed Issues

The application of impedance relays may be compromised in lines with taps due to the in-feed or out-feed problem. The in-feed problem is shown in Figure 9.29. Note that in this case the tapped line contributes to the fault current and makes the impedance "seen" by the relay to be greater than the impedance of the line itself.

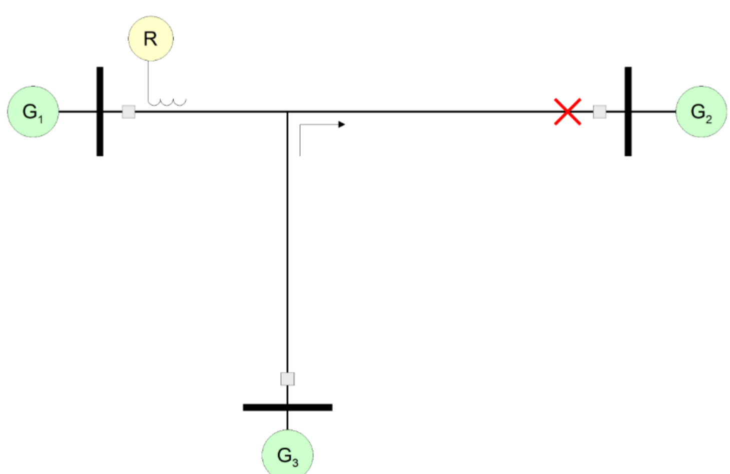


Figure 9.29: Illustration of the In-Feed Problem

9.9.2 Out-Feed Issues

The application of impedance relays may be compromised in lines with taps due to the out-feed problem. The issue of out-feed is shown in Figure 9.30. In this case the electric fault current that out-feeds into the tap line makes the relay "see" an impedance which is lower than the line impedance.

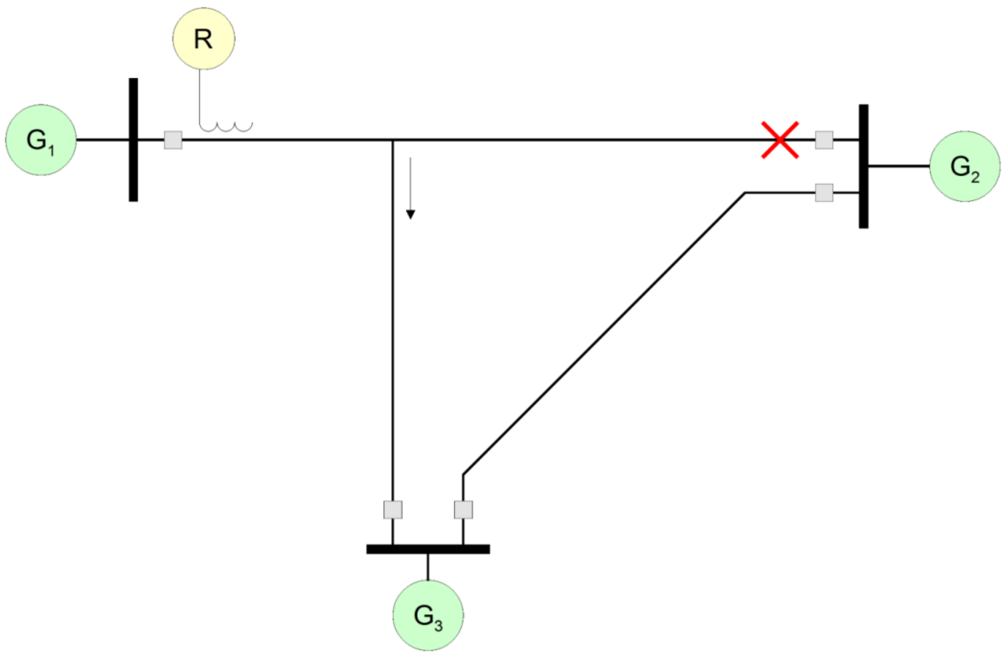


Figure 9.30: Illustration of the Out-Feed Problem

9.10 Protection of UG Transmission Circuits

To be completed.

9.11 Protection of DC Transmission Lines

To be completed.

The present MTDC lines are equipped with 'traveling wave protection (TWP)' as the primary protection and the line current differential protection as the backup protection. The TWP mainly adopts ABB and SIEMENS ideas: to determine whether there are any internal faults from the derivative of the voltage or the current. The line current differential protection is simply a sum of instantaneous currents among terminals of the line. However, the current differential protection is with a long delay (500 - 800 ms) to avoid misoperation during transients (since for DC lines the shunt capacitive currents could be very large during external faults). These protective relays have been installed in practical MTDC systems.

Further details as well as some considerations for practical systems are as follows.

Presently MTDC lines are implemented using VSC topologies instead of CSC topologies due to flexible power flow control, no commutation failure, etc. Among many VSC topologies, MMC topology could be more preferable due to lower switching frequency, lower harmonics, etc. There is now a four terminal MMC-MTDC system near Beijing, China (in the future the system may be expanded with more terminals). The main part of the construction is finished at the end of 2019.

There are two main view points among researchers on how to protect the DC transmission lines in MTDC grids. The first view point is to apply controls to the converters/shut down the faulted pole/shut down the converter station/open the AC side circuit breaker when DC lines are with faults. In fact, these ideas have be applied to two terminal DC lines. However for MTDC grids, the main problem is that the power outage zone will not be minimized with these measures (since faults in MTDC lines may cause overcurrent/undervoltage issues in many converter stations, and in MTDC grids one converter station is connected to multiple transmission lines). Therefore, the second view point might be more preferable: similar as AC grids, one can install DC circuit breakers on terminals of every transmission line and use protective relays to protect each DC transmission line (the development of DC circuit breakers has made it possible). In fact, the practical four terminal MMC-MTDC system in China is using the aforementioned "second view point".

However, protective relaying in MTDC grids is quite challenging since the fault current can rise extremely fast during the fault and the time left for protective relays is very limited (otherwise the DC breakers cannot interrupt such a large fault current and fault current may cause damages of the power electronics switches). Therefore in practice, inductors are usually installed at terminals of the DC lines or inside converter stations to limit the fault current (but the inductors should not be too large to jeopardize the dynamics of the MTDC system). Even so, the time left for protective relays is limited. In the four-terminal MTDC system in China, the system asks the relay to determine the fault within 3 ms after the occurence of the fault and give the DC breaker another 3 ms to interrupt the fault current (6ms in total). That is why protective relays based on local information are very attractive for primary protection of the line. Nevertheless, there are still issues such as limited sensitivity of the relays towards high impedance internal faults if the TWP is utilized, and selectivity issues if the current limiting inductor is too small, etc. Therefore, the protective relaying for MTDC grids is still a hot ongoing research topic.

9.12 Power Line Setting-less Protection

The discussion of the various line protection schemes shows that: (a) the schemes ca be very complex leading to human error an (b) there are cases that the schemes may fail to operate as intended. These two reasons lad to a substantial amount of relay mis-operations. Recent research has introduced a new method for line protection, the dynamic state estimation based protection, a.k.a. setting-less protection.

Dynamic State **E**stimation **B**ased **P**rotection (EBP) (also known as setting-less protection) simplifies the settings of the protection, and does not need coordination with other protection functions. The main idea of this scheme is to monitor the consistency between the measurements and the dynamic model by the health of the components, or the confidence level. Here the dynamic model is built by finding all physical laws that the measurements must satisfy. The widely used current differential protection is a special case of EBP, whose dynamic model describes the zero sum of two-side currents by Kirchhoff's current law. It's also intuitive that the protection will be more effective by combining all physical laws (e.g. Kirchhoff's current law, Kirchhoff's voltage law, Faraday's law, etc.).

To be illustrated with an example. To be added.

9.13 Advanced Fault Locating in Transmission Lines

Accurate fault locating on transmission lines is quite valuable for operators and utility crews, since the amount of time spent searching for the fault can be minimized. Thus, the outage time, labor and operating costs can be reduced. Legacy fault locating techniques can be mainly classified into two groups: fundamental frequency phasor based methods and traveling wave based methods. Details of these methods are introduced next.

Impedance relays are based on the computation of the distance of a fault from the relay location. The primary use of this information is to decide on protective action, i.e. to trip or not to trip the appropriate breaker(s). However, this information is also useful for the purpose of finding the fault location and performing the necessary repairs. In particular, microprocessor implementations of distance relays can provide this information on a display or send it to the control center without substantial additional expense.

The problem of fault locating in power circuits has been long ago recognized as an important one for two reasons: (a) minimization of downtime by quick repair and therefore increased system reliability (especially for cable circuits) and (b) improved selectivity of protection schemes by virtue of knowledge of fault location (for example distance relays are based on evaluation of the fault distance). Recent trends towards automation have accentuated the importance of fault locating. Over the years, several technologies for fault locating have been developed. These technologies can be categorized into the following:

a) Methods based on audible noise of fault recreation (thumpers), this method requires an actual crew to go on location and perform the tests,
b) Methods based on strategic placement of faulted circuit indicators, this method requires either visual inspection f the faulted circuit indicator, or if the FCI are equipment with communications, to bring all the information to a central location and determine the location by analysis,

c) Detection and measurement of travel time of first transient, this method required equipment that can record waveforms with very high sampling rates (recall that transients travel with close to the speed of light and in order to measure the travel time it is necessary to capture the transients with sufficient time resolution), and
d) Methods based on estimation of the circuit impedance to the fault and extraction of the fault location from the known impedance values per unit length

The first two methods are extensively used in distribution circuits, especially on URD cable systems. The thumper technology consists of injecting an impulse to the faulted cable. The fault in the cable is reignited under the impulse and the generated noise is utilized to determine the location of the fault. Application of the thumper requires that the cable is out of service and in general it is time consuming. A criticism of the thumper technology is that it subjects the cable to additional surges and therefore may affect the life of the cable.

The faulted circuit indicators are devices which are triggered by the flow of the fault current. Basically, a faulted circuit indicator is a two state device: state one is normal and state two indicates that an electric current above a threshold value has been detected. Application of many faulted circuit indicators at strategic locations along a circuit, i.e. one at each transformer, provides means for determining the location of the fault between two locations. Models with manual or automatic reset are available. Also models with/without communications are available.
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The third method is based on specialized hardware that record the travel time to the fault and extract the distance to the fault from the known speed of propagation of EM waves on the circuit. Specifically, the travel time from the monitoring location to the fault is measured with rather sophisticated hardware. For a given circuit (circuit parameters) the speed of propagation of surges along the circuit is known and is utilized to estimate the location of the fault. This technology is complex, requiring sophisticated and expensive hardware.

The forth method requires recording of voltages and currents at any location along the faulted circuit. From the recorded voltages and currents and the known impedance per unit length of the system, the distance to the fault can be estimated. The introduction of numerical relays and digital fault recording equipment have made this method very attractive. Specifically, the mentioned equipment provides recordings of the voltage and current during fault. This data can be processed in the relay or fault recording equipment to provide the fault location. In addition this recordings are typically stored and/or can be transmitted to central locations via a variety of communication media, i.e. telephone, fiber, microwave, etc. The data can be processed at the central location to estimate the distance to the fault. This approach provides the additional benefit that recordings from multiple devices can be used to estimate the fault location thus increasing the accuracy of the fault locating method. There are many ways to estimate the fault location from the recorded voltages and currents. This section reviews the existing methods and provides a discussion of the issues and limitations of the approach.

The basic fault locating method is explained with the aid of Figure 10.x. A relay, disturbance recorder, etc. is placed at one end of the circuit. This device records the voltage and current at that location of the circuit.

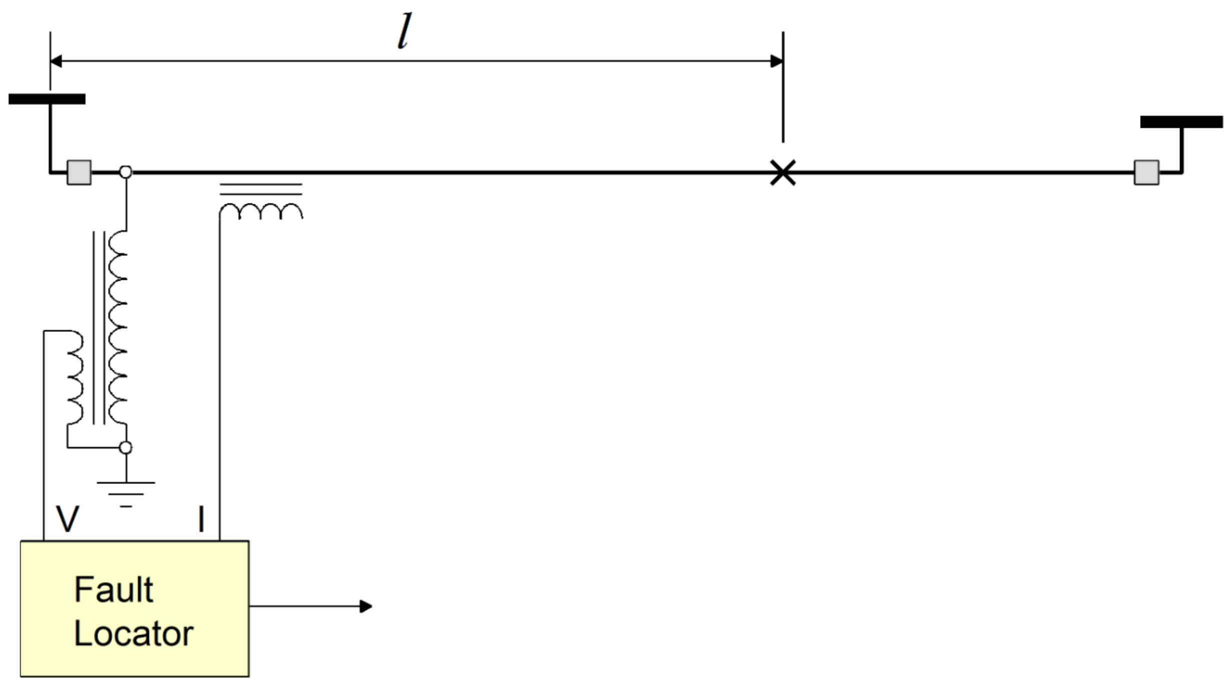


Figure 9.31: Principle of Fault Locating From Circuit Impedance to the Fault

Assume that a fault occurs at a point located ℓ meters away from the recording device. During the fault conditions, the recorded voltages and currents are processed to obtain the phasors $\tilde{V}$ *and* $\tilde{I}$ of the voltage and current. The impedance "seen" at the location of the recorder is:

$$Z_f = \frac{\tilde{V}}{\tilde{I}} = z\ell$$

where z is the circuit impedance per unit length.
Thus:

$$\ell = \frac{Z_f}{z} = \frac{\tilde{V}}{z\tilde{I}}$$

The above approach is simplistic and it is valid for simple circuits with specific impedance per unit length. In reality, circuits are much more complicated, i.e. three phase circuits with 3, 4 or 5 wires, or single phase circuits with multiple fault current return paths, such as neutrals, ground wires and soil. We will examine the application of above principle to typical power circuits.

Assume that fault data has been recorded at Bus 1 of a line of total length L during a fault at some point of the line (ℓ miles from Bus 1) as it is illustrated in Figure 9.x. The fault recorder or relay (DFR) has captured the voltage and current waveforms at all three phases at the Bus 1 terminal.

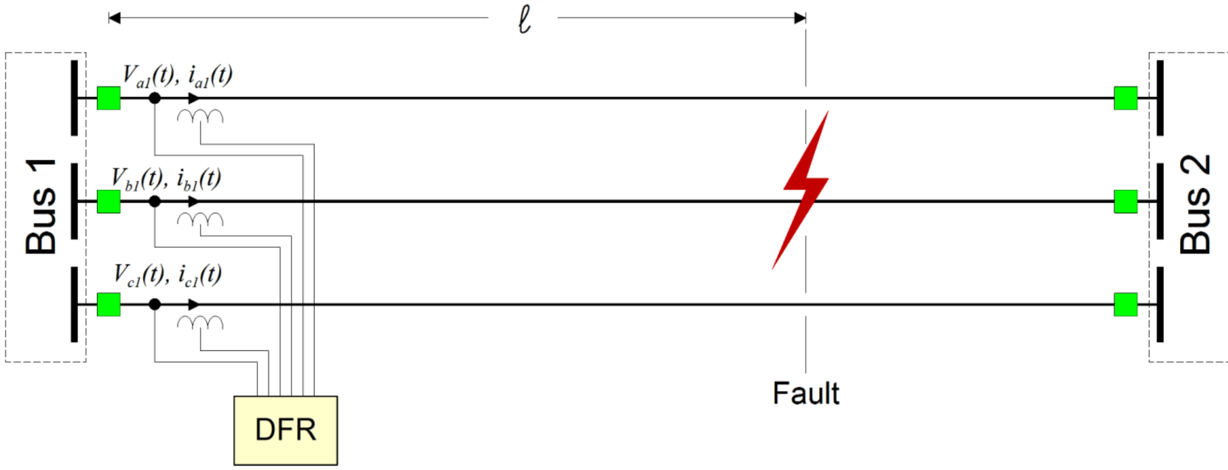


Figure 9.32: Illustration of a Faulted Circuit with Single End Monitoring

The recorded voltage at the faulted phase is:

$$\tilde{V}_f = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)\ell + v_{arc}$$

expressing the arc voltage as a conductance times the fault current yields:

$$\tilde{V}_f = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)\ell + g\tilde{I}_f$$

Splitting real and imaginary parts:

$$V_{fr} = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)_r\ell + g\tilde{I}_{fr}$$
$$V_{fi} = (x_{fa}\tilde{I}_a + x_{fb}\tilde{I}_b + x_{fc}\tilde{I}_c)_i\ell + g\tilde{I}_{fi}$$

Upon solution of above equations for the distance to the fault:
Note that in this case the fault arc voltage is also computed.

Fundamental frequency phasor based methods

These methods use voltage and current phasors of fundamental frequencies to calculate the impedance and afterwards the distance between the fault location and the terminals of the line [1-3]. They can be further classified into single-ended and dual-ended algorithms. (a) **Single-ended algorithms** calculate the distance to the fault from voltage and current phasors at one end. The main advantage is that it does not require communication channels from the remote end. The accuracy of these methods largely depends on the fault type and fault impedance, as well as the line model accuracy, presence of mutually coupled circuits, and grounding parameters of the line [4-5]. (b) **Dual-ended algorithms** use voltage and current phasors at both ends of the line. They can be further subdivided into methods that use GPS synchronized measurements or non-synchronized measurements. These methods are in general more accurate than single ended methods as the influence of fault impedance and mutually coupled circuits is mitigated. [6-7].

There is a variety of algorithms depending on the available data, for example, dual-ended voltages and single-ended currents, dual-ended voltages and currents [8-10]. The main disadvantages of fundamental frequency phasor based single-ended or dual-ended methods are as follows. First, the algorithms are based on fundamental frequency phasors, so fault locating results may not be accurate if the system is experiencing transients. Second, some of these methods utilize a sequence line model (positive, negative and zero sequence) which is an approximate model and increases the fault locating error. Third, the grounding of the line is typically neglected and this can potentially generate larger errors depending on the fault type.

Traveling wave based methods: When a fault initiates in a transmission line, high frequency traveling waves are generated and propagate away towards both terminals of the line approximately with the speed of light. At each discontinuity, for example a bus with multiple lines, the fault, etc., they reflect and transmit generating more traveling waves. Traveling wave based methods [11-12] monitor the traveling waves and the traveling time of these waves to determine the fault location. They can be classified into two groups: single-ended algorithms and dual-ended algorithms. (a) **Single-ended algorithms** use subsequent time signals at one terminal to identify the fault location, including Type A, C, E and F fault locator [13-15]. **Type A** measures the time difference between the arrival of the first wavefront and its reflection; **Type C** injects a high frequency impulse from one terminal and measures the time difference between the arrival of the reflection and the injection; **Type E** and **F** are quite similar with Type C expect that the injected impulses are generated by closing/opening the circuit breaker at one terminal. (b) **Dual-ended algorithms** use the time differences between the arrival of first wavefront at both terminals to identify fault location, including Type B and D fault locator [14, 16-17]. **Type B** measures the time differences by sending stop signals from one end to another; **Type D** is quite similar with Type B except using synchronized time tagging to increase accuracy. Because the traveling waves are modulated by the frequency dependent characteristics of the line, digital signal processing is required to identify the actual arrival time of the wave. Different methods are pplied for this, including Wavelet Transformation (WT) [17-19]. Traveling wave based methods have the following disadvantages. First, the intensity of traveling waves is greatly influenced by the fault initiation time relative to the power frequency cycle. This generates the issue of detection reliability of the traveling waves. Second, they require special instrumentation since the usual CTs will filter out the high frequency content of the traveling waves. Related to this issue is the need to sample at very high rates. Third, the accuracy of the fault locator is dependent upon the sampling rate of the measurements [20]. For example, a system with a 100 kilo-samples per second (ks/s) sampling rate, could cause up to 0.93 miles systematic error with Type D fault locator.

We present a method that uses existing instrumentation, CTs and VTs and GPS synchronized sampled values with improved accuracy in locating faults on transmission lines. The method uses a high fidelity line dynamic model (physically based three phase model with explicit grounding representation), and dynamic state estimation. We named this method the **EBFL** (Dynamic State **E**stimation (**DSE**) **B**ased **F**ault **L**ocating). The main idea of EBFL is to treat the fault location as a parameter of the dynamic line model and estimate the fault location via a DSE algorithm [21-22]. The paper describes the basics of the method, the line dynamic model and presents numerous numerical experiments for quantifying the performance of the method. While the method is equally applicable to cable transmission systems, the discussions is limited to overhead lines.

9.14 Summary and Conclusions

To be completed.

9.15 Problems

Problem P9.1: An impedance relay is used to protect the 115 kV transmission line illustrated in Figure P9.1 with the following phase conductor parameters: Radius = 0.691 inches, Geometric Mean Radius = 0.0456 feet, and Resistance = 0.085 ohms per mile. The length of the line is 45 miles.

The CT ratio of the impedance relay is 2000:5 and the VT ratio is 69kV:69V. The relay engineer decides that the impedance relay will be set as follows: (a) zone 1 will be set to reach 80% of the length of the line, (b) zone 2 will be set to reach 135% of the line and (c) zone 3 will be set to reach 100% of the line plus 125% of the next line. The longest length of the next line is 35 miles and the design is identical to the protected line.

Compute the settings of the impedance relay in ohms (on the relay side).

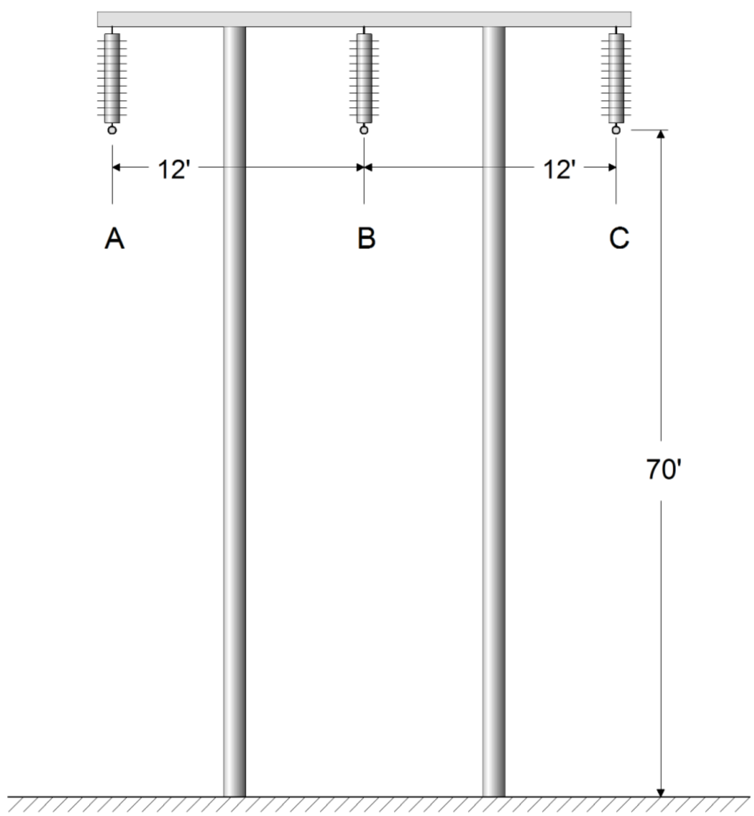


Figure P9.1: 115 kV Line Configuration

Solution: Line impedance:

$$z_s = 0.185 + j1.3756 \quad ohms/mile$$

$$z_m = \frac{1}{3}(z_{ac} + z_{ab}) = 0.1 + j0.6708\, ohms/mile$$

$$z_1 = z_s - z_m = 0.085 + j0.7048\, ohms/mile$$

$$z_{tot} = 45 z_1 = 31.95 e^{j83.12^0} \; ohms$$

Thus the impedance settings on the power line side are:

$$z_{zone1} = 25.56 e^{j83.12^0} \; ohms$$

$$z_{zone2} = 43.13 e^{j83.12^0} \; ohms$$

$$z_{zone3} = 63.01 e^{j83.12^0} \; ohms$$

The relaying settings on the relay (secondary) side are computed as follows. First the transformation of the impedance through the instrument transformers is computed:

$$z_{relay} = \frac{\dfrac{69}{69{,}000}}{\dfrac{5}{2000}} z_{line} = 0.40 z_{line}$$

Then the relay settings on the secondary side are:

$$z_{relay,zone1} = 10.220 e^{j83.12^0} \; ohms$$

$$z_{relay,zone2} = 17.252 e^{j83.12^0} \; ohms$$

$$z_{relay,zone3} = 25.204 e^{j83.12^0} \; ohms$$

Problem P9.2: A distance relay is used to protect the 42.5 mile-long 115 kV transmission line illustrated in Figure P9.2. The conductor parameters can be found in the data base of the program WinIGS. The line is connected to a three phase source with the following parameters:

Rated Power: 100 MVA, Rated Voltage: 115 kV,
$Z_1 = j0.035\; pu, \quad Z_2 = j0.035\; pu, \quad Z_0 = j0.04\; pu$

The CT ratio of the impedance relay is 2000:5 and the VT ratio is 69kV:69V. The relay engineer decides that the impedance relay will be set as follows: (a) zone 1 will be set to reach 80% of the line, (b) zone 2 will be set to reach 135% of the line and (c) zone 3 will be set to reach 100% of the line plus 125% of the next line. The longest length of the next line is 35 miles and the design is identical to the protected line.

(a) Compute the settings of the impedance relay in ohms (rounded to the first decimal point) and the phase of the impedance in degrees (rounded to integer number of degrees). Select the compensation factor (rounded to the first decimal point) and the time delays for each zone. For these computations assume soil resistivity of 100 ohm.meters.

(b) If the soil resistivity varies from 10 ohm.meters to 3000 ohm.meters, what will be the difference in distance calculations when the actual line to ground fault is at a distance of 34 miles from the relay. Hint: determine what the relay will compute as "distance to fault" when the soil resistivity is 10, 500 and 3000 ohm.meters and graph the results.

The use of program WinIGS is encouraged in solving this problem.

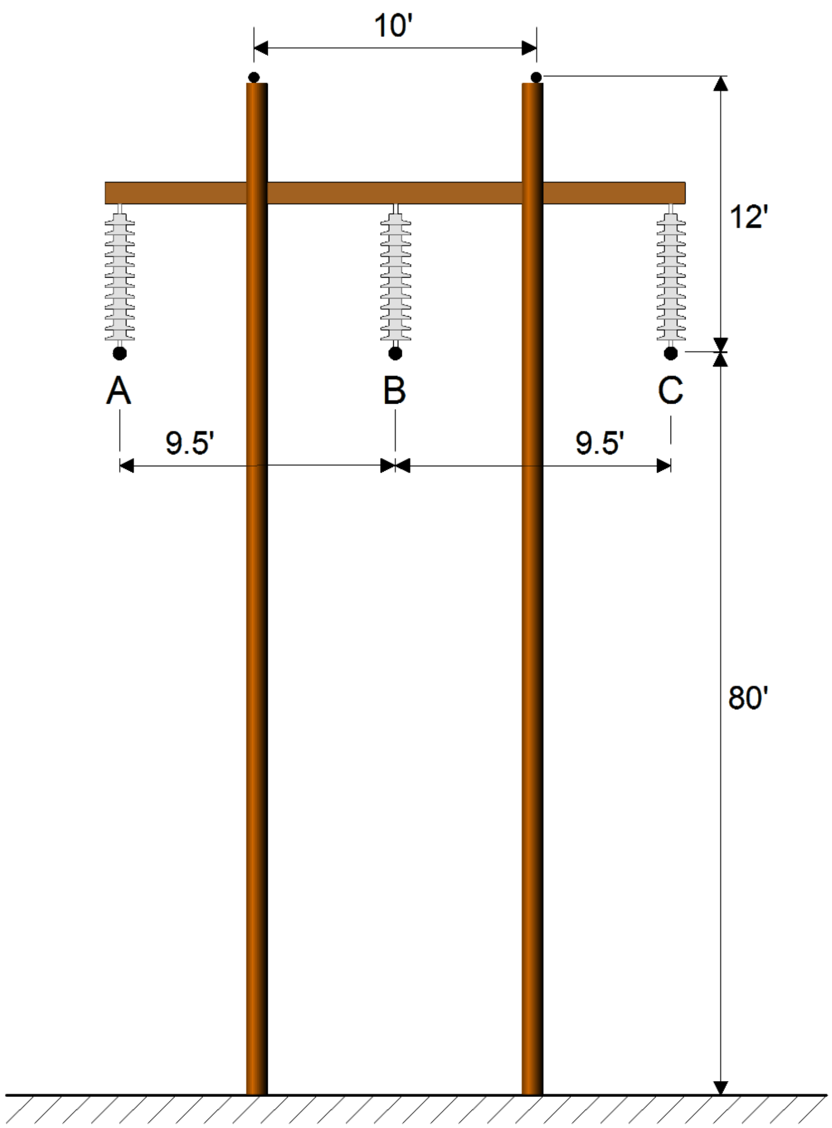
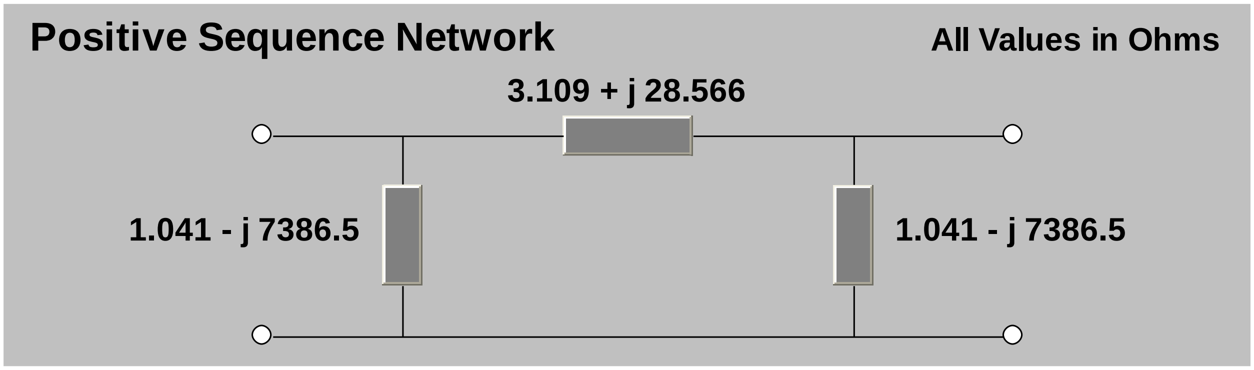
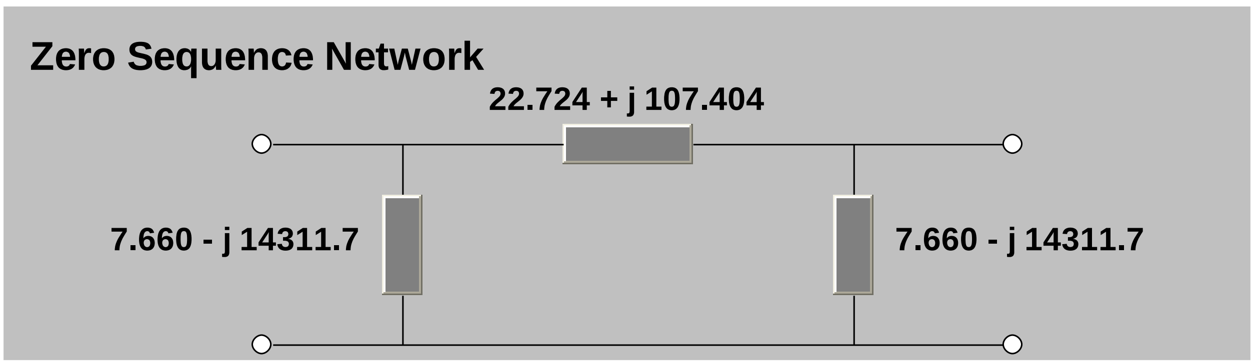


Figure P9.2: 115 kV Overhead Transmission Line
(Phase Conductors: ACSR, BITTERN, Shield Wires: HS, 5/16, tower ground resistance=0.25xsoil resistivity, Span length=0.075 miles)

Solution: (a) First the system is modeled in WinIGS format and the sequence impedances of the line are:





The compensation factor is:

$$m = \frac{22.724 + j107.404 - 3.109 - j28.566}{3.109 + j28.566} = 2.8236e^{-j7.752^0}$$

The relay settings are:

Zone 1: impedance setting: $z_1 = (0.8)(28.7719e^{j83.78^0})\left(\frac{0.001}{0.0025}\right) = 9.207e^{j83.78^0}\ ohms$. Select 9.2 ohms, 84 degrees, compensation factor=2.8, and three cycle delay.

Zone 2: impedance setting: $z_1 = (1.35)(28.7719e^{j83.78^0})\left(\frac{0.001}{0.0025}\right) = 15.5368e^{j83.78^0}\ ohms$. Select 15.5 ohms, 84 degrees, compensation factor=2.8, and 15 cycle delay.

Zone 3: impedance setting: $z_1 = (2.25)(28.7719e^{j83.78^0})\left(\frac{0.001}{0.0025}\right) = 25.895e^{j83.78^0}\ ohms$. Select 25.9 ohms, 84 degrees, compensation factor=2.8, and thirty cycle delay.

(b) Using the WinIGS model, the fault currents were computed for the phase to ground fault (on phase A) at a distance of 34 miles from the relay and for the following soil resistivities: 10, 100, 500 and 3000 ohm.meters. The resulting distances (or impedances computed by the relay are:

10 ohm.meter case: $z = \dfrac{\tilde{V}_{an}}{\tilde{I}_a + m\tilde{I}_z} = \dfrac{59.51e^{-j1.11}}{1.440e^{-j80.49} + (2.8)0.486e^{-j80.61}} = 21.25e^{j79.44^0}\ ohms$

On relay side: $z = 8.5e^{j79.44^0}\ ohms$. This point is in zone 1 and the distance is 31.39 miles.

100 ohm.meter case: $z = \dfrac{\tilde{V}_{an}}{\tilde{I}_a + m\tilde{I}_z} = \dfrac{60.12e^{-j1.21}}{1.323e^{-j78.57} + (2.8)0.447e^{-j78.74}} = 23.35e^{j77.44^0}\ ohms$

On relay side: $z = 9.34e^{j77.44^0}\ ohms$. This point is in zone 2 and the distance is 34.49 miles.

500 ohm.meter case: $z = \dfrac{\tilde{V}_{an}}{\tilde{I}_a + m\tilde{I}_z} = \dfrac{60.54e^{-j1.35}}{1.246e^{-j76.33} + (2.8)0.422e^{-j76.55}} = 24.94e^{j75.09^0}\ ohms$

On relay side: $z = 9.976e^{j75.09^0}\ ohms$. This point is in zone 2 and the distance is 36.84 miles.

3000 ohm.meter case: $z = \dfrac{\tilde{V}_{an}}{\tilde{I}_a + m\tilde{I}_z} = \dfrac{61.08e^{-j1.67}}{1.161e^{-j71.57} + (2.8)0.393e^{-j71.87}} = 27.01e^{j70.04^0}\ ohms$

On relay side: $z = 10.804e^{j70.04^0}\ ohms$. This point is in zone 2 and the distance is 39.89 miles.

Problem P9.3 Consider the electric power system of Figure P9.3a. Each transmission line is protected with two three-zone mho relays (one at each terminal of the line). All indicated lines are identical as it is illustrated in Figure P9.3b. The two parallel lines are at a distance of 107 feet center to center. The lengths of the lines are: 72 (line on the left hand side), 58 (the two parallel lines), 72 and 82 miles respectively. The operating voltage of the system is 230 kV. The transformation ratio of the CTs is 2000:5 while the ratio for the potential transformers is 135,000:115.

(a) Determine the settings for the three zones of the mho relay R_1. Use: zone 1: 80%, zone 2: 125%, zone 3: line length plus 125% of next line.

(b) For a single line to ground fault (SLGF) at the indicated location determine whether the relay R_1 will trip the breaker. The SLGF is located 51 miles from relay R_1.

(c) The 10 minute emergency rating of line 2 is 1600 A. In a specific emergency condition, the line is loaded at 1600 A and the voltage at the location of relay R_1 has dipped to 0.92 pu. Determine whether the mho relay will trip in this case. The load of the line has a power factor of 0.90 (current lagging).

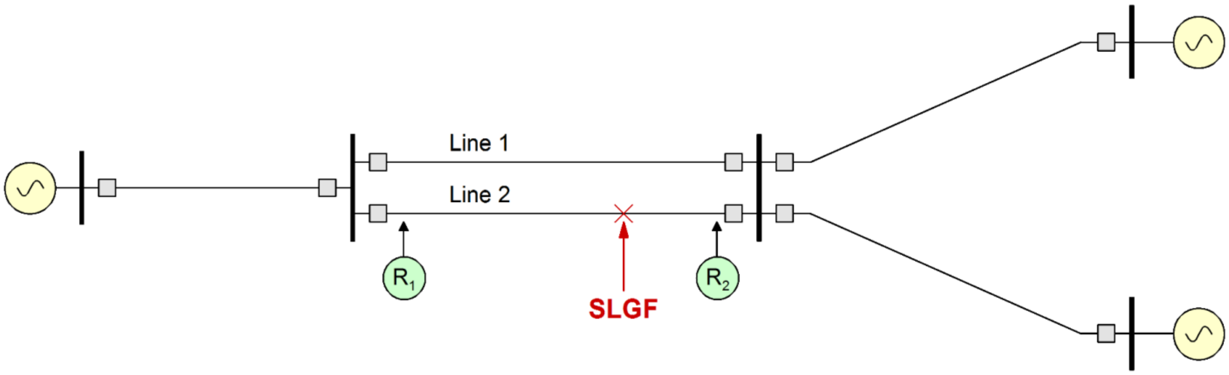


Figure P9.3a

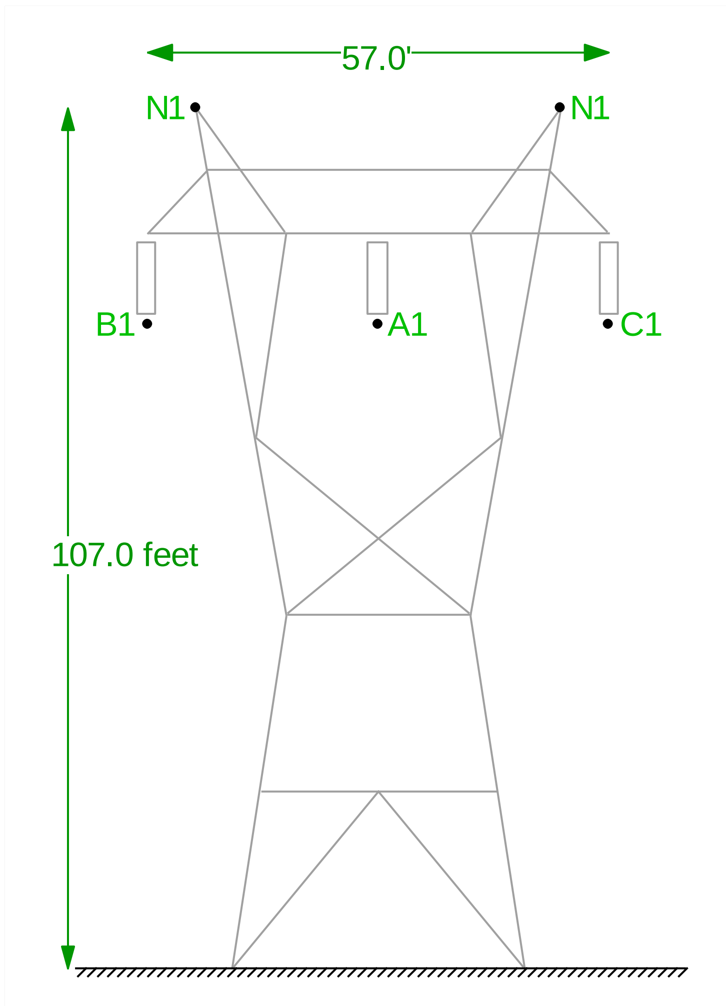
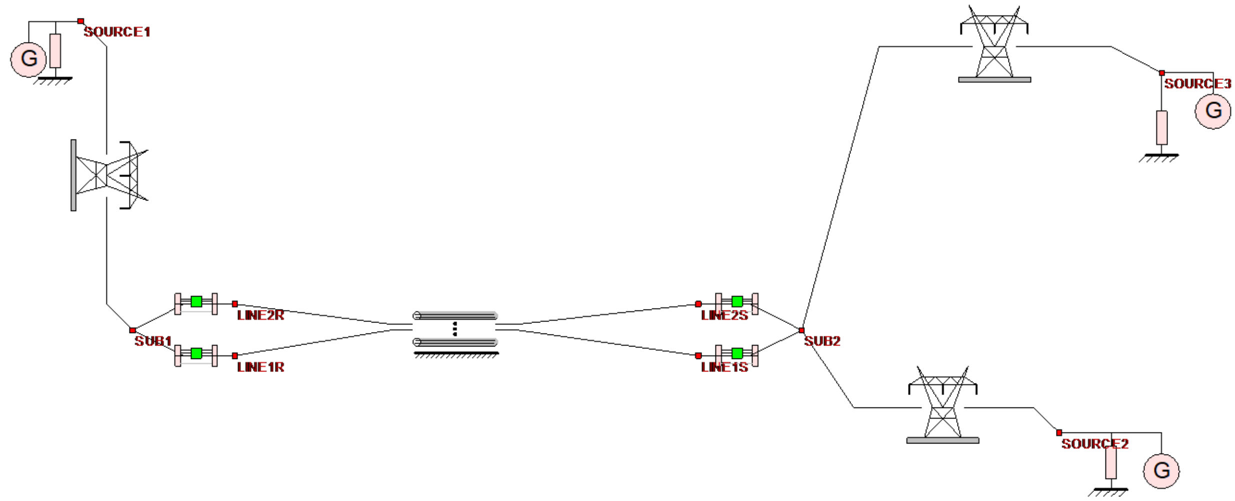
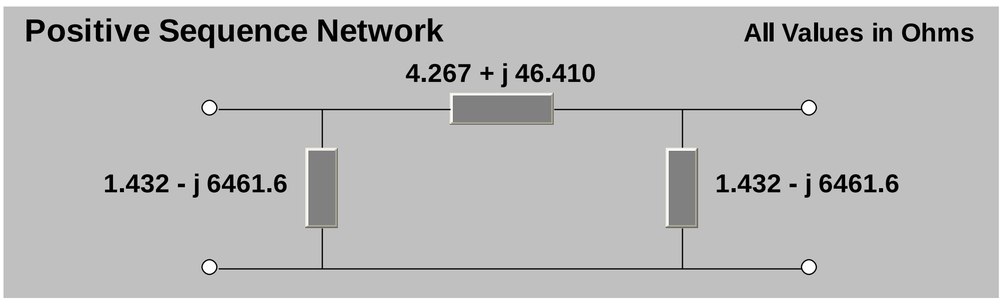


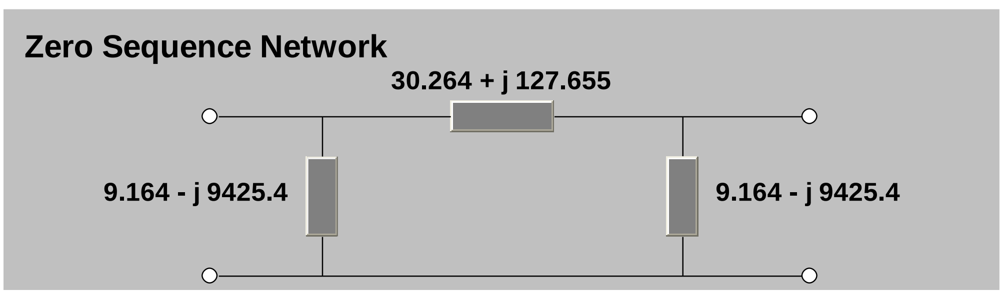
Figure P9.3b
(Phase Conductors: ACSR, BITTERN, Shield Wires: HS, 5/16)

Solution: This problem is solved with the help of the program WinIGS. The system is modeled and the WinIGS model is shown in the figure below.

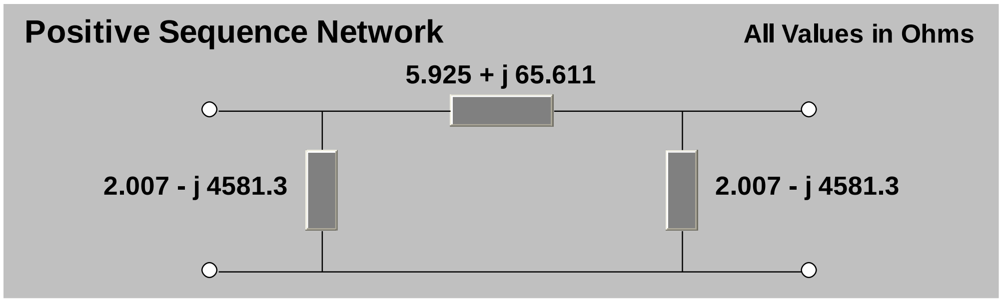


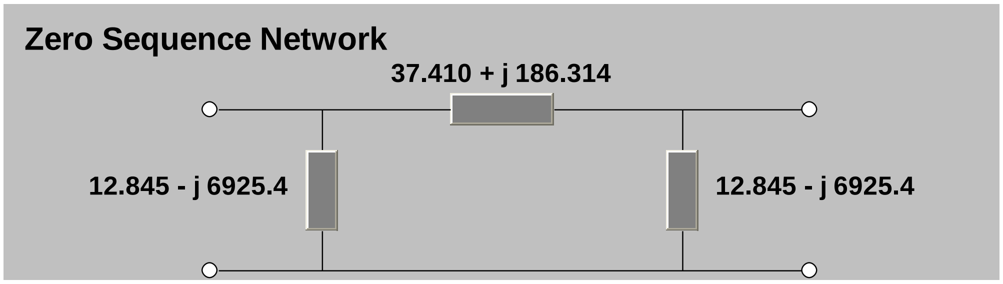
The sequence parameters of the line are:



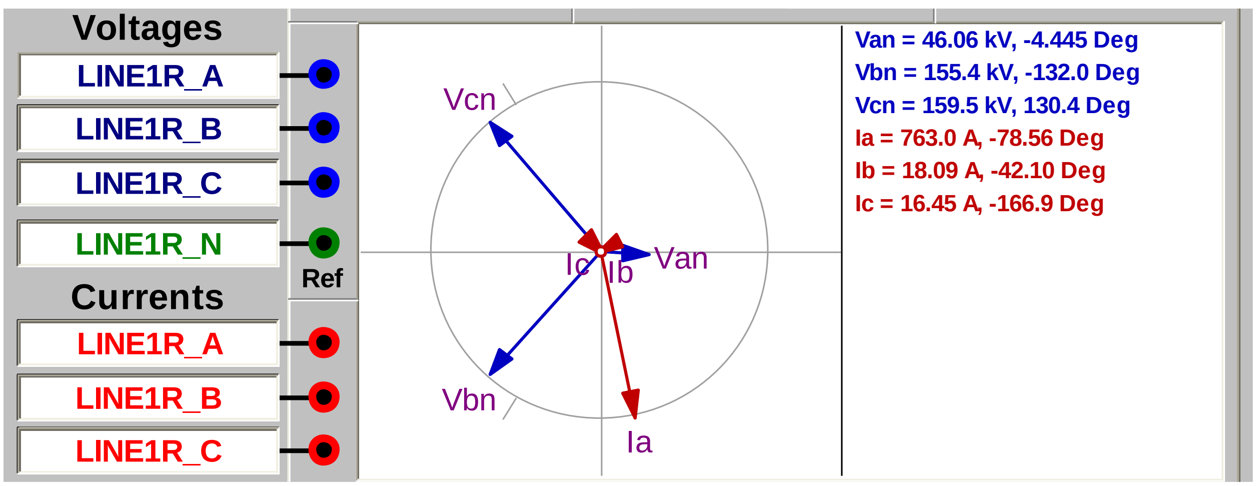


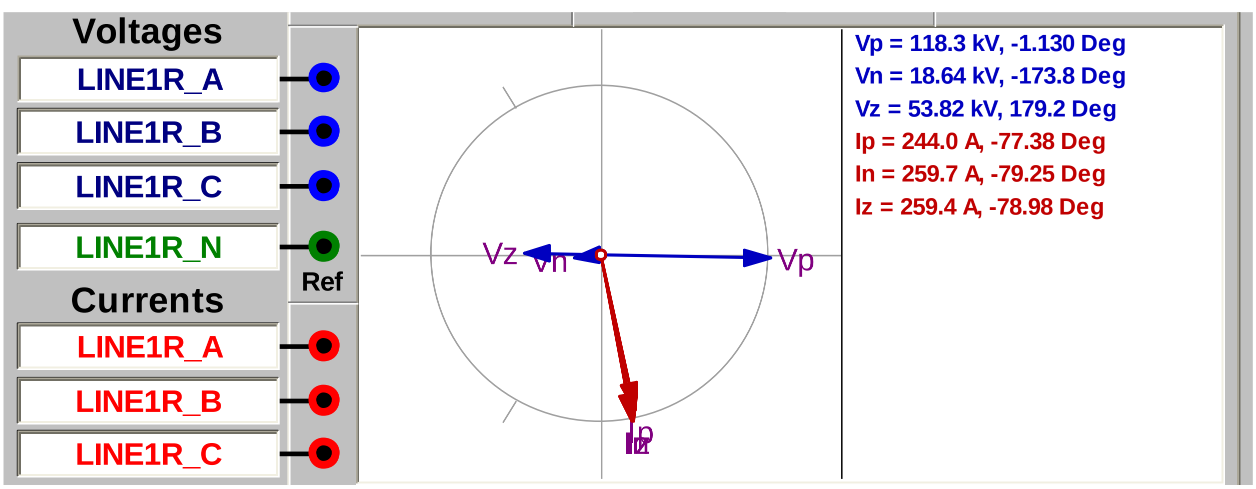
The settings are:
Zone 1: 37.28 ohms on line side, 12.7043 ohms on relay, select 13 ohms. Phase angle is 84.7469 degrees, select 85 degrees. Delay: Use: 4 cycles. Compensation factor is 1.815, select m=1.8
Zone 2: 58.25 ohms on line side, 19.8505 ohms on relay, select 20 ohms. Phase angle is 84.7469 degrees, select 85 degrees. Delay: Use: 15 cycles. Compensation factor is 1.815, select m=1.8.
Zone 3: 46.6057+(1.25)(65.878)=128.9532 ohms on line side, 43.9393 ohms on relay, select 45 ohms. Phase angle, select 85 degrees. Delay: Use: 30 cycles. Compensation factor is 1.815, select m=1.8. The impedances of the next longest line are given below.





(b) The fault analysis for this case is shown below.





The impedance seen by the relay will be:

$$(2.9348)Z_{relay} = \frac{\tilde{V}_a}{\tilde{I}_a + (1.8)\tilde{I}_0} = \frac{46.06e^{-j4.445^0}}{0.763e^{-j78.56^0} + (1.8)(0.2594e^{-j78.98^0})} = 45.05e^{j74.21^0}\ ohms$$

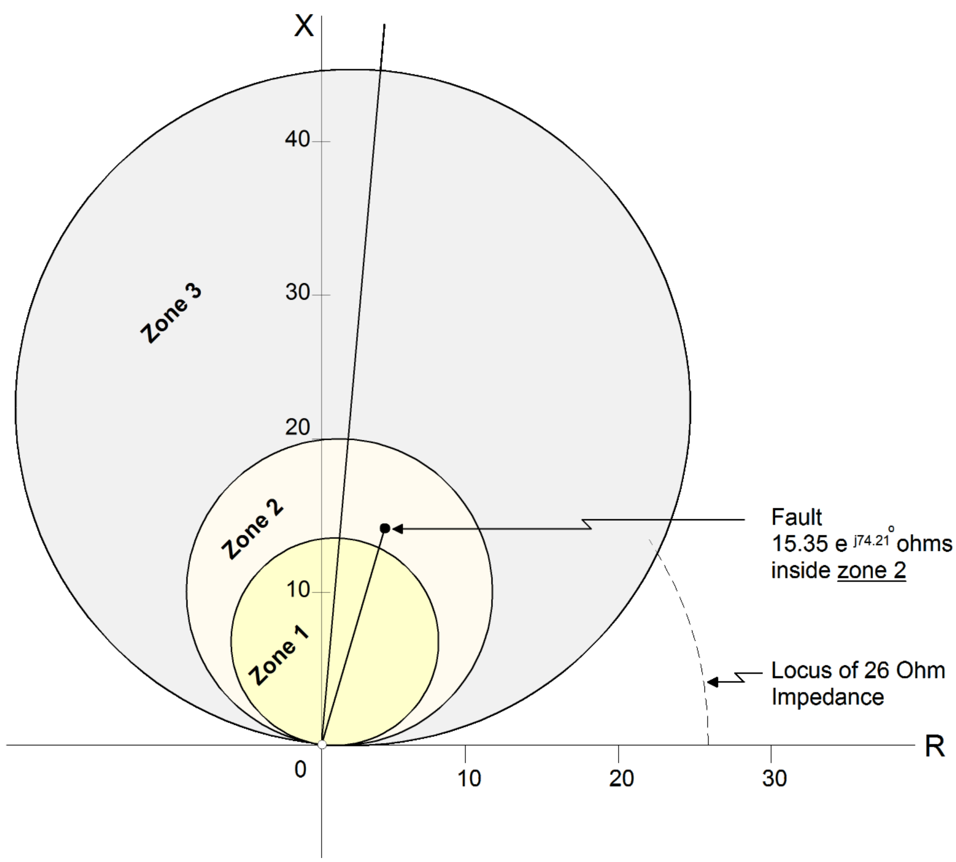
Or 15.35 ohms. The relay will trip in zone 2 (see the figure below). Note that for this distance it should have tripped on zone 1. Note also the phase of the impedance "seen" by the relay is different than the line impedance (74 degrees versus 85 degrees).

(c) In this case the relay will see an impedance of

$$(2.9348)Z_{relay} = \frac{(0.92)(230)/\sqrt{3}}{1.6} = 76.35e^{j25^0}\ ohms$$

$$Z_{relay} = 26.016e^{j25^0}\ ohms$$

Or 26 ohms. Depending on the power factor, this impedance may fall within the zone 3. The figure below shows the locus of the 26 ohm impedance. Note that when the power factor is approximately 0.5 (very low) the impedance will enter zone 3 (load encroachment). For power factors above 0.5 the impedance will be outside the operating region.



Problem P9.x: Consider the electric power system of Figure P9.x. Assume that a time overcurrent relay is located at the indicated location of the transmission line 1. The CT ratio of the relay is 1500:5. The relay settings are: characteristic: very inverse, pickup current = 8 A, time dial = 1.

For a specific near fault condition the electric currents in the transmission line at the location of the relay are: I_a=845A, I_b=931A, and I_c=16,429A. Compute the time to trip of the time overcurrent relay.

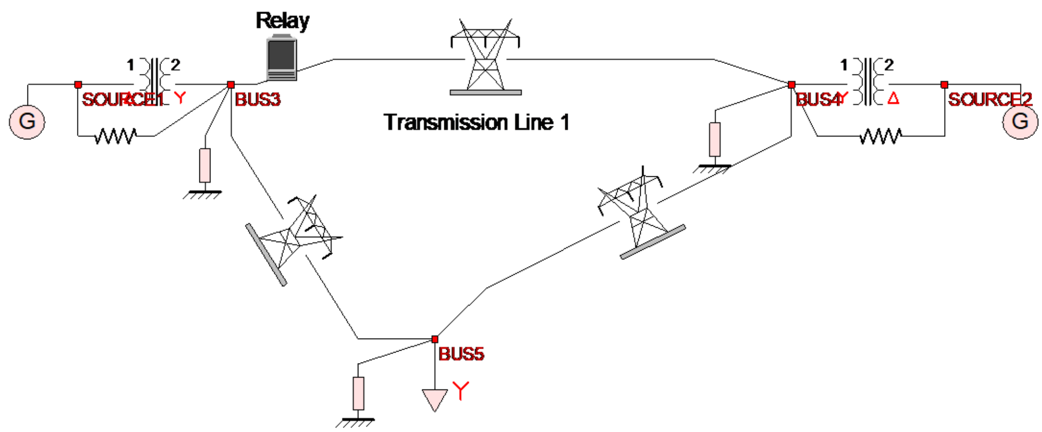


Figure P9.x

Solution: The time to trip will be determined by phase C (highest current). The current on the relay side is:

$$I_{c,relay} = 16,429\frac{5}{1500} = 54.76\ A. \quad \text{Thus:} \quad I_r = \frac{54.76\ A}{8.0\ A} = 6.845$$

Using the very inverse curve the time to trip is:

$$t = \left(\frac{19.61}{I_r^2 - 1} + 0.491\right)t_d = 0.918\ \text{sec}$$

Problem P9.x: Consider the electric power system of Figure P9.xa. Each transmission line is protected with three-zone mho relays (one at each end of the line). All indicated lines are identical – the design of the lines is shown in Figure P9.xb. The phase conductors are ACSR, BITTERN and the shield wires are ALUMOWELD, 3#7AW. The tower ground resistance is 35 ohms. The soil resistivity is 185 ohms. The two lines 1 and 2 are on the same right of way separated by 60 feet (centerline to centerline). The lengths of the lines are: 72, 58, 72 and 82 miles as indicated in Figure P9.xa. The operating voltage of the system is 230 kV. The transformation ratio of the CTs is 2000:5 while the ratio for the potential transformers is 135,000:115.

(a) Determine the settings for the three zones of the mho relay R_1. Use the typical practice as described in the notes.

(b) For a single phase to ground fault at the indicated location determine which zone of the relay R_1 will trip the breaker. The fault is located 45.1 miles from relay R_1.
(c) The 10 minute emergency rating of line 2 is 1,600 A. In a specific emergency condition, the line is loaded at 1600 A and the voltage at the location of relay R_1 has dipped to 0.92 pu. Determine whether the mho relay will trip in this case.

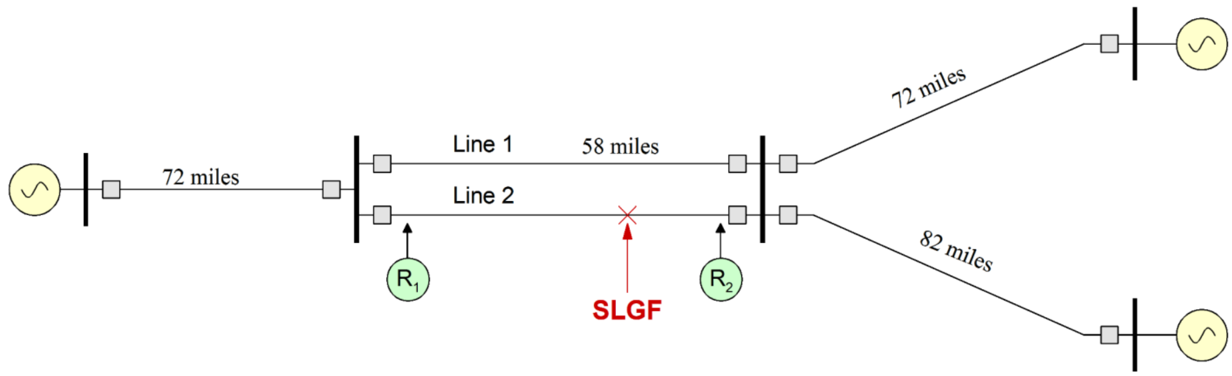


Figure P1a

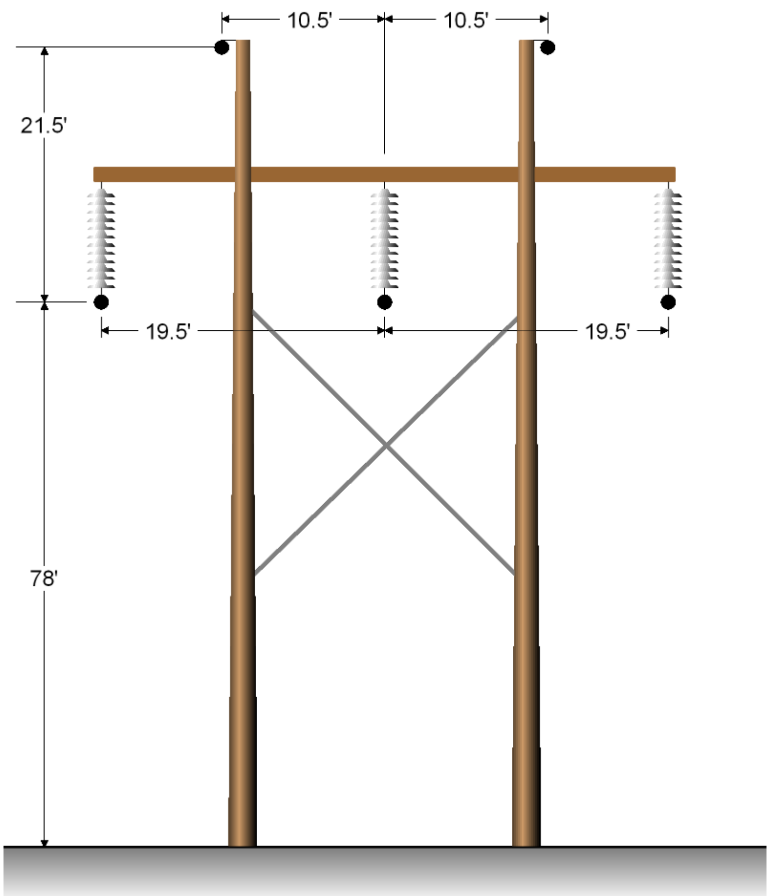
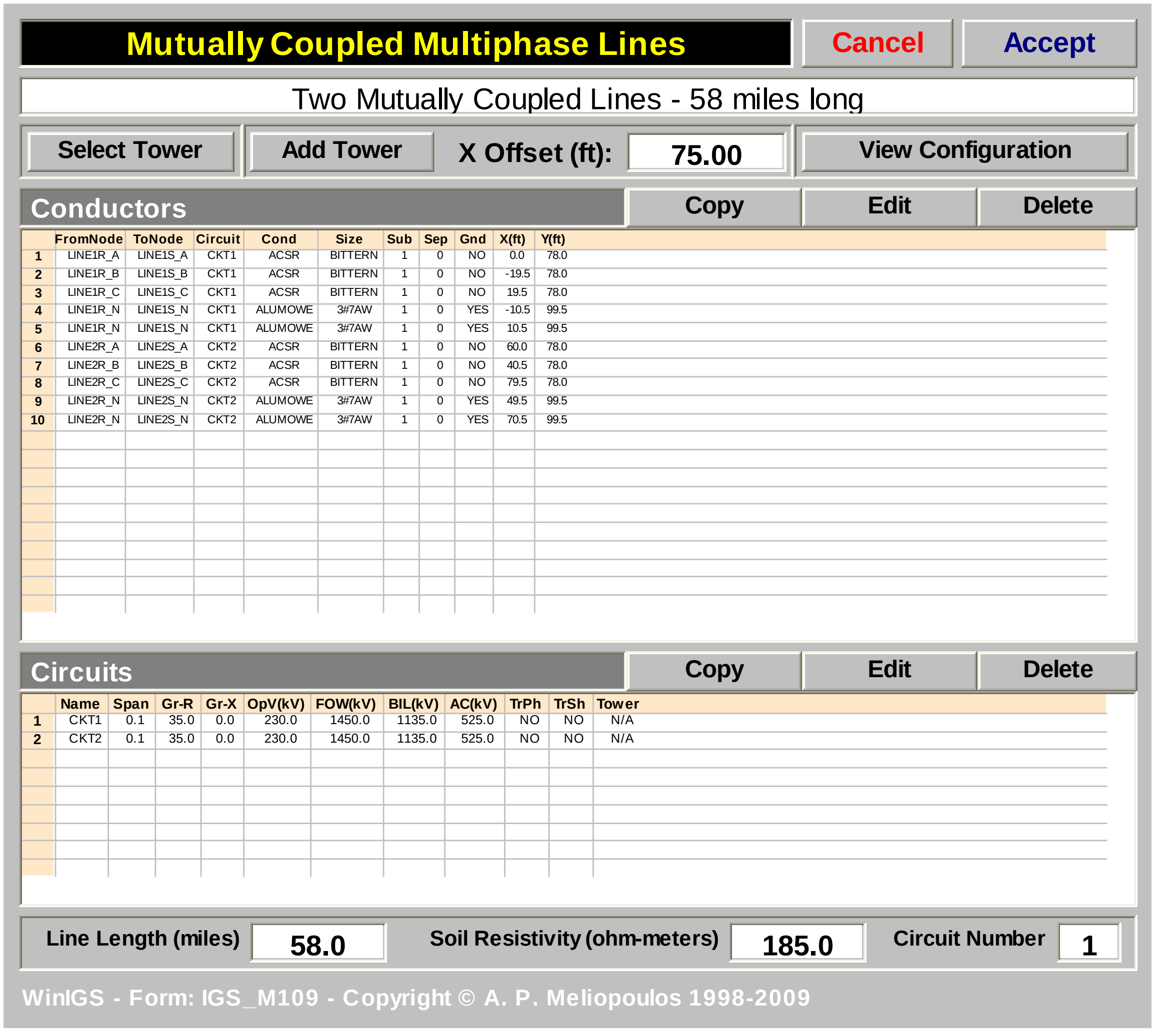


Figure P1b

Solution: Line model and sequence parameters



Mutually Coupled Multiphase Lines

Cancel Accept

Two Mutually Coupled Lines - 58 miles long

Select Tower | Add Tower | X Offset (ft): 75.00 | View Configuration

Conductors — Copy | Edit | Delete

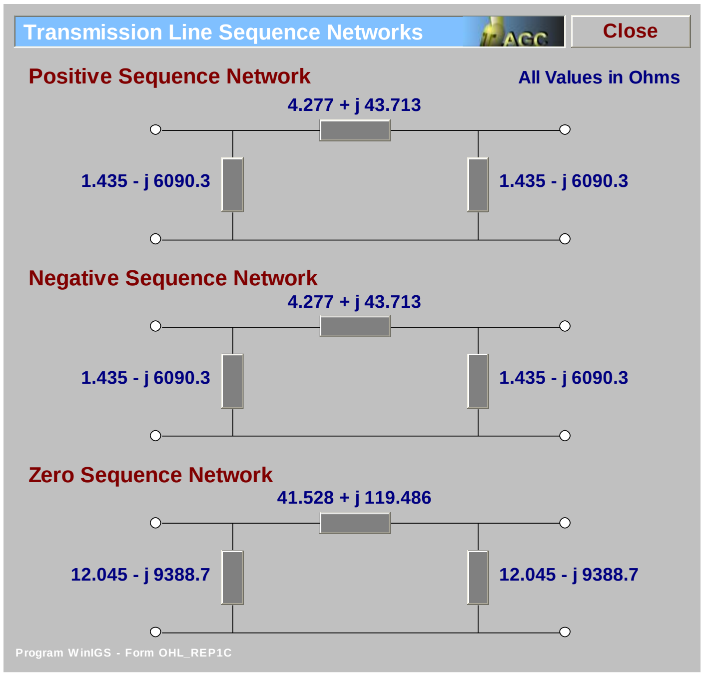
	FromNode	ToNode	Circuit	Cond	Size	Sub	Sep	Gnd	X(ft)	Y(ft)
1	LINE1R_A	LINE1S_A	CKT1	ACSR	BITTERN	1	0	NO	0.0	78.0
2	LINE1R_B	LINE1S_B	CKT1	ACSR	BITTERN	1	0	NO	-19.5	78.0
3	LINE1R_C	LINE1S_C	CKT1	ACSR	BITTERN	1	0	NO	19.5	78.0
4	LINE1R_N	LINE1S_N	CKT1	ALUMOWE	3#7AW	1	0	YES	-10.5	99.5
5	LINE1R_N	LINE1S_N	CKT1	ALUMOWE	3#7AW	1	0	YES	10.5	99.5
6	LINE2R_A	LINE2S_A	CKT2	ACSR	BITTERN	1	0	NO	60.0	78.0
7	LINE2R_B	LINE2S_B	CKT2	ACSR	BITTERN	1	0	NO	40.5	78.0
8	LINE2R_C	LINE2S_C	CKT2	ACSR	BITTERN	1	0	NO	79.5	78.0
9	LINE2R_N	LINE2S_N	CKT2	ALUMOWE	3#7AW	1	0	YES	49.5	99.5
10	LINE2R_N	LINE2S_N	CKT2	ALUMOWE	3#7AW	1	0	YES	70.5	99.5

Circuits — Copy | Edit | Delete

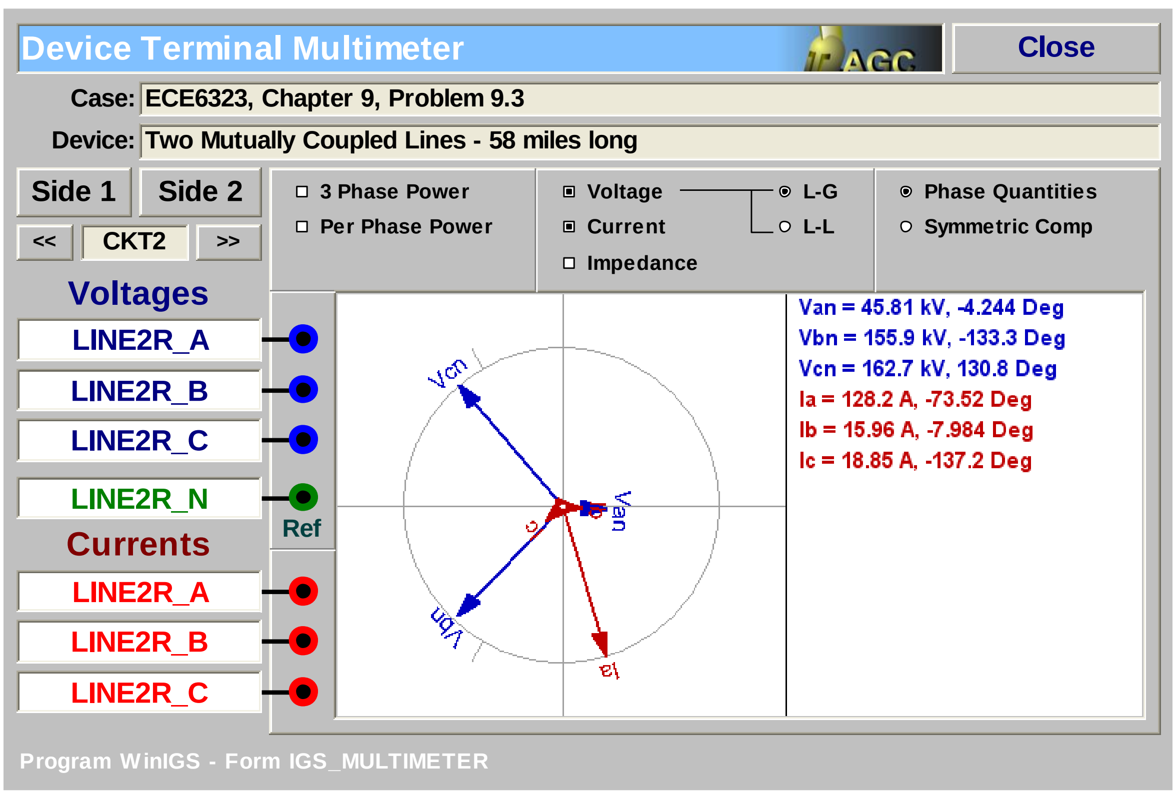
	Name	Span	Gr-R	Gr-X	OpV(kV)	FOW(kV)	BIL(kV)	AC(kV)	TrPh	TrSh	Tower
1	CKT1	0.1	35.0	0.0	230.0	1450.0	1135.0	525.0	NO	NO	N/A
2	CKT2	0.1	35.0	0.0	230.0	1450.0	1135.0	525.0	NO	NO	N/A

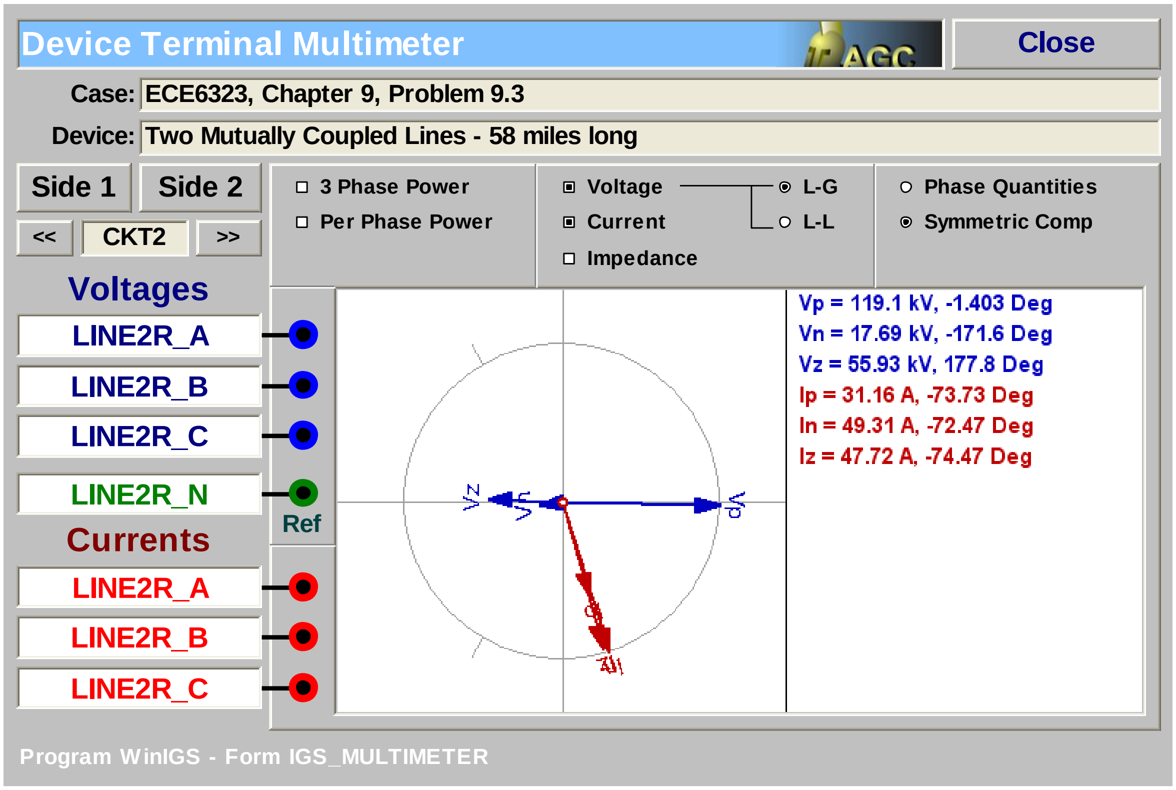
Line Length (miles) 58.0 | Soil Resistivity (ohm-meters) 185.0 | Circuit Number 1

WinIGS - Form: IGS_M109 - Copyright © A. P. Meliopoulos 1998-2009



Voltages and Currents at the location of the relay for a fault at 45 miles from relay.





(a)

$$Z_{relay} = \frac{V_R}{I_R} = \frac{V_L}{I_L}\frac{K_V}{K_I} = 0.3407\, Z_{Line}$$

$$Z_{1,LineTolal} = 4.277 + j43.713 = 43.921 e^{j84.412^0}\,\Omega$$

$$m = \frac{Z_0 - Z_1}{Z_1} = 1.922\, e^{-j20.59^0}$$

Zone 1: Select 80%, round-off to ohms & degrees

$Z = 12\, e^{-j85^0}$, $m = 2.0$, instantaneous

Zone 2: Select 125%, round-off to ohms & degrees

$Z = 19\, e^{-j85^0}$, $m = 2.0$, time delay = 12 cycles

Zone 3: Select 100% + 125% of next shortest line

$Z = 37\, e^{-j85^0}$, $m = 2.0$, time delay = 30 cycles

(b) SLG fault at 45 miles. See WinIGS Results. The relay will see:

$$Z = \frac{45,800e^{-j4.244^0}}{128.2e^{-j73.52^0} + (2.0)47.72\,e^{-j74.47^0}}(0.3407) = 69.78\,e^{-j78.2485^0}$$

Relay will not see the fault

(c) In this case, the relay will see:

$$Z = \frac{(0.92)(230kV)\sqrt{3}}{1.6kA}(0.3407)e^{j\varphi} = 26.01e^{j\varphi}$$

It is possible that the relay will trip on zone 3 for specific power factor

Problem P9.x: A distance relay is used to protect the 235 mile-long 500 kV transmission line illustrated in Figure P9.x. The conductor parameters can be found in the data base of the program WinIGS. The sequence parameters of the line are provided in Figure P9.xa. The line is connected to a three phase source with the following parameters:

Rated Power: 100 MVA, Rated Voltage: 500 kV,
$Z_1 = j0.006\ pu, \quad Z_2 = j0.006\ pu, \quad Z_0 = j0.004\ pu$
Ground Impedance = 1.0 ohms

The CT ratio of the impedance relay is 3000:5 and the PT ratio is 298kV:115V. The compensation factor is selected to be:

$$m = \frac{Z_0 - Z_1}{Z_1} = \frac{(91.162 + j373.977) - (7.545 + j130.219)}{7.545 + j130.219} = 1.9027 - j0.5319 = 1.9756e^{-j15.618^0}$$

The relay engineer decides to set the impedance relay as follows:

(a) zone 1: z=24 ohms, m=1.975, angle=-15.62 degrees, instantaneous
(b) zone 2: z=38 ohms, m=1.975, angle=-15.62 degrees, 12 cycle delay
(c) zone 3: z=75 ohms, m=1.975, angle=-15.62 degrees, 30 cycle delay.

(a) A single phase to ground fault at a distance of 180 miles along the line from the location of the relay yields the fault current indicated in figure P9.xb. The voltages and currents at the location of the relay are provided in Figure P9.xc. Determine what the relay will "see", whether it will trip and if it trips in what zone.

(b) Assume an operating condition at which the phase conductors carry an electric current equal to 115% of the ampacity of the phase conductors (obtain the ampacity of the conductor from the data base in program WinIGS), the voltage at the location of the distance relay is 93% and the power factor is 0.9 current lagging. Determine whether the distance relay will trip.

The use of program WinIGS is encouraged in solving this problem.

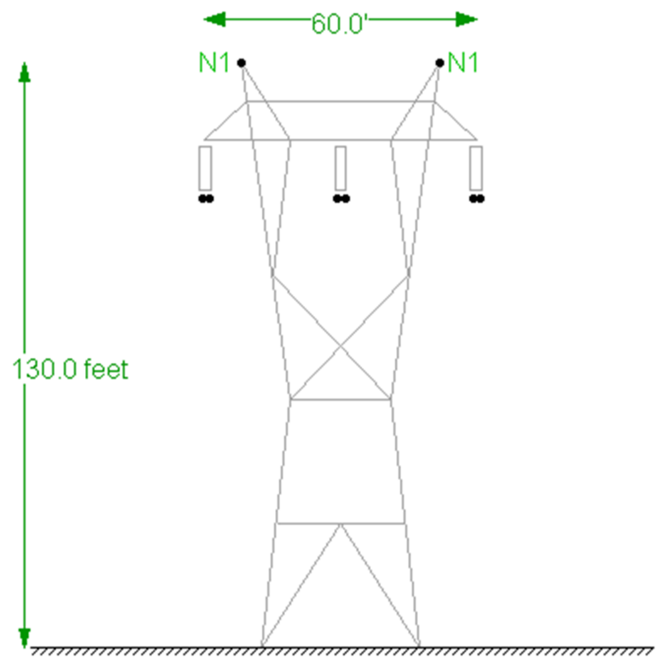
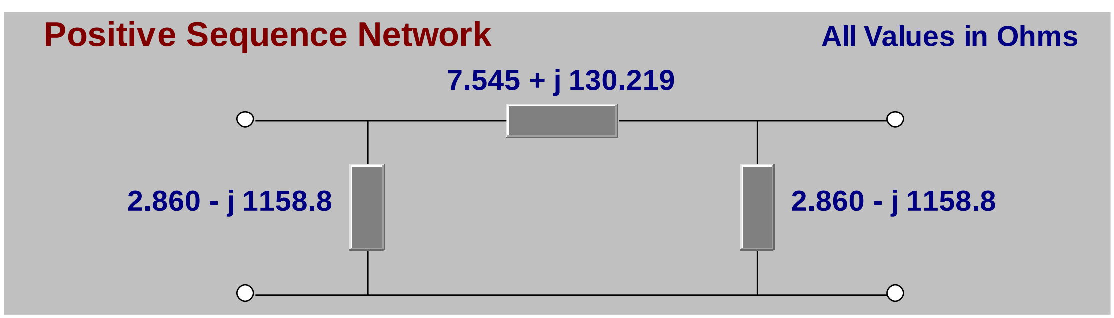


Figure P9.x

(Tower: AGC-T-500B, Phase Conductors: Bundle ACSR, BITTERN, two subconductors per phase, 18 inch separation, Shield Wires: HS, 5/8, tower ground resistance=25 ohms, Soil resistivity=175 ohm.meters, Span length=0.12 miles)



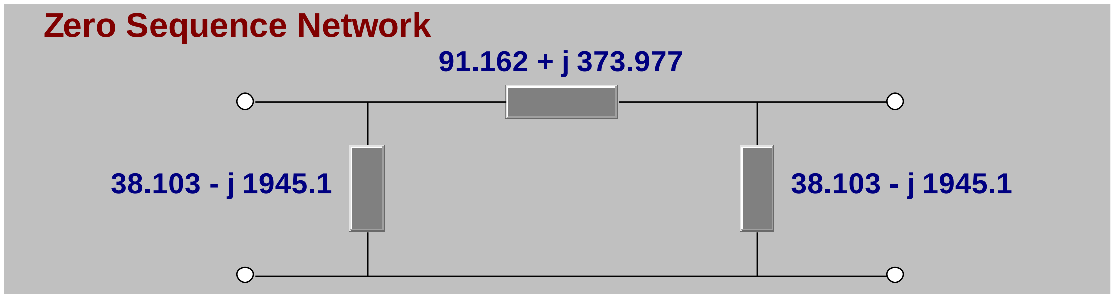
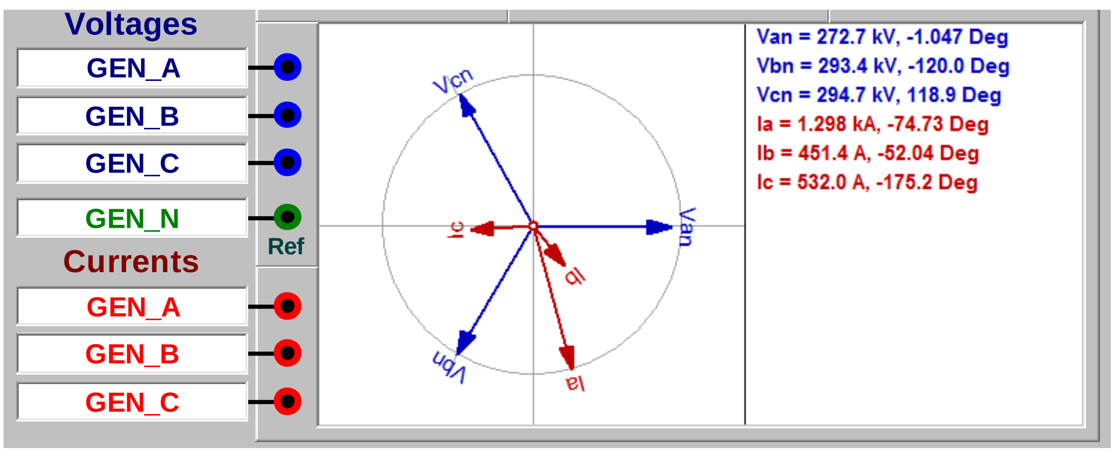


Figure P9.xa

Fault Current	Magnitude (kA)	Phase (deg)
FAULTBUS_A	1.5410	104.0058
X/R Ratio	3.2500	Diagram
Frequency (Hz)	60.0000	Close

Figure P9.xb



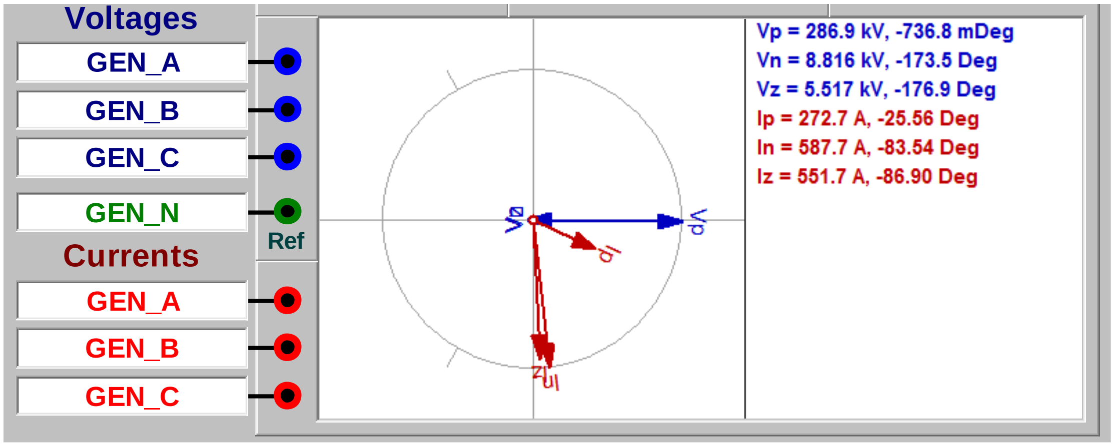


Figure P9.xc

Solution: (a) Using the provided data, the following is computed.

The impedance at relay location is

$$Z = \frac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0} = \frac{272.7e^{-j1.047^\circ}\,kV}{\left[1.298e^{-j74.73^\circ} + (1.9027 - j0.5319)\times 0.5517e^{-j86.9^\circ}\right]kA}$$

$$= 7.5025 + j117.37 = 117.6095e^{j86.3425^\circ}\,\Omega$$

The impedance at the relay side is

$$Z_{relay} = Z\frac{PT^{-1}}{CT^{-1}} = 117.6095e^{j86.3425^\circ}\,\Omega\cdot\frac{115/298k}{5/3000} = 27.2317e^{j86.3425^\circ}\,\Omega$$

Therefore, the relay will trip at zone 2.

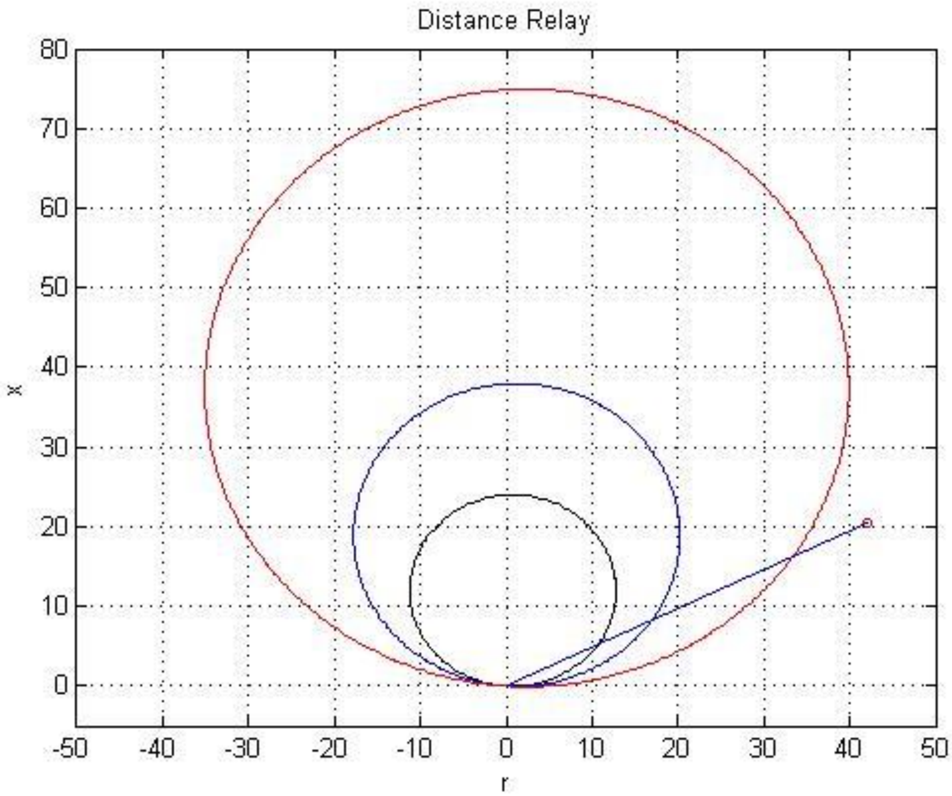
(b) The ampacity of the transmission line is 1155A.
The impedance at relay location is

$$Z = \frac{\tilde{V}_a}{\tilde{I}_a} = \frac{0.93\times 500e^{j0^\circ}\,/\,\sqrt{3}kV}{115\%\times 1.155e^{-j\arccos 0.9}kA} = 202.1215e^{j25.8419^\circ}\,\Omega$$

The impedance at the relay side is

$$Z_{relay} = Z\frac{PT^{-1}}{CT^{-1}} = 202.1215e^{j25.8419^\circ}\,\Omega\cdot\frac{115/298k}{5/3000} = 46.7999e^{j25.8419^\circ}\,\Omega$$

The relationship of the protection zone and impedance is as follows,



Therefore, the relay will not trip.

Problem P9.x: For stability issues and power transfer capability of long overhead transmission lines, many times the line is compensated with series capacitors. The value of the series capacitors is normally expressed as percent compensation. A 30% compensated line means that the absolute value of the series capacitor impedance is 30% of the absolute value of the line impedance (positive sequence).

Consider a 500 kV overhead line of total length 120 miles and 35% compensated. Assume that the line has distance relays at the two terminals. Describe and comment on the operation of distance relaying for this line and what problems may exist.

Problem P9.x: A line differential scheme is applied to a 72 mile transmission line using fiber optic communications. The relays at the two ends digitize the current measurements and transmit the data over the fiber optic. Each relay performs the line differential function. Determine the settings of the line differential function.

Given data: (1) time latency due to relay processing of data: 0.1 ms, (2) transmission time over fiber: 1.6 ms, and (3) the operating voltage of the line is 230 kV and the design is shown in the figure below.

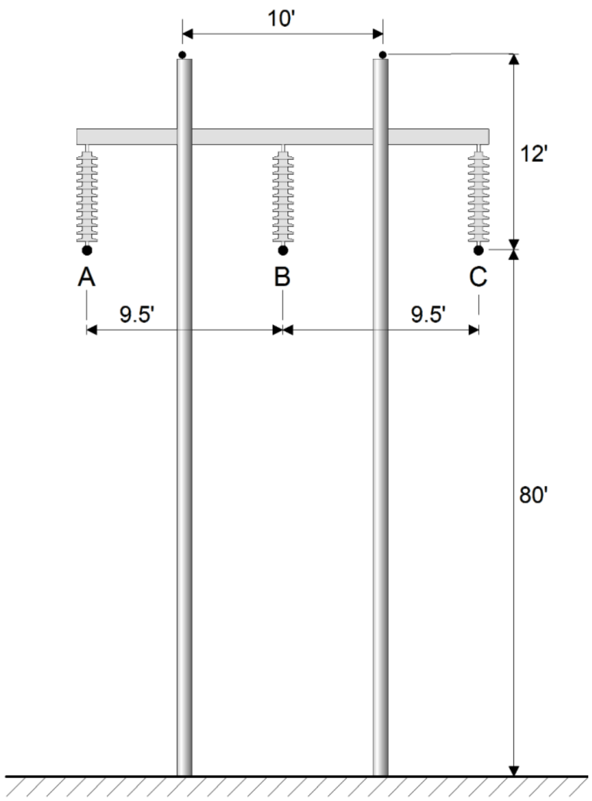


Figure P9.x
(Phase Conductors: ACSR, BITTERN, Shield Wires: HS, 5/16, tower ground resistance=0.25xsoil resistivity, Span length=0.075 miles)

Problem P9.x: Consider the 3-phase 500 kV line, series compensated line as it is illustrated in Figure P9.x. A two-zone distance relay is used to protect this line at the indicated side A of the line. It is desired to set the zone 1 of the relay to trip instantaneous for faults along the line but not to trip for any faults at point B or beyond. It is also desired to set the zone 2 of the relay to trip for any faults anywhere on the line and at least 20% beyond point B.

The CT ratio of the impedance relay is 3000:5 and the PT ratio is 300kV:115V.

(a) Select the settings for the two zones. The following settings should be determined (on relay side):

(1) zone 1: z=_____ ohms, phase angle=_____ degrees, m=_____, instantaneous,

(2) zone 2: z=_____ ohms, phase angle=_____ degrees, m=_____, 12 cycle delay.

(b) Assume a phase A to neutral fault at bus B. Compute the impedance “seen” by the relay. Provide a sketch of the characteristics of the relay (with your selections in (a)) and superimpose the impedance “seen” by the relay for this fault. The source at A has the following parameters: voltage: 500 kV, L-L, pos and neg seq impedance = j12 ohms, zero seq impedance = j16 ohms.

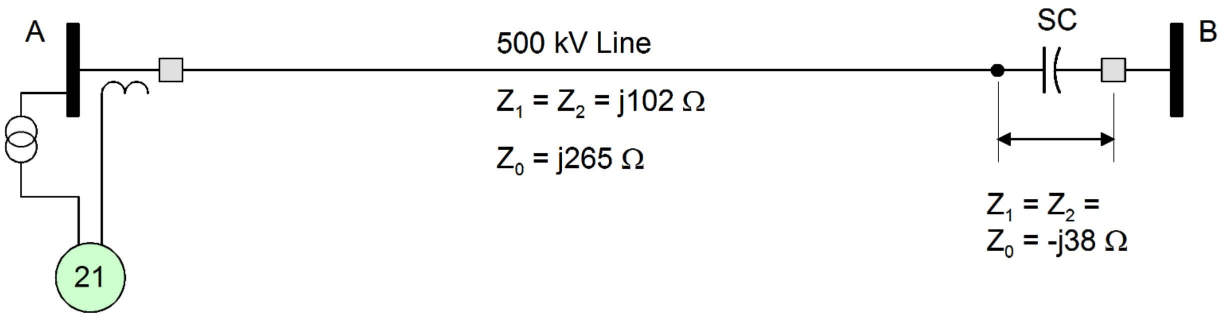


Figure P9.x

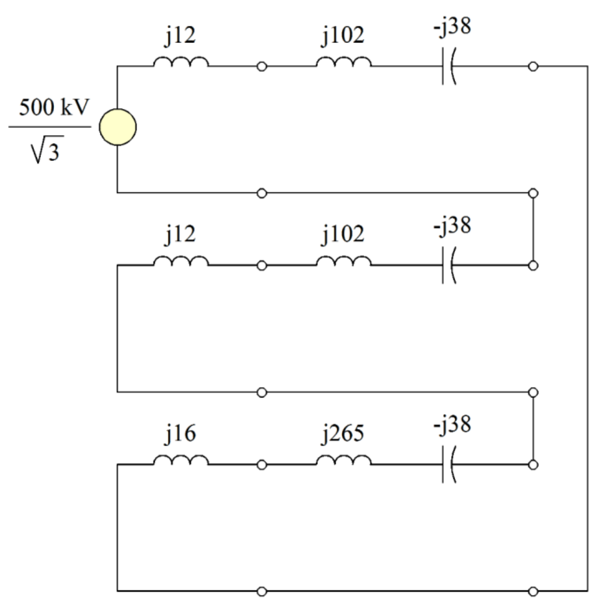
Solution: Part a:

In order to avoid instantaneous tripping for faults near bus B, we set zone 1 to reach to a distance of (102-38)/102x100=62.75%. Zone 2 will be set to detect faults anywhere on the line and at least 20% beyond point B. Then the settings will be:

(1) zone 1: z=14.72 ohms, phase angle=90 degrees, m=1.598, instantaneous,

(2) zone 2: z=23.46 ohms, phase angle=90 degrees, m=2.547, 12 cycle delay.

Part b:



$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{500/\sqrt{3}kV}{j395\Omega} = -j0.7308\,kA$$

$$\tilde{I}_a = -j2.1925\,kA, \quad \tilde{I}_b = \tilde{I}_c = 0$$

At source (relay location):

$$\tilde{V}_1 = \frac{500kV}{\sqrt{3}} - j12\,\tilde{I}_1 = 279.9055\,kV$$
$$\tilde{V}_2 = -j12\,\tilde{I}_2 = -8.7696\,kV$$
$$\tilde{V}_0 = -j16\,\tilde{I}_0 = -11.6928\,kV$$

$$\tilde{V}_a = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0 = \frac{500kV}{\sqrt{3}} - j40(-\mathrm{j}0.7308) = 259.44\,kV$$

Relay:

$$Z_{Line} = \frac{V_a}{I_a + mI_0} = \frac{259.44kV}{-j2.1925 + (2.547)(-j0.7308)} = 63.99e^{j90^0}\ ohms$$

$$Z_{\mathrm{Re}lay} = 0.23Z_{Line} = 14.71e^{j90^0}\ ohms$$

Problem P9.x: A distance relay is used to protect a certain 230 kV transmission line. The perunit length positive sequence impedance of the line is:

$$z_1 = 0.0378 + j0.5673\,ohms/mile$$

The CT ratio of the impedance relay is 3000:5 and the PT ratio is 200kV:115V.

The relay engineer decides to set the impedance relay as follows (impedances on relay side):

(a) zone 1: z=14.8 ohms, phase angle=86 degrees, m=1.9, instantaneous,

(b) zone 2: z=23.0 ohms, phase angle=86 degrees, m=1.9, 12 cycle delay,

(c) zone 3: z=42.0 ohms, phase angle=86 degrees, m=1.9, 30 cycle delay.

During a fault event the line experienced a single line to ground fault initially which then evolved into a line to line fault. The line currents and voltages during this event and for the two fault conditions are shown in Figures P9.5a and P9.5b.

(a) Determine what distance in miles the relay will "see" during the single phase to ground fault. Determine whether it will trip and if it trips in what zone.

(b) Determine what distance in miles the relay will "see" during the line to line fault. Determine whether it will trip and if it trips in what zone.

(c) Compute the difference in fault distance between cases (a) and (b).

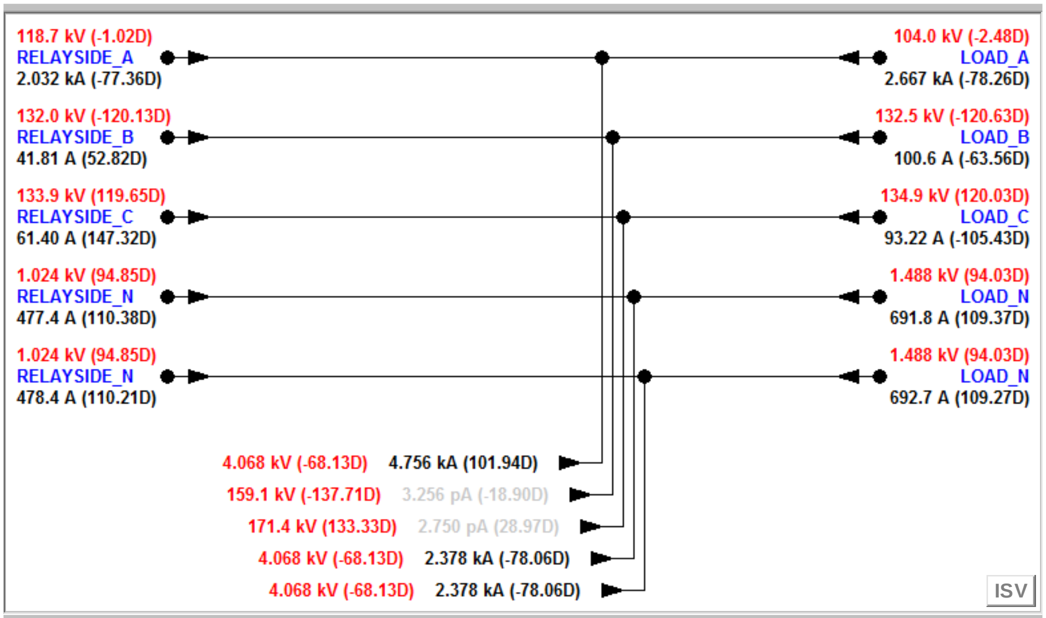


Figure P9.5a: Relay is Located at Bus RELAYSIDE

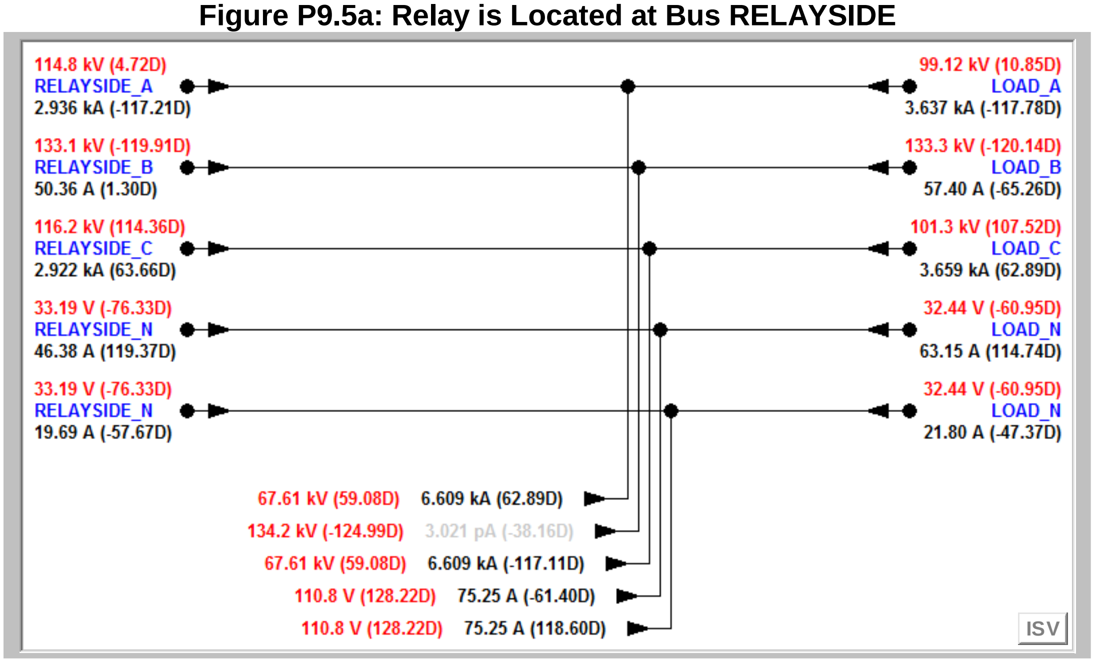


Figure P9.5b: Relay is Located at Bus RELAYSIDE

Solution:

a) $\tilde{V}_{an} = 118.7\ kV\ e^{-j1.02^\circ} \longleftrightarrow 68.2525\, e^{-j1.02^\circ}$

$\tilde{I}_a = 2.032\ kA\ e^{-j77.36^\circ} \longleftrightarrow 3.3867\, e^{-j77.36}$

$\tilde{I}_0 = \tilde{I}_a / 3 \longleftrightarrow \tilde{I}_a / 3$

$$\ell_1 = \frac{68.2525\, e^{-j1.02} / (0.0378 + j0.5673)}{3.3867\, e^{-j77.36} \left(1 + \frac{1.9}{3}\right)} = 62.905\, e^{-j9.8^\circ}\ mi$$

b) $\tilde{V}_{an} = 114.8 kV\ e^{j4.72^\circ} \longleftrightarrow 66.01\, e^{j4.72^\circ}$

$\tilde{V}_{cn} = 116.2\ kV\ e^{j114.36^\circ} \longleftrightarrow 66.815\, e^{j114.36^\circ}$

$\tilde{I}_a = 2.936\ kA\ e^{-j117.21^\circ} \longleftrightarrow 4.8\, e^{-j117.21^\circ}$

$\tilde{I}_c = 2.922\ kA\ e^{j63.66^\circ} \longleftrightarrow 4.87\, e^{j63.66^\circ}$

$$\ell_2 = \frac{66.01\, e^{j4.72^\circ} - 66.815\, e^{j114.36^\circ}}{(4.8\, e^{-j117.21^\circ} - 4.87\, e^{j63.66})(0.0378 + j0.5677)} = 56.69\, e^{-j0.11^\circ}$$

c) $|\ell_1 - \ell_2| = 6.21\ mi$ [8]

Problem P9.3: A 115 kV, 42 mile long three-phase transmission line connects two electrical power systems as in Figure P9.3. Each of the power systems is represented as an equivalent source with the following sequence impedances:

$Z_1 = j0.1\ pu @ 100\, MVA, \quad Z_2 = j0.1\ pu @ 100\, MVA, \quad Z_0 = j0.06\ pu @ 100\, MVA$

The voltage sources behind the equivalent impedances are in phase. The parameters of the transmission line are

$z_1 = z_2 = j0.8\, ohms/mi\ , \quad z_0 = j2.1\, ohms/mi$

(a) Compute the fault current for a single line-to-ground fault at the middle of the line. Assume that a ground distance relay with a compensation factor of m=1.6 is located on the left terminal of the line. What is the impedance "seen" by this relay? (The ground relay "sees" the following impedance: $z_g = \dfrac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0}$)

(b) Compute the fault current for a line-to-line fault at the middle of the line. Assume that a line distance relay is located on the left terminal of the line. What is the impedance "seen" by this relay? (The line relay "sees" the following impedance: $z_{line} = \dfrac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2}$)

(c) Compute the voltage of the un-faulted phases at the location of the fault and for the conditions in (a) and (b).

(d) If the fault is interrupted in 6 cycles after the fault initiation, what will be the maximum rms value of the fault current at the indicated breakers for the cases (a) and (b)?

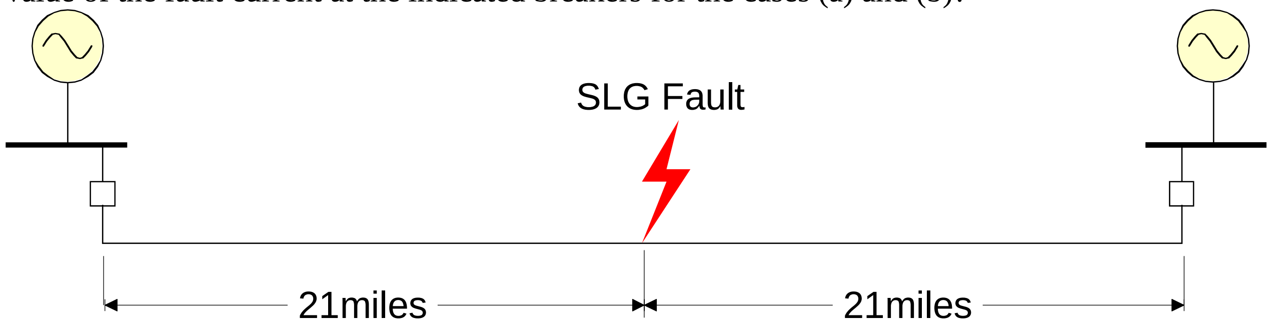
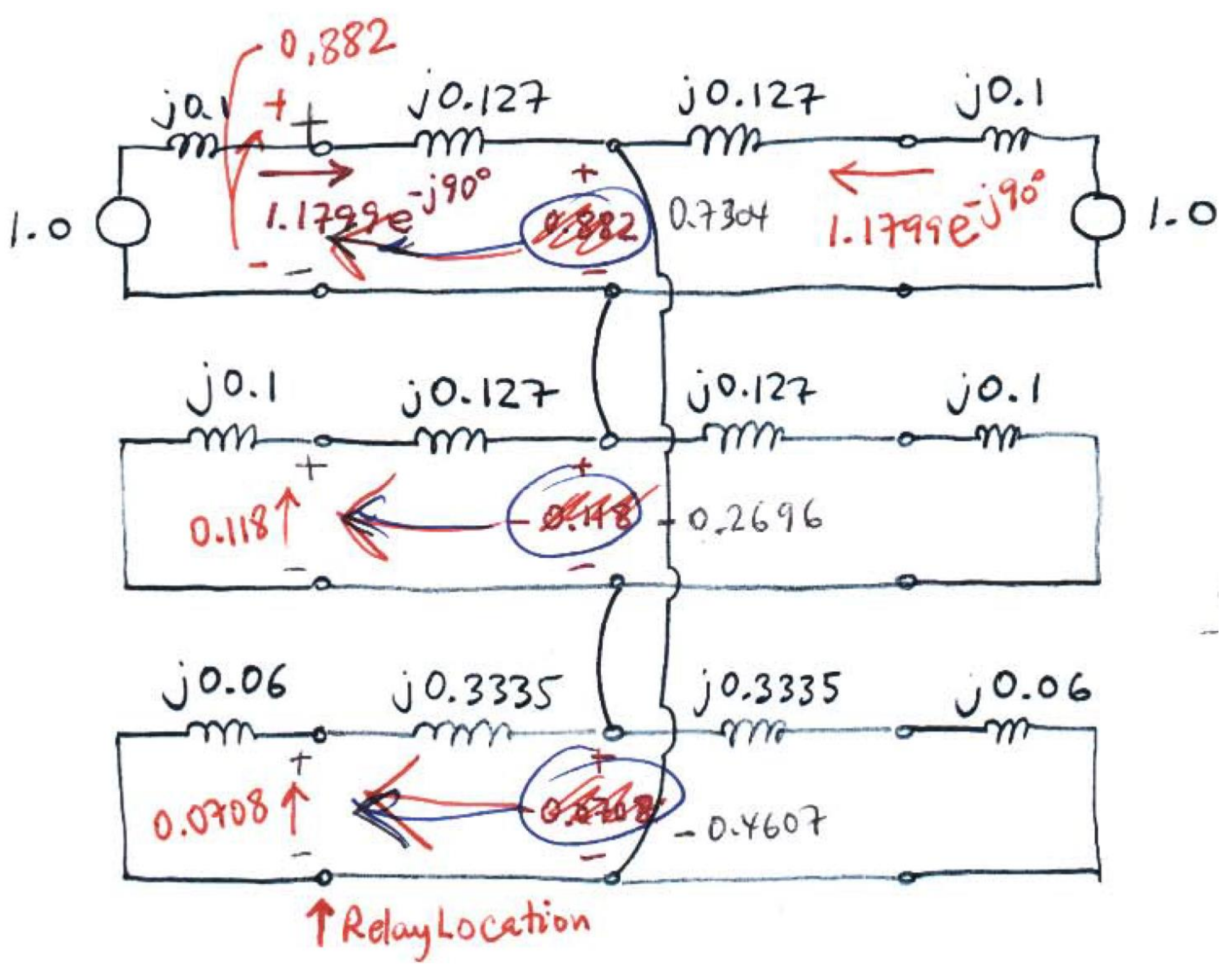


Figure P9.3

Solution: Because of symmetry, one can consider only half of the system for the fault analysis.



(a) Converting the line parameters to per unit, the fault currents are:

$$\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_0 = \frac{1}{j0.8475} = 1.1799e^{-j90^0} \ \ pu$$

The phase A current is:

$$\tilde{I}_a = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_0 = 3.5397e^{-j90^0} \ \ pu$$

The voltage at the location of the relay is:

$$\tilde{V}_1 = 1.0 - (j0.1)\tilde{I}_1 = 0.8820\, pu$$
$$\tilde{V}_2 = -(j0.1)\tilde{I}_2 = -0.1180\, pu$$
$$\tilde{V}_0 = -(j0.06)\tilde{I}_0 = -0.0708\, pu$$

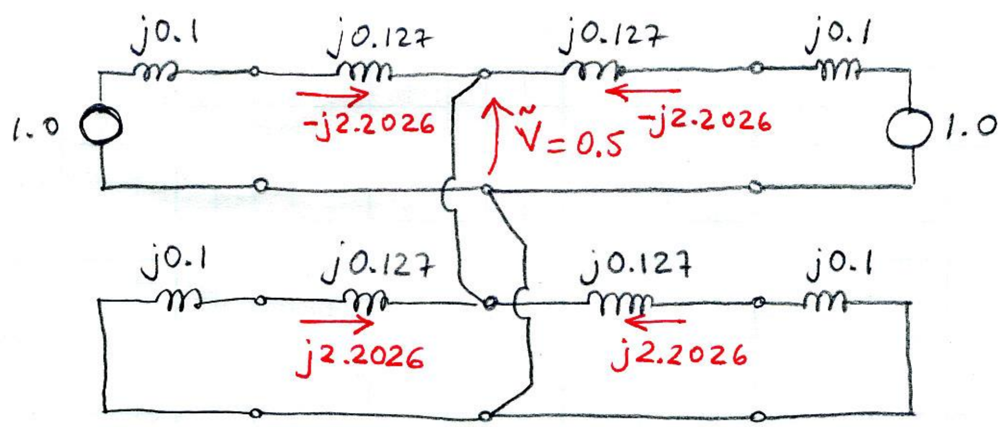
The voltage of the unfaulted phases is:

$$\tilde{V}_a = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0 = 0.6932\, pu$$

The impedance seen by the relay (on the line side) is:

$$Z_{relay(line)} = \frac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0} = \frac{0.6932}{-j3.5397 + 1.6(-j1.1799)} = j0.1277\, pu \quad OR \quad j16.8908\, ohms$$

(b) The equivalent circuit for the line to line fault is given below. The solution is also indicated in the drawing.



The voltages and currents at the location of the relay are:

$$\tilde{I}_1 = -j2.2026\, pu, \qquad \tilde{I}_2 = j2.2026\, pu$$

$$\tilde{V}_1 = 1.0 - (j0.1)(-j2.2026) = 0.77974\, pu, \qquad \tilde{V}_2 = -(j0.1)(j2.2026) = 0.22026\, pu$$

Therefore:

$$z_{line} = \frac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2} = \frac{0.77974 - 0.22026}{-j2.2026 - j2.2026} = j0.127\ pu = j16.79\ ohms$$

(c) The voltages are:

$$\tilde{V}_a = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_0 = 0.0 + j0.0$$
$$\tilde{V}_b = a^2\tilde{V}_1 + a\tilde{V}_2 + \tilde{V}_0 = 0.7304e^{-j120^\circ} - 0.2696e^{j120^0} - 0.4607 = -0.6911 - j0.8660 = 1.1080e^{j\alpha}$$
$$\tilde{V}_c = a\tilde{V}_1 + a^2\tilde{V}_2 + \tilde{V}_0 = 0.7304e^{j120^\circ} - 0.2696e^{-j120^0} - 0.4607 = -0.6911 + j0.8660 = 1.1080e^{j\alpha}$$

(d) The Thevenin equivalent circuit at the location of the fault is:

$z = j0.4238 = R + j\omega L, \quad R = 0, \quad L = 0.00112$. The maximum rms value of the fault current will be:

$$I_{f,rms}(t) = 3\sqrt{I^2 + 2I^2 e^{-\frac{2R}{L}t}} = 6.1309\ pu$$

Problem P9.13: Consider the electric power system of Figure P9.13. The parameters of the various system components are given in the figure. Consider a line to line fault at location A. The location A is indicated in the Figure. Location A is very close to the 230 kV bus of the transformer and practically the impedance between the bus and location A is zero. It is given that prior to the fault the unit operates under nominal voltage at its terminals, rated power and unity power factor.

(a) Compute the negative sequence current in the generator during this fault.

(b) Compute the line impedance "seen" at the terminals of the generator by a line distance relay, i.e. compute the ratio: $z = \frac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2}$

The positive, negative and zero sequence impedances of each line are: $Z_1 = Z_2 = j45.5\,ohms$ and $Z_0 = j106.4\,ohms$. The mutual impedance between the two lines (zero sequence only) is: $Z_{0m} = j24.5\,ohms$.

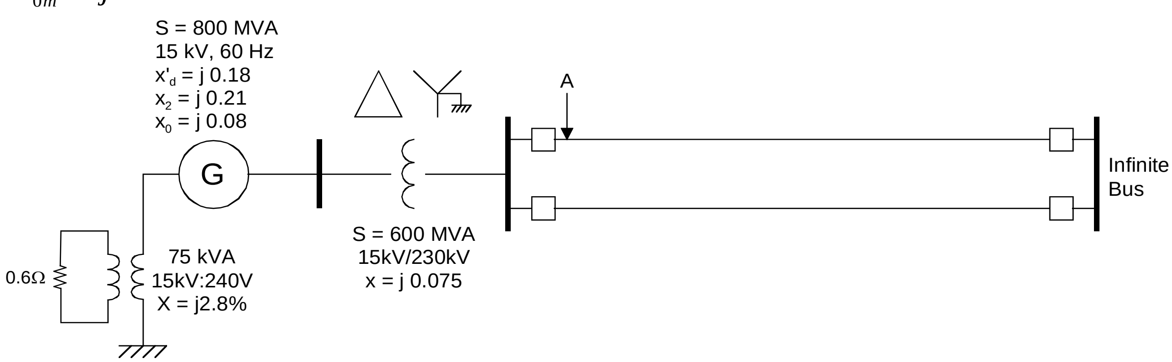
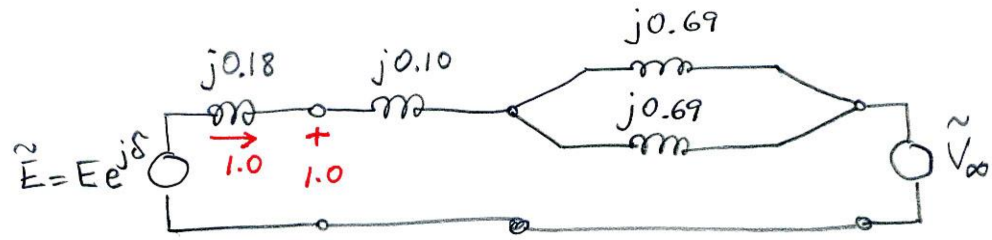


Figure P9.13

Solution: First we compute the operating conditions prior to the fault. The circuit below is the positive sequence network before the fault (all impedances have been referred to an 800 MVA basis).

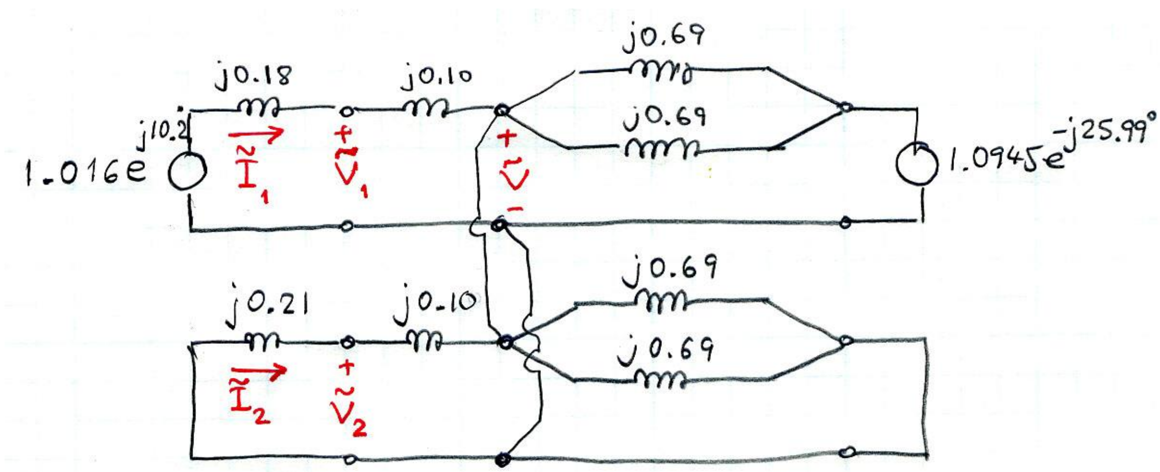


The generator and infinite bus voltages are:

$$Ee^{j\delta} = 1.0 + j0.18 = 1.016e^{j10.2^0} \ pu$$

$$\tilde{V}_\infty = V_\infty e^{j\alpha} = 1.0 - j0.10 - j(0.69/2) = 1.0945e^{-j25.99^0} \ pu$$

(a) Since we have only a line to line fault only the positive and negative sequence equivalent networks will be involved. The figure below illustrates the equivalent circuits and the connections for a line to line fault.



Circuit analysis yields: $\tilde{V} = 0.450e^{-j6.64^0} \ pu$ and $\tilde{I}_2 = 2.250e^{-j96.64^0} \ pu$

(b) From the circuit solution we obtain:

$$\tilde{I}_1 = 2.141e^{-j67.24^0} \ pu$$

$$\tilde{V}_1 = \tilde{V} + (j0.1)\tilde{I}_1 = 0.6452e^{j2.737^0} \ pu$$

$$\tilde{V}_2 = -(j0.21)\tilde{I}_2 = 0.225e^{-j6.64^0} \ pu$$

Thus, the impedance "seen" by the relay is:

$$z = \frac{\tilde{V}_1 - \tilde{V}_2}{\tilde{I}_1 - \tilde{I}_2} =$$

Problem P9.16: A three-phase transmission line connects two electrical power systems as in Figure P4.11. Each of the power systems is represented as an equivalent source with the following sequence impedances:

$Z_1 = j0.08\,pu, \quad Z_2 = j0.08\,pu, \quad Z_0 = j0.06\,pu \quad at\,100\,MVA$

The voltage sources behind the equivalent impedances are in phase. The parameters of the transmission line are

$Z_1 = Z_2 = j0.8\,ohms/mile, \quad Z_0 = j2.1\,ohms/mile$

The operating voltage of the line is 115 kV line to line. The total line length is 40 miles.
(a) Consider a Line-to-Line fault on the transmission line at the indicated location (the involved phases are B and C). Assume that an impedance relay is located at the breaker B1 location of the line. Compute the apparent impedance that the phase B relay will "see".
(b) Consider a Line-to-Ground fault on the transmission line at the indicated location (the involved phase is A). Compute the negative sequence current of generator G1 and the negative sequence current of generator G2 during this fault.

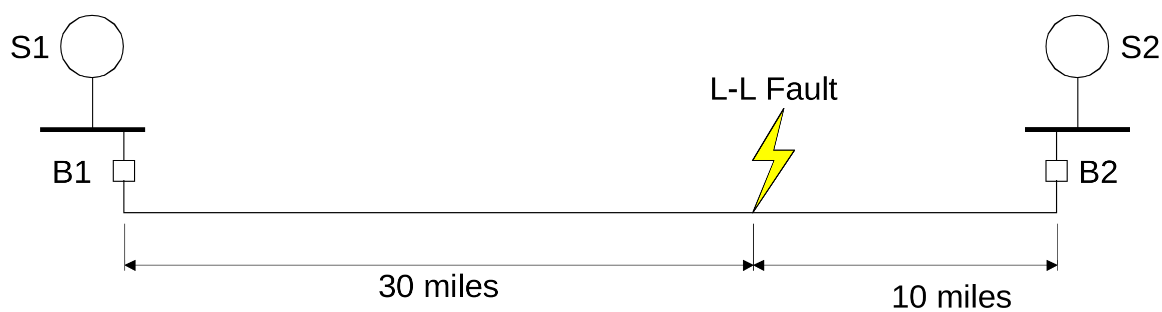
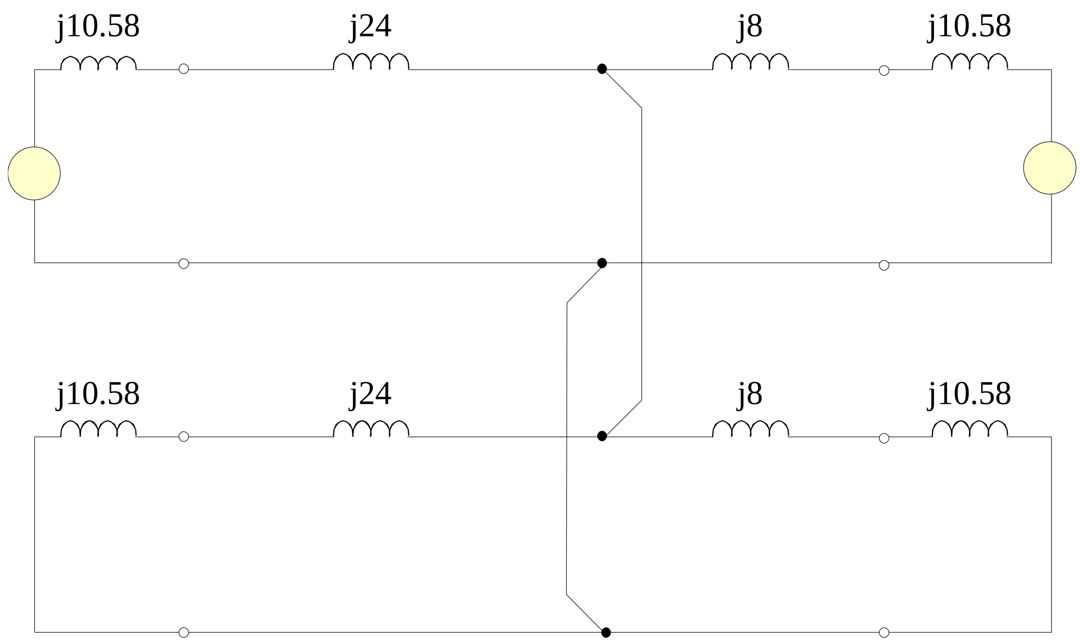
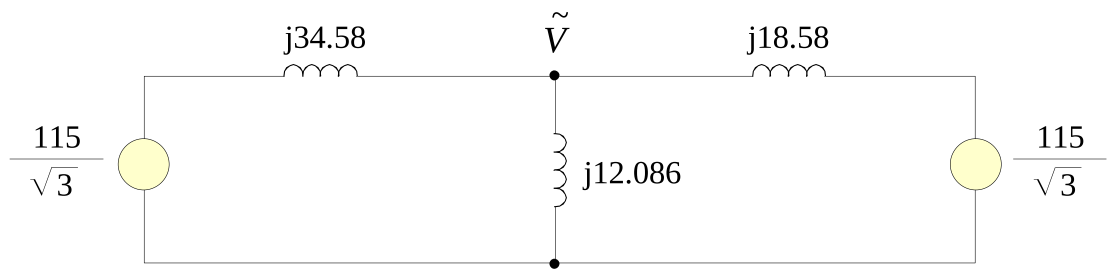


Figure P9.16

Solution: (a) The positive and negative sequence equivalent circuits are connected in parallel at the fault location, as follows:



The above circuit is reduced to:



Circuit analysis of this circuit yields.

$$\frac{\tilde{V}-66.395}{j34.58}+\frac{\tilde{V}}{j12.086}+\frac{\tilde{V}-66.395}{j18.58}=0$$

(b) The sequence networks are shown in Figure P4.16b. Solution of the network yields:

$$\tilde{I}_{2,S2}=1.923e^{-j90^0}\ \ pu$$

$$\tilde{I}_{2,S1}=1.033e^{-j90^0}\ \ pu$$

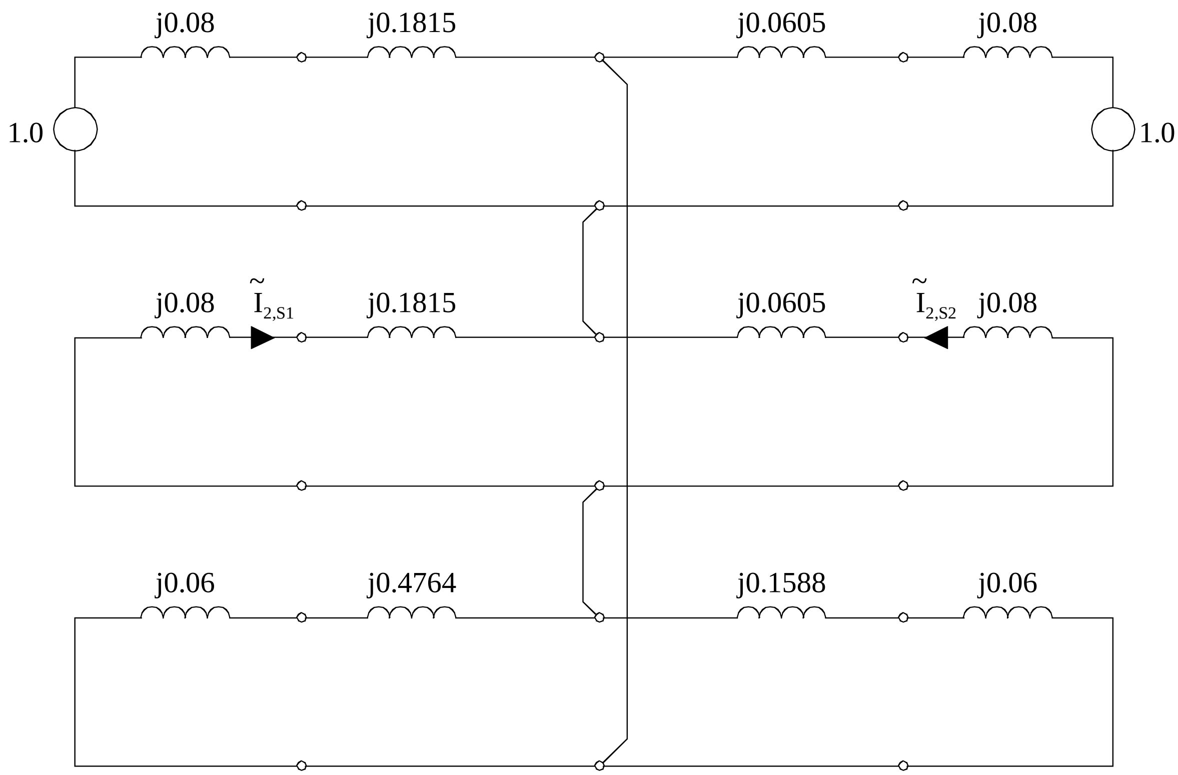


Figure P9.16b

Problem P9.17: Consider the two-unit, two-transformer, and two-transmission line power system of Figure P4.12a. The following data apply to the units and transformers.
Unit G1:
Rated Power: 300 MVA, Rated Voltage: 18 kV,
$Z_1 = j0.18\ pu, \quad Z_2 = j0.21\ pu, \quad Z_0 = j0.09\ pu$
Transformer T1
Rated Power: 300 MVA, Rated Voltage: 18 kV/230 kV,
$Z_1 = j0.08\ pu, \quad Z_2 = j0.08\ pu, \quad Z_0 = j0.08\ pu$
Unit G2:
Rated Power: 800 MVA, Rated Voltage: 18 kV,
$Z_1 = j0.18\ pu, \quad Z_2 = j0.21\ pu, \quad Z_0 = j0.09\ pu$
Rated Power: 800 MVA, Rated Voltage: 18 kV/230 kV,
$Z_1 = j0.08\ pu, \quad Z_2 = j0.08\ pu, \quad Z_0 = j0.08\ pu$
All pu values are referred to the corresponding device rated power and voltage.

The two transmission lines have the configuration shown in the Figure. The two lines parallel for the entire length of the line which is 56 miles. The phase conductors and shield wire sizes are:
Phase Conductors: ACSR, BLUEJAY
Shield Wires: HS steel, 5/16.
Assume a line to line fault on line 1 at a distance of 14.2 miles from transformer 2. Assume also that there is an overcurrent ground fault relay on line 2 at the transformer 2 terminal. The relay is fed via a set of CTs with the transformation ratio of 1200:5. The Overcurrent ground relay is set to trip at 0.2A. Determine whether it will trip during the line to line fault specified above.

Hint: Perform the short circuit analysis and determine the zero sequence current "seen" by the relay. Use of the program WinIGS is encouraged.

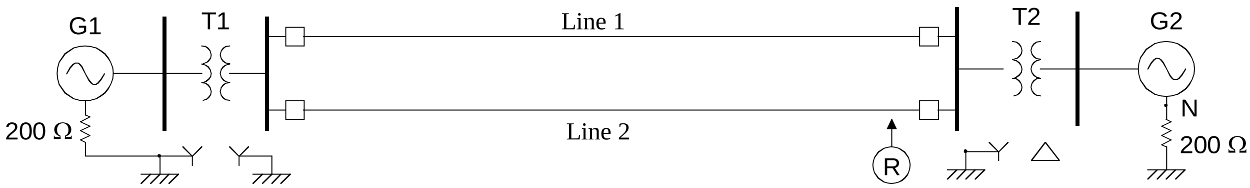


Figure P9.12a

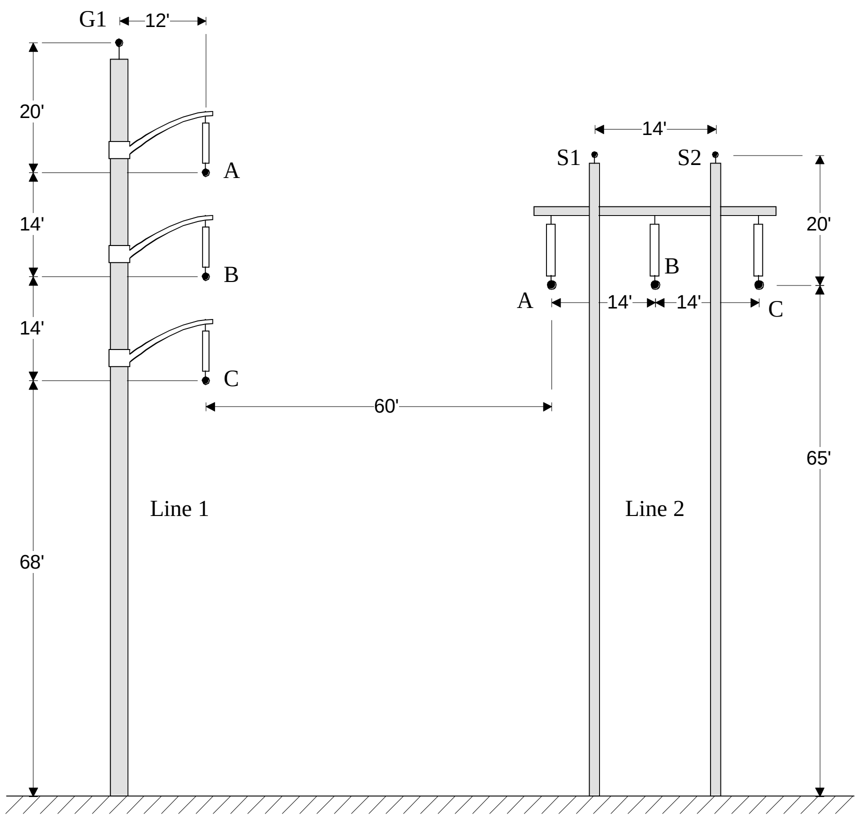
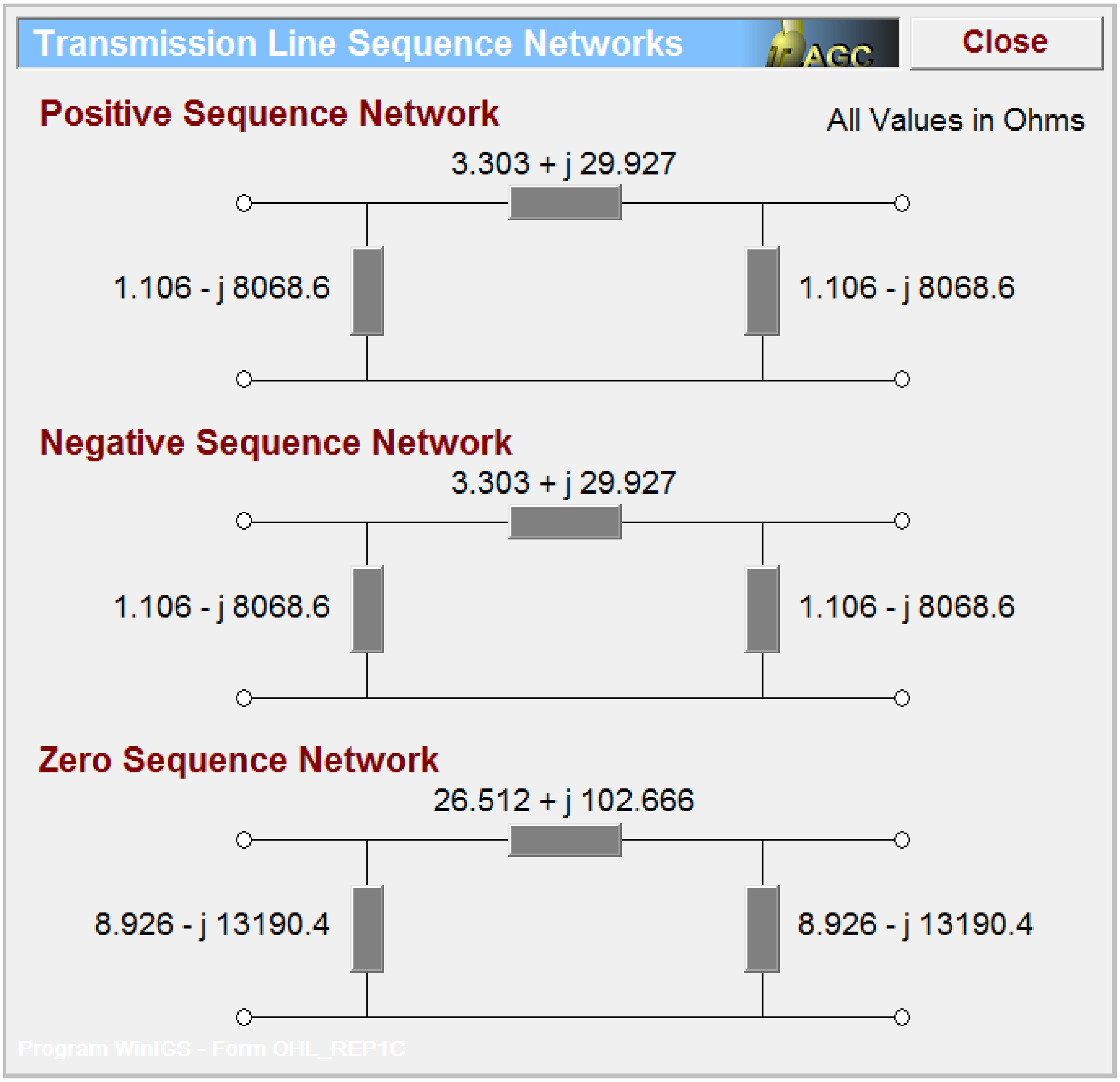


Figure P9.12b

Problem P9.x: Consider a 115 kV transmission line, 41.6 miles long. The positive/negative and zero sequence model of the line is given in the figure.



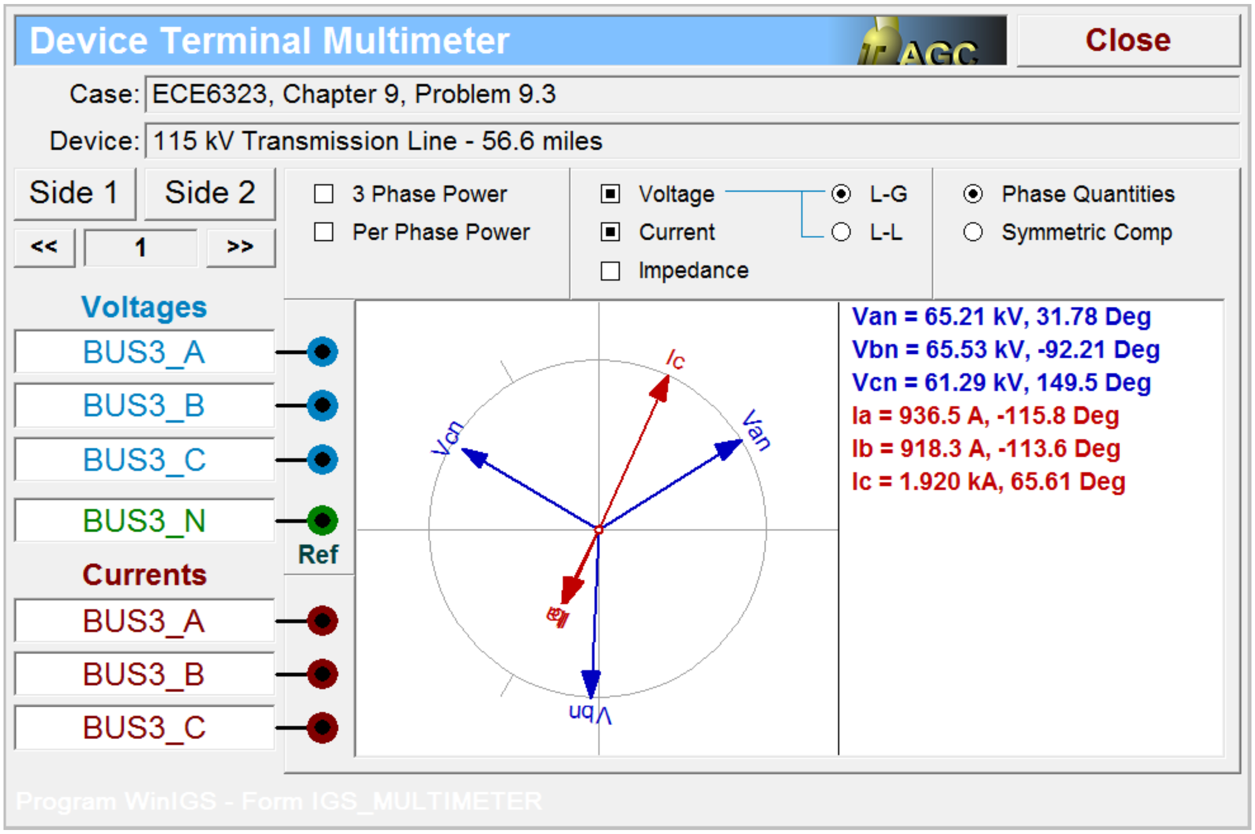
The line is protected with a distance relay at either end. The CT ratio is 1200A:5A and the VT ratio is 69kV:69V.

Determine the settings of the relay to protect the line as follows:

(a) Instantaneous trip if fault is within 75% of the length of the line (zone 1)
(b) Delayed trip by 12 cycles for any fault up to 125% of the line length (zone 2)
(c) Delayed trip by 30 cycle for any fault up to 250% of the line length (zone 3)

Compute and graph the relay characteristics in secondary quantities.

During a specific fault, the distance relay at one of the line ends recorded the following values:



Determine (a) the type of fault and (b) the location of the fault.

Problem P5.24: Consider the electric power system of Figure P5.18. Assume that a distance relay is located at the left terminal of Line 1. The line is 59.5 miles long. The CT ratio of the distance relay is 1800:5 and the PT ratio is 142kV:69V.

Zone 1 of the relay should be set to reach 70% of the line. The available compensation factor settings (m) of the relay are in the range 2.4 to 4.0 in increments of 0.1. The impedance settings are: 5 ohms to 30 ohms in increments of 1 ohm. The parameters of the line are:

$$Z_1 = 2.222 + j39.134\, ohms, \quad Z_2 = 2.222 + j39.134\, ohms, \quad Z_0 = 24.564 + j158.775\, ohms$$

Question 1: Determine the settings of zone 1 of the relay. Specify impedance, angle, compensation factor and time delay. The impedance should be on the relay side.

Question 2: During a fault condition, the relay measures the following voltages and currents.

$$\tilde{V}_{an} = 83.36 e^{j3.389^0}\ kV, \quad \tilde{V}_{bn} = 81.77 e^{-j63.97^0}\ kV, \quad \tilde{V}_{cn} = 137.4 e^{j150.1^0}\ kV$$

$$\tilde{I}_a = 1.858 e^{-j27.23^0}\ kA, \quad \tilde{I}_b = 1.857 e^{j151.6^0}\ kA, \quad \tilde{I}_c = 0.03798 e^{-j98.59^0}\ kA$$

Determine the response of the relay, i.e. determine whether the relay will see the fault inside zone 1 or outside.

Question 3: During another fault condition, the relay measures the following voltages and currents.

$$\tilde{V}_{an} = 85.44e^{j25.19^0}\ kV,\quad \tilde{V}_{bn} = 124.9e^{-j82.56^0}\ kV,\quad \tilde{V}_{cn} = 130.9e^{j141.6^0}\ kV$$

$$\tilde{I}_a = 1.708e^{-j49.97^0}\ kA,\quad \tilde{I}_b = 0.1138e^{j113.0^0}\ kA,\quad \tilde{I}_c = 0.1163e^{j147.0^0}\ kA$$

Determine the response of the relay, i.e. determine whether the relay will see the fault inside zone 1 or outside.

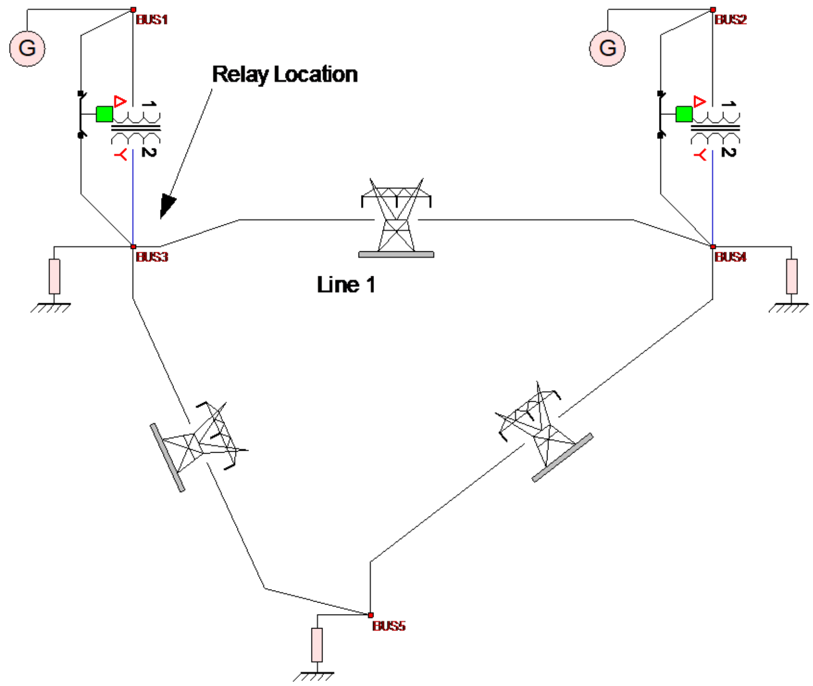


Figure P5.18

Using the selected settings for this relay, compute the distance "seen" by this relay for the following faults on the line:

(a) a line to line fault (phase A to phase B) at a distance of 38.5 miles from the location of the relay.
(b) a line to line fault (phase A to phase C) at a distance of 38.5 miles from the location of the relay.
(c) a line to neutral fault (phase A) at a distance of 38.5 miles from the location of the relay.
(d) a line to neutral fault (phase B) at a distance of 38.5 miles from the location of the relay.

What are your observations about the accuracy of the relay?

Solution: The short circuit analysis is performed with the program WinIGS. First we select the relay settings. For this purpose we compute the sequence parameters of the circuit to be protected. Using the program WinIGS the sequence parameters are:

$$Z_1 = 2.222 + j39.134\,ohms,\quad Z_2 = 2.222 + j39.134\,ohms,\quad Z_0 = 24.564 + j158.775\,ohms$$

$$Z_1^{'} = 0.750 - j5{,}144.6\,ohms, \quad Z_2^{'} = 0.750 - j5{,}144.6\,ohms, \quad Z_0^{'} = 8.331 - j10{,}527.3\,ohms$$

The compensation factor is:

$m = \dfrac{z_0 - z_1}{z_1} = 3.0989 e^{ja}$, Select $m = 3.0$.

The impedance should be set equal to 80% of the positive sequence impedance of the line. This means:

$(0.8) Z_1 = \dfrac{\tilde{V}}{\tilde{I}} = \dfrac{\frac{139{,}000}{115}\tilde{V}_R}{\frac{2{,}000}{5}\tilde{I}_R}$. Upon solution for the impedance on the relay side:

$\dfrac{\tilde{V}_R}{\tilde{I}_R} = 10.3773 e^{j86.75^0}\ ohms$, Select $Z_{zone1} = 10.0\,ohms$ and Phase = 85 degrees

(a) The fault currents and voltages at the location of the relay for a line to line fault (phases A and B), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 83.36 e^{j3.389^0}\ kV, \quad \tilde{V}_{bn} = 81.77 e^{-j63.97^0}\ kV, \quad \tilde{V}_{cn} = 137.4 e^{j150.1^0}\ kV$$
$$\tilde{I}_a = 1.858 e^{-j27.23^0}\ kA, \quad \tilde{I}_b = 1.857 e^{j151.6^0}\ kA, \quad \tilde{I}_c = 0.03798 e^{-j98.59^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_a - \tilde{V}_b}{\tilde{I}_a - \tilde{I}_b} \right| = 37.4211\,miles$$

(b) The fault currents and voltages at the location of the relay for a line to line fault (phases A and C), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 84.11 e^{j54.05^0}\ kV, \quad \tilde{V}_{bn} = 137.8 e^{-j90.07^0}\ kV, \quad \tilde{V}_{cn} = 85.33 e^{j125.2^0}\ kV$$
$$\tilde{I}_a = 1.767 e^{-j88.63^0}\ kA, \quad \tilde{I}_b = 0.04276 e^{-j197.4^0}\ kA, \quad \tilde{I}_c = 1.768 e^{j92.74^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_a - \tilde{V}_c}{\tilde{I}_a - \tilde{I}_c} \right| = 42.333\,miles$$

(c) The fault currents and voltages at the location of the relay for a line to ground fault (phase A to ground), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 85.44e^{j25.19^0}\ kV, \quad \tilde{V}_{bn} = 124.9e^{-j82.56^0}\ kV, \quad \tilde{V}_{cn} = 130.9e^{j141.6^0}\ kV$$
$$\tilde{I}_a = 1.708e^{-j49.97^0}\ kA, \quad \tilde{I}_b = 0.1138e^{j113.0^0}\ kA, \quad \tilde{I}_c = 0.1163e^{j147.0^0}\ kA$$

The phase currents, transformed into symmetrical components, are:

$$\tilde{I}_1 = 0.5866e^{-j49.89^0}\ kA, \quad \tilde{I}_2 = 0.6255e^{-j50.03^0}\ kA, \quad \tilde{I}_0 = 0.496e^{-j50.0^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_a}{\tilde{I}_a + m\tilde{I}_0} \right| = 40.579\ miles$$

(d) The fault currents and voltages at the location of the relay for a line to ground fault (phase B to ground), computed with the program WinIGS, are:

$$\tilde{V}_{an} = 130.5e^{j21.51^0}\ kV, \quad \tilde{V}_{bn} = 85.20e^{-j94.87^0}\ kV, \quad \tilde{V}_{cn} = 124.5e^{j157.40^0}\ kV$$
$$\tilde{I}_a = 0.1261e^{j24.81^0}\ kA, \quad \tilde{I}_b = 1.713e^{-j170.2^0}\ kA, \quad \tilde{I}_c = 0.1151e^{-j3.54^0}\ kA$$

The phase currents, transformed into symmetrical components, are:

$$\tilde{I}_1 = 0.593e^{-j49.81^0}\ kA, \quad \tilde{I}_2 = 0.6272e^{j69.65^0}\ kA, \quad \tilde{I}_0 = 0.4932e^{-j170.4^0}\ kA$$

The fault distance is computed from the equation:

$$\ell = \left| \frac{1}{z_1} \frac{\tilde{V}_b}{\tilde{I}_b + m\tilde{I}_0} \right| = 40.5081\ miles$$

The accuracy of the relay (on the basis of the above four cases) is:

$$Error = \frac{Maximum\ Difference}{\text{Fault Distance}} = \frac{4.9119}{38.5} = 0.1276\ pu$$